Rough Neutrosophic Multisets Relation with Application in Marketing Strategy

Suriana Alias[1], Daud Mohamad[2], and Adibah Shuib[3]

[1]Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (UiTM) Kelantan, Campus Machang, Kelantan, 18500, Malaysia. E-mail: suria588@kelantan.uitm.edu.my

[2]Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (UiTM) Shah Alam, Shah Alam, 40450, Malaysia. E-mail: daud@tmsk.uitm.edu.my

[3]Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (UiTM) Shah Alam, Shah Alam, 40450, Malaysia. E-mail: adibah@tmsk.uitm.edu.my

Abstract. The concepts of rough neutrosophic multisets can be easily extended to a relation, mainly since a relation is also a set, i.e. a subset of a Cartesian product. Therefore, the objective of this paper is to define the definition of rough neutrosophic multisets relation of Cartesian product over a universal set. Some of the relation properties of rough neutrosophic multisets such as max, min, the composition of two rough neutrosophic multisets relation, inverse rough neutrosophic multisets relation, and reflexive, symmetric and transitive rough neutrosophic multisets relation over the universe are defined. Subsequently, their properties are successfully proven. Finally, the application of rough neutrosophic multisets relation for decision making in marketing strategy is presented.

Keywords: Neutrosophic Multisets, relation, rough set, rough neutrosophic multisets

1 Introduction

Imperfect information resulted in an incomplete, imprecision, inconsistency and uncertainty information whereby all the condition must be overcome to represent the perfect information. A relation between each information from the same universe or object is also an important criterion of the information to explain the strong relationship element between them. Fuzzy sets as defined by Zadeh [1] has been used to model the imperfect information especially for uncertainty types of information by representing the membership value between [0, 1]. This indicates the human thinking opinion by replacing the information of linguistic value. Many theories were later introduced with the aim of establishing a fuzzy relation structure [2]. Attanassov introduced an intuitionistic fuzzy set by generalizing the theory of fuzzy sets and introducing two grades of the membership function, namely the degree of membership function and degree of non-membership function [3]. This theory has made the uncertainty decision more interesting. Meanwhile, Burillo et al. studied the intuitionistic fuzzy relation with properties [4]. There are also another theory introduced for solving uncertainty condition such as rough set [5] and soft set [6]. All these studies have extended to rough relation [7] and soft set relation [8].

Smarandache introduced a neutrosophic set as a generalization of the intuitionistic fuzzy set theory [9]. He believed that somehow in a life situation, especially for uncertainty condition, there also exist in-between (indeterminacy) opinion or unexpected condition that cannot be controlled. Instead of two grades of the membership function, neutrosophic set introduced in-between (indeterminacy) function where there exists an element which consists of a set of truth membership function (T), indeterminacy function (I) and falsity membership function (F). Compared to other uncertainty theories, the neutrosophic set can deal with indeterminacy situation. The study in neutrosophic relation with properties are also discussed [10], [11].

Later, Smarandache et al. refined T, I, F to $T_1, T_2, \ldots, T_m$ and $I_1, I_2, \ldots, I_n$ and $F_1, F_2, \ldots, F_r$ was also known as a neutrosophic refined set or neutrosophic multisets [12], [13]. Instead of one-time occurring for each element of T, I, F, the neutrosophic refined set allowed an element of T, I, F to occur more than once with possibly the same or different truth membership values, indeterminacy values, and falsity membership values. The study of the neutrosophic refined set is a generalization of a multi fuzzy set [14] and intuitionistic fuzzy multisets [15]. Later, Deli et al. have studied the relation on neutrosophic refined set with properties [16], [17]. Latest, Smarandache has discussed in detail about neutrosophic perspectives in theory and application parts for neutrosophic triplets, neutrosophic duplets, neutrosophic multisets, hybrid operators and modal logic [18]. The successful application of the neutrosophic refined set in multi criteria decision making problem such as in medical diagnosis and selection problem [13], [19]–[25] has made this theory more applicable in decision making area.

Hybrid theories of uncertainty and imprecision condition were introduced, especially with rough set theory, such as rough fuzzy set and fuzzy rough set [26], rough intuitionistic fuzzy set [27], intuitionistic rough fuzzy set [28], rough neutrosophic set [29], neutrosophic rough set [30], interval rough neutrosophic set [31], rough neutrosophic soft set [32], rough bipolar neutrosophic set [33], single valued neutrosophic rough set model [34] and rough neutrosophic multiset [35]. This is because a rough set theory can handle the imprecision condition from the existence of a value which cannot be measured with suitable precision. Samanta et al. have discussed the fuzzy rough relation on universe set and their properties [36]. Then, Xuan Thao et. al have extended that concept by introducing the rough fuzzy relations on the Cartesian product of two universal sets [37].

The hybrid theory of a rough set also gives a contribution for solving a problem in decision making area. Some researchers already proved that hybrid theory such as rough neutrosophic set can handle the decision making problem in order to get the best solution according to three membership degree (truth, indeterminate and falsity) [38]–[44].

The objective of this paper is to define a rough neutrosophic multisets relation properties as a novel notion. This study also generalizes relation properties of a rough fuzzy relation, rough intuitionistic fuzzy relation and rough neutrosophic relation over universal. Subsequently, their properties are examined.

The remaining parts of this paper are organized as follows. In section 2, some mathematical preliminary concepts were recalled for a deeper understanding of rough neutrosophic multisets relations. Section 3 introduces the definition of rough neutrosophic multisets relation of Cartesian product on a universe set with some examples. Related properties and operations are also investigated. Section 3 also defined the composition of two rough neutrosophic multisets relation, inverse rough neutrosophic multisets relation and the reflexive, symmetric and transitive rough neutrosophic multisets relation. Subsequently, their properties are examined. In section 4, the rough neutrosophic multisets relation is represented as a marketing strategy by evaluating the quality of the product. Finally, section 5 concludes the paper.

2 Preliminaries

In this section, some mathematical preliminary concepts were recalled to understanding more about rough neutrosophic multisets relations.

Definition 2.1 ([10]) Let U be a non-empty set of objects, $\mathcal{R}$ is an equivalence relation on U. Then the space $(U,\mathcal{R})$ is called an approximation space. Let X be a fuzzy set on U. We define the lower and upper approximation set and upper approximation of X, respectively

$$\underline{\mathcal{R}_U}(X) = \{x \in U: [x]_{\mathcal{R}} \subset X\},$$
$$\overline{\mathcal{R}_U}(X) = \{x \in U: [x]_{\mathcal{R}} \cap X \neq 0\}$$

where

$$T_{\underline{\mathcal{R}_U}}(X) = inf_{y\in U}\{T_X(y): y \in [x]_{\mathcal{R}}\},$$
$$T_{\overline{\mathcal{R}_U}}(X) = sup_{y\in U}\{T_X(y): y \in [x]_{\mathcal{R}}\}.$$

The boundary of X, $BND(X) = \overline{\mathcal{R}_U}(X) - \underline{\mathcal{R}_U}(X)$. The fuzzy set X is called the rough fuzzy set if $BND(X) \neq 0$.

Definition 2.2 ([18]) Let U be a non-empty set of objects, $\mathcal{R}$ is an equivalence relation on U. Then the space $(U,\mathcal{R})$ is called an approximation space. Let X be an intuitionistic fuzzy set on U. We define the lower and upper approximation set and upper approximation of X, respectively

$$\underline{\mathcal{R}_U}(X) = \{x \in U: [x]_{\mathcal{R}} \subset X\}$$
$$\overline{\mathcal{R}_U}(X) = \{x \in U: [x]_{\mathcal{R}} \cap X \neq 0\},$$

where

$T_{\underline{\mathcal{R}_U}}(X) = inf_{y \in U}\{T_X(y): y \in [x]_{\mathcal{R}}\}$,
$T_{\overline{\mathcal{R}_U}}(X) = sup_{y \in U}\{T_X(y): y \in [x]_{\mathcal{R}}\}$,
$F_{\underline{\mathcal{R}_U}}(X) = sup_{y \in U}\{F_X(y): y \in [x]_{\mathcal{R}}\}$,
$F_{\overline{\mathcal{R}_U}}(X) = inf_{y \in U}\{T_X(y): y \in [x]_{\mathcal{R}}\}$.

The boundary of X, $BND(X) = \overline{\mathcal{R}_U}(X) - \underline{\mathcal{R}_U}(X)$. The intuitionistic fuzzy set X is called the rough intuitionistic fuzzy set if $BND(X) \neq 0$.

Definition 2.3 ([6]) Let U be a non-null set and R be an equivalence relation on U. Let A be neutrosophic set in U with the membership function T_A, indeterminacy function I_A and non-membership function F_A. The lower and the upper approximations of A in the approximation (U, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A) = \{\langle x, (T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x),)\rangle | y \in [x]_R, x \in U\},$$
$$\overline{N}(A) = \{\langle x, (T_{\overline{N}(A)}(x), I_{\overline{N}(A)}(x), F_{\overline{N}(A)}(x),)\rangle | y \in [x]_R, x \in U\}$$

where

$T_{\underline{N}(A)}(x) = \wedge_{y \in [x]_R} T_A(y),\ I_{\underline{N}(A)}(x) = \vee_{y \in [x]_R} I_A(y), F_{\underline{N}(A)}(x) = \vee_{y \in [x]_R} F_A(y)$,
$T_{\overline{N}(A)}(x) = \vee_{y \in [x]_R} T_A(y), I_{\overline{N}(A)}(x) = \wedge_{y \in [x]_R} I_A(y),\ F_{\overline{N}(A)}(x) = \wedge_{y \in [x]_R} F_A(y)$.

such that,

$T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x), T_{\overline{N}(A)}(x), I_{\overline{N}(A)}(x), F_{\overline{N}(A)}(x): A \in [0, 1]$,
$0 \leq T_{\underline{N}(A)}(x) + I_{\underline{N}(A)}(x) + F_{\underline{N}(A)}(x) \leq 3$ and
$0 \leq T_{\overline{N}(A)}(x) + I_{\overline{N}(A)}(x) + F_{\overline{N}(A)}(x) \leq 3$

Here $\wedge$ and $\vee$ denote "min" and "max'' operators respectively, and $[x]_R$ is the equivalence class of the x. $T_A(y)$, $I_A(y)$ and $F_A(y)$ are the membership sequences, indeterminacy sequences and non-membership sequences of y with respect to A.

Since $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic sets in U, thus the neutrosophic set mappings $\underline{N}, \overline{N}: N(U) \to N(U)$ are respectively referred as lower and upper rough neutrosophic set approximation operators, and the pair of $\left(\underline{N}(A), \overline{N}(A)\right)$ is called the rough neutrosophic set in $(U, \mathcal{R})$.

Definition 2.4 ([1]) Let U be a non-null set and R be an equivalence relation on U. Let A be neutrosophic multisets in U with the truth-membership sequence T_A^i, indeterminacy-membership sequences I_A^i and falsity-membership sequences F_A^i. The lower and the upper approximations of A in the approximation $(U, \mathcal{R})$ denoted by $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are respectively defined as follows:

$$\underline{Nm} = \left\{\langle x, \left(T^i_{\underline{Nm}(A)}(x), I^i_{\underline{Nm}(A)}(x), F^i_{\underline{Nm}(A)}(x), \right)\rangle \middle| y \in [x]_R, x \in U\right\},$$
$$\overline{Nm}(A) = \left\{\langle x, \left(T^i_{\overline{Nm}(A)}(x), I^i_{\overline{Nm}(A)}(x), F^i_{\overline{Nm}(A)}(x), \right)\rangle \middle| y \in [x]_R, x \in U\right\}$$

where

$i = 1, 2, \ldots, p$ and positive integer

$T^i_{\underline{Nm}(A)}(x) = \wedge_{y \in [x]_R} T^i_A(y)$,
$I^i_{\underline{Nm}(A)}(x) = \vee_{y \in [x]_R} I^i_A(y)$,
$F^i_{\underline{Nm}(A)}(x) = \vee_{y \in [x]_R} F^i_A(y)$,
$T^i_{\overline{Nm}(A)}(x) = \vee_{y \in [x]_R} T^i_A(y)$,
$I^i_{\overline{Nm}(A)}(x) = \wedge_{y \in [x]_R} I^i_A(y)$,
$F^i_{\overline{Nm}(A)}(x) = \wedge_{y \in [x]_R} F^i_A(y)$.

Here $\wedge$ and $\vee$ denote "min" and "max'' operators respectively, and $[x]_R$ is the equivalence class of the x. $T_A^i(y)$, $I_A^i(y)$ and $F_A^i(y)$ are the membership sequences, indeterminacy sequences and non-membership sequences of y with respect to A.

It can be said that $T_{\underline{Nm}(A)}^i(x), I_{\underline{Nm}(A)}^i(x), F_{\underline{Nm}(A)}^i(x) \in [0,1]$ and $0 \leq T_{\underline{Nm}(A)}^i(x) + I_{\underline{Nm}(A)}^i(x) + F_{\underline{Nm}(A)}^i(x) \leq 3$. Then, $\underline{Nm}(A)$ is a neutrosophic multisets. Similarly, we have $T_{\overline{Nm}(A)}^i(x), I_{\overline{Nm}(A)}^i(x), F_{\overline{Nm}(A)}^i(x) \in [0,1]$ and $0 \leq T_{\overline{Nm}(A)}^i(x) + I_{\overline{Nm}(A)}^i(x) + F_{\overline{Nm}(A)}^i(x) \leq 3$. Then, $\overline{Nm}(A)$ is neutrosophic multisets.

Since $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are two neutrosophic multisets in U, the neutrosophic multisets mappings $\underline{Nm}, \overline{Nm}: Nm(U) \to Nm(U)$ are respectively referred to as lower and upper rough neutrosophic multisets approximation operators, and the pair of $\left(\underline{Nm}(A), \overline{Nm}(A)\right)$ is called the rough neutrosophic multisets in $(U, \mathcal{R})$, respectively.

3 Rough Neutrosophic Multisets Relation

The concept of a rough set can be easily extended to a relation since the relation is also a set, i.e. a subset of the Cartesian product. This concept is also used to define the rough neutrosophic multisets relation over the universe.

In the following section, the Cartesian product of two rough neutrosophic multisets is defined with some examples. We only considered the case where T, I, F are refined into the same number of subcomponents 1, 2, …, p, and T_A^i, I_A^i and F_A^i are a single valued neutrosophic number. Some of the concepts are quoted from [2], [10], [12], [36]

Definition 3.1 ([7]) Let $A = (U, R)$ be an approximation space. Let $X \subseteq U$. A relation T on X is said to be a rough relation on X if $\underline{T} \neq \overline{T}$, where $\underline{T}$ and $\overline{T}$ are a lower and upper approximation of T, respectively defined by;

$$\underline{T} = \{(x,y) \in U \times U: [x,y]_R \subseteq X\}$$
$$\overline{T} = \{(x,y) \in U \times U: [x,y]_R \cap X \neq \emptyset$$

Definition 3.2 Let U be a non-empty set and X and Y be the rough neutrosophic multisets in U . Then, Cartesian product of X and Y is rough neutrosophic multisets in $U \times U$, denoted by $X \times Y$, defined as

$$X \times Y = \left\{< (x,y), \left(T_{X\times Y}^i(x,y)\right)\left(I_{X\times Y}^i(x,y)\right), \left(F_{X\times Y}^i(x,y)\right) >: (x,y) \in U \times U\right\}$$

where

$$T_{X\times Y}^i(x,y) = \min\{T_X^i(x), T_Y^i(y)\},$$
$$I_{X\times Y}^i(x,y) = \max\{I_X^i(x), I_Y^i(y)\},$$
$$F_{X\times Y}^i(x,y) = \max\{F_X^i(x), F_Y^i(y)\},$$
$T_{X\times Y}^i, I_{X\times Y}^i, F_{X\times Y}^i: U \to [0,1]$, and $i = 1, 2, \dots, p$.

Definition 3.3 Let U be a non-empty set and X and Y be the rough neutrosophic multisets in U. We call $\mathfrak{R} \subseteq U \times U$ is a rough neutrosophic multisets relation on $U \times U$ based on the $X \times Y$, where $X \times Y$ is characterized by truth-membership sequence $T_{\mathfrak{R}}^i$, indeterminacy-membership sequences $I_{\mathfrak{R}}^i$ and falsity-membership sequences $F_{\mathfrak{R}}^i$, defined as

$$\mathfrak{R} = \left\{< (x,y), \left(T_{\mathfrak{R}}^i(x,y)\right)\left(I_{\mathfrak{R}}^i(x,y)\right), \left(F_{\mathfrak{R}}^i(x,y)\right) >: (x,y) \in U \times U\right\}$$

with a condition if it satisfies:

(1) i) $T_{\mathfrak{R}}^i(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$ where $\underline{X \times Y} = \underline{\mathcal{R}_U}(X) \times \underline{\mathcal{R}_U}(Y)$,

ii) $T_{\mathfrak{R}}^i(x,y) = 0$, for all $(x,y) \in U \times U - \overline{X \times Y}$ where $\overline{X \times Y} = \overline{\mathcal{R}}_U(X) \times \overline{\mathcal{R}}_U(Y)$,

iii) $0 < T_{\mathfrak{R}}^i(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.

(2) i) $I^i_{\mathfrak{R}}(x,y)=0$, for all $(x,y)\in \underline{X\times Y}$ where $\underline{X\times Y}=\underline{\mathcal{R}_U}(X)\times\underline{\mathcal{R}_U}(Y)$,

ii) $I^i_{\mathfrak{R}}(x,y)=1$, for all $(x,y)\in U\times U-\overline{X\times Y}$ where $\overline{X\times Y}=\overline{\mathcal{R}}_U(X)\times\overline{\mathcal{R}}_U(Y)$,

iii) $0<I^i_{\mathfrak{R}}(x,y)<1$, for all $(x,y)\in\overline{X\times Y}-\underline{X\times Y}$.

(3) i) $F^i_{\mathfrak{R}}(x,y)=0$, for all $(x,y)\in \underline{X\times Y}$ where $\underline{X\times Y}=\underline{\mathcal{R}_U}(X)\times\underline{\mathcal{R}_U}(Y)$,

ii) $F^i_{\mathfrak{R}}(x,y)=1$, for all $(x,y)\in U\times U-\overline{X\times Y}$ where $\overline{X\times Y}=\overline{\mathcal{R}}_U(X)\times\overline{\mathcal{R}}_U(Y)$,

iii) $0<F^i_{\mathfrak{R}}(x,y)<1$, for all $(x,y)\in\overline{X\times Y}-\underline{X\times Y}$.

Remark 3.4: The rough neutrosophic multisets relation is a relation on neutrosophic multisets, so we can consider that is a rough neutrosophic multisets relation over the universe. The rough neutrosophic multisets relation follows the condition of relation on neutrosophic multisets which is $T^i_{\mathfrak{R}}(x,y)\le T^i_{X\times Y}(x,y),\ I^i_{\mathfrak{R}}(x,y)\ge I^i_{X\times Y}(x,y), F^i_{\mathfrak{R}}(x,y)\ge F^i_{X\times Y}(x,y)$ for all $(x,y)\in U\times U$, and $0\le T^i_{\mathfrak{R}}(x,y)+I^i_{\mathfrak{R}}(x,y)+F^i_{\mathfrak{R}}(x,y)\le 3$.

Therefore, the rough neutrosophic multisets relation will generalize the following relation:

(1) Rough Neutrosophic Set Relation
When $i=1$ for all element T, I, F in definition 3.2, we obtain the relation for rough neutrosophic set over universe;
$\mathfrak{R}=\{<(x,y),\big(T_{\mathfrak{R}}(x,y)\big)\big(I_{\mathfrak{R}}(x,y)\big),\big(F_{\mathfrak{R}}(x,y)\big)>:(x,y)\in U\times U\}$.

(2) Rough Intuitionistic Fuzzy Set Relation
When $i=1$ for element T and F, and properties (2) in definition 3.3 is also omitted, we obtain the relation for rough intuitionistic fuzzy set over universe;
$\mathfrak{R}=\{<(x,y),\big(T_{\mathfrak{R}}(x,y)\big),\big(F_{\mathfrak{R}}(x,y)\big)>:(x,y)\in U\times U\}$.

(3) Rough Fuzzy Set Relation
When $i=1$ for element T and properties (2) and (3) in definition 3.3 is also omitted, we obtain the relation for rough fuzzy set over universe;
$\mathfrak{R}=\{<(x,y),\big(T_{\mathfrak{R}}(x,y)\big)>:(x,y)\in U\times U\}$.

The rough neutrosophic multisets relation can be presented by relational tables and matrices, like a representation of fuzzy relation. Since the triple $\big(T^i_A,I^i_A,F^i_A\big)$ has values within the interval $[0,1]$, the elements of the neutrosophic matrix also have values within $[0,1]$. Consider the following example:

Example 3.5: Let $U=\{u_1,u_2,u_3\}$ be a universal set and $\mathcal{R}_U=(x,y):x\mathcal{R}_U y$ is equivalent relations on U. Let

$$X=\frac{(1,0.3),(0.4,0.7),(0.6,0.8)}{u_1}+\frac{(0.5,0.7),(0.1,0.3),(0.4,0.5)}{u_2}+\frac{(1,0.6),(0.4,0.5),(0.6,0.7)}{u_3}\text{, and}$$

$$Y=\frac{(0.4,0.6),(0.3,0.5),(0.1,0.7)}{u_1}+\frac{(0.5,0.4),(0.1,0.7),(0.3,0.8)}{u_2}+\frac{(1,0.7),(0.2,0.5),(0.1,0.7)}{u_3}$$

are rough neutrosophic multisets on U.

Here we can define a rough neutrosophic multisets relation $\mathfrak{R}$ by a matrix. $x\mathcal{R}_U y$ is composed by $\mathcal{R}_U=\{\{u_1,u_3\},\{u_2\}$. Based on definition 2.4 and 3.2, we solve for;

$$\underline{\mathcal{R}_U}(X)=\frac{(1,0.3),(0.4,0.7),(0.6,0.8)}{u_1}+\frac{(0.5,0.7),(0.1,0.3),(0.4,0.5)}{u_2}+\frac{(1,0.3),(0.4,0.7),(0.6,0.8)}{u_3},$$

$$\overline{\mathcal{R}_U}(X)=\frac{(1,0.6),(0.4,0.5),(0.6,0.7)}{u_1}+\frac{(0.5,0.7),(0.1,0.3),(0.4,0.5)}{u_2}+\frac{(1,0.6),(0.4,0.5),(0.6,0.7)}{u_3},$$

$$\underline{\mathcal{R}_U}(Y)=\frac{(0.4,0.6),(0.3,0.5),(0.1,0.7)}{u_1}+\frac{(0.5,0.4),(0.1,0.7),(0.3,0.8)}{u_2}+\frac{(0.4,0.6),(0.3,0.5),(0.1,0.7)}{u_3},$$

$$\overline{\mathcal{R}_U}(Y)=\frac{(1,0.7),(0.2,0.5),(0.1,0.7)}{u_1}+\frac{(0.5,0.4),(0.1,0.7),(0.3,0.8)}{u_2}+\frac{(1,0.7),(0.2,0.5),(0.1,0.7)}{u_3}$$

We have $\underline{X \times Y} = \underline{\mathcal{R}_U}(X) \times \underline{\mathcal{R}_U}(Y)$ and $\overline{X \times Y} = \overline{\mathcal{R}}_U(X) \times \overline{\mathcal{R}}_U(Y)$. Then, by satisfied all the condition in definition 3.3, we defined $\Re \subseteq U \times U$ as a rough neutrosophic multisets relation on $U \times U$ based on the $X \times Y$ by a matrix form:

$$M(\Re) = \begin{bmatrix} (0.3,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \\ (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) \\ (0.4,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \end{bmatrix}$$

Now, we consider some properties of a rough neutrosophic multisets relation.

Proposition 3.6 *Let $\Re_1, \Re_2$ be two rough neutrosophic multisets relation on $U \times U$ based on the $X \times Y$. Then $\Re_1 \wedge \Re_2$ where*

$$T^i_{\Re_1 \wedge \Re_2}(x,y) = \min\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\},$$
$$I^i_{\Re_1 \wedge \Re_2}(x,y) = \max\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\},$$
$$F^i_{\Re_1 \wedge \Re_2}(x,y) = \max\{F^i_{\Re_1}(x,y), F^i_{\Re_2}(x,y)\}$$

for all $(x,y) \in U \times U$, is a rough neutrosophic multisets on $U \times U$ based on the $X \times Y$ and $i = 1,2,\dots,p$.

Proof: We show that $\Re_1 \wedge \Re_2$ satisfy definition 3.3.

(1) i) Since $T^i_{\Re_1}(x,y) = T^i_{\Re_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$ then $T^i_{\Re_1 \wedge \Re_2}(x,y) = \min\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\} = 1$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $T^i_{\Re_1}(x,y) = T^i_{\Re_2}(x,y) = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$ then $T^i_{\Re_1 \wedge \Re_2}(x,y) = \min\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\} = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then $0 < T^i_{\Re_1 \wedge \Re_2}(x,y) = \min\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.

(2) i) Since $I^i_{\Re_1}(x,y) = I^i_{\Re_2}(x,y) = 0$ for all $(x,y) \in \underline{X \times Y}$ then $I^i_{\Re_1 \wedge \Re_2}(x,y) = \max\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\} = 0$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $I^i_{\Re_1}(x,y) = I^i_{\Re_2}(x,y) = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$ then $I^i_{\Re_1 \wedge \Re_2}(x,y) = \max\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\} = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then $0 < I^i_{\Re_1 \wedge \Re_2}(x,y) = \max\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.

Proof (3) for falsity function are similarly to proving (2) for indeterminate function.

Proposition 3.7 *Let $\Re_1, \Re_2$ be two rough neutrosophic multisets relation on $U \times U$ based on the $X \times Y$. Then $\Re_1 \vee \Re_2$ where*

$$T^i_{\Re_1 \vee \Re_2}(x,y) = \max\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\},$$
$$I^i_{\Re_1 \vee \Re_2}(x,y) = \min\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\},$$
$$F^i_{\Re_1 \vee \Re_2}(x,y) = \min\{F^i_{\Re_1}(x,y), F^i_{\Re_2}(x,y)\}$$

for all $(x,y) \in U \times U$, is a rough neutrosophic multisets on $U \times U$ based on the $X \times Y$ and $i = 1,2,\dots,p$.

Proof: We show that $\Re_1 \vee \Re_2$ satisfy definition 3.3.

(1) i) Since $T^i_{\Re_1}(x,y) = T^i_{\Re_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$ then $T^i_{\Re_1 \vee \Re_2}(x,y) = \max\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\} = 1$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $T^i_{\Re_1}(x,y) = T^i_{\Re_2}(x,y) = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$ then $T^i_{\Re_1 \vee \Re_2}(x,y) = \max\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\} = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then $0 < T^i_{\Re_1 \vee \Re_2}(x,y) = \max\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.

(2) i) Since $I^i_{\mathfrak{R}_1}(x,y) = I^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in \underline{X \times Y}$ then
$I^i_{\mathfrak{R}_1 \vee \mathfrak{R}_2}(x,y) = \min\{I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(x,y)\} = 0$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $I^i_{\mathfrak{R}_1}(x,y) = I^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$
then $I^i_{\mathfrak{R}_1 \vee \mathfrak{R}_2}(x,y) = \min\{I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(x,y)\} = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then
$0 < I^i_{\mathfrak{R}_1 \vee \mathfrak{R}_2}(x,y) = \min\{I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.

Proof (3) for falsity function are similarly to proving (2) for indeterminate function.

Lemma 3.8: If $0 < x, y < 1$, then

(i) $0 < xy < 1$,
(ii) $0 < x + y - xy < 1$.

Since $0 < x, y < 1$ then $x + y \geq 2\sqrt{xy} > 2xy > xy > 0$, therefore $x + y - xy > 0$. On the other hand, $1 - (x + y - xy) = (1 - x)(1 - y) > 0$ then $x + y - xy < 1$. The following properties of a rough neutrosophic multisets relation are obtained by using these algebraic results.

Proposition 3.9 *Let $\mathfrak{R}_1, \mathfrak{R}_2$ be two rough neutrosophic multisets relation on $U \times U$ based on the $X \times Y$. Then $\mathfrak{R}_1 \otimes \mathfrak{R}_2$ where*

$T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y)$,
$I^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) + I^i_{\mathfrak{R}_2}(x,y) - I^i_{\mathfrak{R}_1}(x,y). I^i_{\mathfrak{R}_2}(x,y)$
$F^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = F^i_{\mathfrak{R}_1}(x,y) + F^i_{\mathfrak{R}_2}(x,y) - F^i_{\mathfrak{R}_1}(x,y). F^i_{\mathfrak{R}_2}(x,y)$

for all $(x,y) \in U \times U$, is a rough neutrosophic multisets on $U \times U$ based on the $X \times Y$ and $i = 1, 2, \dots, p$.

Proof: The relation $\mathfrak{R}_1 \otimes \mathfrak{R}_2$satisfied definition 3.3. Indeed:

(1) i) Since $T^i_{\mathfrak{R}_1}(x,y) = T^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$ then
$T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $T^i_{\mathfrak{R}_1}(x,y) = T^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$
then $T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then
$0 < T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y) < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ (Lemma 3.8 (i)).

(2) i) Since $I^i_{\mathfrak{R}_1}(x,y) = I^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in \underline{X \times Y}$ then
$I^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) + I^i_{\mathfrak{R}_2}(x,y) - I^i_{\mathfrak{R}_1}(x,y) \cdot I^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $I^i_{\mathfrak{R}_1}(x,y) = I^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$ then $I^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) + I^i_{\mathfrak{R}_2}(x,y) - I^i_{\mathfrak{R}_1}(x,y) \cdot I^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then
$0 < I^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) + I^i_{\mathfrak{R}_2}(x,y) - I^i_{\mathfrak{R}_1}(x,y) \cdot I^i_{\mathfrak{R}_2}(x,y) < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ (Lemma 3.8 (ii)).

Proof (3) for falsity function are similarly to proving (2) for indeterminate function.

Proposition 3.10 *Let $\mathfrak{R}_1, \mathfrak{R}_2$ be two rough neutrosophic multisets relation on $U \times U$ based on the $X \times Y$. Then $\mathfrak{R}_1 \oplus \mathfrak{R}_2$ where*

$T^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) + T^i_{\mathfrak{R}_2}(x,y) - T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y)$,
$I^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) \cdot I^i_{\mathfrak{R}_2}(x,y)$,
$F^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = F^i_{\mathfrak{R}_1}(x,y) \cdot F^i_{\mathfrak{R}_2}(x,y)$

for all $(x,y) \in U \times U$, is a rough neutrosophic multisets on $U \times U$ based on the $X \times Y$ and $i = 1, 2, \dots, p$.

Proof: The relation $\mathfrak{R}_1 \oplus \mathfrak{R}_2$ satisfied definition 3.3. Indeed:

(1) i) Since $T^i_{\mathfrak{R}_1}(x,y) = T^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$ then $T^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) + T^i_{\mathfrak{R}_2}(x,y) - T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $T^i_{\mathfrak{R}_1}(x,y) = T^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$ then $T^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) + T^i_{\mathfrak{R}_2}(x,y) - T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then $0 < T^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = T^i_{\mathfrak{R}_1}(x,y) + T^i_{\mathfrak{R}_2}(x,y) - T^i_{\mathfrak{R}_1}(x,y) \cdot T^i_{\mathfrak{R}_2}(x,y) < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ (Lemma 3.8 (ii)).

(2) i) Since $I^i_{\mathfrak{R}_1}(x,y) = I^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in \underline{X \times Y}$ then $I^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) \cdot I^i_{\mathfrak{R}_2}(x,y) = 0$ for all $(x,y) \in \underline{X \times Y}$.

ii) Since $I^i_{\mathfrak{R}_1}(x,y) = I^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$ then $I^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) \cdot I^i_{\mathfrak{R}_2}(x,y) = 1$ for all $(x,y) \in U \times U - \overline{X \times Y}$.

iii) Since $0 < I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ then $0 < I^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x,y) = I^i_{\mathfrak{R}_1}(x,y) \cdot I^i_{\mathfrak{R}_2}(x,y) < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$ (Lemma 3.8 (i)).

Proof (3) for falsity function are similarly to proving (2) for indeterminate function.

3.1 Composition of Two Rough Neutrosophic Multisets Relation

The composition of relation is important for applications because if a relation on X and Y is known and if a relation on Y and Z is known, then the relation on X and Z could be computed over a universe with the useful significance.

Definition 3.1.1 Let U be a non-empty set and X, Y and Z are the rough neutrosophic multisets in U. Let $\mathfrak{R}_1, \mathfrak{R}_2$ are two rough neutrosophic multisets relations on $U \times U$, based on $X \times Y, Y \times Z$, respectively. The composition of $\mathfrak{R}_1, \mathfrak{R}_2$ denote as $\mathfrak{R}_1 \circ \mathfrak{R}_2$ which defined on $U \times U$ based on $X \times Z$ where

$$T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(y,z)]\},$$
$$I^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \min_{y \in U}\{\max[I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(y,z)]\},$$
$$F^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \min_{y \in U}\{\max[F^i_{\mathfrak{R}_1}(x,y), F^i_{\mathfrak{R}_2}(y,z)]\}.$$

for all $(x,z) \in U \times U$ and $i = 1, 2, \ldots, p$.

Proposition 3.1.2 $\mathfrak{R}_1 \circ \mathfrak{R}_2$ *is a rough neutrosophic multisets relation on* $U \times U$ *based on* $X \times Z$.

Proof: Since $\mathfrak{R}_1, \mathfrak{R}_2$ are two rough neutrosophic multisets relations on $U \times U$ based on $X \times Y, Y \times Z$ respectively;

(1) i) Then $T^i_{\mathfrak{R}_1}(x,z) = 1 = T^i_{\mathfrak{R}_2}(x,z)$ for all $(x,z) \in \underline{X \times Z}$. Let $(x,z) \in \underline{X \times Z}$, now $T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(y,z)]\} = 1$. This holds for all $(x,z) \in \underline{X \times Z}$.

ii) Let $(x,z) \in U \times U - \overline{X \times Z}$. So, $T^i_{\mathfrak{R}_1}(x,z) = 0 = T^i_{\mathfrak{R}_2}(x,z)$ for all $(x,z) \in U \times U - \overline{X \times Z}$. Then $T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(y,z)]\} = 0$ for all $(x,z) \in U \times U - \overline{X \times Z}$.

iii) Again, since $0 < T^i_{\mathfrak{R}_1}(x,z), T^i_{\mathfrak{R}_2}(x,z) < 1$, for all $(x,z) \in \overline{X \times Z} - \underline{X \times Z}$, then $0 < \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(y,z)]\} < 1$ such that $0 < T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) < 1$ for all $(x,z) \in \overline{X \times Z} - \underline{X \times Z}$.

(2) i) Then $I^i_{\mathfrak{R}_1}(x,z) = 0 = I^i_{\mathfrak{R}_2}(x,z)$ for all $(x,z) \in \underline{X \times Z}$. Let $(x,z) \in \underline{X \times Z}$, now $I^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \min_{y \in U}\{\max[I^i_{\mathfrak{R}_1}(x,y), I^i_{\mathfrak{R}_2}(y,z)]\} = 0$. This holds for all $(x,z) \in \underline{X \times Z}$.

ii) Let $(x, z) \in U \times U - \overline{X \times Z}$. So, $I^i_{\mathfrak{R}_1}(x, z) = 1 = I^i_{\mathfrak{R}_2}(x, z)$ for all $(x, z) \in U \times U - \overline{X \times Z}$. Then $I^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x, z) = \min_{y \in U}\{\max[I^i_{\mathfrak{R}_1}(x, y), I^i_{\mathfrak{R}_2}(y, z)]\} = 1$ for all $(x, z) \in U \times U - \overline{X \times Z}$.

iii) Again, since $0 < I^i_{\mathfrak{R}_1}(x, z), I^i_{\mathfrak{R}_2}(x, z) < 1$, for all $(x, z) \in \overline{X \times Z} - \underline{X \times Z}$, then $0 < \min_{y \in U}\{\max[I^i_{\mathfrak{R}_1}(x, y), I^i_{\mathfrak{R}_2}(y, z)]\} < 1$ such that $0 < I^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x, z) < 1$ for all $(x, z) \in \overline{X \times Z} - \underline{X \times Z}$.

Proof (3) for falsity function are similarly to proving (2) for indeterminate function.

Proposition 3.1.3 *Let U be a non-empty set. $\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3$ are rough neutrosophic multisets relations on $U \times U$ based on $X \times Y, Y \times Z, Z \times Z'$, respectively. Then $(\mathfrak{R}_1 \circ \mathfrak{R}_2) \circ \mathfrak{R}_3 = \mathfrak{R}_1 \circ (\mathfrak{R}_2 \circ \mathfrak{R}_3)$*

Proof: We only proof for truth function. For all $x, y, z, t \in U$ we have

$$\begin{aligned}
T^i_{\mathfrak{R}_1 \circ (\mathfrak{R}_2 \circ \mathfrak{R}_3)}(x, t) &= \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x, y), T^i_{(\mathfrak{R}_2 \circ \mathfrak{R}_3)}(y, t)]\} \\
&= \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x, y), \max_{z \in U}\{\min[T^i_{\mathfrak{R}_2}(y, z), T^i_{\mathfrak{R}_3}(z, t)]]\}\} \\
&= \max_{z \in U}\{\min\{\max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x, y), T^i_{\mathfrak{R}_2}(y, z)], T^i_{\mathfrak{R}_3}(z, t)]\}\}\} \\
&= \max_{z \in U}\{\min[T^i_{(\mathfrak{R}_1 \circ \mathfrak{R}_2)}(x, y), T^i_{\mathfrak{R}_3}(y, t)]\} \\
&= T^i_{(\mathfrak{R}_1 \circ \mathfrak{R}_2) \circ \mathfrak{R}_3}(x, t);
\end{aligned}$$

Similarly proof for indeterminate function and falsity function.

Note that $\mathfrak{R}_1 \circ \mathfrak{R}_2 \neq \mathfrak{R}_2 \circ \mathfrak{R}_1$, since the composition of two rough neutrosophic multisets relations $\mathfrak{R}_1, \mathfrak{R}_2$ exists, the composition of two rough neutrosophic multisets relations $\mathfrak{R}_2, \mathfrak{R}_1$ does not necessarily exist.

3.2 Inverse Rough Neutrosophic Multisets Relation

Definition 3.2.1 Let U be a non-empty set and X and Y be the rough neutrosophic multisets in U. $\mathfrak{R} \subseteq U \times U$ is a rough neutrosophic multisets relation on $U \times U$ based on the $X \times Y$. Then, we define $\mathfrak{R}^{-1} \subseteq U \times U$ is the rough neutrosophic multisets relation on $U \times U$ based on $Y \times X$ as follows:

$$\mathfrak{R}^{-1} = \left\{< (y, x), \left(T^i_{\mathfrak{R}^{-1}}(y, x)\right)\left(I^i_{\mathfrak{R}^{-1}}(y, x)\right), \left(F^i_{\mathfrak{R}^{-1}}(y, x)\right) >: (y, x) \in U \times U\right\}$$

where

$$T^i_{\mathfrak{R}^{-1}}(y, x) = T^i_{\mathfrak{R}}(x, y), I^i_{\mathfrak{R}^{-1}}(y, x) = I^i_{\mathfrak{R}}(x, y), F^i_{\mathfrak{R}^{-1}}(y, x) = F^i_{\mathfrak{R}}(x, y)$$

for all $(y, x) \in U \times U$ and $i = 1, 2, \dots, p$.

Definition 3.2.2 The relation $\mathfrak{R}^{-1}$ is called the inverse rough neutrosophic multisets relation of $\mathfrak{R}$.

Proposition 3.2.3 $(\mathfrak{R}^{-1})^{-1} = \mathfrak{R}$.

Proof: From definition 3.3;

(1) i) $T^i_{(\mathfrak{R}^{-1})^{-1}}(x, y) = T^i_{\mathfrak{R}^{-1}}(y, x) = T^i_{\mathfrak{R}}(x, y) = 1$ for all $(x, y) \in \underline{X \times Y}$

ii) $T^i_{(\mathfrak{R}^{-1})^{-1}}(x, y) = T^i_{\mathfrak{R}^{-1}}(y, x) = T^i_{\mathfrak{R}}(x, y) = 0$ for all $(x, y) \in U \times U - \overline{X \times Y}$

iii) $0 < T^i_{(\mathfrak{R}^{-1})^{-1}}(x, y) = T^i_{\mathfrak{R}^{-1}}(y, x) = T^i_{\mathfrak{R}}(x, y) < 1$ for all $(x, y) \in \overline{X \times Y} - \underline{X \times Y}$

(2) i) $I^i_{(\mathfrak{R}^{-1})^{-1}}(x, y) = I^i_{\mathfrak{R}^{-1}}(y, x) = I^i_{\mathfrak{R}}(x, y) = 0$ for all $(x, y) \in \underline{X \times Y}$

ii) $I^i_{(\mathfrak{R}^{-1})^{-1}}(x, y) = I^i_{\mathfrak{R}^{-1}}(y, x) = I^i_{\mathfrak{R}}(x, y) = 1$ for all $(x, y) \in U \times U - \overline{X \times Y}$

iii) $0 < I^i_{(\mathfrak{R}^{-1})^{-1}}(x, y) = I^i_{\mathfrak{R}^{-1}}(y, x) = I^i_{\mathfrak{R}}(x, y) < 1$ for all $(x, y) \in \overline{X \times Y} - \underline{X \times Y}$

Proof (3) for falsity function are similarly to proving (2) for indeterminate function.

It means $(\mathfrak{R}^{-1})^{-1} = \mathfrak{R}$.

Proposition 3.2.4 *Let $\mathfrak{R}_1, \mathfrak{R}_2$ be two rough neutrosophic multisets relations on $U \times U$, based on $X \times Y, Y \times Z$, respectively. Then $(\mathfrak{R}_1 \circ \mathfrak{R}_2)^{-1} = {\mathfrak{R}_2}^{-1} \circ {\mathfrak{R}_1}^{-1}$.*

Proof: For all $x, y, z \in U$, we have

$$T^i_{(\mathfrak{R}_1 \circ \mathfrak{R}_2)^{-1}}(z,x) = T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z)$$
$$= \max_{y\in U}\{\min[T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(y,z)]\} = \max_{y\in U}\left\{\min\left[T^i_{(\mathfrak{R}_1)^{-1}}(y,x), T^i_{(\mathfrak{R}_2)^{-1}}(z,y)\right]\right\}$$
$$= \max_{y\in U}\left\{\min\left[T^i_{(\mathfrak{R}_2)^{-1}}(z,y), T^i_{(\mathfrak{R}_1)^{-1}}(y,x)\right]\right\}$$
$$= T^i_{(\mathfrak{R}_2)^{-1}\circ(\mathfrak{R}_1)^{-1}}(z,x);$$

Similarly, proof for indeterminate function and falsity function.

That means $(\mathfrak{R}_1 \circ \mathfrak{R}_2)^{-1} = {\mathfrak{R}_2}^{-1} \circ {\mathfrak{R}_1}^{-1}$.

The representation of inverse rough neutrosophic multisets relation $\mathfrak{R}^{-1}$ can be represented by rough neutrosophic multisets relation $\mathfrak{R}$ by using a matrix $M(\mathfrak{R})^t$, it is the transposition of a matrix $M(\mathfrak{R})$.

Example 3.2.5: Consider the rough neutrosophic multisets relation $M(\mathfrak{R})$ in example 3.5;

$$M(\mathfrak{R}) = \begin{bmatrix} (0.3,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \\ (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) \\ (0.4,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \end{bmatrix}$$

Then the inverse rough neutrosophic multisets relation $\mathfrak{R}^{-1}$

$$M(\mathfrak{R}^{-1}) = M(\mathfrak{R})^t = \begin{bmatrix} (0.3,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.4,0),(1,1),(1,1) \\ (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0,0),(1,1),(1,1) \\ (0.9,0),(1,1),(1,1) & (0,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \end{bmatrix}$$

3.3 The Reflexive, Symmetric, Transitive Rough Neutrosophic Multisets Relation

In this section, we consider some properties of rough neutrosophic multisets on universe, such as reflexive, symmetric and transitive properties.

Let $(U, \mathcal{R})$ be a crisp approximation space and X is a rough neutrosophic multisets on $(U, \mathcal{R})$. From here onwards, the rough neutrosophic multisets relation $\mathfrak{R}$ is called a rough neutrosophic multisets relation on $(U, \mathcal{R})$ based on the rough neutrosophic multisets X.

Definition 3.3.1 The rough neutrosophic multisets relation $\mathfrak{R}$ is said to be reflexive rough neutrosophic multisets relation if $T^i_{\mathfrak{R}}(x,x) = 1$, $I^i_{\mathfrak{R}}(x,x) = F^i_{\mathfrak{R}}(x,x) = 0$ and for all $(x,x) \in U \times U$, $i = 1,2,\ldots,p$.

Proposition 3.3.2 *Let $\mathfrak{R}_1, \mathfrak{R}_2$ be two rough neutrosophic multisets relation on U based X. If $\mathfrak{R}_1, \mathfrak{R}_2$ are the reflexive rough neutrosophic multisets relations then $\mathfrak{R}_1 \wedge \mathfrak{R}_2$, $\mathfrak{R}_1 \vee \mathfrak{R}_2$, $\mathfrak{R}_1 \otimes \mathfrak{R}_2$, $\mathfrak{R}_1 \oplus \mathfrak{R}_2$ and $\mathfrak{R}_1 \circ \mathfrak{R}_2$ is also reflexive.*

Proof: If $\mathfrak{R}_1, \mathfrak{R}_2$ are reflexive rough neutrosophic multisets relation, then

$T^i_{\mathfrak{R}_1}(x,x) = T^i_{\mathfrak{R}_2}(x,x) = 1$, $I^i_{\mathfrak{R}_1}(x,x) = I^i_{\mathfrak{R}_2}(x,x) = 0$ and $F^i_{\mathfrak{R}_1}(x,x) = F^i_{\mathfrak{R}_2}(x,x) = 0$ for all $(x,x) \in U \times U$. We have

i) $T^i_{\mathfrak{R}_1 \wedge \mathfrak{R}_2}(x,x) = \min\{T^i_{\mathfrak{R}_1}(x,x), T^i_{\mathfrak{R}_2}(x,x)\} = 1$;
$I^i_{\mathfrak{R}_1 \wedge \mathfrak{R}_2}(x,x) = \max\{I^i_{\mathfrak{R}_1}(x,x), I^i_{\mathfrak{R}_2}(x,x)\} = 0$; and
$F^i_{\mathfrak{R}_1 \wedge \mathfrak{R}_2}(x,x) = \max\{F^i_{\mathfrak{R}_1}(x,x), F^i_{\mathfrak{R}_2}(x,x)\} = 0$

for all $(x, x) \in U \times U$ and $\mathfrak{R}_1 \wedge \mathfrak{R}_2$ is reflexive rough neutrosophic multisets relation.

ii) $T^i_{\mathfrak{R}_1 \vee \mathfrak{R}_2}(x, x) = \max\{T^i_{\mathfrak{R}_1}(x, x), T^i_{\mathfrak{R}_2}(x, x)\} = 1$;
$I^i_{\mathfrak{R}_1 \vee \mathfrak{R}_2}(x, x) = \min\{I^i_{\mathfrak{R}_1}(x, x), I^i_{\mathfrak{R}_2}(x, x)\} = 0$; and
$F^i_{\mathfrak{R}_1 \vee \mathfrak{R}_2}(x, x) = \min\{F^i_{\mathfrak{R}_1}(x, x), F^i_{\mathfrak{R}_2}(x, x)\} = 0$
for all $(x, x) \in U \times U$ and $\mathfrak{R}_1 \vee \mathfrak{R}_2$ is reflexive rough neutrosophic multisets relation.

iii) $T^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x, x) = T^i_{\mathfrak{R}_1}(x, x) \cdot T^i_{\mathfrak{R}_2}(x, x) = 1$;
$I^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x, x) = I^i_{\mathfrak{R}_1}(x, x) + I^i_{\mathfrak{R}_2}(x, x) - I^i_{\mathfrak{R}_1}(x, x) \cdot I^i_{\mathfrak{R}_2}(x, x) = 0$; and
$F^i_{\mathfrak{R}_1 \otimes \mathfrak{R}_2}(x, x) = F^i_{\mathfrak{R}_1}(x, x) + F^i_{\mathfrak{R}_2}(x, x) - F^i_{\mathfrak{R}_1}(x, x) \cdot F^i_{\mathfrak{R}_2}(x, x) = 0$

for all $(x, x) \in U \times U$ and $\mathfrak{R}_1 \otimes \mathfrak{R}_2$ is reflexive rough neutrosophic multisets relation.

iv) $T^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x, x) = T^i_{\mathfrak{R}_1}(x, x) + T^i_{\mathfrak{R}_2}(x, x) - T^i_{\mathfrak{R}_1}(x, x) \cdot T^i_{\mathfrak{R}_2}(x, x) = 1$;
$I^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x, x) = I^i_{\mathfrak{R}_1}(x, x) \cdot I^i_{\mathfrak{R}_2}(x, x) = 0$; and
$F^i_{\mathfrak{R}_1 \oplus \mathfrak{R}_2}(x, x) = F^i_{\mathfrak{R}_1}(x, x) \cdot F^i_{\mathfrak{R}_2}(x, x) = 0$

for all $(x, x) \in U \times U$ and $\mathfrak{R}_1 \oplus \mathfrak{R}_2$ is reflexive rough neutrosophic multisets relation.

v) $T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x, x) = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x, y), T^i_{\mathfrak{R}_2}(y, x)]\}$
$= \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x, x), T^i_{\mathfrak{R}_2}(x, x)]\} = 1$;
$I^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x, x) = \min_{y \in U}\{\max[I^i_{\mathfrak{R}_1}(x, y), I^i_{\mathfrak{R}_2}(y, x)]\}$
$= \min_{y \in U}\{\max[I^i_{\mathfrak{R}_1}(x, x), I^i_{\mathfrak{R}_2}(x, x)]\} = 0$; and
$F^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x, x) = \min_{y \in U}\{\max[F^i_{\mathfrak{R}_1}(x, y), F^i_{\mathfrak{R}_2}(y, x)]\}$
$= \min_{y \in U}\{\max[F^i_{\mathfrak{R}_1}(x, x), F^i_{\mathfrak{R}_2}(x, x)]\} = 0$

for all $(x, x) \in U \times U, X \equiv Y$ and $\mathfrak{R}_1 \circ \mathfrak{R}_2$ is reflexive rough neutrosophic multisets relation.

Definition 3.3.3 The rough neutrosophic multisets relation $\mathfrak{R}$ is said to be symmetric rough neutrosophic multisets relation if $T^i_{\mathfrak{R}}(x, y) = T^i_{\mathfrak{R}}(y, x)$, $I^i_{\mathfrak{R}}(x, y) = I^i_{\mathfrak{R}}(y, x)$ and $F^i_{\mathfrak{R}}(x, y) = F^i_{\mathfrak{R}}(y, x)$ for all $(x, y) \in U \times U$, $i = 1, 2, \dots, p$.

We note that if $\mathfrak{R}$ is a symmetric rough neutrosophic multisets relation then the matrix $M(\mathfrak{R})$ is a symmetric matrix.

Proposition 3.3.4 *If $\mathfrak{R}$ is said to be symmetric rough neutrosophic multisets relation, then $\mathfrak{R}^{-1}$ is also symmetric.*

Proof: Assume that $\mathfrak{R}$ is symmetric rough neutrosophic multisets relation, then we have
$T^i_{\mathfrak{R}}(x, y) = T^i_{\mathfrak{R}}(y, x)$, $I^i_{\mathfrak{R}}(x, y) = I^i_{\mathfrak{R}}(y, x)$ and $F^i_{\mathfrak{R}}(x, y) = F^i_{\mathfrak{R}}(y, x)$.

Also, if $\mathfrak{R}^{-1}$ is an inverse rough neutrosophic multisets relation, then we have
$T^i_{\mathfrak{R}^{-1}}(x, y) = T^i_{\mathfrak{R}}(y, x)$, $I^i_{\mathfrak{R}^{-1}}(x, y) = I^i_{\mathfrak{R}}(y, x)$ and $F^i_{\mathfrak{R}^{-1}}(x, y) = F^i_{\mathfrak{R}}(y, x)$ for all $(x, y) \in U \times U$.

To prove $\mathfrak{R}^{-1}$ is symmetric, we must prove that
$T^i_{\mathfrak{R}^{-1}}(x, y) = T^i_{\mathfrak{R}^{-1}}(y, x)$, $I^i_{\mathfrak{R}^{-1}}(x, y) = I^i_{\mathfrak{R}^{-1}}(y, x)$ and $F^i_{\mathfrak{R}^{-1}}(x, y) = F^i_{\mathfrak{R}^{-1}}(y, x)$ for all $(x, y) \in U \times U$.
Therefore,
$T^i_{\mathfrak{R}^{-1}}(x, y) = T^i_{\mathfrak{R}}(y, x) = T^i_{\mathfrak{R}}(x, y) = T^i_{\mathfrak{R}^{-1}}(y, x)$;
$I^i_{\mathfrak{R}^{-1}}(x, y) = I^i_{\mathfrak{R}}(y, x) = I^i_{\mathfrak{R}}(x, y) = I^i_{\mathfrak{R}^{-1}}(y, x)$; and
$F^i_{\mathfrak{R}^{-1}}(x, y) = F^i_{\mathfrak{R}}(y, x) = F^i_{\mathfrak{R}}(x, y) = F^i_{\mathfrak{R}^{-1}}(y, x)$

for all $(x, y) \in U \times U$ and $\Re$ is said to be symmetric rough neutrosophic multisets relation, then $\Re^{-1}$ is also symmetric rough neutrosophic multisets relation.

Proposition 3.3.5 *Let $\Re_1, \Re_2$ be two rough neutrosophic multisets relations on U based rough neutrosophic multisets. If $\Re_1, \Re_2$ are the symmetric rough neutrosophic multisets relations then $\Re_1 \wedge \Re_2$, $\Re_1 \vee \Re_2$, $\Re_1 \otimes \Re_2$ and $\Re_1 \oplus \Re_2$ also symmetric.*

Proof: Since $\Re_1$ is symmetric, then we have;

$T^i_{\Re_1}(x,y) = T^i_{\Re_1}(y,x)$, $I^i_{\Re_1}(x,y) = I^i_{\Re_1}(y,x)$ and $F^i_{\Re_1}(x,y) = F^i_{\Re_1}(y,x)$

Similarly, $\Re_2$ is symmetric, then we have;

$T^i_{\Re_2}(x,y) = T^i_{\Re_2}(y,x)$, $I^i_{\Re_2}(x,y) = I^i_{\Re_2}(y,x)$ and $F^i_{\Re_2}(x,y) = F^i_{\Re_2}(y,x)$

Therefore;

i) $T^i_{\Re_1 \wedge \Re_2}(x,y) = \min\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\}$
$= \min\{T^i_{\Re_1}(y,x), T^i_{\Re_2}(y,x)\} = T^i_{\Re_1 \wedge \Re_2}(y,x)$;
$I^i_{\Re_1 \wedge \Re_2}(x,y) = \max\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\}$
$= \max\{I^i_{\Re_1}(y,x), I^i_{\Re_2}(y,x)\} = I^i_{\Re_1 \wedge \Re_2}(y,x)$; and
$F^i_{\Re_1 \wedge \Re_2}(x,y) = \max\{F^i_{\Re_1}(x,y), F^i_{\Re_2}(x,y)\}$
$= \max\{F^i_{\Re_1}(y,x), F^i_{\Re_2}(y,x)\} = F^i_{\Re_1 \wedge \Re_2}(y,x)$

for all $(x, y) \in U \times U$ and $\Re_1 \wedge \Re_2$ is symmetric rough neutrosophic multisets relation.

ii) $T^i_{\Re_1 \vee \Re_2}(x,y) = \max\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\}$
$= \max\{T^i_{\Re_1}(y,x), T^i_{\Re_2}(y,x)\} = T^i_{\Re_1 \vee \Re_2}(y,x)$;
$I^i_{\Re_1 \vee \Re_2}(x,y) = \min\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\}$
$= \min\{I^i_{\Re_1}(y,x), I^i_{\Re_2}(y,x)\} = I^i_{\Re_1 \vee \Re_2}(y,x)$; and
$F^i_{\Re_1 \vee \Re_2}(x,y) = \min\{F^i_{\Re_1}(x,y), F^i_{\Re_2}(x,y)\}$
$= \min\{F^i_{\Re_1}(y,x), F^i_{\Re_2}(y,x)\} = F^i_{\Re_1 \vee \Re_2}(y,x)$

for all $(x, y) \in U \times U$ and $\Re_1 \vee \Re_2$ is symmetric rough neutrosophic multisets relation.

iii) $T^i_{\Re_1 \otimes \Re_2}(x,y) = T^i_{\Re_1}(x,y) \cdot T^i_{\Re_2}(x,y) = T^i_{\Re_1}(y,x) \cdot T^i_{\Re_2}(y,x) = T^i_{\Re_1 \otimes \Re_2}(y,x)$;
$I^i_{\Re_1 \otimes \Re_2}(x,y) = I^i_{\Re_1}(x,y) + I^i_{\Re_2}(x,y) - I^i_{\Re_1}(x,y) \cdot I^i_{\Re_2}(x,y)$
$= I^i_{\Re_1}(y,x) + I^i_{\Re_2}(y,x) - I^i_{\Re_1}(y,x) \cdot I^i_{\Re_2}(y,x)$
$= I^i_{\Re_1 \otimes \Re_2}(y,x)$; and
$F^i_{\Re_1 \otimes \Re_2}(x,y) = F^i_{\Re_1}(x,y) + F^i_{\Re_2}(x,y) - F^i_{\Re_1}(x,y) \cdot F^i_{\Re_2}(x,y)$
$= F^i_{\Re_1}(y,x) + F^i_{\Re_2}(y,x) - F^i_{\Re_1}(y,x) \cdot F^i_{\Re_2}(y,x)$
$= F^i_{\Re_1 \otimes \Re_2}(y,x)$

for all $(x, y) \in U \times U$ and $\Re_1 \otimes \Re_2$ is symmetric rough neutrosophic multisets relation.

iv) $T^i_{\Re_1 \oplus \Re_2}(x,y) = T^i_{\Re_1}(x,y) + T^i_{\Re_2}(x,y) - T^i_{\Re_1}(x,y) \cdot T^i_{\Re_2}(x,y)$
$= T^i_{\Re_1}(y,x) + T^i_{\Re_2}(y,x) - T^i_{\Re_1}(y,x) \cdot T^i_{\Re_2}(y,x) = T^i_{\Re_1 \oplus \Re_2}(y,x)$;
$I^i_{\Re_1 \oplus \Re_2}(x,y) = I^i_{\Re_1}(x,y) \cdot I^i_{\Re_2}(x,y) = I^i_{\Re_1}(y,x) \cdot I^i_{\Re_2}(y,x) = I^i_{\Re_1 \oplus \Re_2}(y,x)$; and
$F^i_{\Re_1 \oplus \Re_2}(x,y) = F^i_{\Re_1}(x,y) \cdot F^i_{\Re_2}(x,y) = F^i_{\Re_1}(y,x) \cdot F^i_{\Re_2}(y,x) = F^i_{\Re_1 \oplus \Re_2}(y,x)$

for all $(x, y) \in U \times U$ and $\Re_1 \oplus \Re_2$ is symmetric rough neutrosophic multisets relation.

Remark 3.3.6: $\mathfrak{R}_1 \circ \mathfrak{R}_2$ in general is not symmetric, as

$$T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(y,z)]\}$$
$$= \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(y,x), T^i_{\mathfrak{R}_2}(z,x)]\} \neq T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(z,x)$$

The proof is similarly for indeterminate function and falsity function.

But, $\mathfrak{R}_1 \circ \mathfrak{R}_2$ is symmetric if $\mathfrak{R}_1 \circ \mathfrak{R}_2 = \mathfrak{R}_2 \circ \mathfrak{R}_1$, for $\mathfrak{R}_1$ and $\mathfrak{R}_2$ are symmetric relations.

$$T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(x,z) = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(x,y), T^i_{\mathfrak{R}_2}(y,z)]\}$$
$$= \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(y,x), T^i_{\mathfrak{R}_2}(z,y)]\} = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_2}(y,x), T^i_{\mathfrak{R}_2}(z,y)]\}$$
$$= \max_{y \in U}\{\min[T^i_{\mathfrak{R}_2}(z,y), T^i_{\mathfrak{R}_2}(y,x)]\} = \max_{y \in U}\{\min[T^i_{\mathfrak{R}_1}(z,y), T^i_{\mathfrak{R}_2}(y,x)]\}$$
$$= T^i_{\mathfrak{R}_1 \circ \mathfrak{R}_2}(z,x)$$

for all $(x,z) \in U \times U$ and $y \in U$.

The proof is similarly for indeterminate function and falsity function.

Definition 3.3.7 The rough neutrosophic multisets relation $\mathfrak{R}$ is said to be transitive rough neutrosophic multisets relation if $\mathfrak{R} \circ \mathfrak{R} \subseteq \mathfrak{R}$ such that $T^i_{\mathfrak{R}}(y,x) \geq T^i_{\mathfrak{R} \circ \mathfrak{R}}(y,x)$, $I^i_{\mathfrak{R}}(y,x) \leq I^i_{\mathfrak{R} \circ \mathfrak{R}}(y,x)$ and $F^i_{\mathfrak{R}}(y,x) \leq F^i_{\mathfrak{R} \circ \mathfrak{R}}(y,x)$ for all $x,y \in U$.

Definition 3.3.8 The rough neutrosophic multisets relation $\mathfrak{R}$ on *U* based on the neutrosophic multisets *X* is called a rough neutrosophic multisets equivalence relation if it is reflexive, symmetric and transitive rough neutrosophic multisets relation.

Proposition 3.3.9 *If $\mathfrak{R}$ is transitive rough neutrosophic multisets relation, then $\mathfrak{R}^{-1}$ is also transitive.*

Proof: $\mathfrak{R}$ is transitive rough neutrosophic multisets relation if $\mathfrak{R} \circ \mathfrak{R} \subseteq \mathfrak{R}$, hence if $\mathfrak{R}^{-1} \circ \mathfrak{R}^{-1} \subseteq \mathfrak{R}^{-1}$, then $\mathfrak{R}^{-1}$ is transitive.
Consider;

$$T^i_{\mathfrak{R}^{-1}}(x,y) = T^i_{\mathfrak{R}}(y,x) \geq T^i_{\mathfrak{R} \circ \mathfrak{R}}(y,x)$$
$$= \max_{z \in U}\{\min[T^i_{\mathfrak{R}}(y,z), T^i_{\mathfrak{R}}(z,x)]\}$$
$$= \max_{z \in U}\{\min[T^i_{\mathfrak{R}^{-1}}(z,y), T^i_{\mathfrak{R}^{-1}}(x,z)]\} = \max_{z \in U}\{\min[T^i_{\mathfrak{R}^{-1}}(x,z), T^i_{\mathfrak{R}^{-1}}(z,y)]\}$$
$$= T^i_{\mathfrak{R}^{-1} \circ \mathfrak{R}^{-1}}(x,y);$$

The proof is similarly for indeterminate function and falsity function.

Hence, the proof is valid.

Proposition 3.3.10 *Let $\mathfrak{R}_1, \mathfrak{R}_2$ be two rough neutrosophic multisets relations on U based rough neutrosophic multisets. If $\mathfrak{R}_1$ are the transitive rough neutrosophic multisets relation, then $\mathfrak{R}_1 \wedge \mathfrak{R}_2$ is also transitive.*

Proof: As $\mathfrak{R}_1$ and $\mathfrak{R}_2$ are transitive rough neutrosophic multisets relation, $\mathfrak{R}_1 \circ \mathfrak{R}_1 \subseteq \mathfrak{R}_1$ and $\mathfrak{R}_2 \circ \mathfrak{R}_2 \subseteq \mathfrak{R}_2$. Also

$$T^i_{\mathfrak{R}_1 \wedge \mathfrak{R}_2}(x,y) \geq T^i_{(\mathfrak{R}_1 \wedge \mathfrak{R}_2) \circ (\mathfrak{R}_1 \wedge \mathfrak{R}_2)}(x,y);$$
$$I^i_{\mathfrak{R}_1 \wedge \mathfrak{R}_2}(x,y) \leq I^i_{(\mathfrak{R}_1 \wedge \mathfrak{R}_2) \circ (\mathfrak{R}_1 \wedge \mathfrak{R}_2)}(x,y); \text{ and}$$
$$F^i_{\mathfrak{R}_1 \wedge \mathfrak{R}_2}(x,y) \leq F^i_{(\mathfrak{R}_1 \wedge \mathfrak{R}_2) \circ (\mathfrak{R}_1 \wedge \mathfrak{R}_2)}(x,y)$$

implies that $(\mathfrak{R}_1 \wedge \mathfrak{R}_2) \circ (\mathfrak{R}_1 \wedge \mathfrak{R}_2) \subseteq \mathfrak{R}_1 \wedge \mathfrak{R}_2$, hence $\mathfrak{R}_1 \wedge \mathfrak{R}_2$ is transitive.

Proposition 3.3.11 *If $\Re_1$ and $\Re_2$ are transitive rough neutrosophic multisets relations, then $\Re_1 \vee \Re_2$, $\Re_1 \otimes \Re_2$ and $\Re_1 \oplus \Re_2$ are not transitive.*

Proof:

i) As

$$T^i_{\Re_1 \vee \Re_2}(x,y) = \max\{T^i_{\Re_1}(x,y), T^i_{\Re_2}(x,y)\},$$
$$I^i_{\Re_1 \vee \Re_2}(x,y) = \min\{I^i_{\Re_1}(x,y), I^i_{\Re_2}(x,y)\}, \text{ and}$$
$$F^i_{\Re_1 \vee \Re_2}(x,y) = \min\{F^i_{\Re_1}(x,y), F^i_{\Re_2}(x,y)\}$$

and,

$$T^i_{\Re_1 \vee \Re_2}(x,y) \leq T^i_{(\Re_1 \vee \Re_2)\circ(\Re_1 \vee \Re_2)}(x,y);$$
$$I^i_{\Re_1 \vee \Re_2}(x,y) \geq I^i_{(\Re_1 \vee \Re_2)\circ(\Re_1 \vee \Re_2)}(x,y); \text{ and}$$
$$F^i_{\Re_1 \vee \Re_2}(x,y) \geq F^i_{(\Re_1 \vee \Re_2)\circ(\Re_1 \vee \Re_2)}(x,y)$$

ii) As

$$T^i_{\Re_1 \otimes \Re_2}(x,y) = T^i_{\Re_1}(x,y) \cdot T^i_{\Re_2}(x,y),$$
$$I^i_{\Re_1 \otimes \Re_2}(x,y) = I^i_{\Re_1}(x,y) + I^i_{\Re_2}(x,y) - I^i_{\Re_1}(x,y) \cdot I^i_{\Re_2}(x,y), \text{ and}$$
$$F^i_{\Re_1 \otimes \Re_2}(x,y) = F^i_{\Re_1}(x,y) + F^i_{\Re_2}(x,y) - F^i_{\Re_1}(x,y) \cdot F^i_{\Re_2}(x,y)$$

and,

$$T^i_{\Re_1 \otimes \Re_2}(x,y) \leq T^i_{(\Re_1 \otimes \Re_2)\circ(\Re_1 \otimes \Re_2)}(x,y);$$
$$I^i_{\Re_1 \otimes \Re_2}(x,y) \geq I^i_{(\Re_1 \otimes \Re_2)\circ(\Re_1 \otimes \Re_2)}(x,y); \text{ and } F^i_{\Re_1 \otimes \Re_2}(x,y) \geq F^i_{(\Re_1 \otimes \Re_2)\circ(\Re_1 \otimes \Re_2)}(x,y)$$

(iii) As

$$T^i_{\Re_1 \oplus \Re_2}(x,y) = T^i_{\Re_1}(x,y) + T^i_{\Re_2}(x,y) - T^i_{\Re_1}(x,y) \cdot T^i_{\Re_2}(x,y),$$
$$I^i_{\Re_1 \oplus \Re_2}(x,y) = I^i_{\Re_1}(x,y) \cdot I^i_{\Re_2}(x,y), \text{ and}$$
$$F^i_{\Re_1 \oplus \Re_2}(x,y) = F^i_{\Re_1}(x,y) \cdot F^i_{\Re_2}(x,y)$$

and

$$T^i_{\Re_1 \oplus \Re_2}(x,y) \leq T^i_{(\Re_1 \oplus \Re_2)\circ(\Re_1 \oplus \Re_2)}(x,y);$$
$$I^i_{\Re_1 \oplus \Re_2}(x,y) \geq I^i_{(\Re_1 \oplus \Re_2)\circ(\Re_1 \oplus \Re_2)}(x,y); \text{ and } F^i_{\Re_1 \oplus \Re_2}(x,y) \geq F^i_{(\Re_1 \oplus \Re_2)\circ(\Re_1 \oplus \Re_2)}(x,y)$$

Hence, $\Re_1 \vee \Re_2$, $\Re_1 \otimes \Re_2$ and $\Re_1 \oplus \Re_2$ are not transitive.

4 An application to marketing strategy

The aims of multi criteria decision making (MCDM) are to solve the problem involving multi decision by many expert opinions and many alternatives given and MCDM also try to get the best alternative solution based on the multi criteria evaluate by many experts. The study of MCDM with the neutrosophic environment is well established in [38]–[44].

This section gives a situation of solving a real application of the rough neutrosophic multisets relation in marketing strategy.

Assume $J = \{j_1, j_2, j_3\}$ denotes for three jeans showed available to be purchased in a shop G. Let $\mathcal{R}_J$ be a relation defined on the J as $a\mathcal{R}_J b$ if and only if a, b coming from the same continent about quality of the jeans. $a\mathcal{R}_J b$ is composed by $\mathcal{R}_J = \{j_1, j_2, j_3\}$. The relation $\mathcal{R}_J$ is explains the effect of the quality of jeans in shop Z. We now try to get the opinion from two independent customers about the quality of jeans considering whether the jeans are comprised of "good texture", a level of indeterminacy with respect to the customers which is “no comment” and whether they feel that the jean is comprised of "a not all that great texture". In the customers’ opinion, rough neutrosophic multisets, A and B can be defined as follows:

$$A = \{< j_1, (0.9, 0.2), (0.3, 0.6), (0.5, 0.7) >,$$
$$< j_2, (0.4, 0.6), (0.2, 0.4), (0.5, 0.6) >,$$

$< j_3, (1.0, 0.6), (0.4, 0.5), (0.6, 0.7) >\}$ and

$$\begin{aligned} B = \{ &< j_1, (0.5, 0.7), (0.4, 0.6), (0.2, 0.8) >, \\ &< j_2, (0.6, 0.7), (0.2, 0.8), (0.4, 0.5) >, \\ &< j_3, (1.0, 0.8), (0.3, 0.6), (0.2, 0.8) >\} \end{aligned}$$

By satisfied all the condition in definition 3.3, we will define the relation of rough neutrosophic multisets $\mathfrak{R}$ on qualities of jeans $J \times J$ based on customers opinion $A \times B$ as follows:

Step 1: Compute lower and upper approximation values for rough neutrosophic multisets.

$$\begin{aligned} \underline{\mathcal{R}_J}(A) = \{ &< j_1, (0.4, 0.2), (0.4, 0.6), (0.6, 0.7) >, \\ &< j_2, (0.4, 0.2), (0.4, 0.6), (0.6, 0.7) >, \\ &< j_3, (0.4, 0.2), (0.4, 0.6), (0.6, 0.7) >\} \end{aligned}$$

$$\begin{aligned} \underline{\mathcal{R}_J}(B) = \{ &< j_1, (0.5, 0.7), (0.4, 0.8), (0.4, 0.8) >, \\ &< j_2, (0.5, 0.7), (0.4, 0.8), (0.4, 0.8) >, \\ &< j_3, (0.5, 0.7), (0.4, 0.8), (0.4, 0.8) >\} \end{aligned}$$

$$\begin{aligned} \overline{\mathcal{R}_J}(A) = \{ &< j_1, (1.0, 0.6), (0.2, 0.4), (0.5, 0.6) >, \\ &< j_2, (1.0, 0.6), (0.2, 0.4), (0.5, 0.6) >, \\ &< j_3, (1.0, 0.6), (0.2, 0.4), (0.5, 0.6) >\} \end{aligned}$$

$$\begin{aligned} \overline{\mathcal{R}_J}(B) = \{ &< j_1, (1.0, 0.8), (0.2, 0.6), (0.2, 0.5) >, \\ &< j_2, (1.0, 0.8), (0.2, 0.6), (0.2, 0.5) >, \\ &< j_3, (1.0, 0.8), (0.2, 0.6), (0.2, 0.5) >\} \end{aligned}$$

Step 2: Construct the relation of $\underline{A \times B} = \underline{\mathcal{R}_J}(A) \times \underline{\mathcal{R}_J}(B)$, relation of $\overline{A \times B} = \overline{\mathcal{R}_J}(A) \times \overline{\mathcal{R}_J}(B)$, and relation of $J \times J$. All the relation was represented in the Table 1, Table 2 and Table 3, respectively.

$\underline{A \times B}$	j_1	j_2	j_3
j_1	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)
j_2	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)
j_3	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)	(0.4, 0.2), (0.4, 0.8), (0.6, 0.8)

Table 1: Relation of $\underline{\mathrm{A \times B}}$

$\overline{A \times B}$	j_1	j_2	j_3
j_1	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)
j_2	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)
j_3	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)	(1.0, 0.6), (0.4, 0.6), (0.5, 0.6)

Table 2: Relation of $\overline{\mathrm{A \times B}}$

Note that $T^i_{\mathfrak{R}}(a,b) = 1$, $I^i_{\mathfrak{R}}(a,b) = 0$ and $F^i_{\mathfrak{R}}(a,b) = 0$ for all $(a,b) \in \underline{A \times B}$. Therefore, the relation of $J \times J$ is

$J \times J$	j_1	j_2	j_3
j_1	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)
j_2	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)
j_3	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)	(1.0, 1.0), (0.0, 0.0), (0.0, 0.0)

Table 3: Relation of J × J

Step 3: Construct a rough neutrosophic multisets relation $\mathfrak{R}$. Note that, $T^i_{\mathfrak{R}}(a,b) = 0$, $I^i_{\mathfrak{R}}(a,b) = 1$ and $F^i_{\mathfrak{R}}(a,b) = 1$ for all $(a,b) \in J \times J - \overline{A \times B}$. Table 4 represent the rough neutrosophic multisets relation $\mathfrak{R}$.

$\mathfrak{R}$	j_1	j_2	j_3
j_1	(**t**$_1$, 0.0), (1.0, 1.0), (1.0, 1.0)	(**t**$_2$, 0.0), (1.0, 1.0), (1.0, 1.0)	(**t**$_3$, 0.0), (1.0, 1.0), (1.0, 1.0)
j_2	(**t**$_4$, 0.0), (1.0, 1.0), (1.0, 1.0)	(**t**$_5$, 0.0), (1.0, 1.0), (1.0, 1.0)	(**t**$_6$, 0.0), (1.0, 1.0), (1.0, 1.0)
j_3	(**t**$_7$, 0.0), (1.0, 1.0), (1.0, 1.0)	(**t**$_8$, 0.0), (1.0, 1.0), (1.0, 1.0)	(**t**$_9$, 0.0), (1.0, 1.0), (1.0, 1.0)

Table 4: Rough neutrosophic multi relation $\mathfrak{R}$.

Step 4: Compute the values for t_1 until t_9 in Table 4.
Note that $\boldsymbol{t_1}, \boldsymbol{t_2}, .., \boldsymbol{t_9} \in [0,1]$ and $0 < T^i_{\mathfrak{R}}(a,b) < 1$, for all $(a,b) \in \overline{A \times B} - \underline{A \times B}$ and neutrosophic multi relation of $T^i_{\mathfrak{R}}(a,b) \leq T^i_{A\times B}(a,b)\ \forall\ (a,b) \in J \times J$.

(i) $\boldsymbol{t_1} = T^1_{\mathfrak{R}}(j_1, j_1) = T^1_{\overline{A\times B}}(j_1, j_1) - T^1_{\underline{A\times B}}(j_1, j_1) = 1 - 0.4 = 0.6$, $T^1_{\mathfrak{R}}(j_1, j_1) \leq T^1_{A\times B}(j_1, j_1)$ where $0.6 \leq 1$. Therefore, the possible values of $\boldsymbol{t_1}$ is 0.9, 0.8, 0.7 and 0.6.
(ii) $\boldsymbol{t_2} = T^1_{\mathfrak{R}}(j_1, j_2) = T^1_{\overline{A\times B}}(j_1, j_2) - T^1_{\underline{A\times B}}(j_1, j_2) = 1 - 0.4 = 0.6$, $T^1_{\mathfrak{R}}(j_1, j_2) \leq T^1_{A\times B}(j_1, j_2)$ where $0.6 \leq 1$. Therefore, the possible values of $\boldsymbol{t_2}$ is 0.9, 0.8, 0.7 and 0.6.
(iii) The same calculation was used for t_3 until t_9. Therefore, the possible values for t_1 until t_9 is represent in Table 5.

$t_n, n = 1, 2, \ldots, 9$	Possible values	$t_n, n = 1, 2, \ldots, 9$	Possible values
t_1	0.6, 0.7, 0.8, 0.9	t_6	0.6, 0.7, 0.8, 0.9
t_2	0.6, 0.7, 0.8, 0.9	t_7	0.6, 0.7, 0.8, 0.9
t_3	0.6, 0.7, 0.8, 0.9	t_8	0.6, 0.7, 0.8, 0.9
t_4	0.6, 0.7, 0.8, 0.9	t_9	0.6, 0.7, 0.8, 0.9
t_5	0.6, 0.7, 0.8, 0.9		

Table 5: Possible values for $t_1$until t_9

Step 5: We defined $\mathfrak{R} \subseteq J \times J$ as a rough neutrosophic multisets relation on $J \times J$ based on the $A \times B$ by a matrix form. We can have different values for t_1 until t_9 as it is true for all possible values in Table 5. We try to get some pattern of the rough neutrosophic multisets relation matrix of our study by three possible cases.

Case 1: $(j_1, j_n) > (j_2, j_n) > (j_3, j_n)$ for all n, and unknown value. Therefore, there are two rough neutrosophic multisets relation matrix resulted for this case, represented as $M(\mathfrak{R}_1)$ and $M(\mathfrak{R}_2)$, respectively.

$$M(\mathfrak{R}_1) = \begin{bmatrix} (0.9,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \\ (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \\ (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) \end{bmatrix}$$

$$M(\mathfrak{R}_2) = \begin{bmatrix} (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \\ (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) \\ (0.6,0),(1,1),(1,1) & (0.6,0),(1,1),(1,1) & (0.6,0),(1,1),(1,1) \end{bmatrix}$$

Case 2: $(j_1, j_n) < (j_2, j_n) < (j_3, j_n)$ for all n, and unknown value. Therefore, there are two rough neutrosophic multisets relation matrix resulted for this case, represented as $M(\mathfrak{R}_3)$ and $M(\mathfrak{R}_4)$, respectively.

$$M(\mathfrak{R}_3) = \begin{bmatrix} (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) \\ (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \\ (0.9,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \end{bmatrix}$$

$$M(\mathfrak{R}_4) = \begin{bmatrix} (0.6,0),(1,1),(1,1) & (0.6,0),(1,1),(1,1) & (0.6,0),(1,1),(1,1) \\ (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) \\ (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \end{bmatrix}$$

Case 3: $(j_1, j_n) = (j_2, j_n) = (j_3, j_n)$ for all n, and unknown value. Therefor the rough neutrosophic multisets relation matrix resulted as $M(\mathfrak{R}_5)$.

$$M(\mathfrak{R}_5) = \begin{bmatrix} (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \\ (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \\ (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \end{bmatrix}$$

Case 4: Random possible value for all unknown. Therefore, the rough neutrosophic multisets relation matrix resulted as $M(\mathfrak{R}_6)$.

$$M(\mathfrak{R}_6) = \begin{bmatrix} (0.9,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) & (0.9,0),(1,1),(1,1) \\ (0.8,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) & (0.8,0),(1,1),(1,1) \\ (0.7,0),(1,1),(1,1) & (0.6,0),(1,1),(1,1) & (0.7,0),(1,1),(1,1) \end{bmatrix}$$

Step 6: Compute the comparison matrix using the formula $D^i_{\mathfrak{R}} = T^i_{\mathfrak{R}} + I^i_{\mathfrak{R}} - F^i_{\mathfrak{R}}$ for all i, and select the maximum value for comparison table. The result is shown in Table 6 and Table 7, respectively.

	j_1	j_2	j_3
$\mathfrak{R}_1$			
j_1	(0.9, 0)	(0.9, 0)	(0.9, 0)
j_2	(0.8, 0)	(0.8, 0)	(0.8, 0)
j_3	(0.7, 0)	(0.7, 0)	(0.7, 0)
$\mathfrak{R}_2$			
j_1	(0.8, 0)	(0.8, 0)	(0.8, 0)
j_2	(0.7, 0)	(0.7, 0)	(0.7, 0)
j_3	(0.6, 0)	(0.6, 0)	(0.6, 0)
$\mathfrak{R}_3$			
j_1	(0.7, 0)	(0.7, 0)	(0.7, 0)
j_2	(0.8, 0)	(0.8, 0)	(0.8, 0)
j_3	(0.9, 0)	(0.9, 0)	(0.9, 0)
$\mathfrak{R}_4$			
j_1	(0.6, 0)	(0.6, 0)	(0.6, 0)
j_2	(0.7, 0)	(0.7, 0)	(0.7, 0)
j_3	(0.8, 0)	(0.8, 0)	(0.8, 0)
$\mathfrak{R}_5$			
j_1	(0.8, 0)	(0.8, 0)	(0.8, 0)
j_2	(0.8, 0)	(0.8, 0)	(0.8, 0)
j_3	(0.8, 0)	(0.8, 0)	(0.8, 0)
$\mathfrak{R}_6$			
j_1	(0.9, 0)	(0.8, 0)	(0.9, 0)
j_2	(0.8, 0)	(0.7, 0)	(0.8, 0)
j_3	(0.7, 0)	(0.6, 0)	(0.7, 0)

Table 6: Comparison matrix of rough neutrosophic multi relation $\mathfrak{R}$.

J	j_1	j_2	j_3
J_1			
j_1	0.9	0.9	0.9
j_2	0.8	0.8	0.8
j_3	0.7	0.7	0.7
J_2			
j_1	0.8	0.8	0.8
j_2	0.7	0.7	0.7
j_3	0.6	0.6	0.6
J_3			
j_1	0.7	0.7	0.7
j_2	0.8	0.8	0.8
j_3	0.9	0.9	0.9
J_4			
j_1	0.6	0.6	0.6
j_2	0.7	0.7	0.7
j_3	0.8	0.8	0.8
J_5			
j_1	0.8	0.8	0.8
j_2	0.8	0.8	0.8
j_3	0.8	0.8	0.8
J_6			
j_1	0.9	0.8	0.9
j_2	0.8	0.7	0.8
j_3	0.7	0.6	0.7

Table 7: Comparison table for rough neutrosophic multisets, *J*

Step 7: Next we compute the row-sum, column-sum, and the score for all cases as shown in Table 8.

J	Row sum	Column sum	Score
$\boldsymbol{J_1}$			
j_1	2.7	2.4	**0.3**
j_2	2.4	2.4	0
j_3	2.1	2.4	-0.3
$\boldsymbol{J_2}$			
j_1	2.4	2.1	**0.3**
j_2	2.1	2.1	0
j_3	1.8	2.1	-0.3
$\boldsymbol{J_3}$			
j_1	2.1	2.4	-0.3
j_2	2.4	2.4	0
j_3	2.7	2.4	**0.3**
$\boldsymbol{J_4}$			
j_1	1.8	2.1	-0.3
j_2	2.1	2.1	0
j_3	2.4	2.1	**0.3**
$\boldsymbol{J_5}$			
j_1	2.4	2.4	**0**
j_2	2.4	2.4	**0**
j_3	2.4	2.4	**0**
$\boldsymbol{J_6}$			
j_1	2.6	2.4	**0.2**
j_2	2.3	2.1	**0.2**
j_3	2.0	2.4	-0.4

Table 8: Score of three jeans for all cases.

The relation of quality of jeans in shop G is successfully approximate by using rough neutrosophic multisets relation. Jean type j_1 has the highest score of 0.3 for case 1, jean type j_3 has the highest score of 0.3 for case 2, neither choose a jean or not for case 3, and jeans type j_1 and j_2 have a highest score of 0.2 for case 4. The different selection of jeans has resulted in different cases. From the scoring perspective, the highest value for each unknown will resulted in the highest possibility to select the subject. Besides that, it cannot take the same possible values for each unknown at the same time because the score result will be equal to zero (case 3).

Based on the result, the customers should purchase the jeans of type j_1 in the shop G and the manager should sell more jeans of type j_1.

Conclusion

The successful discussion of rough neutrosophic multisets relation with application in marketing strategy is obtained in this paper. Firstly, this paper is defined the rough neutrosophic multisets relation with their properties and operations such as max, min, the composition of two rough neutrosophic multisets, inverse rough neutrosophic multisets, and symmetry, reflexive and transitive of rough neutrosophic multisets. The approximation set boundary of rough neutrosophic multisets was applied for rough neutrosophic multisets relation. This relation theory is useful to apply in marketing strategy problem by getting the real relation of goods sold in the market. Decision matrix analysis is further conducted to get the best result. For further work, the relation of two universe sets can be derived as a rough neutrosophic multisets relation of two universe sets.

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