University of New Mexico

Some operators with IVGSVTrN-numbers and their applications to multiple criteria group decision making

İrfan Deli[1]

[1]Muallim Rıfat Faculty of Education, 7 Aralık University, 79000 Kilis, Turkey.

E-mail: irfandeli@kilis.edu.tr

Abstract: Interval valued generalized single valued neutrosophic trapezoidal number (IVGSVTrN-number), which permits the membership degrees of an element to a set expressed with intervals rather than exact numbers, is considered to be very useful to describe uncertain information for analyzing multiple criteria decision making (MCDM) problems. In this paper, we firstly introduced the concept of IVGSVTrN-number with some operations based on neutrosophic number. Then, we presented some aggregation and geometric operators. Finally, we developed a approaches for multiple criteria group decision making problems based on the proposed operators and we applied the method to a numerical example to illustrate proposed approach.

Keywords: Neutrosophic set, interval eutrosophic set, neutrosophic numbers, IVGSVTrN-numbers, aggregation and geometric operators, multiple criteria group decision making.

1 Introduction

Since the nature of real world and limited knowledge and perception capability of human beings, their real life contain different styles of vagueness, inexact and imprecise information. To handle and analyze various kinds of vagueness, inexact and imprecise information a number of methods and theories have been developed. For example; in 1965, fuzzy set theory [53] has gradually become the mainstream in the field of representing and handling vagueness, inexact and imprecise information in decision-making, pattern recognition, game theory and so on. After fuzzy set theory, various classes of extensions have been defined and extended successively such as; intuitionistic fuzzy sets introduced Atanassov[3], neutrosophic sets by proposed by Smarandache[43], interval neutrosophic sets by developed by Wang et al. [44]. Recently, some studies on the sets have been researched by many authors (e.g. [7, 8, 9, 10, 16, 21, 22, 36, 41, 42, 50, 54]).

In recent years, many researchers have realized the need for a set that has the ability to accurately model and represent intuitionistic information in [48]. As a theory to model different styles of uncertainty, intuitionistic fuzzy set is usually employed to analyze uncertain MCDM problems through intuitionistic fuzzy number. As an important representation of fuzzy numbers, intuitionistic trapezoidal fuzzy numbers in [33]. Some of the recent research done on the MCDM of intuitionistic fuzzy number were presented in [28, 37].

In some real problems, a information can be modelling with intervals rather than exact numbers. Therefore, Wan [46] presented interval-valued intuitionistic trapezoidal fuzzy numbers which is its membership function and non-membership function are intervals rather than exact numbers. After Wan [46] some authors studied on the interval-valued intuitionistic trapezoidal fuzzy numbers in [4, 12, 27, 34, 38, 39]. Then, Wei [47] introduced some aggregating oprators and gave an illustrative example.

To modelling an ill-known quantity some decision making problems Deli and Şubaş [23, 24] defined single valued neutrosophic numbers. Some of the recent researchs done on the MCDM of neutrosophic numbers such as; on triangular neutrosophic numbers [1, 5, 18, 35] and on trapezoidal neutrosophic numbers [6, 13, 15, 17, 19, 26, 30, 31, 32, 40, 49, 51, 52]. Although single valued neutrosophic numbers can characterize possible membership degrees of x into the set A in a exact number way, it may lose some original information. For this, interval valued single valued neutrosophic trapezoidal numbers studied in [2, 11, 14, 25, 29]. This paper is organized as follows; in section 2, we presented a literature review that presents papers about fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, single valued neutrosophic sets and single valued neutrosophic numbers. In section 3, we gave the concept of interval valued generalized single valued neutrosophic trapezoidal number(IVGSVTrN-number) which is a generalization of fuzzy number, intuitionistic fuzzy number, neutrosophic number, and so on. In section 4, we presented some aggregation is called IVGSVTrN ordered weighted aggregation operator, IVGSVTrN ordered hybrid weighted aggregation operator. In section 5 proposed some geometric operators is called IVGSVTrN ordered weighted geometric operator, IVGSVTrN ordered hybrid weighted geometric operator. In section 6, we developed a approaches for multiple criteria decision making problems based on the operator and we applied the method to a numerical example to illustrate the practicality and effectiveness of the proposed approach. In section 7, we concluded the research and determines the future directions of the work.

2 Preliminary

In this section, we recall some of the necessary notions related to fuzzy sets, neutrosophic sets, single valued neutrosophic sets and single valued neutrosophic numbers.

From now on we use $I_n = \{1, 2, ..., n\}$ and $I_m = \{1, 2, ..., m\}$ as an index set for $n \in \mathbb{N}$ and $m \in \mathbb{N}$, respectively.

Definition 2.1. [53] Let E be a universe. Then a fuzzy set X over E is defined by

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where μ_X is called membership function of X and defined by $\mu_X : E \to [0.1]$. For each $x \in E$, the value $\mu_X(x)$ represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [54] t-norm a function such that $t : [0, 1] \times [0, 1] \to [0, 1]$

1. $t(0, 0) = 0$ and $t(\mu_{X_1}(x), 1) = t(1, \mu_{X_1}(x)) = \mu_{X_1}(x),\ x \in E$
2. If $\mu_{X_1}(x) \leq \mu_{X_3}(x)$ and $\mu_{X_2}(x) \leq \mu_{X_4}(x)$, then $t(\mu_{X_1}(x), \mu_{X_2}(x)) \leq t(\mu_{X_3}(x), \mu_{X_4}(x))$
3. $t(\mu_{X_1}(x), \mu_{X_2}(x)) = t(\mu_{X_2}(x), \mu_{X_1}(x))$
4. $t(\mu_{X_1}(x), t(\mu_{X_2}(x), \mu_{X_3}(x))) = t(t(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$

Definition 2.3. [54] s-norm a function such that $s : [0, 1] \times [0, 1] \to [0, 1]$ with the following conditions:

1. $s(1, 1) = 1 \ and \ s(\mu_{X_1}(x), 0) = s(0, \mu_{X_1}(x)) = \mu_{X_1}(x), x \in E$

2. if $\mu_{X_1}(x) \leq \mu_{X_3}(x)$ and $\mu_{X_2}(x) \leq \mu_{X_4}(x)$, then $s(\mu_{X_1}(x), \mu_{X_2}(x)) \leq s(\mu_{X_3}(x), \mu_{X_4}(x))$
3. $s(\mu_{X_1}(x), \mu_{X_2}(x)) = s(\mu_{X_2}(x), \mu_{X_1}(x))$
4. $s(\mu_{X_1}(x), s(\mu_{X_2}(x), \mu_{X_3}(x))) = s(s(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$

t-norm and t-conorm are related in a sense of lojical duality as;

$$t(\mu_{X_1}(x), \mu_{X_2}(x)) = 1 - s(1 - \mu_{X_3}(x), 1 - \mu_{X_4}(x))$$

Some t-norm and t-conorm are given as;

1. Drastic product:

$$t_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} min\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & max\{\mu_{X_1}(x)\mu_{X_2}(x)\} = 1 \\ 0, & otherwise \end{cases}$$

2. Drastic sum:

$$s_w(\mu_{X_1}(x), \mu_{X_2}(x)) = \begin{cases} max\{\mu_{X_1}(x), \mu_{X_2}(x)\}, & min\{\mu_{X_1}(x)\mu_{X_2}(x)\} = 0 \\ 1, & otherwise \end{cases}$$

3. Bounded product:
$$t_1(\mu_{X_1}(x), \mu_{X_2}(x)) = max\{0, \mu_{X_1}(x) + \mu_{X_2}(x) - 1\}$$

4. Bounded sum:
$$s_1(\mu_{X_1}(x), \mu_{X_2}(x)) = min\{1, \mu_{X_1}(x) + \mu_{X_2}(x)\}$$

5. Einstein product:

$$t_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x).\mu_{X_2}(x)}{2 - [\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x).\mu_{X_2}(x)]}$$

6. Einstein sum:
$$s_{1.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x)}{1 + \mu_{X_1}(x).\mu_{X_2}(x)}$$

7. Algebraic product:
$$t_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x).\mu_{X_2}(x)$$

8. Algebraic sum:
$$s_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x).\mu_{X_2}(x)$$

9. Hamacher product:

$$t_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x).\mu_{X_2}(x)}{\mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x).\mu_{X_2}(x)}$$

10. Hamacher sum:

$$s_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x) + \mu_{X_2}(x) - 2.\mu_{X_1}(x).\mu_{X_2}(x)}{1 - \mu_{X_1}(x).\mu_{X_2}(x)}$$

11. Minumum:

$$t_3(\mu_{X_1}(x), \mu_{X_2}(x)) = min\{\mu_{X_1}(x), \mu_{X_2}(x)\}$$

12. Maximum:

$$s_3(\mu_{X_1}(x), \mu_{X_2}(x)) = max\{\mu_{X_1}(x), \mu_{X_2}(x)\}$$

Definition 2.4. [45] Let E be a universe. An single valued neutrosophic set (SVN-set) over E defined by

$$T_A : E \to [0,1], \quad I_A : E \to [0,1], \quad F_A : E \to [0,1]$$

such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.5. [44] Let U be a universe. Then, an interval value neutrosophic set (IVN-sets) A in U is given as;

$$A = \{\langle T_A(u), I_A(u), F_A(u)\rangle / u : u \in U\}$$

In here, $(T_A(u), I_A(u), F_A(u)) = ([infT_A(u), supT_A(u)], [infI_A(u), supI_A(u)]], [infF_A(u), supF_A(u)])$ is called interval value neutrosophic number for all $u \in U$ and all interval value neutrosophic numbers over U will be denoted by $IVN(U)$.

%begindefinition[24]

3 Interval valued generalized SVTrN -numbers

In this section, we give definitions of interval valued generalized SVTrN-numbers with operations. Some of it is quoted from application in [2, 11, 23, 24, 25].

Definition 3.1. [2, 11, 25, 29] A interval valued generalized single valued trapezoidal neutrosophic number (IVGSVTrN-number)

$$\tilde{a} = \langle (a_1, b_1, c_1, d_1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}]\rangle$$

is a special neutrosophic set on the set of real numbers $\mathbb{R}$, whose truth-membership, indeterminacy-membership and falsity-membership functions are respectively defined by

$$T^-_{\tilde{a}}(x) = \begin{cases} (x - a_1)T^-_{\tilde{a}}/(b_1 - a_1) & (a_1 \leq x < b_1) \\ T^-_{\tilde{a}} & (b_1 \leq x \leq c_1) \\ (d_1 - x)T^-_{\tilde{a}}/(d_1 - c_1) & (c_1 < x \leq d_1) \\ 0 & otherwise, \end{cases}$$

$$T^+_{\tilde{a}}(x) = \begin{cases} (x - a_1)T^+_{\tilde{a}}/(b_1 - a_1) & (a_1 \leq x < b_1) \\ T^+_{\tilde{a}} & (b_1 \leq x \leq c_1) \\ (d_1 - x)T^+_{\tilde{a}}/(d_1 - c_1) & (c_1 < x \leq d_1) \\ 0 & otherwise, \end{cases}$$

$$I^-_{\tilde{a}}(x) = \begin{cases} (b_1 - x + I^-_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ I^-_{\tilde{a}} & (b_1 \leq x \leq c_1) \\ (x - c_1 + I^-_{\tilde{a}}(d_1 - x))/(d_1 - c_1) & (c_1 < x \leq d_1) \\ 1 & otherwise \end{cases}$$

$$I^+_{\tilde{a}}(x) = \begin{cases} (b_1 - x + I^+_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ I^+_{\tilde{a}} & (b_1 \leq x \leq c_1) \\ (x - c_1 + I^+_{\tilde{a}}(d_1 - x))/(d_1 - c_1) & (c_1 < x \leq d_1) \\ 1 & otherwise \end{cases}$$

$$F^-_{\tilde{a}}(x) = \begin{cases} (b_1 - x + F^-_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ F^-_{\tilde{a}} & (b_1 \leq x \leq c_1) \\ (x - c_1 + F^-_{\tilde{a}}(d_1 - x))/(d_1 - c_1) & (c_1 < x \leq d_1) \\ 1 & otherwise \end{cases}$$

and

$$F^+_{\tilde{a}}(x) = \begin{cases} (b_1 - x + F^+_{\tilde{a}}(x - a_1))/(b_1 - a_1) & (a_1 \leq x < b_1) \\ F^+_{\tilde{a}} & (b_1 \leq x \leq c_1) \\ (x - c_1 + F^+_{\tilde{a}}(d_1 - x))/(d_1 - c_1) & (c_1 < x \leq d_1) \\ 1 & otherwise \end{cases}$$

If $a_1 \geq 0$ and at least $d_1 > 0$, then $\tilde{a} = \langle (a_1, b_1, c_1, d_1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle$, is called a positive IVGSVTrN, denoted by $\tilde{a} > 0$. Likewise, if $d_1 \leq 0$ and at least $a_1 < 0$, then $\tilde{a} = \langle (a_1, b_1, c_1, d_1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle$, is called a negative IVGSVTrN, denoted by $\tilde{a} < 0$.

Note that the set of all IVGSVTrN-number on $\mathbb{R}$ will be denoted by Ω.

[2, 11, 25, 29] give some operations based algebraic sum-product norms on interval valued generalized SVTrN -numbers . We now give alternative operations based maximum-minimum norms on interval valued generalized SVTrN -numbers as;

Definition 3.2. Let $\tilde{a} = \langle (a_1, b_1, c_1, d_1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_2); [T^-_{\tilde{b}}, T^+_{\tilde{b}}], [I^-_{\tilde{b}}, I^+_{\tilde{b}}], [F^-_{\tilde{b}}, F^+_{\tilde{b}}] \rangle \in \Omega$ and $\Omega \neq 0$ be any real number. Then,

1. sum of $\tilde{a}$ and $\tilde{b}$, denoted by $\tilde{a} + \tilde{b}$, defined as;

$$\begin{aligned} \tilde{a} + \tilde{b} = \ & \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); \\ & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle \end{aligned} \tag{3.1}$$

2.

$$\begin{aligned} \tilde{a} - \tilde{b} = \ & \langle (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); \\ & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle \end{aligned} \tag{3.2}$$

3.

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2); & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle (d_1 > 0, d_2 > 0) \\ \langle (a_1d_2, b_1c_2, c_1b_2, d_1a_2); & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle (d_1 < 0, d_2 > 0) \\ \langle (d_1d_2, c_1c_2, b_1b_2, a_1a_2); & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle (d_1 < 0, d_2 < 0) \end{cases} \tag{3.3}$$

4.

$$\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2); & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle (d_1 > 0, d_2 > 0) \\ \langle (d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2); & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle (d_1 < 0, d_2 > 0) \\ \langle (d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2); & [min\{T^-_{\tilde{a}}, T^-_{\tilde{b}}\}, min\{T^+_{\tilde{a}}, T^+_{\tilde{b}}\}], [max\{I^-_{\tilde{a}} \vee I^-_{\tilde{b}}\}, max\{I^+_{\tilde{a}}, I^+_{\tilde{b}}\}], \\ & [max\{F^-_{\tilde{a}}, F^-_{\tilde{b}}\}, max\{F^+_{\tilde{a}}, F^+_{\tilde{b}}\}] \rangle (d_1 < 0, d_2 < 0) \end{cases} \tag{3.4}$$

5.

$$\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle & (\gamma > 0) \\ \langle (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle & (\gamma < 0) \end{cases} \tag{3.5}$$

6.

$$\tilde{a}^{\gamma} = \begin{cases} \langle (a_1^{\gamma}, b_1^{\gamma}, c_1^{\gamma}, d_1^{\gamma}); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle & (\gamma > 0) \\ \langle (d_1^{\gamma}, c_1^{\gamma}, b_1^{\gamma}, a_1^{\gamma}); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle & (\gamma < 0) \end{cases} \tag{3.6}$$

7.

$$\tilde{a}^{-1} = \langle (1/d_1, 1/c_1, 1/b_1, 1/a_1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle \;\; (\tilde{a} \neq 0). \tag{3.7}$$

Definition 3.3. Let $\tilde{a} = \langle (a, b, c, d); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle \in \Omega$. Then, we defined a method to normalize $\tilde{a}$ as;

$$\langle (\frac{a}{d}, \frac{b}{d}, \frac{c}{d}, 1); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle$$

such that $d \neq 0$.

Definition 3.4. Let $\tilde{a} = \langle (a, b, c, d); [T^-_{\tilde{a}}, T^+_{\tilde{a}}], [I^-_{\tilde{a}}, I^+_{\tilde{a}}], [F^-_{\tilde{a}}, F^+_{\tilde{a}}] \rangle \in \Omega$, then

$$S(\tilde{a}) = \frac{1}{16}[a + b + c + d] \times [4 + (T^-_{\tilde{a}} - I^-_{\tilde{a}} - F^-_{\tilde{a}}) + (T^+_{\tilde{a}} - I^+_{\tilde{a}} - F^+_{\tilde{a}})] \tag{3.8}$$

and

$$A(\tilde{a}) = \frac{1}{16}[a + b + c + d] \times [4 + (T^-_{\tilde{a}} - I^-_{\tilde{a}} + F^-_{\tilde{a}}) + (T^+_{\tilde{a}} - I^+_{\tilde{a}} + F^+_{\tilde{a}})] \tag{3.9}$$

is called the score and accuracy degrees of $\tilde{a}$, respectively.

Example 3.5. Let $\tilde{a} = \langle(0.3, 0.4, 0.8, 0.9); [0.5, 0.7], [0.4, 0.6], [0.3, 0.7]\rangle$ be a IVGSVTrN-number then, based on Equation 3.8 and 3.9, $S(\tilde{a})$ and $A(\tilde{a})$ is computed as;

$$S(\tilde{a}) = \frac{1}{16}[0.3 + 0.4 + 0.8 + 0.9] \times [4 + (0.5 - 0.4 - 0.3) + (0.7 - 0.6 - 0.7)] = 0.533$$

$$A(\tilde{a}) = \frac{1}{16}[0.3 + 0.4 + 0.8 + 0.9] \times [4 + (0.5 - 0.4 + 0.3) + (0.7 - 0.6 + 0.7)] = 0.866$$

Definition 3.6. Let $\tilde{a}_1, \tilde{a}_2 \in \Omega$. Then,

1. If $S(\tilde{a}_1) < S(\tilde{a}_2) \Rightarrow \tilde{a}_1 < \tilde{a}_2$
2. If $S(\tilde{a}_1) > S(\tilde{a}_2) \Rightarrow \tilde{a}_1 > \tilde{a}_2$
3. If $S(\tilde{a}_1) = S(\tilde{a}_2)$;
   (a) If $A(\tilde{a}_1) < A(\tilde{a}_2) \Rightarrow \tilde{a}_1 < \tilde{a}_2$
   (b) If $A(\tilde{a}_1) > A(\tilde{a}_2) \Rightarrow \tilde{a}_1 > \tilde{a}_2$
   (c) If $A(\tilde{a}_1) = A(\tilde{a}_2) \Rightarrow \tilde{a}_1 = \tilde{a}_2$

4 Aggregation operators on IVGSVTrN-numbers

In this section, three IVGSVTrN weighted aggregation operator of IVGSVTrN-numbers is given. Some of it is quoted from application in [2, 11, 23, 24, 25].

Definition 4.1. Let $\tilde{a}_j = \langle(a_j, b_j, c_j, d_j); [T^-_{\tilde{a}_j}, T^+_{\tilde{a}_j}], [I^-_{\tilde{a}_j}, I^+_{\tilde{a}_j}], [F^-_{\tilde{a}_j}, F^+_{\tilde{a}_j}]\rangle \in \Omega$ $(j \in I_n)$. Then IVGSVTrN weighted aggregation operator, denoted by K_{ao}, is defined as;

$$K_{ao} : \Omega^n \to \Omega, \quad K_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{i=1}^{n} \omega_i \tilde{a}_i \tag{4.1}$$

where, $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is a weight vector associated with the K_{ao} operator, for every $j \in I_n$ such that, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Theorem 4.2. *Let $\tilde{a}_j = \langle(a_j, b_j, c_j, d_j); [T^-_{\tilde{a}_j}, T^+_{\tilde{a}_j}], [I^-_{\tilde{a}_j}, I^+_{\tilde{a}_j}], [F^-_{\tilde{a}_j}, F^+_{\tilde{a}_j}]\rangle \in \Omega$ $(j \in I)$, $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$be a weight vector of $\tilde{a}_j$, for every $j \in I_n$ such that $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Then, their aggregated value by using K_{ao} operator is also a IVGSVTrN-number and*

$$\begin{aligned} K_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = & \Big\langle \Big(\textstyle\sum_{j=1}^{n} \omega_j a_j, \sum_{j=1}^{n} \omega_j b_j, \sum_{j=1}^{n} \omega_j c_j, \sum_{j=1}^{n} \omega_j d_j \Big); [\min_{1\le j\le n}\{T^-_{\tilde{a}_j}\}, \min_{1\le j\le n}\{T^+_{\tilde{a}_j}\}], \\ & [\max_{1\le j\le n}\{I^-_{\tilde{a}_j}\}, \max_{1\le j\le n}\{I^+_{\tilde{a}_j}\}], [\max_{1\le j\le n}\{F^-_{\tilde{a}_j}\}, \max_{1\le j\le n}\{F^+_{\tilde{a}_j}\}] \Big\rangle \end{aligned} \tag{4.2}$$

Proof: The proof can be made by using mathematical induction on n as; Assume that,

$$\tilde{a}_1 = \langle(a_1, b_1, c_1, d_1); [T^-_{\tilde{a}_1}, T^+_{\tilde{a}_1}], [I^-_{\tilde{a}_1}, I^+_{\tilde{a}_1}], [F^-_{\tilde{a}_1}, F^+_{\tilde{a}_1}]\rangle$$

and

$$\tilde{a}_2 = \langle (a_2, b_2, c_2, d_2); [T^-_{\tilde{a}_2}, T^+_{\tilde{a}_2}], [I^-_{\tilde{a}_2}, I^+_{\tilde{a}_2}], [F^-_{\tilde{a}_2}, F^+_{\tilde{a}_2}] \rangle$$

be two IVGSVTrN-numbers then, for $n = 2$, we have

$$\begin{aligned}\omega_1\tilde{a}_1 + \omega_2\tilde{a}_2 = \; & \Bigg\langle \Bigg(\textstyle\sum_{j=1}^{2} \omega_j a_j, \sum_{j=1}^{2} \omega_j b_j, \sum_{j=1}^{2} \omega_j c_j, \sum_{j=1}^{2} \omega_j d_j \Bigg); [\min_{1\le j\le 2}\{T^-_{\tilde{a}_j}\}, \min_{1\le j\le 2}\{T^+_{\tilde{a}_j}\}], \\ & [\max_{1\le j\le 2}\{I^-_{\tilde{a}_j}\}, \max_{1\le j\le 2}\{I^+_{\tilde{a}_j}\}], [\max_{1\le j\le 2}\{F^-_{\tilde{a}_j}\}, \max_{1\le j\le 2}\{F^+_{\tilde{a}_j}\}] \Bigg\rangle\end{aligned} \tag{4.3}$$

If holds for $n = k$, that is

$$\begin{aligned}\omega_1\tilde{a}_1 + \omega_2\tilde{a}_2 + \cdots + \omega_k\tilde{a}_k = \; & \Bigg\langle \Bigg(\textstyle\sum_{j=1}^{k} \omega_j a_j, \sum_{j=1}^{k} \omega_j b_j, \sum_{j=1}^{k} \omega_j c_j, \sum_{j=1}^{k} \omega_j d_j \Bigg); \\ & [\min_{1\le j\le k}\{T^-_{\tilde{a}_j}\}, \min_{1\le j\le k}\{T^+_{\tilde{a}_j}\}], [\max_{1\le j\le k}\{I^-_{\tilde{a}_j}\}, \max_{1\le j\le k}\{I^+_{\tilde{a}_j}\}], \\ & [\max_{1\le j\le k}\{F^-_{\tilde{a}_j}\}, \max_{1\le j\le k}\{F^+_{\tilde{a}_j}\}] \Bigg\rangle\end{aligned} \tag{4.4}$$

then, when $n = k + 1$, by the operational laws in Definition 3.2, I have

$$\begin{aligned}\omega_1\tilde{a}_1 + \omega_2\tilde{a}_2 + \cdots + \omega_{k+1}\tilde{a}_{k+1} = \; & \Bigg\langle \Bigg(\textstyle\sum_{j=1}^{k} \omega_j a_j, \sum_{j=1}^{k} \omega_j b_j, \sum_{j=1}^{k} \omega_j c_j, \sum_{j=1}^{k} \omega_j d_j \Bigg); \\ & [\min_{1\le j\le k}\{T^-_{\tilde{a}_j}\}, \min_{1\le j\le k}\{T^+_{\tilde{a}_j}\}], [\max_{1\le j\le k}\{I^-_{\tilde{a}_j}\}, \max_{1\le j\le k}\{I^+_{\tilde{a}_j}\}], \\ & [\max_{1\le j\le k}\{F^-_{\tilde{a}_j}\}, \max_{1\le j\le k}\{F^+_{\tilde{a}_j}\}] \Bigg\rangle + \\ & \Bigg\langle \Bigg(\omega_{k+1}\, a_{k+1}, \omega_{k+1}\, b_{k+1}, \omega_{k+1} c_{k+1}, \omega_{k+1} d_{k+1} \Bigg); \\ & [T^-_{\tilde{a}_{k+1}}, T^+_{\tilde{a}_{k+1}}], [I^-_{\tilde{a}_{k+1}}, I^+_{\tilde{a}_{k+1}}], [F^-_{\tilde{a}_{k+1}}, F^+_{\tilde{a}_{k+1}}] \Bigg\rangle \\ = \; & \Bigg\langle \Bigg(\textstyle\sum_{j=1}^{k+1} \omega_j a_j, \sum_{j=1}^{k+1} \omega_j b_j, \sum_{j=1}^{k+1} \omega_j c_j, \sum_{j=1}^{k+1} \omega_j d_j \Bigg); \\ & [\min_{1\le j\le k+1}\{T^-_{\tilde{a}_j}\}, \min_{1\le j\le k+1}\{T^+_{\tilde{a}_j}\}], [\max_{1\le j\le k+1} I^-_{\tilde{a}_j}, \max_{1\le j\le k+1}\{I^+_{\tilde{a}_j}\}], \\ & [\max_{1\le j\le k+1}\{F^-_{\tilde{a}_j}\}, \max_{1\le j\le k+1}\{F^+_{\tilde{a}_j}\}] \Bigg\rangle\end{aligned} \tag{4.5}$$

Finally, based on Equation 4.3, 4.4 and 4.5, the proof is valid.

Example 4.3. Let

$$\tilde{a}_1 = \big\langle (0.125, 0.439, 0.754, 0.847); [0.5, 0.6], [0.4, 0.7], [0.6, 0.9] \big\rangle,$$

$$\tilde{a}_2 = \big\langle (0.326, 0.427, 0.648, 0.726); [0.8, 0.9], [0.2, 0.5], [0.4, 0.8] \big\rangle,$$

$$\tilde{a}_3 = \big\langle (0.427, 0.524, 0.578, 0.683); [0.4, 0.6], [0.3, 0, 8], [0, 5, 0.7] \big\rangle$$

be three IVGSVTrN-numbers, and $\omega = (0.4, 0.3, 0.3)^T$ be the weight vector of $\tilde{a}_j (j = 1, 2, 3)$. Then, based on

Equation 4.2,

$$K_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \big\langle(0.276, 0.461, 0.669, 0.762); [0.4, 0.6], [0.4, 0.8], [0.6, 0.9]\big\rangle$$

and, based on Equation 3.8, their score is 0.312.

Definition 4.4. Let $\tilde{a}_j = \langle(a_j, b_j, c_j, d_j); [T^-_{\tilde{a}_j}, T^+_{\tilde{a}_j}], [I^-_{\tilde{a}_j}, I^+_{\tilde{a}_j}], [F^-_{\tilde{a}_j}, F^+_{\tilde{a}_j}]\rangle \in \Omega\,(j \in I_n)$.Then IVGSVTrN ordered weighted aggregation operator(K_{oao}) is defined as;

$$K_{oao} : \Omega^n \to \Omega, \;\; K_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{k=1}^{n} \omega_k \tilde{b}_k \tag{4.6}$$

where $\tilde{b}_k = \langle(a_k, b_k, c_k, d_k); [T^-_{\tilde{a}_k}, T^+_{\tilde{a}_k}], [I^-_{\tilde{a}_k}, I^+_{\tilde{a}_k}], [F^-_{\tilde{a}_k}, F^+_{\tilde{a}_k}]\rangle$ is the *k*-th largest of the n IVGSVTrN-numbers $\tilde{a}_j\,(j \in I_n)$ based on Equation 3.6.

Their aggregated value by using K_{oao} operator is also a IVGSVTrN-number and computed as;

$$\begin{aligned} K_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \;& \Bigg\langle\Bigg(\textstyle\sum_{k=1}^{n} \omega_k a_k, \sum_{k=1}^{n} \omega_k b_k, \sum_{k=1}^{n} \omega_k c_k \sum_{k=1}^{n} \omega_k d_k \Bigg); \\ & [\min_{1\le j\le n} T^-_{\tilde{a}_j}, \min_{1\le j\le n} T^+_{\tilde{a}_j}], [\max_{1\le j\le n} I^-_{\tilde{a}_j}, \max_{1\le j\le n} I^+_{\tilde{a}_j}], \\ & [\max_{1\le j\le n} F^-_{\tilde{a}_j}, \max_{1\le j\le n} F^+_{\tilde{a}_j}] \Bigg\rangle \end{aligned} \tag{4.7}$$

Definition 4.5. Let $\tilde{a}_j = \langle(a_j, b_j, c_j, d_j); [T^-_{\tilde{a}_j}, T^+_{\tilde{a}_j}], [I^-_{\tilde{a}_j}, I^+_{\tilde{a}_j}], [F^-_{\tilde{a}_j}, F^+_{\tilde{a}_j}]\rangle \in \Omega \;\; (j \in I_n)$. Then, IVGSVTrN ordered hybrid weighted averaging operator denoted by K_{hao} is defined as;denoted K_{hao}

$$K_{hao} : \Omega^n \to \Omega, \;\; K_{hao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{k=1}^{n} \omega_k \widehat{b}_k \tag{4.8}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is a weight vector associated with the mapping K_{hao} such that $\omega_k \in [0, 1]$ and $\sum_{k=1}^{n} \omega_k = 1$, $\tilde{a}_j \in \Omega$ weighted with $n\varpi_j(j \in I_n)$ is denoted by $\tilde{A}_j$, i.e., $\tilde{A}_j = n\varpi_j \tilde{a}_j$, here n is regarded as a balance factor; $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ is a weight vector of the $\tilde{a}_j \in \Omega\,(j \in I_n)$n such that $\varpi_j \in [0, 1]$ and $\sum_{j=1}^{n} \varpi_j = 1$; $\widehat{b}_k$ is the *k*-th largest of the n IVGSVTrN-number $\tilde{A}_j \in \Omega\,(j \in I_n)$ based on Equation 3.6.

Their aggregated value by using K_{hao} operator is also a IVGSVTrN-number and computed as

$$\begin{aligned} K_{hao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \;& \Bigg\langle\Bigg(\textstyle\sum_{k=1}^{n} \omega_k a_k, \sum_{k=1}^{n} \omega_k b_k, \sum_{k=1}^{n} \omega_k c_k, \sum_{k=1}^{n} \omega_k d_k \Bigg); \\ & [\min_{1\le j\le n} T^-_{\tilde{a}_j}, \min_{1\le j\le n} T^+_{\tilde{a}_j}], [\max_{1\le j\le n} I^-_{\tilde{a}_j}, \max_{1\le j\le n} I^+_{\tilde{a}_j}], \\ & [\max_{1\le j\le n} F^-_{\tilde{a}_j}, \max_{1\le j\le n} F^+_{\tilde{a}_j}] \Bigg\rangle \end{aligned} \tag{4.9}$$

Example 4.6. Let

$$\tilde{a}_1 = \big\langle(0.123, 0.278, 0.347, 0.426); [0.7, 0.8], [0.4, 0.7], [0.1, 0.6]\big\rangle,$$

$$\tilde{a}_2 = \big\langle(0.133, 0.268, 0.357, 0.416); [0.1, 0.6], [0.7, 0.8], [0.4, 0.7]\big\rangle$$

and

$$\tilde{a}_3 = \big\langle (0.143, 0.258, 0.367, 0.406); [0.4, 0.7], [0.1, 0.6], [0.7, 0.8] \big\rangle$$

be three IVGSVTrN-numbers. Assume that $\varpi = (0.2, 0.3, 0.5)^T$ be a weight vector and $\omega = (0.5, 0.3, 0.2)^T$ be a position weight vector.Then evaluation of the three numbers by using the Equation 4.9 is given as;

Solving

$$\tilde{A}_1 = 3 \times 0.2 \times \tilde{a}_1 = \big\langle (0.074, 0.167, 0.208, 0.256); [0.7, 0.8], [0.4, 0.7], [0.1, 0.6] \big\rangle$$

Likewise, we obtain:

$$\tilde{A}_2 = 3 \times 0.3 \times \tilde{a}_2 = \big\langle (0.160, 0.322, 0.428, 0.499); [0.1, 0.6], [0.7, 0.8], [0.4, 0.7] \big\rangle$$

$$\tilde{A}_3 = 3 \times 0.5 \times \tilde{a}_3 = \big\langle (0.286, 0.516, 0.734, 0.812); [0.4, 0.7], [0.1, 0.6], [0.7, 0.8] \big\rangle$$

we obtain the scores of the IVGSVTrN-numbers $\tilde{A}_j$(j=1,2,3), based on Equation 3.8, as follows:

$$S(\tilde{A}_1) = \frac{1}{16}[0.074 + 0.167 + 0.208 + 0.256] \times (4 + (0.7 - 0.4 - 0.1) + (0.8 - 0.7 - 0.6)) = 0.163$$

$$S(\tilde{A}_2) = \frac{1}{16}[0.160 + 0.322 + 0.428 + 0.499] \times (4 + (0.1 - 0.7 - 0.4) + (0.6 - 0.8 - 0.7)) = 0.185$$

$$S(\tilde{A}_3) = \frac{1}{16}[0.286 + 0.516 + 0.734 + 0.812] \times (4 + (0.4 - 0.1 - 0.7) + (0.7 - 0.6 - 0.8)) = 0.426$$

respectively. Obviously, $S(\tilde{A}_3) > S(\tilde{A}_2) > S(\tilde{A}_1)$. Thereby, according to the Equation 3.6, we have

$$\hat{b}_1 = \tilde{A}_3 = \big\langle (0.143, 0.258, 0.367, 0.406); [0.4, 0.7], [0.1, 0.6], [0.7, 0.8] \big\rangle$$

$$\hat{b}_2 = \tilde{A}_2 = \big\langle (0.133, 0.268, 0.357, 0.416); [0.1, 0.6], [0.7, 0.8], [0.4, 0.7] \big\rangle$$

$$\hat{b}_3 = \tilde{A}_1 = \big\langle (0.123, 0.278, 0.347, 0.426); [0.7, 0.8], [0.4, 0.7], [0.1, 0.6] \big\rangle$$

It follows from Equation 4.9 that

$$\begin{aligned} K_{hao}(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) = \ & \big\langle \big(0.143 \times 0.5 + 0.133 \times 0.3 + 0.123 \times 0.2, \\ & 0.258 \times 0.5 + 0.268 \times 0.3 + 0.278 \times 0.2, \\ & 0.367 \times 0.5 + 0.357 \times 0.3 + 0.347 \times 0.2, \\ & 0.406 \times 0.5 + 0.416 \times 0.3 + 0.326 \times 0.2\big); [0.1, 0.6], [0.7, 0.8], [0.7, 0.8] \big\rangle \\ = \ & \big\langle \big(0.1360, 0.2650, 0.3600, 0.4130\big); [0.1, 0.6], [0.7, 0.8], [0.7, 0.8] \big\rangle \end{aligned}$$

5 Geometric operators of the IVGSVTrN-number

In this section, we give three IVGSVTrN weighted geometric operator of IVGSVTrN-numbers. Some of it is quoted from application in [2, 11, 24, 33].

Definition 5.1. Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); [T^-_{\tilde{a}_j}, T^+_{\tilde{a}_j}], [I^-_{\tilde{a}_j}, I^+_{\tilde{a}_j}], [F^-_{\tilde{a}_j}, F^+_{\tilde{a}_j}] \rangle \in \Omega$ $(j \in I_n)$. Then IVGSVTrN weighted geometric operator, denoted by L_{go}, is defined as;

$$L_{go} : \Omega^n \to \Omega, \quad L_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{i=1}^{n} \tilde{a}_i^{\omega_i} \tag{5.1}$$

where, $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is a weight vector associated with the L_{go} operator, for every $j \in I_n$ such that, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Their aggregated value by using L_{go} operator is also a IVGSVTrN-number and computed as;

$$\begin{aligned} L_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = & \Bigg\langle \Bigg(\prod_{j=1}^{n} a_j^{\omega_j}, \prod_{j=1}^{n} b_j^{\omega_j}, \prod_{j=1}^{n} c_j^{\omega_j} \prod_{j=1}^{n} d_j^{\omega_j} \Bigg); \\ & [\min_{1\le j\le n}\{T^-_{\tilde{a}_j}\}, \min_{1\le j\le n}\{T^+_{\tilde{a}_j}\}], [\max_{1\le j\le n}\{I^-_{\tilde{a}_j}\}, \max_{1\le j\le n}\{I^+_{\tilde{a}_j}\}], \\ & [\max_{1\le j\le n}\{F^-_{\tilde{a}_j}\}, \max_{1\le j\le n}\{F^+_{\tilde{a}_j}\}] \Bigg\rangle \end{aligned} \tag{5.2}$$

Example 5.2. Let

$$\tilde{a}_1 = \big\langle (0.125, 0.439, 0.754, 0.847); [0.5, 0.6], [0.4, 0.7], [0.6, 0.9] \big\rangle,$$

$$\tilde{a}_2 = \big\langle (0.326, 0.427, 0.648, 0.726); [0.8, 0.9], [0.2, 0.5], [0.4, 0.8] \big\rangle,$$

$$\tilde{a}_3 = \big\langle (0.427, 0.524, 0.578, 0.683); [0.4, 0.6], [0.3, 0, 8], [0, 5, 0.7] \big\rangle$$

be four IVGSVTrN-numbers, and $w = (0.4, 0.3, 0.3)^T$ be the weight vector of $\tilde{a}_j (j = 1, 2, 3)$. Then, based on Equation 5.2,

$$L_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \big\langle (0.241, 0.459, 0.665, 0.758); [0.4, 0.6], [0.4, 0.8], [0.6, 0.9] \big\rangle$$

and, based on Equation 3.8, their score is 0.305.

Definition 5.3. Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); [T^-_{\tilde{a}_j}, T^+_{\tilde{a}_j}], [I^-_{\tilde{a}_j}, I^+_{\tilde{a}_j}], [F^-_{\tilde{a}_j}, F^+_{\tilde{a}_j}] \rangle \in \Omega\, (j \in I_n)$. Then IVGSVTrN ordered weighted geometric operator denoted by L_{ogo}, is defined as;

$$L_{ogo} : \Omega^n \to \Omega, \; L_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{k=1}^{n} \tilde{b}_k^{w_k} \tag{5.3}$$

where $\omega_k \in [0, 1]$, $\sum_{k=1}^{n} \omega_k = 1$; $\tilde{b}_k = \langle (a_k, b_k, c_k, d_k); [T^-_{\tilde{a}_k}, T^+_{\tilde{a}_k}], [I^-_{\tilde{a}_k}, I^+_{\tilde{a}_k}], [F^-_{\tilde{a}_k}, F^+_{\tilde{a}_k}] \rangle$ is the k-th largest of the n neutrosophic sets $\tilde{a}_j$ $(j \in I_n)$ based on Equation 3.6.

Their aggregated value by using L_{ogo} operator is also a IVGSVTrN-number and computed as;

$$\begin{aligned} L_{ogo} : \Omega^n \to \Omega, \quad & L_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \\ & \Bigg\langle \Bigg(\prod_{k=1}^{n} a_k^{w_k}, \prod_{k=1}^{n} b_k^{w_k}, \prod_{k=1}^{n} c_k^{w_k} \prod_{k=1}^{n} d_k^{w_k} \Bigg); \\ & [\min_{1\le j\le n}\{T^-_{\tilde{a}_j}\}, \min_{1\le j\le n}\{T^+_{\tilde{a}_j}\}], [\max_{1\le j\le n}\{I^-_{\tilde{a}_j}\}, \max_{1\le j\le n}\{I^+_{\tilde{a}_j}\}], \\ & [\max_{1\le j\le n}\{F^-_{\tilde{a}_j}\}, \max_{1\le j\le n}\{F^+_{\tilde{a}_j}\}] \Bigg\rangle \end{aligned} \tag{5.4}$$

Definition 5.4. Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); [T^-_{\tilde{a}_j}, T^+_{\tilde{a}_j}], [I^-_{\tilde{a}_j}, I^+_{\tilde{a}_j}], [F^-_{\tilde{a}_j}, F^+_{\tilde{a}_j}]\rangle \in \Omega\, (j \in I_n)$. Then IVGSVTrN ordered hybrid weighted geometric operator denoted by L_{hgo}, is defined as;

$$L_{hgo} : \Omega^n \to \Omega, \quad L_{hgo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{k=1}^{n} \hat{b}_k^{\omega_k} \tag{5.5}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$. $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$ is a weight vector associated with the mapping L_{hgo}, $a_j \in \Omega$ a weight with $n\varpi(j \in I_n)$ is denoted by $\tilde{A}_j$ i.e., $\tilde{A}_j = n\varpi\tilde{a}_j$, here n is regarded as a balance factor $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ is a weight vector of the $a_j \in \Omega\, (j \in I_n)$; $\hat{b}_k$ is the k-th largest of the n IVGSVTrN-numbers $\tilde{A}_j \in \Omega\, (j \in I_n)$ based on Equation 3.6.

Their aggregated value by using L_{hgo} operator is also a IVGSVTrN-number and computed as

$$\begin{aligned} L_{hgo} : \Omega^n \to \Omega, \quad & L_{hgo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \\ & \Big\langle \Big(\textstyle\prod_{k=1}^n a_k^{w_k}, \prod_{k=1}^n b_k^{w_k}, \prod_{k=1}^n c_k^{w_k} \prod_{k=1}^n d_k^{w_k} \Big); \\ & [\min_{1\le j\le n} T^-_{\tilde{a}_j}, \min_{1\le j\le n} T^+_{\tilde{a}_j}], [\max_{1\le j\le n} I^-_{\tilde{a}_j}, \max_{1\le j\le n} I^+_{\tilde{a}_j}], \\ & [\max_{1\le j\le n} F^-_{\tilde{a}_j}, \max_{1\le j\le n} F^+_{\tilde{a}_j}] \Big\rangle \end{aligned} \tag{5.6}$$

6 IVGSVTrN-multi-criteria decision-making method

In this section, we define a multi-criteria decision making method as follows. Some of it is quoted from application in [2, 11, 23, 24, 25].

Definition 6.1. Let $X = (x_1, x_2, ..., x_m)$ be a set of alternatives, $U = (u_1, u_2, ..., u_n)$ be the set of attributes. If $\tilde{a}_{ij} = \langle (a_{ij}, b_{ij}, c_{ij}, d_{ij}); [T^-_{\tilde{a}_{ij}}, T^+_{\tilde{a}_{ij}}], [I^-_{\tilde{a}_{ij}}, I^+_{\tilde{a}_{ij}}], [F^-_{\tilde{a}_{ij}}, F^+_{\tilde{a}_{ij}}]\rangle \in \Omega$, then

$$[\tilde{a}_{ij}]_{m\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{pmatrix} \end{array} \tag{6.1}$$

is called an IVGSVTrN-multi-criteria decision-making matrix of the decision maker.

Now, we can give an algorithm of the IVGSVTrN-multi-criteria decision-making method as follows;
Algorithm:

Step 1. Construct the decision-making matrix $[\tilde{a}_{ij}]_{m\times n}$ for decision based on Equation 6.1;

Step 2. Compute the IVGSVTrN-numbers $\tilde{A}_{ij} = n\varpi_i\tilde{a}_{ij}\ (i \in I_m;\ j \in I_n)$ and write the decision-making matrix $[\tilde{A}_{ij}]_{m\times n}$;

Step 3. Obtain the scores of the IVGSVTrN-numbers $\tilde{A}_{ij}\ (i \in I_m;\ j \in I_n)$ based on Equation 3.8;

Step 4. Rank all IVGSVTrN-numbers $\tilde{A}_{ij}(i \in I_m;\ j \in I_n)$ by using the ranking method of IVGSVTrN-numbers and determine the IVGSVTrN-numbers $[b_i]_{1\times n} = \tilde{b}_{ik}(i \in I_m;\ k \in I_n)$ where $\tilde{b}_{ik}$ is k-th largest of $\tilde{A}_{ij}$ for $j \in I_n$ based on Equation 3.6 ;

Step 5. Give the decision matrix $[b_i]_{1\times n}$ for $i = 1, 2, 3, 4$;

Step 6. Compute $K_{hao}(\tilde{b}_{i1}, \tilde{b}_{i2}, ..., \tilde{b}_{in})$ for $i \in I_m$ based on Equation 4.9;

Step 7. Compute $L_{hgo}(\tilde{b}_{i1}, \tilde{b}_{i2}, ..., \tilde{b}_{in})$ for $i \in I_m$ based on Equation 5.6;

Step 8. Rank all alternatives x_i by using the Equation 3.6 and determine the best alternative.

Example 6.2. Let us consider the decision-making problem adapted from [24, 52]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with the set of the four alternatives is denoted by $X = \{x_1$= car company, x_2=food company, x_3=computer company, x_4=arms company$\}$ to invest the money. The investment company must take a decision according to the set of the four attributes is denoted by $U = \{u_1 = risk, u_2 = growth, u_3 = environmental\ impact, u_4 = performance\}$. Then, the weight vector of the attributes is $\varpi = (0.2, 0.3, 0.2, 0.3)^T$ and the position weight vector is $\omega = (0.3, 0.2, 0.3, 0.2)^T$ by using the weight determination based on the normal distribution. For the evaluation of an alternative x_i $(i = 1, 2, 3, 4)$ with respect to a criterion u_j $(j = 1, 2, 3, 4)$, it is obtained from the questionnaire of a domain expert. Then, the four possible alternatives are to be evaluated under the above three criteria by corresponding to linguistic values of IVGSVTrN-numbers for linguistic terms (adapted from [24]), as shown in Table 1.

Linguistic terms	Linguistic values of IVGSVTrN-numbers
Absolutely low	$\langle(0.1, 0.2, 0.3, 0.4); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\rangle$
Low	$\langle(0.1, 0.3, 0.4, 0.7); [0.2, 0.4], [0.7, 0.8], [0.6, 0.8]\rangle$
Fairly low	$\langle(0.1, 0.4, 0.5, 0.7); [0.3, 0.5], [0.6, 0.7], [0.5, 0.7]\rangle$
Medium	$\langle(0.2, 0.4, 0.5, 0.8); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\rangle$
Fairly high	$\langle(0.4, 0.5, 0.6, 0.8); [0.6, 0.7], [0.4, 0.5], [0.3, 0.5]\rangle$
High	$\langle(0.5, 0.6, 0.7, 0.9); [0.7, 0.8], [0.3, 0.4], [0.2, 0.3]\rangle$
Absolutely high	$\langle(0.6, 0.7, 0.8, 0.9); [0.8, 0.9], [0.1, 0.2], [0.1, 0.2]\rangle$

Table 1: IVGSVTrN-numbers for linguistic terms

Step 1. The decision maker construct the decision matrix $[\tilde{a}_{ij}]_{4x4}$ based on Equation 6.1 as follows:

$$\left(\begin{array}{cc} \langle(0.2,0.4,0.5,0.8);[0.5,0.6],[0.5,0.6],[0.4,0.5]\rangle & \langle(0.1,0.4,0.5,0.7);[0.3,0.5],[0.6,0.7],[0.5,0.7]\rangle \\ \langle(0.1,0.3,0.4,0.7);[0.2,0.4],[0.7,0.8],[0.6,0.8]\rangle & \langle(0.1,0.2,0.3,0.4);[0.1,0.2],[0.8,0.9],[0.8,0.9]\rangle \\ \langle(0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle & \langle(0.4,0.5,0.6,0.8);[0.6,0.7],[0.4,0.5],[0.3,0.5]\rangle \\ \langle(0.5,0.6,0.7,0.9);[0.7,0.8],[0.3,0.4],[0.2,0.3]\rangle & \langle(0.2,0.4,0.5,0.8);[0.5,0.6],[0.5,0.6],[0.4,0.5]\rangle \end{array}\right.$$
$$\left.\begin{array}{cc} \langle(0.1,0.4,0.5,0.7);[0.3,0.5],[0.6,0.7],[0.5,0.7]\rangle & \langle(0.1,0.3,0.4,0.7);[0.2,0.4],[0.7,0.8],[0.6,0.8]\rangle \\ \langle(0.2,0.4,0.5,0.8);[0.5,0.6],[0.5,0.6],[0.4,0.5]\rangle & \langle(0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle \\ \langle(0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle & \langle(0.1,0.2,0.3,0.4);[0.1,0.2],[0.8,0.9],[0.8,0.9]\rangle \\ \langle(0.1,0.2,0.3,0.4);[0.1,0.2],[0.8,0.9],[0.8,0.9]\rangle & \langle(0.5,0.6,0.7,0.9);[0.7,0.8],[0.3,0.4],[0.2,0.3]\rangle \end{array}\right)$$

Step 2. Compute $\tilde{A}_{ij} = n\varpi_i\tilde{a}_{ij}\,(i = 1, 2, 3, 4;\ j = 1, 2, 3, 4)$ as follows:

$$\begin{aligned}\tilde{A}_{11} = &\ 4 \times 0.2 \times \big\langle (0.2, 0.4, 0.5, 0.8); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5] \big\rangle \\ = &\ \big\langle \big(0.16, 0.32, 0.40, 0.64\big); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5] \big\rangle\end{aligned}$$

Likewise, we can obtain other IVGSVTrN-numbers $\tilde{A}_{ij} = n\varpi_i\tilde{a}_{ij}\ (i = 1, 2, 3, 4;\ j = 1, 2, 3, 4)$ which are given by the IVGSVTrN-decision matrix $[\tilde{A}_{ij}]_{4\times4}$ as follows:

$$\left(\begin{array}{cc} \big\langle(0.16, 0.32, 0.40, 0.64); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\big\rangle & \big\langle(0.12, 0.48, 0.60, 0.84); [0.3, 0.5], [0.6, 0.7], [0.5, 0.7]\big\rangle \\ \big\langle(0.08, 0.24, 0.32, 0.56); [0.2, 0.4], [0.7, 0.8], [0.6, 0.8]\big\rangle & \big\langle(0.12, 0.24, 0.36, 0.48); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle \\ \big\langle(0.48, 0.56, 0.64, 0.72); [0.8, 0.9], [0.1, 0.2], [0.1, 0.2]\big\rangle & \big\langle(0.48, 0.60, 0.72, 0.96); [0.6, 0.7], [0.4, 0.5], [0.3, 0.5]\big\rangle \\ \big\langle(0.40, 0.48, 0.56, 0.72); [0.7, 0.8], [0.3, 0.4], [0.2, 0.3]\big\rangle & \big\langle(0.24, 0.48, 0.60, 0.96); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\big\rangle \end{array}\right.$$

$$\left.\begin{array}{cc} \big\langle(0.08, 0.32, 0.40, 0.56); [0.3, 0.5], [0.6, 0.7], [0.5, 0.7]\big\rangle & \big\langle(0.12, 0.36, 0.48, 0.84); [0.2, 0.4], [0.7, 0.8], [0.6, 0.8]\big\rangle \\ \big\langle(0.16, 0.32, 0.40, 0.64); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\big\rangle & \big\langle(0.72, 0.84, 0.96, 1.08); [0.8, 0.9], [0.1, 0.2], [0.1, 0.2]\big\rangle \\ \big\langle(0.08, 0.16, 0.24, 0.32); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle & \big\langle(0.12, 0.24, 0.36, 0.48); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle \\ \big\langle(0.48, 0.56, 0.64, 0.72); [0.8, 0.9], [0.1, 0.2], [0.1, 0.2]\big\rangle & \big\langle(0.60, 0.72, 0.84, 1.08); [0.7, 0.8], [0.3, 0.4], [0.2, 0.3]\big\rangle \end{array}\right)$$

Step 3. We can obtain the scores of the IVGSVTrN-numbers $\tilde{A}_{ij}$ of the alternatives $x_j\ (j = 1, 2, 3, 4)$ on the four attributes $u_i\ (i = 1, 2, 3, 4)$ based on Equation 3.8 as follows:

$$\begin{array}{llll} S(\tilde{A}_{11}) = 0.295 & S(\tilde{A}_{12}) = 0.293 & S(\tilde{A}_{13}) = 0.196 & S(\tilde{A}_{14}) = 0.191 \\ S(\tilde{A}_{21}) = 0.128 & S(\tilde{A}_{22}) = 0.068 & S(\tilde{A}_{23}) = 0.295 & S(\tilde{A}_{24}) = 1.148 \\ S(\tilde{A}_{31}) = 0.765 & S(\tilde{A}_{32}) = 0.621 & S(\tilde{A}_{33}) = 0.045 & S(\tilde{A}_{34}) = 0.068 \\ S(\tilde{A}_{41}) = 0.581 & S(\tilde{A}_{42}) = 0.442 & S(\tilde{A}_{43}) = 0.765 & S(\tilde{A}_{44}) = 0.871 \end{array}$$

respectively.

Step 4. The ranking order of all IVGSVTrN-numbers $\tilde{A}_{ij}(i = 1, 2, 3, 4;\ j = 1, 2, 3, 4)$ based on Equation 3.6 as follows;

$$\begin{gathered} \tilde{A}_{11} > \tilde{A}_{12} > \tilde{A}_{13} > \tilde{A}_{14} \\ \tilde{A}_{24} > \tilde{A}_{23} > \tilde{A}_{21} > \tilde{A}_{22} \\ \tilde{A}_{31} > \tilde{A}_{32} > \tilde{A}_{34} > \tilde{A}_{33} \\ \tilde{A}_{44} > \tilde{A}_{43} > \tilde{A}_{41} > \tilde{A}_{42} \end{gathered}$$

Thus, we have:

$$\begin{gathered} \tilde{b}_{11} = \tilde{A}_{11},\ \tilde{b}_{12} = \tilde{A}_{12},\ \tilde{b}_{13} = \tilde{A}_{13},\ \tilde{b}_{14} = \tilde{A}_{14} \\ \tilde{b}_{21} = \tilde{A}_{24},\ \tilde{b}_{22} = \tilde{A}_{23},\ \tilde{b}_{23} = \tilde{A}_{21},\ \tilde{b}_{24} = \tilde{A}_{22} \\ \tilde{b}_{31} = \tilde{A}_{31},\ \tilde{b}_{32} = \tilde{A}_{32},\ \tilde{b}_{33} = \tilde{A}_{34},\ \tilde{b}_{34} = \tilde{A}_{33} \\ \tilde{b}_{41} = \tilde{A}_{44},\ \tilde{b}_{42} = \tilde{A}_{43},\ \tilde{b}_{43} = \tilde{A}_{41},\ \tilde{b}_{44} = \tilde{A}_{42} \end{gathered}$$

Step 5. The decision matrix $[b_i]_{1\times n}$ for $i = 1, 2, 3, 4$ are given by;

$$\begin{aligned}\tilde{b}_1 = & \Big(\langle(0.16, 0.32, 0.40, 0.64); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\rangle, \langle(0.12, 0.48, 0.60, 0.84); [0.3, 0.5], [0.6, 0.7], [0.5, 0.7]\rangle,\\ & \langle(0.08, 0.32, 0.40, 0.56); [0.3, 0.5], [0.6, 0.7], [0.5, 0.7]\rangle, \langle(0.12, 0.36, 0.48, 0.84); [0.2, 0.4], [0.7, 0.8], [0.6, 0.8]\rangle\Big)\\ \tilde{b}_2 = & \Big(\langle(0.72, 0.84, 0.96, 1.08); [0.8, 0.9], [0.1, 0.2], [0.1, 0.2]\rangle, \langle(0.16, 0.32, 0.40, 0.64); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\rangle,\\ & \langle(0.08, 0.24, 0.32, 0.56); [0.2, 0.4], [0.7, 0.8], [0.6, 0.8]\rangle, \langle(0.12, 0.24, 0.36, 0.48); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\rangle\Big)\\ \tilde{b}_3 = & \Big(\langle(0.48, 0.56, 0.64, 0.72); [0.8, 0.9], [0.1, 0.2], [0.1, 0.2]\rangle, \langle(0.48, 0.60, 0.72, 0.96); [0.6, 0.7], [0.4, 0.5], [0.3, 0.5]\rangle,\\ & \langle(0.12, 0.24, 0.36, 0.48); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\rangle, \langle(0.08, 0.16, 0.24, 0.32); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\rangle\Big)\\ \tilde{b}_4 = & \Big(\langle(0.60, 0.72, 0.84, 1.08); [0.7, 0.8], [0.3, 0.4], [0.2, 0.3]\rangle, \langle(0.48, 0.56, 0.64, 0.72); [0.8, 0.9], [0.1, 0.2], [0.1, 0.2]\rangle,\\ & \langle(0.40, 0.48, 0.56, 0.72); [0.7, 0.8], [0.3, 0.4], [0.2, 0.3]\rangle, \langle(0.24, 0.48, 0.60, 0.96); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\rangle\Big)\end{aligned}$$

Step 6. We can calculate the IVGSVTrN-numbers based on Equation 4.9 $K_{hao}(b_i) = K_{hao}(\tilde{b}_{i1}, \tilde{b}_{i2}, \tilde{b}_{i3}, \tilde{b}_{i4})$ for $i = 1, 2, 3, 4$ as follows:

$$\begin{aligned}K_{hao}(b_1) = & K_{hao}(\tilde{b}_{11}, \tilde{b}_{12}, \tilde{b}_{13}, \tilde{b}_{14})\\ = & \Big\langle\Big(0.16 \times 0.3 + 0.72 \times 0.2 + 0.48 \times 0.3 + 0.60 \times 0.2,\\ & 0.32 \times 0.3 + 0.84 \times 0.2 + 0.56 \times 0.3 + 0.72 \times 0.2,\\ & 0.40 \times 0.3 + 0.96 \times 0.2 + 0.64 \times 0.3 + 0.84 \times 0.2,\\ & 0.64 \times 0.3 + 1.08 \times 0.2 + 0.72 \times 0.3 + 1.08 \times 0.2\Big); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\Big\rangle\\ = & \Big\langle\Big(0.456, 0.576, 0.672, 0.840\Big); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\Big\rangle\end{aligned}$$

$$\begin{aligned}K_{hao}(b_2) = & K_{hao}(\tilde{b}_{21}, \tilde{b}_{22}, \tilde{b}_{23}, \tilde{b}_{24})\\ = & \Big\langle\Big(0.12 \times 0.3 + 0.16 \times 0.2 + 0.48 \times 0.3 + 0.48 \times 0.2,\\ & 0.48 \times 0.3 + 0.32 \times 0.2 + 0.60 \times 0.3 + 0.56 \times 0.2,\\ & 0.60 \times 0.3 + 0.40 \times 0.2 + 0.72 \times 0.3 + 0.64 \times 0.2,\\ & 0.84 \times 0.3 + 0.64 \times 0.2 + 0.96 \times 0.3 + 0.72 \times 0.2\Big); [0.3, 0.5], [0.6, 0.7], [0.7, 0.8]\Big\rangle\\ = & \Big\langle\Big(0.308, 0.500, 0.604, 0.812\Big); [0.3, 0.5], [0.6, 0.7], [0.7, 0.8]\Big\rangle\end{aligned}$$

$$\begin{aligned}K_{hao}(b_3) = & K_{hao}(\tilde{b}_{31}, \tilde{b}_{32}, \tilde{b}_{33}, \tilde{b}_{34})\\ = & \Big\langle\Big(0.08 \times 0.3 + 0.08 \times 0.2 + 0.12 \times 0.3 + 0.40 \times 0.2,\\ & 0.32 \times 0.3 + 0.24 \times 0.2 + 0.24 \times 0.3 + 0.48 \times 0.2,\\ & 0.40 \times 0.3 + 0.32 \times 0.2 + 0.36 \times 0.3 + 0.56 \times 0.2,\\ & 0.56 \times 0.3 + 0.56 \times 0.2 + 0.48 \times 0.3 + 0.72 \times 0.2\Big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\Big\rangle\\ = & \Big\langle\Big(0.156, 0.312, 0.404, 0.568\Big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\Big\rangle\end{aligned}$$

and

$$\begin{aligned}
K_{hao}(b_4) = \; & K_{hao}(\tilde{b}_{41}, \tilde{b}_{42}, \tilde{b}_{43}, \tilde{b}_{44}) \\
= \; & \big\langle \big(0.12 \times 0.3 + 0.12 \times 0.2 + 0.08 \times 0.3 + 0.24 \times 0.2, \\
& 0.36 \times 0.3 + 0.24 \times 0.2 + 0.16 \times 0.3 + 0.48 \times 0.2, \\
& 0.48 \times 0.3 + 0.36 \times 0.2 + 0.24 \times 0.3 + 0.60 \times 0.2, \\
& 0.84 \times 0.3 + 0.48 \times 0.2 + 0.32 \times 0.3 + 0.96 \times 0.2\big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle \\
= \; & \big\langle \big(0.132, 0.300, 0.408, 0.636\big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle
\end{aligned}$$

Step 7. We can calculate the IVGSVTrN-numbers $L_{hgo}(b_i) = L_{hgo}(\tilde{b}_{i1}, \tilde{b}_{i2}, \tilde{b}_{i3}, \tilde{b}_{i4})$ for $i = 1, 2, 3, 4$ based on Equation 5.6 as follows:

$$\begin{aligned}
L_{hao}(b_1) = \; & L_{hao}(\tilde{b}_{11}, \tilde{b}_{12}, \tilde{b}_{13}, \tilde{b}_{14}) \\
= \; & \big\langle \big(0.16^{0.3} + 0.72^{0.2} + 0.48^{0.3} + 0.60^{0.2}, \\
& 0.32^{0.3} + 0.84^{0.2} + 0.56^{0.3} + 0.72^{0.2}, \\
& 0.40^{0.3} + 0.96^{0.2} + 0.64^{0.3} + 0.84^{0.2}, \\
& 0.64^{0.3} + 1.08^{0.2} + 0.72^{0.3} + 1.08^{0.2}\big); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\big\rangle \\
= \; & \big\langle \big(0.391, 0.540, 0.636, 0.817\big); [0.5, 0.6], [0.5, 0.6], [0.4, 0.5]\big\rangle \\
L_{hao}(b_2) = \; & L_{hao}(\tilde{b}_{21}, \tilde{b}_{22}, \tilde{b}_{23}, \tilde{b}_{24}) \\
= \; & \big\langle \big(0.12^{0.3} + 0.16^{0.2} + 0.48^{0.3} + 0.48^{0.2}, \\
& 0.48^{0.3} + 0.32^{0.2} + 0.60^{0.3} + 0.56^{0.2}, \\
& 0.60^{0.3} + 0.40^{0.2} + 0.72^{0.3} + 0.64^{0.2}, \\
& 0.84^{0.3} + 0.64^{0.2} + 0.96^{0.3} + 0.72^{0.2}\big); [0.3, 0.5], [0.6, 0.7], [0.7, 0.8]\big\rangle \\
= \; & \big\langle \big(0.254, 0.488, 0.592, 0.803\big); [0.3, 0.5], [0.6, 0.7], [0.7, 0.8]\big\rangle \\
L_{hao}(b_3) = \; & L_{hao}(\tilde{b}_{31}, \tilde{b}_{32}, \tilde{b}_{33}, \tilde{b}_{34}) \\
= \; & \big\langle \big(0.08^{0.3} + 0.08^{0.2} + 0.12^{0.3} + 0.40^{0.2}, \\
& 0.32^{0.3} + 0.24^{0.2} + 0.24^{0.3} + 0.48^{0.2}, \\
& 0.40^{0.3} + 0.32^{0.2} + 0.36^{0.3} + 0.56^{0.2}, \\
& 0.56^{0.3} + 0.56^{0.2} + 0.48^{0.3} + 0.72^{0.2}\big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle \\
= \; & \big\langle \big(0.125, 0.301, 0.396, 0.562\big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle
\end{aligned}$$

and

$$\begin{aligned}
L_{hao}(b_4) = \; & L_{hao}(\tilde{b}_{41}, \tilde{b}_{42}, \tilde{b}_{43}, \tilde{b}_{44}) \\
= \; & \big\langle \big(0.12^{0.3} + 0.12^{0.2} + 0.08^{0.3} + 0.24^{0.2}, \\
& 0.36^{0.3} + 0.24^{0.2} + 0.16^{0.3} + 0.48^{0.2}, \\
& 0.48^{0.3} + 0.36^{0.2} + 0.24^{0.3} + 0.60^{0.2}, \\
& 0.84^{0.3} + 0.48^{0.2} + 0.32^{0.3} + 0.96^{0.2}\big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle \\
= \; & \big\langle \big(0.122, 0.276, 0.385, 0.577\big); [0.1, 0.2], [0.8, 0.9], [0.8, 0.9]\big\rangle
\end{aligned}$$

Step 8. The scores of $K_{hao}(\tilde{b}_i)$ for $i = 1, 2, 3, 4$ can be obtained based on Equation 3.8 as follows:

$$S(K_{hao}(b_1)) = 0.493$$

$$S(K_{hao}(b_2)) = 0.320$$

$$S(K_{hao}(b_3)) = 0.081$$

$$S(K_{hao}(b_4)) = 0.083$$

respectively. It is obvious based on Equation 3.6 that

$$K_{hao}(b_1) > K_{hao}(b_2) > K_{hao}(b_4) > K_{hao}(b_3)$$

Therefore, the ranking order of the alternatives x_j $(j = 1, 2, 3, 4)$ is generated as follows:

$$x_1 \succ x_2 \succ x_4 \succ x_3$$

The best supplier for the enterprise is x_1.

Similarly, the scores of $L_{hgo}(\tilde{b}_i)$ for $i = 1, 2, 3, 4$ can be obtained based on Equation 3.8 as follows:

$$S(L_{hgo}(b_1)) = 0.462$$

$$S(L_{hgo}(b_2)) = 0.307$$

$$S(L_{hgo}(b_3)) = 0.078$$

$$S(L_{hgo}(b_4)) = 0.077$$

respectively. It is obvious that

$$L_{hgo}(b_1) > L_{hgo}(b_2) > L_{hgo}(b_3) > L_{hgo}(b_4)$$

Therefore, the ranking order of the alternatives x_j $(j = 1, 2, 3, 4)$ is generated based on Equation 3.6 as follows:

$$x_1 \succ x_2 \succ x_3 \succ x_4$$

The best supplier for the enterprise is x_1.

7 Conclusion

The paper gave the concept of interval valued generalized single valued neutrosophic trapezoidal number (IVGSVTrN-number) which is a generalization of fuzzy number, intuitionistic fuzzy number, neutrosophic number, and so on. An IVGSVTrN-number is a special interval neutrosophic set on the set of real numbers $\mathbb{R}$. To aggregating the information with IVGSVTrN-numbers, we give some operations on IVGSVTrN-numbers.

Also, we presented some aggregation and geometric operators is called IVGSVTrN weighted aggregation operator, IVGSVTrN ordered weighted aggregation operator, IVGSVTrN ordered hybrid weighted aggregation operator, IVGSVTrN weighted geometric operator, IVGSVTrN ordered weighted geometric operator, IVGSVTrN ordered hybrid weighted geometric operator. Furtermore, for these operators, we examined some desirable properties and special cases. Finally, we developed a approach for multiple criteria decision making problems based on the operator and we applied the method to a numerical example to demonstrate its practicality and effectiveness. In the future, we shall focus on the multiple criteria group decision making problems problems with IVGSVTrNs in which the information of attributes weights is partially unknown in advance.

References

[1] S. I. A. Aal, M. M. A. A. Ellatif, M. M. Hassan, Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality, Neutrosophic Sets and Systems, 19 (2018) 132–141.

[2] M. C. J. Anand and J. Bharatraj, Interval-Valued Neutrosophic Numbers with WASPAS, Fuzzy Multicriteria Decision Making Using Neutrosophic Sets, Studies in Fuzziness and Soft Computing 369 (2019) Springer Nature Switzerland AG.

[3] K. T. Atanassov, Intuitionistic Fuzzy Sets, Pysica-Verlag A Springer-Verlag Company, (1999) New York.

[4] A. I. Ban, and D. A. Tuse, Trapezoidal/triangular intuitionistic fuzzy numbers versus interval-valued trapezoidal/triangular fuzzy numbers and applications to multicriteria decision making methods . 18th Int. Conf. on IFSs, Sofia, 1011 May 2014, Notes on Intuitionistic Fuzzy Sets 20(2) 2(014) 43-51.

[5] M. A. Basset, M. Mohamed, A. N. Hussien, A.K. Sangaiah, A novel group decision-making model based on triangular neutrosophic numbers, Soft Comput., 22 (2018) 6629-6643.

[6] M. A. Basset, M. Mohamed, A. K. Sangaiah, Neutrosophic AHP-Delphi Group decision making model based on trapezoidal neutrosophic numbers, J. Ambient Intell. Human Comput., 9 (2018) 1427-1443.

[7] M. A. Baset, V. Chang, A. Gamal, F. Smarandache, An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field, Computers in Industry, 106 (2019) 94-110.

[8] M. A. Baset, M. Saleh, A. Gamal, F. Smarandache, An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number, Applied Soft Computing, 77 (2019)438-452.

[9] M. A. Baset, G. Manogaran, A. Gamal, F. Smarandache, A Group Decision Making Framework Based on Neutrosophic TOPSIS Approach for Smart Medical Device Selection, Journal of Medical Systems, 43(2) (2019) 38.

[10] M. A. Baset, G. Manogaran, A. Gamal, F. Smarandache, A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria, Design Automation for Embedded Systems, 22/3 (2018) 257278.

[11] P. Biswas, S. Pramanik, and B. C. Giri, Distance Measure Based MADM Strategy with Interval Trapezoidal Neutrosophic Numbers, Neutrosophic Sets and Systems, 19 (2018) 40–46.

[12] P. Biswas, S. Pramanik, B. C. Giri, Multi-attribute Group decision Making Based on Expected Value of Neutrosophic Trapezoidal Numbers, New Trends in Neutrosophic Theory and Applications, 2 (2018) 105-124.

[13] P. Biswas, S. Pramanik, B. C. Giri, Cosine Similarity Measure Based Multi-attribute Decision-making with Trapezoidal Fuzzy Neutrosophic Numbers, Neutrosophic Sets and Systems, 8 (2014) 46–56.

[14] P. Biswas, S. Pramanik, B. C. Giri, TOPSIS strategy for MADM with trapezoidal neutrosophic numbers, Neutrosophic Sets and Systems, 19 (2018) 29–39.

[15] P. Biswas, S. Pramanik, and B. C. Giri, Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making, Neutrosophic Sets and Systems, 12 (2016) 127-138.

[16] S. Broumi and F. Smarandache, Single valued neutrosophic trapezoid linguistic aggregation operators based multi-attribute decision making, Bulletin of Pure and Applied Sciences Mathematics and Statistics, 33(2) (2014) 135-155.

[17] S. Broumi, M. Talea,, A. Bakali, F. Smarandache and S. K. Patro, Minimum Spanning Tree Problem with Single- Valued Trapezoidal Neutrosophic Numbers, Studies in Fuzziness and Soft Computing, Soft Comput., 22 (2019) Springer Nature Switzerland AG.

[18] S. Broumi, M. Talea, A. Bakali, F. Smarandache,Shortest Path Problem Under Triangular Fuzzy Neutrosophic Information, 10th International Conference on Software, Knowledge, Information Managementand Applications (SKIMA) (2016).

[19] S. Broumi, M. Talea, A. Bakali, F. Smarandache, Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, (2016) Australia.

[20] T. Chen, Multiple criteria group decision-making with generalized interval-valued fuzzy numbers based on signed distances and incomplete weights. Applied Mathematical Modelling, 36 (7) (2012) 3029-3052.

[21] I. Deli and S. Broumi, Neutrosophic Soft Matrices and NSM-decision Making, Journal of Intelligent and Fuzzy Systems, 28 (5) (2015) 2233-2241.

[22] I. Deli, and N. Çağman, Intuitionistic fuzzy parameterized soft set theory and its decision making, Applied Soft Computing 28 (2014) 109–113.

[23] I. Deli, Y. Şubaş, A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems, International Journal of Machine Learning and Cybernetics, 8 (2017) 1309-1322.

[24] I. Deli, Y. SUBAS, Some weighted geometric operators with SVTrN-numbers and their application to multi-criteria decision making problems, Journal of Intelligent and Fuzzy Systems, 32(1) (2017) 291-301.

[25] I. Deli, Y. SUBAS, and N. Çağman, Single valued interval valued trapezoidal neutrosophic numbers and SVIVTN mult attribute decision making method, International Conference on Mathematics and Mathematics Education (ICMME-2016) May 19-20, (2016) Elazığ.

[26] I. Deli, Operators on Single Valued Trapezoidal Neutrosophic Numbers and SVTN-Group Decision Making, Neutrosophic Sets and Systems, 22 (2018) 130–150.

[27] P. P. Dey, S. Pramanik, B.C. Giri, TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-Attribute Decision Making Problems, Neutrosophic Sets and Systems, 10 (2015) 88-95.

[28] B. Farhadinia, A. I. Ban, Developing new similarity measures of generalized intuitionistic fuzzy numbers and generalized interval-valued fuzzy numbers from similarity measures of generalized fuzzy numbers. Mathematical and Computer Modelling 57 (2013) 812-825.

[29] B. C. Giri, M. U. Molla, P. Biswas, TOPSIS Method for MADM based on Interval Trapezoidal Neutrosophic Number, Neutrosophic Sets and Systems, 22 (2018) 151–167.

[30] R. X. Liang, J. Q. Wang and L. Li, Multi-criteria group decision-making method based on interdependent inputs of single-valued trapezoidal neutrosophic information, Neural Comput. and Applic., 30 (2018) 241-260.

[31] R. X. Liang, J. Q. Wang, H. Y. Zhang, A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information, Neural Comput. and Applic., 30 (2018) 3383–3398.

[32] P. Liu, X.H. Zhang, Some Maclaurin Symmetric Mean Operators for Single-Valued Trapezoidal Neutrosophic Numbers and Their Applications to Group Decision Making, Int. J. Fuzzy Syst., 20(1) (2018) 45-61.

[33] D. F. Li, Decision and Game Theory in Management With Intuitionistic Fuzzy Sets. Studies in Fuzziness and Soft Computing Volume 308, (2014) springer.

[34] J. Li, W. Zeng and P. Guo, Interval-valued Intuitionistic Trapezoidal Fuzzy Number and its Application. 2014 IEEE International Conference on Systems, Man, and Cybernetics October 5-8, (2014), San Diego, CA, USA 734-737.

[35] M. Mohamed, Y. Q. Zhou, M. A. Baset, F. Smarandache, A Critical Path Problem Using Triangular Neutrosophic Number, Neutrosophic Operational Research I, section X, (2017) Pons Brussels.

[36] F. Karaaslan, Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making, Neutrosophic Sets and Systems, 22 (2018) 101-117.

[37] G. S. Mahapatra and T. K. Roy, Intuitionistic Fuzzy Number and Its Arithmetic Operation with Application on System Failure. Journal of Uncertain Systems 7(2) (2013) 92–107.

[38] S. Pramanik, P. P. Dey, B. C. Giri, F. Smarandache, An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information, Neutrosophic Sets and Systems, 17 (2017) 20-28.

[39] S. Pramanik, P. Biswas, B. C. Giri, Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications, 28 (5) (2017) 1163-1176.

[40] S. Pramanik and R. Mallick, VIKOR based MAGDM Strategy with Trapezoidal Neutrosophic Numbers, Neutrosophic Sets and Systems, 22 (2018) 118–129.

[41] A. A. Salama, and S. A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces. IOSR Journal of Mathematics, 3(4) (2012) 31–35.

[42] F. Smarandache, Neutrosophy and Neutrosophic Logic , First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, (2002) Gallup, NM 87301, USA.

[43] F. Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic". Rehoboth (1998) American Research Press.

[44] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman, Interval Neutrosophic Sets and Logic: Theory and Applications in Computing, Hexis, (2005) Phoenix, AZ.

[45] H. Wang, F. Smarandache, F. Q. Zhang and R. Sunderraman, Single valued neutrosophic sets. Multispace and Multistructure, 4 (2010) 410–413.

[46] S. P. Wan, Multi-attribute decision making method based on interval-valued intuitionistic trapezoidal fuzzy number, Control and Decision, 26 (2011) 857–860.

[47] G. W. Wei, X. F. Zhao, and H. J. Wang, An approach to multiple attribute group decision making with interval intuitionistic trapezoidal fuzzy information. Technological and Economic Development of Economy, 8 (2012) 317–330.

[48] J. Wu,and Y. Liu, An approach for multiple attribute group decision making problems with interval-valued intuitionistic trapezoidal fuzzy numbers. Computers and Industrial Engineering 66 (2013) 311-324.

[49] X. Wu, J. Qian, J. J. Peng and C.C. Xue,(2018). A Multi-Criteria Group Decision-Making Method with Possibility Degree and Power Aggregation Operators of Single Trapezoidal Neutrosophic Numbers, Symmetry, 10(590) (2018) 1-21.

[50] Z. S. Xu, Intuitionistic fuzzy aggregation operators, IEEE Trans. Fuzzy Syst., 15 (6) (2007) 1179-1187.

[51] J. Ye, Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method, Informatica, 28(2) (2017) 387–402.

[52] J. Ye, Trapezoidal neutrosophic set and its application to multiple attribute decision making, Neural Computing and Applications, 26(5) (2015) 1157-1166

[53] L.A. Zadeh, Fuzzy Sets, Information and Control, 8 (1965) 338-353.

[54] H.J. Zimmermann, Fuzzy Set Theory and its applications. Kluwer Academic, 2th edition, (1999) Dordrecht.

Received: December 26, 2018. Accepted: March 21, 2019.