Optimization of EOQ Model with Limited Storage Capacity by Neutrosophic Geometric Programming

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Abstract: In this article, we present deterministic single objective economic order quantity model with limited storage capacity in neutrosophic environment. We consider variable limit production cost and time dependent holding cost into account. Here we minimize total average cost of proposed model by applying neutrosophic geometric programming, which is obtained by extending existing fuzzy and intuitionistic fuzzy geometric programming for solving resultant non-linear optimization model. Next we consider numerical application to show that optimal solution obtained by neutrosophic geometric programming is more desirable than that of crisp, fuzzy and intuitionistic fuzzy geometric programming. Also we perform sensitivity analysis of parameters and present key managerial insights. Finally we draw the conclusions.

Keywords: Economic Order Quantity, Neutrosophic geometric programming, Non-linear optimization, Limited storage capacity, Shape parameter.

1 Introduction

We define inventory as an idle resource of any enterprise. Although idle, a certain amount of inventory is essential for smooth conduction of organisational activities. We find control of inventory as one of the key areas for operational management. We observe that an adequate control of inventory significantly brings down operating cost and increases efficiency [1, 2]. So we determine Economic Order Quantity (EOQ) to minimize total cost of inventory e.g., holding cost, order cost, and shortage cost. In most cases, optimization of corrosponding mathematical model requires Non-Linear Programming(NLP). And one of the most popular and constructive method for solving NLP problem is Geometric Programming (GP). It is convenient in applications of variety of optimization models and is under general class of signomial problems. We employ it to solve large scale, real life based models by quantifying them into an equivalent optimization problem. Also GP allows sensitivity analysis to be performed efficiently.

Historically, F. Harries [3, 4] first presented concept of EOQ and subsequently Wilson applied it. Hoon Jung et al. [5,6] discussed optimal inventory policies for maximizing profit of EOQ models under various cost functions. N. K. Mondal et al. [7] considered a model with deteriorating items. S. Islam [8] formulated multi-objective inventory model with capacity constraint and shortage cost. S. Sadjadi et al. [2] considered a model with cubic demand function. L. Janseen et al. [9] presented one extensive literature review of deteriorating inventory models. In recent era, development of EOQ model is primarily based on several constraints, among which budgetary and limited storage capacitys generate considerable attention of researchers.

Again through investigation of cost minimization techniques for engineering and designing problem [10, 11] Zenner introduced notion of GP. Later, Duffin et al. [12,13] presented the mathematical formulations of GP. Kochenberger [14] was the first scientist to solve non-linear EOQ problem by GP. Beightler and Phillips [15] studied advantages of applying GP to real life based problems. Afterwards, Cheng [16, 17] formulated an EOQ model with unit production cost. Lee [18] proposed GP formulations for optimal order quantities and prices with storage capacity limitations. Nezami et al. [19] determined optimal demand rate and production quantity by GP. Sadjadi et al. [1,2] investigated the integrated pricing, lot sizing and marketing planning model and reviewed literature of last two decades. Tabatabaei et al. [20] discussed optimal pricing and marketing planning for deteriorating items. Again total inventory cost of an EOQ model is controlled by constraints in real life based imprecise environment. Among numerous constraints that affect optimal inventory cost, e.g. ceiling on storage capacity, number of orders and production cost, In this article, we consider upper limit on storage capacity to be imprecise in nature. The much needed paradigm shift to bring impreciseness in math-ematics was formaly acted by Zadeh [21]. Next Bellman and Zadeh [22] used fuzzy set in decision making problems. Tanaka et al. [23] proposed objectives as fuzzy goals. Zimmerman [24] presented solution method for multi-objective linear programming problem in fuzzy environment. In subsequent years, mathematicians developed various optimization methods and employed them in different directions. Sommer [25] employed fuzzy concept to inventory and production-scheduling problem. Park [26] examined fuzzy EOQ model. Roy and Maiti [27] solved single objective EOQ model using GP technique in fuzzy environment. Islam and Mon-dal [28] formulated one fuzzy Eeconomic Production Quantity (EPQ) model having flexibility and reliability considerations. Mahapatra et al. [29] considered fuzzy EPQ model and solved by applying parametric GP technique.

On the other hand, fuzzy set theory has been widely developed and recently several modifications have appeared. Atanassov presented Intuitionistic Fuzzy (IF) set theory, where we consider non-membership function along with membership function of imprecise information. Whereas Atanassov and Gargov [30] listed optimization in IF environment as an open problem, Angelov [31] developed optimization technique in IF environment. Pramanik and Roy [32] analyzed vector operational problem using IF goal programming. A transportation model was elucidated by Jana and Roy [33] by using multi-objective IF linear programming. Chakraborty et al. [34] applied IF optimization technique for Pareto optimal solution of manufacturing inventory model with shortages. Garai et al. [35, 36] worked on T-Sets based on optimization technique in air quality strategies and supply chain management respectively. Pramanik and Roy [37–39] applied IF goal programming approach to solve quality control problem and multi objective transportation problem also they investigated bilevel programming in said environment.

Again F. Smarandache [40, 41] introduced Neutrosophic (NS) Set, by combining nature with philosophy. It is the study of neutralities as an extension of dialectics. Interestingly, whereas IF sets can only handle incomplete information but failed in case of indeterminacy, NS set can manipulate both incomplete and imprecise information [40]. We characterize NS set by membership function (or, truth membership degree), hesitancy function (or, indeterminacy membership degree) and non-membership function (or, falsity membership degree). In NS environment, decision maker maximizes degree of membership function, minimizes both degree of indeterminacy and degree of non-membership function. Whereas we find application of NS in different directions of research, in this article, we concentrate on optimization in NS environment. Roy and Das [42] solved multi-criteria production planning problem by NS linear programming approach. Baset et al. [43] presented NS Goal Programming (NSGP) problem. Pramanik et al. [44] presented TOPSIS method for multi-attribute group decision-making under single-valued NS environment Basset et al. [45] used Analytic Hierarchy Process (AHP) in multi-criteria group decision making problems in NS environment. Also they extended AHP-SWOT analysis in NS environment [46]. Sarkar et al. [47] used NS optimization technique in truss design and multi-objective cylindrical skin plate design problem. S. Pramanik [48, 49] discussed multi-objective linear goal programming problem in neutrosophic number environment.

Recently Several researcher has worked on Multi-Criteria Decision Making (MCDM) or Multi-Attribute Decision Making (MADM) problem using neutrosophic environment. Biswas et al. [50] discussed neutrosophic MADM with unknown weight information. Mondal and Pramanik [51] extended Multi-Criteria Group Decision Making (MCGDM) approach for teacher recruitment in higher education in neutrosophic environment. Also, Biswas et al. [52] discussed MADM using entropy based grey relational analysis method under SVNSs environment. Afterwards, Mondal and Pramanik [53] explained neutrosophic decision making model for school choice. Pramanik et al. [54] investigated the contribution of some indian researchers to MADM in neutrosophic environment. Later on, Mondal and Pramanik [55] applied tangent similarity measure to neutrosophic MADM process. Mondal et al. [56] developed MADM process for SVNSs using similarity measures based on hyperbolic sine functions. Mondal et al. [57, 58] used hybrid binary logarithm similarity measure and refined similarity measure based on cotangent function to solve Multi-Attribute Group Decision Making (MAGDM) problem under SVNSs environment. Recently mondal et al. [59] analyzed interval neutrosophic tangent similarity measure based MADM strategy and its application to MADM problems. In recent era, Pramanik et al. [60–62] solved MAGDM problem using NS and IN cross entropy, also they investigate MAGDM problem for logistic center location selection. Recently Biswas et al. [63–69] discussed distance measure based MADM and TOPSIS strategies with interval trapeziodal neutrosophic numbers, also they worked on aggrega-tion of triangular fuzzy neutrosophic set, value and ambiguity index based ranking method of SVTNs, hybrid vector similarity measures and their application to MADM problem respectively.

Although we have performed extensive literature reviews and have found case studies of EOQ models in NS environment, we observe that in most cases, models are optimized through various existing software packages only. In this article, we consider one EOQ model with limited storage capacity. Next we solve it by using NSGP method.

We organize the rest of the article as follows. In Section 2, we present elementary definitions. In Section 3, we construct single objective EOQ model with limited storage capacity. In Section 4, we solve the model in crisp environment by applying classical GP. In Section 5, we present optimal solution of proposed model in fuzzy GP. In Section 6, we present optimal solution of proposed model in IFGP. In Section 7, we consider the model in NS environment and solve it by applying NSGP. Next numerical application in Section 8.1 shows that optimal solution in NS environment is more preferable than crisp, fuzzy and IF environment. Also we perform sensitivity analysis and present key managerial insights. Finally in Section 9, we draw conclusions and discuss future scopes of research.

2 Definitions

2.1 Intuitionistic fuzzy set

Let X be an universal set. An intuitionistic fuzzy set A in X is an object of the form:

$$A = \left\{ (x, \mu_A(x), \nu_A(x)) : x \in X \right\}.$$

Here $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ are membership function and non-membership function of A in X respectively and satisfy the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$.

2.2 Neutrosophic set

Let X be an universal set. A neutrosophic (NS) set $A \in X$ is defined by:

$$A = \left\{ (x, \mu_A(x), \sigma_A(x), \nu_A(x)) : x \in X \right\}.$$

Here $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ are called membership function, hesitancy function and non-membership function respectively. They are respectively defined by:

$$\mu_A(x) : X \to \left]0^-, 1^+\right[, \ \sigma_A(x) : X \to \left]0^-, 1^+\right[, \ \nu_A(x) : X \to \left]0^-, 1^+\right[$$

subject to $0^- \leq sup\ \mu_A(x) + sup\ \sigma_A(x) + sup\ \nu_A(x) \leq 3^+$.

2.3 Single valued NS set

Let X be an universal set. A single valued NS set $A \in X$ is defined by:

$$\mu_A(x) : X \to [0,1], \sigma_A(x) : X \to [0,1], \nu_A(x) : X \to [0,1]$$

subject to $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$. here $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ are called membership function, hesitancy function and non-membership function respectively.

2.4 Union of two NS sets

Let X be an universal set and A and B are any two subsets of X. Here $\mu_A(x) : X \to [0,1]$, $\sigma_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ are membership function, hesitancy function and non-membership function of A respectively. Then union of A and B is denoted by $A \cup B$ and is definded as:

$$A \cup B = \left\{ (x, max(\mu_A(x), \mu_B(x)), max(\sigma_A(x), \sigma_B(x)), min(\nu_A(x), \nu_B(x))) : x \in X \right\}.$$

2.5 Intersection of two NS sets

Let X be an universal set and A and B are any two subsets of X. Here $\mu_A(x) : X \to [0,1]$, $\sigma_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ are membership function, hesitancy function and non-membership function of A

respectively. Then intersection of A and B is denoted by $A \cap B$ and is definded as:

$$A \cap B = \{(x, min(\mu_A(x), \mu_B(x)), min(\sigma_A(x), \sigma_B(x)), max(\nu_A(x), \nu_B(x))) : x \in X\}.$$

3 Formulation of single objective EOQ model with limited storage capacity

In this article, we take a single objective EOQ model, along with limited storage capacity. Here we take following unit production cost:

$$P(D, S) = \theta D^{-x} S^{-1}$$

We note that shape parameter (x) should lie within pre-determined values so as to satisfy positivity conditions of Dual Geometric Programming Problem (DGPP). We present the notations and assumptions of proposed model, for which explanations are given in Table 9, as follows:

3.1 Assumptions

To specify scopes of study and to further simplify the proposed EOQ model, we consider following assumptions

(i) proposed EOQ model shall involve exactly one item;
(ii) we consider infinite rate for instantaneously replenishment;
(iii) lead time is negligible;
(iv) we take demand rate as constant;
(v) the holding cost of proposed model is a funtion of time, i.e. we take $H(t) = at$;
(vi) upgradation to modern machineries involves higher costs, which is a part of set up cost. Since these machineries have higher production rates and other advantages, large scale production can bring down the unit production cost and it is generally adopted when demand is high. Therefore we find that unit production cost is inversely releated to set-up cost and rate of demand. Hence we get as follows:

$$P(D, S) = \theta D^{-x} S^{-1}; \;\; \theta, x \in R^{+}$$

(vii) We do not allow any shortage in inventory.

3.2 Formulation of model

In this article, we take initial inventory level at $t = 0$ as Q. Also inventory level gradually decreases in [0, T] and it is zero at time T. Since we do not allow shortage, the cycle is repeated over time period T. We illustrate the proposed inventory model graphically in Fig.1. Here inventory level at any time t in [0, T] is denoted by Q(t). Hence differential equation for instantaneous inventory level Q(t) at time t in [0, T] is as follows:

$$\frac{dI(t)}{dt} = -D \qquad for \;\; 0 \leqslant t \leqslant T$$

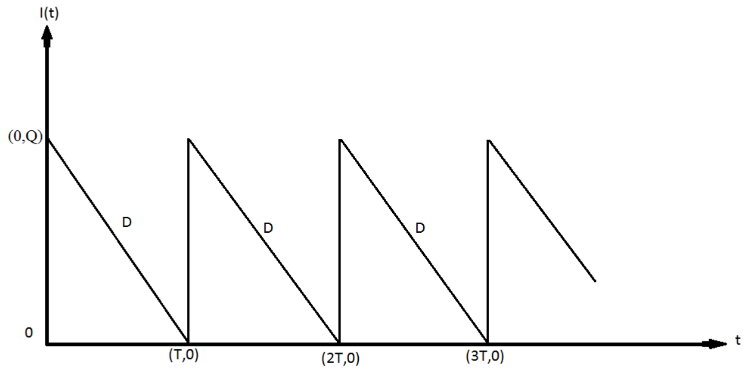

Figure 1: Inventory model

with boundary conditions as $I(0) = Q$, $I(T) = 0$.
By applying those conditions, we obtain as follows:

$$I(t) = D(T - t)$$

Therefore inventory holding cost becomes as follows:

$$\int_0^T H(t)I(t)d(t) = \frac{aQ^3}{6D^2}$$

Hence total average inventory cost per cycle [0, T] is as follows:

$$TAC(D, S, Q) = \frac{SD}{Q} + \frac{aQ^2}{6D} + \theta D^{1-x} S^{-1}$$

Here maximum floor capacity for storing items in warehouse is W. So storage area $w_0 Q$ for production quantity Q can never go beyond maximum floor capacity in warehouse for storing items at any time t. Therefore limited storage capacity is as follows:

$$w_0 Q \leqslant W$$

Finally we have inventory model in crisp environment as follows:

$$\min TAC(D, S, Q) = \frac{SD}{Q} + \frac{aQ^2}{6D} + \theta D^{1-x} S^{-1} \tag{3.1}$$

subject to

$S(Q) \equiv w_0 Q \leqslant W$

$D, S, Q > 0.$

4 Solution of EOQ model by crisp GP

We apply classical or crisp GP to solve proposed EOQ model. Here DD is 0. We apply Duffin and Peterson theorem [13] of GP on equation (3.1) and obtain DGPP as follows:

$$\max \mathrm{d(w)} = \left(\frac{1}{w_{01}}\right)^{w_{01}} \left(\frac{a}{6w_{02}}\right)^{w_{02}} \left(\frac{\theta}{w_{03}}\right)^{w_{03}} \left(\frac{w_0}{Ww_{11}}\right)^{w_{11}} {w_{11}}^{w_{11}}$$

subject to

$$w_{01} + w_{02} + w_{03} = 1,$$
$$w_{01} - w_{02} + (1-x)w_{03} = 0,$$
$$w_{01} - w_{03} = 0,$$
$$-w_{01} + 2w_{02} + w_{11} = 0,$$
$$w_{01}, w_{02}, w_{03}, w_{11} \geq 0.$$

The optimal solution in crisp environment is as follows:

$$w_{01}^* = w_{03}^* = \frac{1}{4-x}, \;\; w_{02}^* = \frac{2-x}{4-x}, \;\; w_{11}^* = \frac{2x-3}{4-x}.$$

Since value of shape paremeter x has to lie in interval $[1.5, 2]$, all dual variables remain positive. Thus optimal values of primal variables are as follows:

$$D^* = \left\{\frac{1}{\theta}\left(\frac{a}{6(2-x)}\right)^2 \left(\frac{W}{w_0}\right)^5\right\}^{\frac{1}{4-x}},$$

$$S^* = \left\{\left(\frac{\theta W}{w_0}\right)^2 \left(\frac{6w_0^3(2-x)}{aW^3}\right)^x\right\}^{\frac{1}{4-x}},$$

$$Q^* = \frac{W}{w_0}.$$

with optimal TAC as follows:

$$TAC^*(D^*, S^*, Q^*) = (4-x)\left\{\theta \left(\frac{w_0}{W}\right)^{(2x-3)} \left(\frac{a}{6(2-x)}\right)^{(2-x)}\right\}^{\frac{1}{4-x}} = T_1 \;\; (say)$$

5 Solution of EOQ model by fuzzy GP

We apply max-additive operator to solve proposed EOQ model in fuzzy environment. Here we compute individual optimum values of objective function: TAC and constraint: limited storage capacity of model (3.1), as given in Table 1. Also DM supplies goal and goal plus tolerance values for membership functions of objective function and constraint. For sake of simplicity, we consider linear membership function for TAC and limited storage capacity as follows:

Table 1: Individual maximum and minimum values of decision variables and TAC

	Maximum value	Minimum value
Demand per unit time (D)	$\left\{\frac{1}{\theta}\left(\frac{a}{6(2-x)}\right)^2\left(\frac{W}{w_0}\right)^5\right\}^{\frac{1}{4-x}}$	$\left\{\frac{1}{\theta}\left(\frac{a}{6(2-x)}\right)^2\left(\frac{W+w_p}{w_0}\right)^5,\right\}^{\frac{1}{4-x}}$
Set up cost (S)	$\left\{\left(\frac{\theta W}{w_0}\right)^2\left(\frac{6w_0^3(2-x)}{aW^3}\right)^x\right\}^{\frac{1}{4-x}}$	$\left\{\left(\frac{\theta(W+w_p)}{w_0}\right)^2\left(\frac{6w_0^3(2-x)}{a(W+w_p)^3}\right)^x\right\}^{\frac{1}{4-x}}$
Production quantity per batch (Q)	$\frac{W}{w_0}$	$\frac{W+w_p}{w_0}$
Total Average Cost TAC(D, S, Q)	$(4-x)\left\{\theta\left(\frac{w_0}{W}\right)^{(2x-3)}\left(\frac{a}{6(2-x)}\right)^{(2-x)}\right\}^{\frac{1}{4-x}}$	$(4-x)\left\{\theta\left(\frac{w_0}{W+w_p}\right)^{(2x-3)}\left(\frac{a}{6(2-x)}\right)^{(2-x)}\right\}^{\frac{1}{4-x}}$

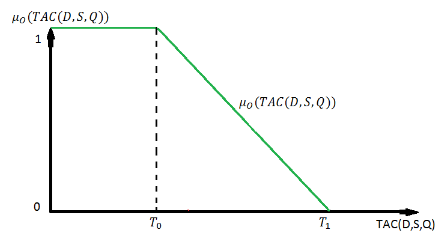

Figure 2: Membership function of fuzzy objective function

$$\mu_{\tilde{O}}(TAC(D,S,Q)) = \begin{cases} 1 & \text{if } TAC(D,S,Q) \leqslant T_0 \\ \frac{T_1 - TAC(D,S,Q)}{T_1 - T_0} & \text{if } T_0 \leqslant TAC(D,S,Q) \leqslant T_1 \\ 0 & \text{if otherwise.} \end{cases}$$

$$\mu_{\tilde{C}}(S(Q)) = \begin{cases} 1 & \text{if } w_0 Q \leqslant W \\ \frac{W + w_p - w_0 Q}{w_p} & \text{if } W \leqslant w_0 Q \leqslant W + w_p \\ 0 & \text{if otherwise.} \end{cases}$$

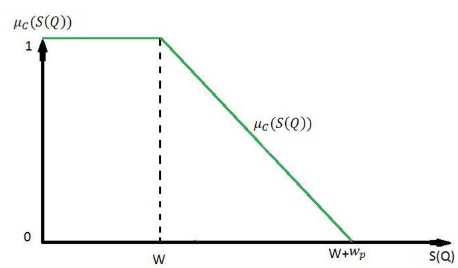

Figure 3: Membership function of fuzzy constraint

Next we formulate the mathematical model as follows:

$$\begin{aligned} &\max \ \{\mu_{\tilde{O}}(TAC(D,S,Q))\mu_{\tilde{C}}(S(Q))\} \\ &\text{subject to} \\ &0 < \mu_{\tilde{O}}(TAC(D,S,Q)) + \mu_{\tilde{C}}(S(Q)) < 1, \\ &D, S, Q > 0. \end{aligned}$$

By applying max-additive operator, we get crisp Primal Geometric Programming Problem (PGPP) and use convex combination operator to obtain as follows:

$$\max VF_{FA}(D,S,Q) = F_K - VF_{FA1}(D,S,Q)$$

Here $F_K = \frac{T_0}{T_1-T_0} + \frac{W+w_p}{w_p}$ and $VF_{FA1}(D,S,Q) = \frac{TAC(D,S,Q)}{T_1-T_0} + \frac{w_0 Q}{w_p}$.
Therefore the problem reduces to the following model:

$$\min VF_{FA1}(D,S,Q) = \frac{SD}{Q(T_1-T_0)} + \frac{aQ^2}{6D(T_1-T_0)} + \frac{\theta D^{1-x}}{(T_1-T_0)S} + \frac{w_0 Q}{w_p}$$

subject to

$$D, S > 0, Q \in \left[\frac{W}{w_0}, \frac{W+w_p}{w_0}\right], TAC(D,S,Q) \in [T_0, T_1]. \tag{5.1}$$

It is unconstrained PGPP with DD = 0. Hence optimal values for primal variables of model (5.1) are as follows:

$$D^* = \frac{3}{2}\left\{\theta\, 6^{(x-2)} \left(\frac{a}{2-x}\right)^3 \left(\frac{w_0}{w_p}\right)^{(2x-3)} \left(\frac{2x-3}{T_1-T_0}\right)^5\right\}^{\frac{1}{x+1}},$$

$$S^* = 2\left\{\theta\, 6^{(x-2)} \left(\frac{T_1-T_0}{2x-3}\right)^{(3x-2)} \left(\frac{w_0}{w_p}\right)^{(2x-3)} \left(\frac{a}{2-x}\right)^{(1-2x)}\right\}^{\frac{1}{x+1}},$$

$$Q^* = 3(2x-3)\left\{\theta \left(\frac{1}{T_1-T_0}\right)^{(4-x)} \left(\frac{a}{6(2-x)}\right)^{(2-x)} \left(\frac{w_0}{w_p(2x-3)}\right)^{(2x-3)}\right\}^{\frac{1}{x+1}},$$

with optimal TAC as follows:

$$TAC^*(D^*,S^*,Q^*) = \left[\left\{\theta \left(\frac{a}{6(2-x)}\right)^{(2-x)} \left(\frac{w_0(T_1-T_0)}{w_p(2x-3)}\right)^{(2x-3)}\right\}^{\frac{1}{x+1}} \left\{1 + \left(\frac{2-x}{6^x}\right)^{\frac{1}{x+1}} + \left(\frac{w_p}{3w_0}\right)^{(2x-3)}\right\}\right]$$

provided $Q^* \in \left[\frac{W}{w_0}, \frac{W+w_p}{w_0}\right]$, $\text{TAC}^*(D^*,S^*,Q^*) \in [T_0, T_1]$.

6 Solution of EOQ model by IFGP

We employ IF optimization method and solve proposed EOQ model (3.1). Goal and goal plus tolerance values of non-membership functions of TAC and limited storage capacity, as obtained from DM, are given in Table 1. Based on these values, we construct following linear non-membership functions of TAC and limited storage capacity:

$$\nu_{\tilde{O}}(TAC(D,S,Q)) = \begin{cases} 0 & \text{if } TAC(D,S,Q) \leqslant T_0 + \epsilon_o \\ \frac{TAC(D,S,Q) - T_0 - \epsilon_o}{T_1 - T_0 - \epsilon_o} & \text{if } T_0 + \epsilon_o \leqslant TAC(D,S,Q) \leqslant T_1 \\ 1 & \text{otherwise.} \end{cases}$$

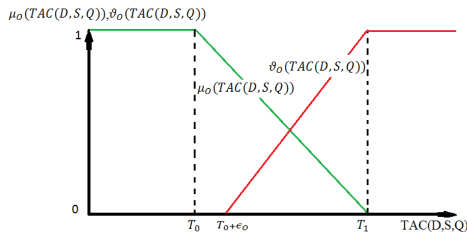

Figure 4: Membership and non-membership function of IF objective function

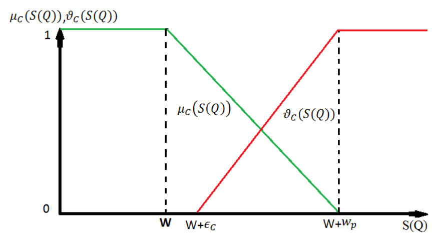

Figure 5: Membership and non-membership function for IF constraint

$$\nu_{\tilde{C}}(S(Q)) = \begin{cases} 0 & \text{if} \quad w_0 Q \leqslant W + \epsilon_C \\ \frac{w_0 Q - W - \epsilon_C}{w_p - \epsilon_C} & \text{if} \quad W + \epsilon_C \leqslant w_0 Q \leqslant W + w_p \\ 1 & \text{if} \quad w_0 Q \geq W + w_p \end{cases}$$

Next we formulate EOQ model as follows:

$$\begin{aligned} &\max \;\; \{\mu_{\tilde{O}}(TAC(D,S,Q))\mu_{\tilde{C}}(S(Q))\} \\ &\min \;\; \{\nu_{\tilde{O}}(T(D,S,Q)), \nu_{\tilde{C}}(S(Q))\} \\ &\text{subject to} \\ &0 < \mu_{\tilde{O}}(TAC(D,S,Q)) + \nu_{\tilde{O}}(TAC(D,S,Q)) < 1; \\ &0 < \mu_{\tilde{C}}(S(Q)) + \nu_{\tilde{C}}(S(Q)) < 1; \\ &D, S, Q > 0. \end{aligned}$$

By applying max-additive operator and then GP in IF environment, we obtain optimal decision variables as follows:

$$D^* = \left\{ \theta \left(\frac{a}{6(2-x)} \right)^3 \left(\frac{I_{K1}\,(2x-3)}{I_{K2}\,w_0} \right)^5 \right\}^{\frac{1}{x+1}},$$

$$S^* = \left\{ \theta \, \left(\frac{I_{K2}\,w_0}{I_{K1}\,(2x-3)} \right)^{(3x-2)} \left(\frac{a}{6(2-x)} \right)^{(1-2x)} \right\}^{\frac{1}{x+1}},$$

$$Q^* = \left\{ \theta \, \left(\frac{I_{K1}\,(2x-3)}{I_{K2}\,w_0} \right)^{(4-x)} \left(\frac{a}{6(2-x)} \right)^{(2-x)} \right\}^{\frac{1}{x+1}}$$

with optimal TAC as follows:

$$TAC^*(D^*,S^*,Q^*) = \left[\left\{\theta\left(\frac{a}{6}\right)^{(2-x)}\left(\frac{IK_1(2x-3)}{Ik_2\, w_0}\right)^{(3-2x)}\right\}^{\frac{1}{x+1}}\left\{2\left(\frac{1}{2-x}\right)^{\left(\frac{2-x}{x+1}\right)}+\left(\frac{1}{2-x}\right)^{\left(\frac{1-2x}{x+1}\right)}\right\}\right]$$

provided$Q^* \in \left[\frac{W+\epsilon_C}{w_0}, \frac{W+w_p}{w_0}\right]$, $\mathrm{TAC^*(D^*, S^*, Q^*)} \in [\mathrm{T_0} + \epsilon_\mathrm{O}, \mathrm{T_1}]$.

7 Solution of EOQ model by NSGP

The world is full of indeterminacy and hence we require more precise imprecision. Thus the concept of NS set comes into picture. We consider membership function, hesitancy function, non-membership function for each objective function and constraint of proposed model. we consider same memebership function, as given in Section 5 and same non-membership function, as given in Section 6. We take hesitancy functions for objective function and constraint as follows:

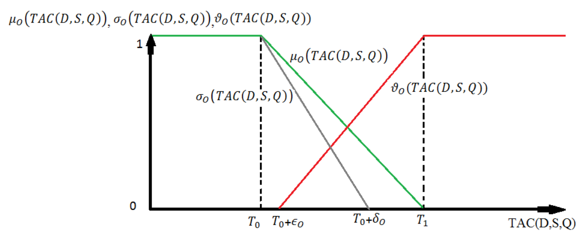

Figure 6: Membership, hesitancy and non-membership function of objective function in NS environment.

$$\sigma_{\tilde{O}}(TAC(D,S,Q)) = \begin{cases} 1 & \text{if } \ TAC(D,S,Q) \leqslant T_0 \\ \frac{T_0+\delta_o-TAC(D,S,Q)}{\delta_o} & \text{if } \ T_0 \leqslant TAC(D,S,Q) \leqslant T_0+\delta_o \\ 0 & \text{if } \ TAC(D,S,Q) \geq T_0+\delta_o \end{cases}$$

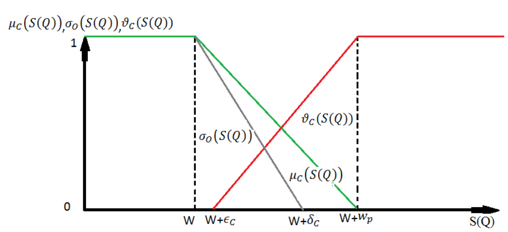

Figure 7: Membership, hesitancy and non-membership function of constraint in NS environment.

$$\sigma_{\tilde{C}}(S(Q)) = \begin{cases} 1 & \text{if } \ w_0Q \leqslant W \\ \frac{W+\delta_c-w_0Q}{\delta_c} & \text{if } \ W \leqslant w_0Q \leqslant W+\delta_c \\ 0 & \text{if } \ w_0Q \geq W+\delta_C \end{cases}$$

We note that $0 < \epsilon_C$, $\delta_c < w_p$. Here we consider the case when hesitancy function behaves like non-membership function. We present several more cases in Table 2. Then linear hesitancy functions of objective function and constraint are as follows:

Table 2: On different natures of hesitancy function

Nature of hesitancy function in		Value of parameter	
Objective function	Constraint	N_{k1}	N_{k2}
non-increasing	non-increasing	$\left(\frac{1}{T_1-T_0}+\frac{1}{\delta_o}+\frac{1}{T_1-T_0-\epsilon_O}\right)$	$\left(\frac{1}{w_p}+\frac{1}{\delta_c}+\frac{1}{w_p-\epsilon_C}\right)$
non-decreasing	non-decreasing	$\left(\frac{1}{T_1-T_0}+\frac{1}{T_1-T_0-\delta_o}+\frac{1}{T_1-T_0-\epsilon_O}\right)$	$\left(\frac{1}{w_p}+\frac{1}{w_p-\delta_c}+\frac{1}{w_p-\epsilon_C}\right)$
non-increasing	non-decreasing	$\left(\frac{1}{T_1-T_0}+\frac{1}{\delta_o}+\frac{1}{T_1-T_0-\epsilon_O}\right)$	$\left(\frac{1}{w_p}+\frac{1}{w_p-\delta_c}+\frac{1}{w_p-\epsilon_C}\right)$
non-decreasing	non-increasing	$\left(\frac{1}{T_1-T_0}+\frac{1}{T_1-T_0-\delta_o}+\frac{1}{T_1-T_0-\epsilon_O}\right)$	$\left(\frac{1}{w_p}+\frac{1}{\delta_c}+\frac{1}{w_p-\epsilon_C}\right)$

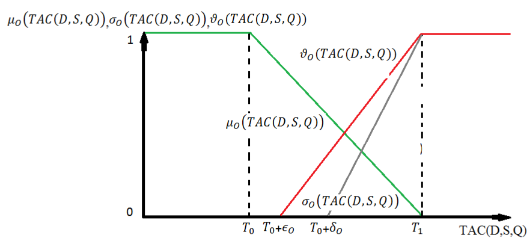

Figure 8: Membership, hesitancy and non-membership function of objective function in NS environment.

$$\sigma_{\tilde{O}}(TAC(D,S,Q)) = \begin{cases} 0 & \text{if } \ TAC(D,S,Q) \leqslant T_0+\delta_o \\ \frac{TAC(D,S,Q)-(T_0+\delta_o)}{T_1-T_0-\delta_o} & \text{if } \ T_0+\delta_o \leqslant TAC(D,S,Q) \leqslant T_1 \\ 1 & \text{if } \ TAC(D,S,Q) \geq T_1 \end{cases}$$

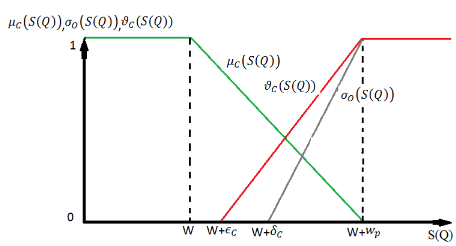

Figure 9: Membership, hesitancy and non-membership function of constraint in NS environment.

$$\sigma_{\tilde{C}}(S(Q)) = \begin{cases} 0 & \text{if } \ w_0Q \leqslant W+\delta_c \\ \frac{w_0Q-(W+\delta_c)}{w_p-\delta_c} & \text{if } \ W+\delta_c \leqslant w_0Q \leqslant W+w_p \\ 1 & \text{if } \ w_0Q \geq W+w_p \end{cases}$$

Then we obtain following optimization model in NS environment:

$$\begin{aligned}
&\max\ \{\mu_{\tilde{O}}(TAC(D,S,Q))\mu_{\tilde{C}}(S(Q))\}\\
&\max\ \{\sigma_{\tilde{O}}(T(D,S,Q)),\sigma_{\tilde{C}}(S(Q))\}\\
&\min\ \{\nu_{\tilde{O}}(T(D,S,Q)),\nu_{\tilde{C}}(S(Q))\}\\
&\text{subject to}\\
&\mu_{\tilde{O}}(TAC(D,S,Q))\geq\sigma_{\tilde{O}}(TAC(D,S,Q)),\mu_{\tilde{C}}(S(Q))\geq\sigma_{\tilde{C}}(S(Q))\\
&\mu_{\tilde{O}}(TAC(D,S,Q))\geq\nu_{\tilde{O}}(TAC(D,S,Q)),\mu_{\tilde{C}}(S(Q))\geq\nu_{\tilde{C}}(S(Q))\\
&0\leqslant\mu_{\tilde{O}}(TAC(D,S,Q)),\sigma_{\tilde{O}}(TAC(D,S,Q)),\nu_{\tilde{O}}(TAC(D,S,Q))\leq 1\\
&0\leq\mu_{\tilde{C}}(S(Q)),\sigma_{\tilde{C}}(S(Q)),\nu_{\tilde{C}}(S(Q))\leq 1\\
&D,S,Q>0.
\end{aligned}$$

The corrosponding single objective optimization model is as follows:

$$\begin{aligned}
Max\,VF_{NFA}(D,S,Q) = {} & \mu_{\tilde{O}}(TAC(D,S,Q))+\mu_{\tilde{C}}(S(Q))+\sigma_{\tilde{O}}(TAC(D,S,Q))\\
& +\sigma_{\tilde{C}}(S(Q)) \quad -\nu_{\tilde{O}}(TAC(D,S,Q))-\nu_{\tilde{C}}(S(Q))\\
\text{subject to} &\\
\mu_{\tilde{O}}(TAC(D,S,Q)) & \geq\sigma_{\tilde{O}}(TAC(D,S,Q)),\mu_{\tilde{C}}(S(Q))\geq\sigma_{\tilde{C}}(S(Q))\\
\mu_{\tilde{O}}(TAC(D,S,Q)) & \geq\nu_{\tilde{O}}(TAC(D,S,Q)),\mu_{\tilde{C}}(S(Q))\geq\nu_{\tilde{C}}(S(Q))\\
0\leqslant\mu_{\tilde{O}}(TAC(D,S,Q)) & ,\sigma_{\tilde{O}}(TAC(D,S,Q)),\nu_{\tilde{O}}(TAC(D,S,Q))\leq 1\\
0\leq\mu_{\tilde{C}}(S(Q)) & ,\sigma_{\tilde{C}}(S(Q)),\nu_{\tilde{C}}(S(Q))\leq 1;\\
D,S,Q>0. &
\end{aligned}$$

We rewrite the above model as follows:

$$\begin{aligned}
&\max \mathrm{VF_{NFA}(D,S,Q)=N_K-VF_{NFA1}(D,S,Q)}\\
&\text{subject to}\\
&D,S>0,Q\in\left[\frac{W+\epsilon_C}{w_0},\frac{W+w_p}{w_0}\right],\mathrm{TAC(D,S,Q)\in[T_0+\epsilon_O,T_1]}.
\end{aligned}$$

Here $N_K=\left(\frac{T_1}{T_1-T_0}+\frac{T_0+\delta_o}{\delta_o}+\frac{T_0+\epsilon_O}{T_1-T_0-\epsilon_O}\right)+\left(\frac{W+w_p}{w_p}+\frac{W+\delta_c}{\delta_c}+\frac{W+\epsilon_C}{w_p-\epsilon_C}\right)$,
$VF_{NFA1}(D,S,Q)=\frac{N_{K1}\,SD}{Q}+\frac{N_{K1}\,aQ^2}{6D}+N_{K1}\,\theta D^{1-x}S^{-1}+N_{K2}\,w_0Q$,
with $N_{K1}=\left(\frac{1}{T_1-T_0}+\frac{1}{\delta_o}+\frac{1}{T_1-T_0-\epsilon_O}\right)$ *and* $N_{K2}=\left(\frac{1}{w_p}+\frac{1}{\delta_c}+\frac{1}{w_p-\epsilon_C}\right)$.
Hence unconstrainted PGPP is as follows:

$$\begin{aligned}
&\min \mathrm{VF_{NFA1}(D,S,Q)=\frac{N_{K1}\,SD}{Q}+\frac{N_{K1}\,aQ^2}{6D}+\frac{N_{K1}\,\theta D^{1-x}}{S}+N_{K2}\,w_0Q}\\
&\text{subject to}\\
&D,S>0,Q\in\left[\frac{W+\epsilon_C}{w_0},\frac{W+w_p}{w_0}\right],\mathrm{TAC(D,S,Q)\in[T_0+\epsilon_O,T_1]}. \qquad (7.1)
\end{aligned}$$

Here DD=0. we solve above model by NSGP [8, 14] and obtain as follows:

$$\max d(w) = \left(\frac{N_{K1}}{w_{01}}\right)^{w_{01}} \left(\frac{aN_{K1}}{6w_{02}}\right)^{w_{02}} \left(\frac{\theta N_{K1}}{w_{03}}\right)^{w_{03}} \left(\frac{w_0 N_{K2}}{w_{04}}\right)^{w_{04}}$$

subject to

$$w_{01} + w_{02} + w_{03} + w_{04} = 1,$$
$$w_{01} - w_{02} + (1-x)w_{03} = 0,$$
$$w_{01} - w_{03} = 0,$$
$$-w_{01} + 2w_{02} + w_{04} = 0,$$
$$w_{01}, w_{02}, w_{03}, w_{04} \geq 0.$$

Therefore optimal dual variables are as follows:

$$w_{01}^* = \frac{1}{4-x}, w_{02}^* = \frac{2-x}{4-x}, w_{03}^* = \frac{1}{4-x}, w_{04}^* = \frac{2x-3}{4-x}.$$

Hence optimal decision variables are as follows:

$$D^* = \left\{\theta \left(\frac{a}{6(2-x)}\right)^3 \left(\frac{N_{K1}\,(2x-3)}{N_{K2}\,w_0}\right)^5\right\}^{\frac{1}{x+1}}$$

$$S^* = \left\{\theta \left(\frac{N_{K2}\,w_0}{N_{K1}\,(2x-3)}\right)^{(3x-2)} \left(\frac{a}{6(2-x)}\right)^{(1-2x)}\right\}^{\frac{1}{x+1}}$$

$$Q^* = \left\{\theta \left(\frac{N_{K1}\,(2x-3)}{N_{K2}\,w_0}\right)^{(4-x)} \left(\frac{a}{6(2-x)}\right)^{(2-x)}\right\}^{\frac{1}{x+1}}$$

with optimal TAC as follows:

$$TAC^*(D^*, S^*, Q^*) = \left[\left\{\theta \left(\frac{a}{6}\right)^{(2-x)} \left(\frac{N_{KI}(2x-3)}{N_{K2}\,w_0}\right)^{(3-2x)}\right\}^{\frac{1}{x+1}} \left\{2\left(\frac{1}{2-x}\right)^{\left(\frac{2-x}{x+1}\right)} + \left(\frac{1}{2-x}\right)^{\left(\frac{1-2x}{x+1}\right)}\right\}\right]$$

provided$Q^* \in \left[\frac{W+\epsilon_C}{w_0}, \frac{W+w_p}{w_0}\right]$, $\mathrm{TAC}^*(\mathrm{D}^*, \mathrm{S}^*, \mathrm{Q}^*) \in [\mathrm{T}_0 + \epsilon_{\mathrm{O}}, \mathrm{T}_1]$.

8 Numerical application

We consider a simple numerical application to solve proposed model in NS environment as follows:

A manufacturing company produces machines PBA_{597}. The inventory carrying cost for the machines is $Rs.105$ per unit per year. The production cost of this machine varies inversely with the demand and set-up cost. From the past experiences, we can consider the production cost of the machine PBA_{597} at about $120D^{-0.75}S^{-1}$, where D is the demand rate and S is the set-up cost. The company has storage capacity area per unit time (w_0) and total storage capacity area (W) as 100 sq. ft. and 2000 sq. ft. respectively. The task is

to determine the optimal demand rate (D), set-up cost (S), production quantity (Q) and hence optimal TAC of the production system.
Here mathematical model is of the following form:

$$\min TAC(D,S,Q) = \frac{SD}{Q} + \frac{105Q^2}{6D} + 120D^{-0.75}S^{-1} \tag{8.1}$$

subject to

$S(Q) \equiv 100Q \leqslant 2000,$

$D, S, Q > 0.$

We consider goal and goal plus tolerance values for TAC and limited storage capacity as given in Table 3. Based on these values, we construct following linear membership, hesitancy and non-membership functions of TAC and limited storage capacity:

Table 3: Goal and goal plus tolerance values of TAC and variables

	Demand (D)	Set-up cost (S)	Production quantity (Q)	Total Average Cost (TAC(D,S,Q))
Goal	4047.477	0.034	20.000	15.565
Goal plus tolerance	5521.645	0.028	23.000	15.089

$$\mu_{\tilde{O}}(TAC(D,S,Q)) = \begin{cases} 1 & \text{if } \ TAC(D,S,Q) \leqslant 15.089 \\ \frac{15.565 - TAC(D,S,Q)}{0.476} & \text{if } \ 15.089 \leqslant TAC(D,S,Q) \leqslant 15.565 \\ 0 & \text{if } \ \text{otherwise.} \end{cases}$$

$$\mu_{\tilde{C}}(S(Q)) = \begin{cases} 1 & \text{if } \ 100Q \leqslant 2000 \\ \frac{2300 - 100Q}{300} & \text{if } \ 2000 \leqslant 100Q \leqslant 2300 \\ 0 & \text{if } \ \text{otherwise.} \end{cases}$$

$$\sigma_{\tilde{O}}(TAC(D,S,Q)) = \begin{cases} 1 & \text{if } \ TAC(D,S,Q) \leqslant 15.089 \\ \frac{15.389 - TAC(D,S,Q)}{0.3} & \text{if } \ 15.089 \leqslant TAC(D,S,Q) \leqslant 15.389 \\ 0 & \text{if } \ TAC(D,S,Q) \geq 15.389 \end{cases}$$

$$\sigma_{\tilde{C}}(S(Q)) = \begin{cases} 1 & \text{if } \ 100Q \leqslant 2000 \\ \frac{2170 - 100Q}{170} & \text{if } \ 2000 \leqslant 100Q \leqslant 2170 \\ 0 & \text{if } \ 100Q \geq 2170 \end{cases}$$

$$\nu_{\tilde{O}}(TAC(D,S,Q)) = \begin{cases} 0 & \text{if } \ TAC(D,S,Q) \leqslant 15.306 \\ \frac{TAC(D,S,Q) - 15.306}{0.259} & \text{if } \ 15.306 \leqslant TAC(D,S,Q) \leqslant 15.565 \\ 1 & \text{if } \ \text{otherwise.} \end{cases}$$

$$\nu_{\tilde{C}}(S(Q)) = \begin{cases} 0 & \text{if } \ 100Q \leqslant 2070 \\ \frac{100Q - 2070}{230} & \text{if } \ 2070 \leqslant 100Q \leqslant 2300 \\ 1 & \text{if } \ 100Q \geq 2300 \end{cases}$$

Therefore single objective EOQ model with limited storage capacity is as follows:

$$\min TAC(D,S,Q) = \frac{9.295SD}{Q} + \frac{162.663Q^2}{D} + 1115.4D^{-0.75}S^{-1} + 1.356Q$$
subject to
$$D,S>0, Q\in[20.5,23], \text{TAC}(D,S,Q)=[15.089,15.565] \quad (8.2)$$

We solve the model (8.2) by GP. Here DD = 0. Hence DGPP of (8.2) is as follows:

$$\max d(w) = \left(\frac{9.295}{w_{01}}\right)^{w_{01}}\left(\frac{162.663}{w_{02}}\right)^{w_{02}}\left(\frac{1115.4}{w_{03}}\right)^{w_{03}}\left(\frac{1.356}{w_{04}}\right)^{w_{04}}$$
subject to
$$w_{01}+w_{02}+w_{03}+w_{04}=1,$$
$$w_{01}-w_{02}+(1-x)w_{03}=0,$$
$$w_{01}-w_{03}=0,$$
$$-w_{01}+2w_{02}+w_{04}=0,$$
$$w_{01},w_{02},w_{03},w_{04}\geq 0.$$

Therefore optimal values of dual variables are as follows:

$$w_{01}^*=0.444, w_{02}^*=0.111, w_{03}^*=0.444, w_{04}^*=0.222.$$

Hence optimal values of decision variables are as follows:

$$D^*=5575.110, S^*=0.028, Q^*=22.998, TAC^*(D^*,S^*,Q^*)=15.094.$$

We note that optimal TAC is 15.094 units with demand as 5575.110 units, set-up cost as 0.030 units and production quantity as 22.998 units. Also the optimal order quantity and TAC satify the necessary conditions. Next we compare the relative performance of proposed model by comparing its result with that obtained by employing crisp GP, fuzzy GP and IFGP and present it in Table 4. We find that optimal TAC is more preferable in NS environment than that of crisp, fuzzy and IF environments. Also NS environment yields higher demand for the machine PBA_{597} with lower set-up cost. Moreover production quantity increases in NS environment.

Table 4: Optimal solutions of model (3.1) in different environments

Environment	Demand (D)	Set-up cost (S)	Production quantity (Q)	Total Average Cost (TAC(D,S,Q))
Crisp	4047.477	0.034	20.000	15.565
Fuzzy	4742.869	0.031	21.479	15.320
IF	4998.630	0.030	21.993	15.240
NS	*5575.110*	*0.028*	*22.998*	*15.094*

8.1 Sensitivity analysis

In this article, we investigate optimal policy of DM of proposed model in real life based NS environment. We perform sensitivity analysis of following key parameters
(i) storage capacity per machine 'w_0' (Table 5)
(ii) shape parameter 'x' (Table 6)
(iii) variational parameter 'a' (Table 7)
(iv) shape parameter 'θ' (Table 8)
and present corresponding optimal solution in NS environment.

8.1.1 Managerial insights

We present phenomenon of change of storage capacity per machine 'w_0' in Table 5 . We observe that optimal TAC is most preferable to DM in NS environment, which is well explained in Fig.10 . Also we find that each reduction in storage capacity per machine reduces TAC not only in NS environment but also in other environ-ments. Hence the management should trim down the size of packet of finished goods to reduce TAC.

Table 5: Sensitivity analysis in different environments of storage capacity per machine $'w_0'$

TAC in	Storage capacity $'w_0'$				
	80	90	100	110	120
Crisp environment	14.812	15.205	15.565	15.898	16.209
Fuzzy environment	14.718	15.032	15.320	15.595	15.840
IF environment	14.513	14.826	15.240	15.379	15.623
NS environment	*14.494*	*14.805*	*15.094*	*15.355*	*15.600*

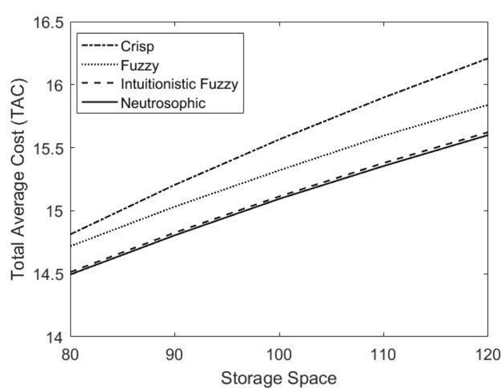

Figure 10: Effect on TAC in different environment due to change in storage space per machine 'w_0'.

Next we consider change of shape parameter 'x' in Table 6. Here we find that optimal TAC rapidly reduces for every increment in value of shape parameter and hence for every rise in demand in each of the said environments. It is consistent with common knowledge. Also in nearly all cases, we get most preferable optimal TAC in NS environment. This can be observed in Fig. 11. Again we perform sensitivity analysis of variational parameter 'a' and present in Table 7. Here in all cases, we obtain most desirable TAC in NS environment among said environments. It can be visualized in Fig. 12. Also optimal TAC reduces as holding cost decreases in all said environments.

Table 6: Sensitivity analysis in different environments of shape parameter $'x'$

TAC in	Shape parameter $'x'$				
	1.6	1.7	1.75	1.8	1.9
Crisp environment	*25.799*	18.612	15.565	12.860	8.385
Fuzzy environment	26.582	18.554	15.320	12.585	8.408
IF environment	26.429	18.349	15.240	12.385	8.198
NS environment	26.413	*18.328*	*15.094*	*12.366*	*8.179*

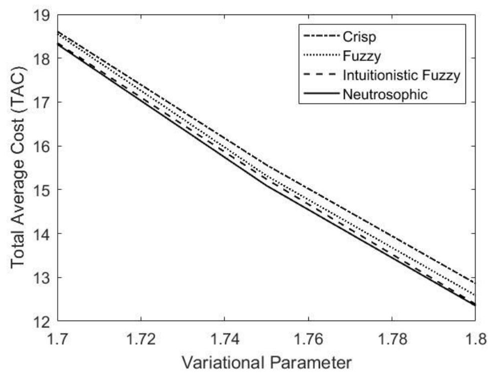

Figure 11: Effect on TAC in different environments due to change in shape parameter'x'.

Table 7: Sensitivity analysis in different environments of variational parameter $'a'$

TAC in	Variational parameter $'a'$				
	95	100	105	110	115
Crisp environment	15.393	15.481	15.565	15.646	15.723
Fuzzy environment	15.182	15.253	15.320	15.387	15.448
IF environment	15.103	15.174	15.240	15.307	15.368
NS environment	*14.956*	*15.024*	*15.094*	*15.155*	*15.217*

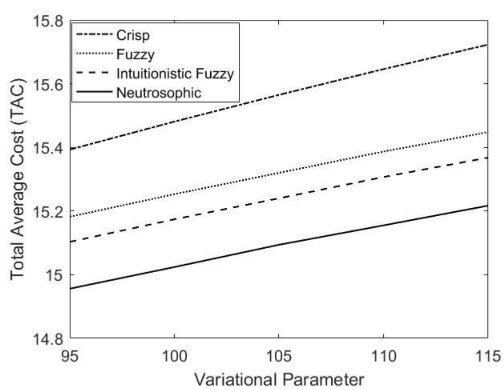

Figure 12: Effect on TAC in different environments due to change in variational parameter 'a'.

Also we consider change of shape parameter 'θ' and present result in Table 8. As before, we find that optimal TAC is most favourable to DM in NS environment among said environments. Fig.13 brings clarity to this phenomenon. Additionally, we observe that optimal TAC can be further reduced by decreasing the value of shape parameter.

Table 8: Sensitivity analysis in different environments of shape parameter $'\theta'$

TAC in	Shape parameter $'x'$				
	100	110	120	130	140
Crisp environment	14.354	14.975	15.565	16.129	16.669
Fuzzy environment	14.338	14.843	15.320	15.773	16.037
IF environment	14.146	14.643	15.240	15.560	15.985
NS environment	*14.123*	*14.622*	*15.094*	*15.536*	*15.962*

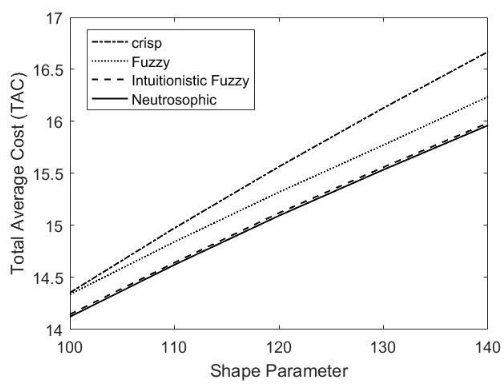

Figure 13: Effect on TAC in different environments due to change in shape parameter 'θ'.

9 Conclusions

In this article, we consider deterministic single objective EOQ model with limited storage capacity and solve it by applying GP in NS environment. We know it well that fuzzy set can better represent real life cases than crisp set. Again Ranjit Biswas [70] has shown how IF set can better represent real life cases than fuzzy set in many cases. Next Smarandache introduced NS set by generalizing IF set and at which we consider hesitancy function along with membership and non-membership function with appropriate constraints. Again advantages of GP among non-linear optimization methods are manifold. As per Cao [71], GP provides us with a systematic approach for solving a class of non-linear optimization problems by determining optimal values of decision variables and objective functions.

Whereas existing literature survey finds that GP is extended and thereby employed to solve mathematical models in fuzzy and IF environment, we can find very few articles, where EOQ models with limited storage capacity are solved by GP in NS environment. In this article, we employ max-additive operator to convert EOQ model with limited storage capacity to single objective PGPP and thereby solve it by applying NSGP. In numerical application, we find that optimal solution, obtained by NSGP is more preferable to DM than those obtained in crisp GP, fuzzy GP and IFGP. Next we perform sensitivity analysis of key parameters of proposed model and list several key managerial insights. Also we explain them graphically.

Future scopes of research
We locate lot of scopes for further research and enlist few of them as follows:
(i) We can consider multiple products scenario. In this case, we can employ modified GP in NS environment.
(ii) Shape parameters can be neutrsophic in nature.
(iii) We can allow shortage of items in inventory and update the mathematical model accordingly.
(iv) We can use other optimization methods to solve non-linear models in NS environment.
(v) And last but not the least, we can discuss present model in other imprecise environments.

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Table 9: Notations and their explanations

D	Demand per unit time, which is constant
H(t)	Holding cost per unit item, which is time (t) depended
I(t)	Inventory level at any time, $t \geq 0$
P(D, S)	Unit demand (D) and set-up cost (S) dependent production cost
Q	Production quantity per batch
S	Set-up cost per unit time
T	Period of cycle
TAC(D, S, Q)	Total average cost per unit time
W	Total storage capacity area
w_0	capacity area per unit quantity

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