Multi-objective Mathematical Model for Asset Portfolio Selection using Neutrosophic Goal Programming Technique

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Abstract: In this paper we have considered a multi-objective asset portfolio selection optimization model with the objectives maximization of the expected return of the portfolio and simultaneously minimizing the overall risk of the asset portfolio. Our model is an improved and enlarged version in a particular direction. In our model we had incorporated transaction cost in the first objective. We had considered absolute deviation as risk measure. Our portfolio optimization model had been solved by generalized neutrosophic goal programming method.

For applicability of this technique and demonstration of the methodology we have illustrated it numerically by data taken from National Stock Exchange (NSE). And finally the result obtained using generalized neutrosophic goal programming approach is compared with that of the result obtained different method of aggregation for objective functions.

Keywords: Portfolio; Generalized Neutrosophic Goal Programming, Arithmetic Aggregation, Geometric Aggregation.

1. Introduction

Portfolio management is one of the most important aspects of economic management. Essentially, portfolio management is the process of building a portfolio with the goal of satisfying an investor's risk and return expectations. The primary goal of portfolio management is to select a proper combination of assets in order to provide the best predicted return while maintaining a suitable level of risk.

An investor's goal in portfolio optimization is to maximise portfolio return while maintaining a reasonable level of risk at the same time. Because risk will repay the return, investors will need to manage the risk-return trade-off for their investments. As a result, a single optimization portfolio is ineffective. As a result, when determining the best portfolio, one must consider the investor's risk-reward preferences.

The Mean-Variance (MV) model, established by Markowitz[1] in 1952, is considered the first model in the field of portfolio management. Markowitz trade-off between expected return and portfolio risk in the basic mean-variance model of the portfolio framework, where mean is represented by the average mean of the past performances, i.e. the mean of asset's return and the dispersion of the return as risk, respectively.

Over the last few years, the pioneer model proposed by Markowitz, mathematical programming approaches have grown to be vital tools to guide financial decision-making systems and have been widely deployed in real-world scenarios. There are numbers of well-known mathematical tools that are used to find the best solution in portfolio optimization. Forecasting, simulation, statistical models, and mathematical programming models are some examples. Among these approaches, mathematical programming is a good option for a decision maker looking for the best solution.

According to the existing literatures, a mathematical model for portfolio addressing transaction cost generally seeks to generate a changed portfolio from cash, i.e., preferring to pass from a present portfolio to a new one. The majority of the models add at least one more binary variable to the portfolio, as well as new constraints will be added. As a result, the majority of these transaction pricing components will add complexity to the problems. Let us now have a look at the available literature of the transaction cost. Angelelli et al. [2] used a mixed integer linear programming model that included transaction cost and cardinality constraints with CVaR and MAD model. In the generalised MV Markowitz model, Chen and Cai [3] added transaction cost. According to the assumptions, transaction costs are a V-shaped function that is known at the beginning of the period and paid at the conclusion. In the transaction cost model, Baule [4] took transaction cost into account as a non-convex function. In the mixed quadratic portfolio optimization model technique, Adcock and Meade [5] included a weighting factor to account for variable transaction costs. There are also a few additional journals, as well as the concept of transaction price in portfolio optimization.

Integer programming technique [6], goal programming technique [7], lexiographic goal programming technique [8], and other precise method based techniques were used to solve portfolio optimization models. Simulated annealing [9], genetic algorithm [10], particle swarm optimization [11], and ant colony optimization [12] are some of the meta-heuristics-based techniques used. However, in practice, if you want to make good portfolio decisions, you'll need to use a few vaguely defined financial characteristics like the return is greater than 20%, the risk is less than 10%, and so on. It's difficult to put together satisfying portfolios using crisp or interval numbers when the language is so hazy. In such a situation, the decision maker must enlist the help of fuzzy set theory in order to build portfolio selection models. Fuzzy set theory not only manages uncertainty and ambiguity, but it also helps decision makers make flexible choices by considering the choices of investors.

Financial risks are the component of the uncertainty that pertains to asset returns as a result of unforeseeable and unpredictable events. Risks cannot be quantified in portfolio selection or asset assessment for a variety of reasons, including a lack or plenty of information, subjective estimation and perception, insufficient knowledge, the complexity of the researched systems, and so on. In these instances, language judgements rather than numerical values are a more realistic approach.But there is a lot of uncertainty and ambiguity related with these linguistic expressions, such as, "high", "low", "moderate". So traditional two valued logic of probability is not enough to handle the dual presence of uncertainty and ambiguity. In this scenario, fuzzy set theory proposed by Professor L.A. Zadeh becomes a natural choice since it can define the linguistic information in a more logical and meaningful fashion. It is also quite impossible for decision maker to determine or estimate the movement in financial markets. So the decision maker faces the dilemma of guessing the market direction in order to meet the return target for asset under management. Under these circumstances, an uncertainty may be included in their estimation. Because of some uncertainty and ambiguity present in the Asset Liability Management and portfolio optimization, concept of fuzzy set theory is used in this area. Watada [13] had used fuzzy computational intelligence in portfolio selection problem. Yager [14] contributed in taking decisions on uncertain issue like portfolio selection using fuzzy mathematics. In [15] the authors described the selection of fuzzy portfolio using the concept like expected value of fuzzy numbers and ranking .

Bellmann and Zadeh [16] proposed the concept of fuzzy decision theory, which was based on Zadeh's 1965 [17] presentation of fuzzy sets. Several writers had also used the fuzzy framework to select the most efficient portfolio using the mean-variance model.

This is also a tough procedure due to elements like insufficient information that is frequently offered in real-life decision-making scenarios. Our major goal in this decision-making process is to identify a value from the chosen set that has the maximum degree of membership in the decision set and that agrees with the goals only under certain constraints. However, there may be many times when some of the selected values from the set are incompatible with the aim, i.e., those values are strongly opposed to the purpose due to limitations that cannot be accepted. Such values may be found in this case from the selected set with the lowest degree of non-membership in the choice set. In such instances, intuitionistic fuzzy can help the decision maker deal with partial data, but it is unable to deal with indeterminate and inconsistent data, which are also common in the systems. Atanassov [18],[19] developed the concept of intuitionistic fuzzy sets. Truth membership, falsity membership, and indeterminacy membership are all independent in the neutrosophic set presented by Smarandache [20], and indeterminacy can be quantified directly. As a result, it is evident that the value in the decision set from the chosen set with the highest degree of truth membership, falsity membership, and indeterminacy membership should be considered. As a result, we have chosen a neutrosophic environment to deal with asset liability management decisions for commercial banks. Different authors have used the concept of neutrosophic optimization in a variety of fields. This approach was used to the reliability problem by Sahidul Islam and Tanmay Kundu [21], to the multi-objective welded beam optimization by M. Sarkar and T.K. Roy [22], to the riser design problem by Pintu Das and T.K. Roy [23], and to optimization problems in a variety of other domains. S.Islam and Partha Ray [24] created a multi-objective portfolio selection model with entropy using the Neutrosophic optimization technique for portfolio selection.

With the above observation in mind, we will attempt to propose a multi-objective portfolio optimization model in this paper. In a specific direction, our model is a better and larger version. One of the objectives of our approach was to include transaction costs. We used absolute deviation as a risk indicator. The generalised neutrosophic goal programming method which is just a generalisation of Neutrosophic Goal programming method proposed by M.Abdel-Baset, I.M.Heza, and F.Smarandache [28] was used to solve our portfolio optimization model. The portfolio optimization model was validated in this research using data from the National Stock Exchange (NSE).

2. Mathematical Model:

In this section we will discuss about proposed optimization model for selection of portfolio. The notations used for this model are listed below:

n : the number of assets which are available for investment.

x_i :the proportion of the total fund invested in i-th asset, for $i = 1,2, \dots \dots \dots .., n$.

x_i^0 : the proportion of the total funds had been invested in i-th asset, for $i = 1,2, \dots \dots \dots .., n$.

R_i : the rate of return of i-th asset which is basically a random variable for $i = 1,2, \dots \dots \dots \dots \dots .., n$.

r_i : the expected rate of return on the i-th asset, for $i = 1,2, \dots \dots \dots .., n$. $r_i = E[R_i]$

r_{n+1} : the rate of return for the risk free asset.

λ_i : the rate of transaction cost on i-th asset , for $i = 1,2, \dots \dots \dots \dots \dots .., n$.

L_i : The lower limit of the fund that can be invested on the i-th asset for $i = 1,2, \dots \dots \dots \dots \dots .., n$.

U_i : The upper limit of the fund that can be invested on the i-th asset for $i = 1,2, \dots \dots \dots \dots \dots .., n$.

In this model we had considered absolute deviation as risk measure. Before introducing the mathematical model let us give some introduction to this measure of risk.

2.1 Absolute deviation

The main aim of every investor in portfolio selection is to get portfolio return $r(x_1, x_2, \dots \dots \dots, x_n)$ as high as possible. Also an investor would also prefer to have minimum variation or dispersion in the portfolio return. Variance is the most common measure to quantify risk of portfolio, which measures the variation from the expected return. Despite its shortcomings, researchers continue to choose variance as a prominent risk metric. The biggest disadvantage of utilizing variance as a risk indicator is that it penalizes extreme upside and downside deviations from the expected return. As a result, the variance will be a less appropriate measure of portfolio risk in the case of an asymmetric probability distribution of asset return. This is due to the fact that, in exchange for a larger predicted return, the obtained portfolio may provide a risk. As a result, a downside risk metric may be preferable to variance. Only negative deviations from a reference return level are included in this risk assessment. Another downside risk metric, known as semi variance, was established by Markowitz.

Both the above mentioned risk measure have some advantages and simultaneously have some limitations. In order to improve both the theoretical and computational performance of the mean-variance model or mean-Semi variance model Konno and Yamazaki [27] had considered an alternative risk measure namely absolute deviation to quantify risk and introduced a linear programming portfolio selection model. So far the formulation of the risk function was based on the notion of L_2 metric, we had discussed these earlier. The risk function namely absolute deviation is defined based on the notion of L_1 metric on $\mathbb{R}^n$. Normally this risk measure is applicable to the problems having a-symmetric distributions of the rate of return. L_1 risk function draw much attention of the researcher since a portfolio selection model with L_1 risk function can easily be converted into a scalar parametric linear programming problem. Another benefit of using absolute deviation in a portfolio optimization model is computational ease and simplicity even for large number of assets also.

The expected absolute for the difference between the random variables and its mean is known as absolute deviation of a random variable. This measure of portfolio risk is denoted by $m(x_1, x_2, \dots\dots\dots, x_n)$ and is expressed as:

$$m(x_1, x_2, \dots\dots\dots, x_n) = E[|\sum_{i=1}^{n} R_i x_i - E[\sum_{i=1}^{n} R_i x_i]|].$$

Since we shall approximate expected value of the random variable by the average derived from the past data, so we shall use $r_i = E[R_i] = \frac{\sum_{t=1}^{T} r_{it}}{T}$, the absolute deviation is approximated as

$$m(x_1, x_2, \dots\dots\dots, x_n) = E[|\sum_{i=1}^{n} R_i x_i - E[\sum_{i=1}^{n} R_i x_i]|] = \frac{1}{T}\sum_{t=1}^{T}|\sum_{i=1}^{n}(r_{it} - r_i)x_i|.$$

2.2 The proposed Mathematical model:

(P 1.1)

$$Maximize\ Z_1 = \sum_{i=1}^{n+1}\left(r_i x_i - \lambda_i\left|x_i - x_i^0\right|\right)$$

$$Minimize\ Z_2 = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{i=1}^{n}(r_{it} - r_i)x_i\right|$$

$subject\ to:$

$$\sum_{i=1}^{n} x_i = 1,$$

$$x_i \geq 0\,,$$

$$L_i \leq x_i \leq U_i$$

$$i = 1,2, \dots\dots\dots\dots\dots\dots\dots\dots., n$$

Because of the existence of the absolute value function the above mathematical model is non-linear and non-smooth. For elimination the absolute value function the above mathematical model had been transformed into the following form

(P 1.2)

$$Maximize\ \ Er = \sum_{i=1}^{n+1}(r_i x_i - \lambda_i q_i)$$

$$Minimize\ \ Ad = \frac{1}{T}\sum_{t=1}^{T} p_t$$

$subject\ to:$

$$\sum_{i=1}^{n} x_i = 1,$$

$$q_i \geq (x_i - x_i^0)$$
$$q_i \geq -(x_i - x_i^0)$$
$$p_t \geq \sum_{i=1}^{n} (r_{it} - r_i)x_i$$

$$p_t \geq -\sum_{i=1}^{n} (r_{it} - r_i)x_i$$

$$x_i \geq 0\,,$$
$$p_t \geq 0$$
$$q_i \geq 0$$
$$L_i \leq x_i \leq U_i$$
$$i = 1,2, \dots \dots \dots \dots \dots \dots \dots ., n$$

2.3 Descriptions of the Objectives and the Constraints

The first objective is maximization of expected return of the portfolio, which is difference between the rate of expected return of the portfolio and the transaction cost of the portfolio. In the first objective $\sum_{i=1}^{n+1}\left(r_i x_i - \lambda_i\left|x_i - x_i^0\right|\right)$, $\sum_{i=1}^{n+1} r_i x_i$ is the rate of expected return , and $\sum_{i=1}^{n+1} \lambda_i\left|x_i - x_i^0\right|$ is the transaction cost of the portfolio. And the second objective is minimization of absolute deviation. $\sum_{i=1}^{n} x_i = 1$ is the capital budget constraint. $L_i \leq x_i \leq U_i, i = 1,2, \dots \dots, n$ is the maximal and minimal fraction of the total capital to be invested in each asset.

3. Mathematical Analysis

In this section we will discuss about some preliminary concepts of the neutrosophic set and then the Neutrosophic goal programming technique which will be used in this paper to deal with the portfolio selection model.

3.1 Some definitions

Fuzzy Sets

Let $\tilde{B}$ is a fuzzy set and X be considered as universe of discourse. Then fuzzy set $\tilde{B}$-can be defined as follow-$\tilde{B} = \{< x, \mu_{\tilde{B}}(x) >: x \in X\}$; where $\mu_{\tilde{B}}(x)$ is a mapping from X to [0, 1], which is the membership function of the corresponding fuzzy set$\tilde{B}$.

Intuitionistic Fuzzy Sets

An intuitionistic fuzzy sets (IFS) $\tilde{B}^i$ in the universe of discourse X is defined by $\tilde{B}^i = \{\langle x, \mu_{\tilde{B}^i}(x), \nu_{\tilde{B}^i}(x)\rangle | x \in X\}$

Where, $\mu_{\tilde{B}^i}(x): X \to [0,1]$ is the degree of membership of $x \in X$ and $\nu_{\tilde{B}^i}(x): X \to [0,1]$ is the degree of non-membership of $x \in X$. Also for every-$x \in X$, $0 \leq \mu_{\tilde{B}^i}(x) + \nu_{\tilde{B}^i}(x) \leq 1$.

Now for each element-$x \in X$, the value of $\Pi_{\tilde{B}^i}(x) = 1 - \mu_{\tilde{B}^i}(x) - \nu_{\tilde{B}^i}(x)$ is said to be the degree of uncertainty of the element $x \in X$ to the IFS $\tilde{B}^i$.

Neutrosophic Sets

Let X be the universe of discourse and x be a generic element of this set. A neutrosophic set (NS) denoted by $\tilde{B}^N$ in X is characterized by a truth membership function $\mu_B(x)$, a falsity membership function $\nu_B(x)$and an indeterminacy membership function $\sigma_B(x)$ and having the form

$\tilde{B}^N = \{\langle x, \mu_B(x), \nu_B(x), \sigma_B(x)\rangle | x \in X\}$

Where,

$\mu_B(x): X \to]0^-, 1^+[$

$\nu_B(x): X \to]0^-, 1^+[$

$\sigma_B(x): X \to]0^-, 1^+[$

i.e. $\mu_B(x), \nu_B(x), \sigma_B(x)$ are real standard or non standard subsets of $]0^-, 1^+[$.

Also $0^- \leq \text{Sup}\ \mu_B(x) + Sup\ \nu_B(x) + Sup\ \sigma_B(x) \leq 3^+$.

The NS takes the value from the real standard or non-standard subsets of $]0^-, 1^+[$ from the philosophical point of view, but in application of real life in engineering and scientific problems it is difficult to use NS with value from the subsets of$]0^-, 1^+[$.

3.2 Neutrosophic Goal Programming

Let us consider a goal programming problem as

To find $X = (x_1, x_2, \dots \dots, x_{n-1}, x_n)^T$

to achieve :

$f_i = t_i, i = 1,2, \dots \dots, k$

Under the conditions, $x \in X$

where X is a feasible set of all the constraints,t_i are scalars representing level of achievement for the objective functions, which the decision maker want to attain in the feasible set.

More generally a non-linear goal programming problem can be expressed as

(P 1.3)

To find $X = (x_1, x_2, \dots \dots, x_{n-1}, x_n)^T$

In order to $Minimize\ f_i$,having the target value t_i , acceptance tolerance a_i, rejection tolerance c_i, and indeterminacy tolerance d_i

$g_j(x) \leq b_j,\ j = 1,2, \dots \dots \dots, m$

$x_i \geq 0, i = 1,2, \dots \dots, n$

The truth-membership functions, falsity-membership functions and indeterminacy-membership-functions as given by Mohamed Abdel-Baset et all [28] are respectively

$$T_i(f_i) = \begin{cases} 1 & if\ f_i \leq t_i \\ \left(\frac{t_i + a_i - f_i}{a_i}\right) & if\ t_i \leq f_i \leq t_i + a_i \\ 0 & if\ \ f_i \geq t_i + a_i \end{cases}$$

$$F_i(f_i) = \begin{cases} 0 & if\ f_i \leq t_i \\ \left(\frac{f_i - t_i}{c_i}\right) & if\ t_i \leq f_i \leq t_i + c_i \\ 1 & if\ \ f_i \geq t_i + c_i \end{cases}$$

$$I_i(f_i) = \begin{cases} 0 & if\ f_i \leq t_i \\ \left(\frac{f_i - t_i}{d_i}\right) & if\ t_i \leq f_i \leq t_i + a_i \\ \left(\frac{t_i + a_i - f_i}{a_i - d_i}\right) & if\ t_i + d_i \leq f_i \leq t_i + a_i \\ 0 & if\ \ f_i \geq t_i + a_i \end{cases}$$

Now the formulation to minimize the degree of rejection and maximize the degree of acceptance as well as the degree of the indeterminacy of objectives and constraints for a given nonlinear goal programming is as follow:

(P 1.4)

$Maximize\ T_{f_i}(f_i)\ , i = 1,2, \dots \dots , k$

$Maximize\ I_{f_i}(f_i)\ , i = 1,2, \dots \dots , k$

$Minimize\ F_{f_i}(f_i)\ , i = 1,2, \dots \dots , k$

Subject to

$0 \leq T_{f_i}(f_i) + I_{f_i}(f_i) + F_{f_i}(f_i) \leq 3, i = 1,2, \dots \dots , k$

$T_{f_i}(f_i) \geq 0, I_{f_i}(f_i) \geq 0, F_{f_i}(f_i) \geq 0\ , i = 1,2, \dots \dots , k$

$T_{f_i}(f_i) \geq I_{f_i}(f_i), i = 1,2, \dots \dots , k$

$T_{f_i}(f_i) \geq F_{f_i}(f_i), i = 1,2, \dots \dots , k$

$g_j(x) \leq b_j,\ j = 1,2, \dots \dots \dots , m$

$x_i \geq 0, i = 1,2, \dots \dots , n$

Here the truth-membership function, falsity-membership function and indeterminacy-membership function of the corresponding neutrosophic decision set are respectively $T_{f_i}(f_i), F_{f_i}(f_i)$ and $I_{f_i}(f_i)$.

Now using the truth-membership function, falsity-membership function and indeterminacy-membership function in generating the corresponding crisp programming model of P(1.4) which is non-linear goal programming problem be expressed as follow

(P 1.5)

$Maximize\ A$

$Maximize\ C$

$Minimize\ B$

$T_{f_i}(f_i) \geq A,\ i = 1,2, \dots \dots , k$

$I_{f_i}(f_i) \geq C,\ i = 1,2, \dots \dots , k$

$F_{f_i}(f_i) \leq B, i = 1,2, \dots \dots , k$

$f_i \leq t_i, i = 1,2, \dots \dots , k$

$0 \leq A + B + C \leq 3;$

$A \geq 0, C \geq 0, B \leq 1;$

$g_j(x) \leq b_j,\ j = 1,2, \dots \dots \dots , m$

$x_i \geq 0, i = 1,2, \dots \dots , n$

3.3 Generalized Neutrosophic Goal Programming

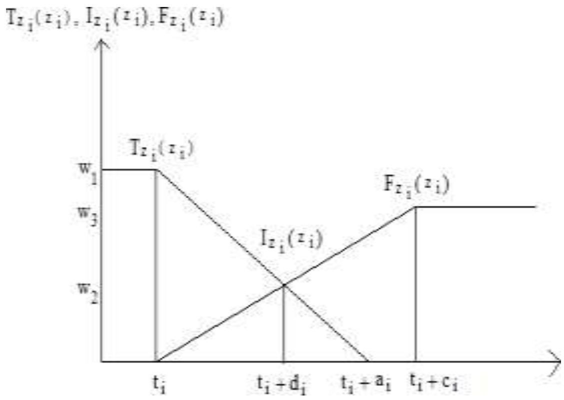
In the case of generalized neutrosophic goal programming, the truth-membership functions, falsity-membership functions and the indeterminacy-membership-functions as defined by Mridula Sarkar et all [29] are defined respectively as

$$T_i^{w_1}(f_i) = \begin{cases} w_1 & if\ f_i \leq t_i \\ w_1\ \left(\frac{t_i + a_i - f_i}{a_i}\right) & if\ t_i \leq f_i \leq t_i + a_i \\ 0 & if\ \ f_i \geq t_i + a_i \end{cases}$$

$$F_i^{w_2}(f_i) = \begin{cases} 0 & if\ f_i \leq t_i \\ w_2\ \left(\frac{f_i - t_i}{c_i}\right) & if\ t_i \leq f_i \leq t_i + c_i \\ w_2 & if\ \ f_i \geq t_i + c_i \end{cases}$$

$$I_i^{w_3}(f_i)=\begin{cases}0 & if\ f_i\le t_i\\ w_3\left(\frac{f_i-t_i}{d_i}\right) & if\ t_i\le f_i\le t_i+a_i\\ w_3\left(\frac{t_i+a_i-f_i}{a_i-d_i}\right) & if\ t_i+d_i\le f_i\le t_i+a_i\\ 0 & if\ \ f_i\ge t_i+a_i\end{cases}$$

where w_1, w_2, w_3 are degree of gradations of the truth-membership functions, falsity-membership functions and the indeterminacy-membership-functions respectively. Also the target value is t_i , acceptance tolerance is a_i, rejection tolerance c_i, and indeterminacy tolerance is d_i



The general formulation of Neutrosophic goal programming is as follow:

(P 1.6)

$Maximize\ T_{f_i}(f_i)\ , i=1,2,\dots\dots,k$

$Maximize\ I_{f_i}(f_i)\ , i=1,2,\dots\dots,k$

$Minimize\ F_{f_i}(f_i)\ , i=1,2,\dots\dots,k$

Subject to

$0\le T_{f_i}(f_i)+I_{f_i}(f_i)+F_{f_i}(f_i)\le w_1+w_2+w_3, i=1,2,\dots\dots,k$

$T_{f_i}(f_i)\ge 0, I_{f_i}(f_i)\ge 0, F_{f_i}(f_i)\ge 0\ , i=1,2,\dots\dots,k$

$T_{f_i}(f_i)\ge I_{f_i}(f_i), i=1,2,\dots\dots,k$

$T_{f_i}(f_i)\ge F_{f_i}(f_i), i=1,2,\dots\dots,k$

$0\le w_1+w_2+w_3\le 3$

$w_1, w_2, w_3\in[0,1]$

$g_j(x)\le b_j,\ j=1,2,\dots\dots\dots,m$

$x_i\ge 0, i=1,2,\dots\dots,n$

The above problem is equivalent to

(P 1.7)

$Maximize\ A$

$Minimize\ B$

$Maximize\ C$

$T_{f_i}(f_i)\ge A,\ i=1,2,\dots\dots,k$

$I_{f_i}(f_i)\ge C,\ i=1,2,\dots\dots,k$

$F_{f_i}(f_i) \leq B, i = 1,2, \dots \dots, k$

$f_i \leq t_i, i = 1,2, \dots \dots, k$

$0 \leq A + B + C \leq w_1 + w_2 + w_3$;

$A \in [0, w_1],\ B \in [0, w_2]\ , C \in [0, w_3]$;

$0 \leq w_1 + w_2 + w_3 \leq 3$

$w_1, w_2, w_3 \in [0,1]$

$g_j(x) \leq b_j,\ j = 1,2, \dots \dots \dots, m$

$x_i \geq 0, i = 1,2, \dots \dots, n$

Again using the corresponding membership function, finally this problem is equivalent to

(P 1.8)

$Maximize\ A$

$Minimize\ B$

$Maximize\ C$

$f_i \leq t_i + a_i \left(1 - \frac{A}{w_1}\right),\ i = 1,2, \dots \dots, k$

$f_i \leq t_i + \frac{c_i}{w_2} B,\ i = 1,2, \dots \dots, k$

$f_i \geq t_i + \frac{d_i}{w_3} C,\ i = 1,2, \dots \dots, k$

$f_i \leq t_i + a_i - \frac{1}{w_3}(a_i - d_i)C, i = 1,2, \dots \dots, k$

$f_i \leq t_i, i = 1,2, \dots \dots, k$

$0 \leq A + B + C \leq w_1 + w_2 + w_3$;

$A \in [0, w_1],\ B \in [0, w_2]\ , C \in [0, w_3]$;

$0 \leq w_1 + w_2 + w_3 \leq 3$

$w_1, w_2, w_3 \in [0,1]$

$g_j(x) \leq b_j,\ j = 1,2, \dots \dots \dots, m$

$x_i \geq 0, i = 1,2, \dots \dots, n$

Now using generalized truth, falsity and indeterminacy membership function and under the consideration of arithmetic aggregation operator the generalized neutrosophic goal programming can be formulated as

(P 1.9)

$Minimize\ \left\{\frac{(1-A)+B+(1-C)}{3}\right\}$

Under the same set of constraints as of (P 1.8)

Also using geometric aggregation operator same generalized neutrosophic goal programming can be formulated as :

(P 1.10)

$Minimize\ \sqrt[3]{(1-A)B(1-C)}$

Under the same set of constraints as of (P 1.8)

Finally to get the solution of multi-objective non-linear programming problem by generalized neutrosophic goal programming approach, we can take help of some appropriate mathematical programming to solve the non linear programming problem (P 1.8 or P 1.9 or P 1.10).

4. Solution of Multi-Objective Portfolio Optimization Model by Generalized Neutrosophic Goal Programming

Multi-objective neutrosophic portfolio optimization model can be expressed as

$Maximize\ Er(X)$, with target value E_0 , acceptance tolerance a_E, indeterminacy tolerance d_E , and rejection tolerance c_E.

$Minimize\ Ad(X)$, with target value A_0 , acceptance tolerance a_A, indeterminacy tolerance d_A , and rejection tolerance c_A.

$subject\ to$:

$\sum_{i=1}^{n} x_i = 1,$

$x_i \geq 0$,

$L_i \leq x_i \leq U_i$

$i = 1,2, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots, n$

Where $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix}$ are the decision variables.

In case of generalized neutrosophic goal programming the truth-membership functions, falsity-membership functions and indeterminacy-membership-functions for the objective functions are defined respectively as

$$T_{Er(X)}^{w_1}(Er(X)) = \begin{cases} w_1 & if\ Er(X) \geq E_0 \\ w_1\ \left(\frac{Er(X)-E_0+a_E}{a_E}\right) & if\ E_0 - a_E \leq Er(X) \leq E_0 \\ 0 & if\ \ Er(X) \leq E_0 - a_E \end{cases}$$

$$F_{Er(X)}^{w_2}(Er(X)) = \begin{cases} 0 & if\ Er(X) \geq E_0 \\ w_2\ \left(\frac{E_0-Er(X)}{c_E}\right) & if\ E_0 - c_E \leq Er(X) \leq E_0 \\ w_2 & if\ \ Er(X) \leq E_0 - c_E \end{cases}$$

$$I_{Er(X)}^{w_3}(Er(X)) = \begin{cases} 0 & if\ Er(X) \leq E_0 - a_E \\ w_3\ \left(\frac{a_E+Er(X)-E_0}{a_E-d_E}\right) & if\ E_0 - a_E \leq Er(X) \leq E_0 - d_E \\ w_3\ \left(\frac{-Er(X)+E_0}{d_E}\right) & if\ E_0 - d_E \leq Er(X) \leq E_0 \\ 0 & if\ \ Er(X) \geq E_0 \end{cases}$$

Where $d_E = \frac{w_1}{\frac{w_1}{a_E}+\frac{w_2}{c_E}}$

And

$$T_{Ad(X)}^{w_1}(Ad(X)) = \begin{cases} w_1 & if\ Ad(X) \leq A_0 \\ w_1\ \left(\frac{A_0+a_A-Ad(X)}{a_A}\right) & if\ A_0 \leq Ad(X) \leq A_0 + a_A \\ 0 & if\ \ Ad(X) \geq A_0 + a_A \end{cases}$$

$$F_{Ad(X)}^{w_2}(Ad(X)) = \begin{cases} 0 & if\ Ad(X) \le A_0 \\ w_2\left(\frac{Ad(X)-A_0}{c_A}\right) & if\ A_0 \le Ad(X) \le A_0 + c_A \\ w_2 & if\ \ Ad(X) \ge A_0 + c_A \end{cases}$$

$$I_{Ad(X)}^{w_3}(Ad(X)) = \begin{cases} 0 & if\ Ad(X) \le A_0 \\ w_3\left(\frac{Ad(X)-A_0}{d_A}\right) & if\ A_0 \le Ad(X) \le A_0 + a_A \\ w_3\left(\frac{A_0+a_A-Ad(X)}{a_A-d_A}\right) & if\ A_0 + d_A \le Ad(X) \le A_0 + a_A \\ 0 & if\ \ Ad(X) \ge A_0 + a_A \end{cases}$$

Where $d_A = \frac{w_1}{\frac{w_1}{a_A}+\frac{w_2}{c_A}}$

Now using generalized neutrosophic goal programming technique and incorporating truth, falsity and indeterminacy membership functions the problem (P 1.2) can be formulated as the following (P 1.11)

(P 1.11)

$Maximize\ A$

$Minimize\ B$

$Maximize\ C$

$Er(X) \ge E_0 + a_E\left(\frac{A}{w_1} - 1\right),$

$Er(X) \ge E_0 - \frac{c_E}{w_2}B,$

$Er(X) \le E_0 - \frac{d_E}{w_3}C,$

$Er(X) \ge E_0 - a_E + \frac{C}{w_3}(a_E - d_E),$

$Er(X) \ge E_0,$

$\mathrm{Ad}(X) \le A_0 + a_A\left(1 - \frac{A}{w_1}\right),$

$\mathrm{Ad}(X) \le A_0 + \frac{c_A}{w_2}B,$

$\mathrm{Ad}(X) \ge A_0 + \frac{d_A}{w_3}C,$

$Ad(X) \le A_0 + a_A - \frac{C}{w_3}(a_A - d_A),$

$Ad(X) \le A_0,$

$0 \le A + B + C \le w_1 + w_2 + w_3\ ;$

$A \in [0, w_1],\ B \in [0, w_2]\ , C \in [0, w_3];$

$0 \le w_1 + w_2 + w_3 \le 3$

$w_1, w_2, w_3 \in [0,1]$

$g_j(x) \le b_j,\ j = 1,2, \dots \dots \dots, m$

$x_i \ge 0, i = 1,2, \dots \dots, n$

$\sum_{i=1}^{n} x_i = 1,$

$q_i \geq (x_i - x_i^0)$

$q_i \geq -(x_i - x_i^0)$

$p_t \geq \sum_{i=1}^{n}(r_{it} - r_i)x_i$

$p_t \geq -\sum_{i=1}^{n}(r_{it} - r_i)x_i$

$x_i \geq 0$,

$p_t \geq 0$

$q_i \geq 0$

$L_i \leq x_i \leq U_i$

$i = 1,2, \dots \dots \dots \dots \dots \dots \dots ., n$

5. Numerical Illustration

Our portfolio optimization model had been solved by generalized neutrosophic goal programming method. In this paper the portfolio optimization model had been validated by data taken from National Stock Exchange (NSE). For demonstration a data set of 10 randomly selected assets had been considered from NSE for an entire financial year i.e. 12 months, here each rows are data of any companies like ABL, ALL, etc for the entire financial year and columns are data for 1st month, 2nd month, etc of the financial year. The data is given below

Table1: Return of assets of some companies taken from National stock exchange

Company	1	2	3	4	5	6	7	8	9	10	11	12
ABL	0.072	0.32032	0.2971	0.236	-0.05161	0.50633	-0.02516	0.90484	0.03214	0.45968	0.227	-0.87871
ALL	-0.14433	0.19032	0.75032	0.03433	-0.33581	0.247	0.49968	0.27032	-0.32786	0.31968	0.11933	-0.50903
BHL	0.08667	1.05613	0.05516	0.27567	-0.21839	0.49233	1.11516	0.57613	0.17143	0.92258	0.22367	-0.67903
CGL	-0.18567	0.76774	0.16194	0.48633	-0.2071	0.47833	0.2571	0.59484	-0.02321	0.55387	0.07333	-0.11871
HHM	0.18233	0.33	0.13677	0.46533	-0.12774	0.56067	0.10839	0	0.14321	0.00968	-0.15767	-0.27258
HCC	-0.157	0.61226	1.23548	0.56067	-0.71065	0.97333	0.32839	0.61581	0.03286	0.49935	-0.03733	-0.59452
KMB	0.18567	0.27806	0.55097	0.02733	-0.46613	0.73333	0.20581	0.17065	-0.05286	0.6671	0.373	-0.08355
MML	0.37533	0.65903	0.1929	0.16533	-0.15226	0.80867	0.39097	0.29	0.1975	0.21839	0.031	-0.06548
SIL	-0.10467	0.200552	0.31161	0.43333	-0.3171	1.104	0.37194	0.73097	0.03321	0.75903	0.09467	-0.44903
UNL	0.26367	0.41581	0.24484	0.12967	-0.0829	0.54	0.93258	0.61871	0.2275	0.68968	0.65433	0.65258

Using this data set the problem reduces to $Maximize\ Er(X)$ with target value 0.28745, truth tolerance 0.1295, and indeterminacy tolerance $\frac{w_1}{7.72\, w_1 + 20\, w_2}$ and rejection tolerance 0.05.

and $Minimize\ \ Ad(X)$with target value 0.0877, truth tolerance 0.08, and indeterminacy tolerance $\frac{w_1}{12.5\, w_1 + 6.67\, w_2}$ and rejection tolerance 0.15.

Solving the portfolio optimization model by the above mentioned methods using LINGO the solutions so obtained is given below in tabular form.

Table2: Optimal solutions using different methods

	$Z_1(x)$	$Z_2(x)$
Generalized neutrosophic goal programming $w_1 = 0.3, w_2 = 0.5, w_3 = 0.7$	0.3159	0.0784
Generalized neutrosophic optimization based on arithmetic aggregation operator $w_1 = 0.3, w_2 = 0.5, w_3 = 0.7$	0.3255	0.0781
Generalized neutrosophic optimization based on geometric aggregation operator $w_1 = 0.3, w_2 = 0.5, w_3 = 0.7$	0.3491	0.0698

For different value of w_1, w_2, w_3 using different method of aggregation for objective functions the solutions so obtained are almost same. Although the best solutions have been obtained using geometric aggregation method for objective functions for different value of w_1, w_2, w_3.

It is clear from the above table that in neutrosophic goal programming method based upon distinct aggregation operator, all the objective functions attained their respective goal and also the restrictions of truth, falsity and indeterminacy membership functions. The sum of truth, falsity and indeterminacy membership function of each of the objective is less than sum of degree of gradiation $w_1 + w_2 + w_3$, which in turn satisfies the condition of neutrosophic set.

6. Conclusion

It was explored in this study that, when the neutrosophic goal programming considered as a method for determining the best portfolio the the best result obtained utilizing different aggregation methods for the mathematical model of this study was obtained by employing geometric aggregation method. The degree of truth membership function is defined using the neutrosophic optimization technique; however, it is not simply a complement of degree of falsehood; rather, these two degrees of membership are independent of degree of indeterminacy. Because we used the neutrosophic goal programming technique to optimize portfolios, it may also be applied to solve other optimization problems of several fields.

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