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Optimal Neutrosophic Assignment and the Hungarian Method

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Abstract:

Operations research methods focus on formulating decision models, models that approximate the real work environment to mathematical models, and their optimal solution helps the decision maker to make optimal decisions that guarantee the greatest profit or the lowest cost. These models were built based on the data collected on the practical issue under study, and these data are classic values, specific values that are correct and accurate during the period in which these data are collected, which means that they remain ideal if the surrounding conditions remain the same as those in which they were collected. Data, and since the aim of this study is to make ideal decisions and to develop future plans through which we achieve the greatest profit or the least cost, It was necessary to search for a study that fits all the conditions in which the work environment passes in the present and the future. If we take data, neural values, indeterminate values - uncertain - leave nothing to chance, and take into account all conditions from worst to best. In this paper, we will present a complementary study to what we have done in previous research, the purpose of which was to reformulate some operational research methods using neutrosophic data. We will reformulate the assignment problem and one of the methods for its solution, which is the Hungarian method, using neutrosophic concepts, and we will explain the difference between using classical and neutrosophic data through examples.

Key words:

The optimal assignment problem - The Neutrosophic optimal assignment problem - The Hungarian method for solving assignment problems - linear models - Neutrosophic logic.

Introduction:

Assignment issues are a special case of linear programming issues that are concerned with the optimal assignment of various economic, productive and human resources for the various works to be accomplished, and we encounter them frequently in practical life in educational institutions - hospitals - construction projects…..etc. In order to obtain an optimal assignment that achieves the greatest profit and the least loss in all conditions that the work environment can pass through, it was necessary to reformulate the issue of optimal assignment and one of the ways to solve it using the concepts of the science neutrosophic , the science that has proven its ability to provide ideal and appropriate solutions in all circumstances. And in many areas, as shown in research and studies presented by researchers interested in scientific development [1-15].Since the optimal assignment model is one of the important models in the field of operations research, where we have a machines or people, and we need to assignment them to do the required work, and the number of works is equal to the number of machines or people. We need an optimal assignment for each machine so that it performs only one work and achieves the greatest profit or the lowest cost depending on the nature of the issue. This issue was studied using classical data by building the mathematical models, when we solving it, we get the desired. In this research, we will reformulate the issue using neutrosophic data that takes into account all changes that may occur in the work environment by taking costs or profit neutrosophic values, meaning that the cost (or return profit) of assignment the machine or person i to perform work j is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$ where ε_{ij} is the indeterminacy and $\varepsilon_{ij} \in [\lambda_{\text{ij1}}, \lambda_{\text{ij2}}]$, it is any neighborhood to the value c_{ij} that we get while collecting data on the problem then the cost (or profit) matrix becomes $NC_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

Discussion:

Assignment issues are considered a special case of linear programming issues and are concerned with the optimal assignment of various economic, human and productive resources for the different work to be accomplished, based on what we have presented in the research Mysterious Neutrosophic Linear Models [16] and the classical formulation contained in the two references [17,18]. In this research, we will reformulate the problem of optimal assignment and the Hungarian method that is used to solve these problems using the concepts of neutrosophic science, that is, we will take the costs or profit from the neutrosophic values, so that the cost of doing job j by the machine or the person i is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$, where ε_{ij} is indeterminacy and $\varepsilon_{ij} \in [\lambda_{\text{ij1}}, \lambda_{\text{ij2}}]$, which is any neighborhood of the value c_{ij} which we get it while collecting the data and then the cost matrix becomes equal to $NC_{ij} = [c_{ij} \pm \varepsilon_{ij}]$ and the problem text is as follows:

Standard assignment issues:

In these issues, the number of machines or people equals the number of works, which we will address in this research.

Text of the minimum cost type neutrosophic normative assignment problem:

If we have $\boldsymbol{n}$ machines, we denote them by $M_1 , M_2 , \dots \dots , M_n$ and we have a set of works consisting of $\boldsymbol{n}$ different work we denote them by $N_1 , N_2 , \dots \dots , N_n$ we want to designate the machines to do these jobs, cost of doing any work j on the device i, it is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$. Assuming that any machine can do only one job, it is required to find the optimal assignment so that the cost is as small as possible.

Formulation of the mathematical model:
To formulate the linear mathematical model, we assume:

$$\boldsymbol{x_{ij}} = \begin{cases} \mathbf{1} & \boldsymbol{if\ job\ j\ was\ given\ to\ machine\ i} \\ \mathbf{0} & \boldsymbol{otherwise} \end{cases}$$

Then write the target function as follows:

$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} \pm \varepsilon_{ij}) x_{ij}$$

Conditions for machines:
Since each machine accepts only one action, we find:

$$\sum_{j=1}^{n} \boldsymbol{x_{ij}} = \mathbf{1} \quad ; \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \dots \dots , \boldsymbol{n}$$

Business terms:
Since each work is assigned to only one machine, we find:

$$\sum_{i=1}^{n} \boldsymbol{x_{ij}} = \mathbf{1} \quad ; \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots \dots , \boldsymbol{n}$$

Accordingly, the neutrosophic mathematical model is written as follows:
Find the minimum value:

$$\boldsymbol{Z} = \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} \pm \varepsilon_{ij}) \boldsymbol{x_{ij}}$$

Machine terms:

$$\sum_{j=1}^{n} \boldsymbol{x_{ij}} = \mathbf{1} \quad ; \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \dots \dots , \boldsymbol{n}$$

Business terms:

$$\sum_{i=1}^{n} \boldsymbol{x_{ij}} = \mathbf{1} \quad ; \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \dots \dots , \boldsymbol{n}$$

Example 1: (The data are classic values).

Formulation of the mathematical model for the problem of standard assignment of minimum cost:

We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table:

Business / The machines	N_1	N_2	N_3	N_4
M_1	10	9	8	7
M_2	3	4	5	6
M_3	2	1	1	2
M_4	4	3	5	6

Table No. (1) Table of Distribution cost table and classic values

To formulate the linear mathematical model:
We impose:

$$x_{ij} = \begin{cases} 1 & if\ job\ j\ was\ given\ to\ machine\ i \\ 0 & otherwise \end{cases} \quad ;\ i,j = 1,2,3,4$$

Using the problem data, we get the following objective function:

$$Z = 10x_{11} + 9x_{12} + 8x_{13} + 7x_{14} + 3x_{21} + 4x_{22} + 5x_{23} + 6x_{24} + 2x_{31} + x_{32} + x_{33} + 2x_{34} + 4x_{41} + 3x_{42} + 5x_{43} + 6x_{44}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:
Find the minimum value of the function:

$$Z = 10x_{11} + 9x_{12} + 8x_{13} + 7x_{14} + 3x_{21} + 4x_{22} + 5x_{23} + 6x_{24} + 2x_{31} + x_{32} + x_{33} + 2x_{34} + 4x_{41} + 3x_{42} + 5x_{43} + 6x_{44}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$

$$\boldsymbol{x_{21} + x_{22} + x_{23} + x_{24} = 1}$$
$$\boldsymbol{x_{31} + x_{32} + x_{33} + x_{34} = 1}$$
$$\boldsymbol{x_{11} + x_{21} + x_{31} + x_{41} = 1}$$
$$\boldsymbol{x_{12} + x_{22} + x_{32} + x_{42} = 1}$$
$$\boldsymbol{x_{13} + x_{23} + x_{33} + x_{43} = 1}$$
$$\boldsymbol{x_{14} + +x_{24} + x_{34} + x_{44} = 1}$$

Where $\boldsymbol{x_{ij}}$ it is either equal to zero or one.

In the previous model, there is some indeterminacy in the assignment process, as we do not know which machine will perform a certain work. In addition to that, we will also use neutrosophic data. We will take the cost of neutrosophic values, i.e. the cost of assignment machine i to perform work j is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$, where ε_{ij} is the indeterminacy and $\varepsilon_{ij} \in [\lambda_{\text{ij1}}, \lambda_{\text{ij2}}]$, which is any neighborhood to the value c_{ij} then the cost matrix becomes $NC_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

Example 2: (Cost is neutrosophic values):
We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table**:**

Business / The machines	$\boldsymbol{N_1}$	$\boldsymbol{N_2}$	$\boldsymbol{N_3}$	$\boldsymbol{N_4}$
$\boldsymbol{M_1}$	$\mathbf{10} + \varepsilon_{11}$	$\mathbf{9} + \varepsilon_{12}$	$\mathbf{8} + \varepsilon_{13}$	$\mathbf{7} + \varepsilon_{14}$
$\boldsymbol{M_2}$	$\mathbf{3} + \varepsilon_{21}$	$\mathbf{4} + \varepsilon_{22}$	$\mathbf{5} + \varepsilon_{23}$	$\mathbf{6} + \varepsilon_{24}$
$\boldsymbol{M_3}$	$\mathbf{2} + \varepsilon_{31}$	$\mathbf{1} + \varepsilon_{32}$	$\mathbf{1} + \varepsilon_{33}$	$\mathbf{2} + \varepsilon_{34}$
$\boldsymbol{M_4}$	$\mathbf{4} + \varepsilon_{41}$	$\mathbf{3} + \varepsilon_{42}$	$\mathbf{5} + \varepsilon_{43}$	$\mathbf{6} + \varepsilon_{44}$

Table No. (2) Table of allocation cost of neutrosophic values

Where ε_{ij} is the limitation on the costs of assignment and it can be any neighborhood of the values contained in Table No. (1)

To formulate the linear mathematical model we assume:

$$\boldsymbol{x_{ij}} = \begin{cases} \mathbf{1} & \boldsymbol{if\ job\ j\ was\ given\ to\ machine\ i} \\ \mathbf{0} & \boldsymbol{otherwise} \end{cases} \qquad ; \boldsymbol{i, j} = \mathbf{1, 2, 3, 4}$$

Using the problem data, we get the following objective function:

$$Z \in \{(\mathbf{10} + \varepsilon_{11})x_{11} + (\mathbf{9} + \varepsilon_{12})x_{12} + (\mathbf{8} + \varepsilon_{13})x_{13} + (\mathbf{7} + \varepsilon_{14})x_{14} + (\mathbf{3} + \varepsilon_{21})x_{21} + (\mathbf{4} + \varepsilon_{22})x_{22} + (\ \mathbf{5} + \varepsilon_{23})x_{23} + (\mathbf{6} + \varepsilon_{24})x_{24} + (\mathbf{2} + \varepsilon_{31})x_{31} + (\mathbf{1} + \varepsilon_{32})x_{32} + (\mathbf{1} + \varepsilon_{33})x_{33} + (\mathbf{2} + \varepsilon_{34})x_{34} + (\mathbf{4} + \varepsilon_{41})x_{41} + (\mathbf{3} + \varepsilon_{42})x_{42} + (\mathbf{5} + \varepsilon_{43})x_{43} + (\mathbf{6} + \varepsilon_{44})x_{44}\}$$

Machine terms:

$$\boldsymbol{x_{11} + x_{12} + x_{13} + x_{14} = 1}$$
$$\boldsymbol{x_{21} + x_{22} + x_{23} + x_{24} = 1}$$
$$\boldsymbol{x_{31} + x_{32} + x_{33} + x_{34} = 1}$$
$$\boldsymbol{x_{41} + x_{42} + x_{43} + x_{44} = 1}$$

Business terms:

$$\boldsymbol{x_{11} + x_{21} + x_{31} + x_{41} = 1}$$

$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:
Find the minimum value of the function:

$$Z \in \{(\mathbf{10} + \varepsilon_{11})x_{11} + (\mathbf{9} + \varepsilon_{12})x_{12} + (\mathbf{8} + \varepsilon_{13})x_{13} + (\mathbf{7} + \varepsilon_{14})x_{14} + (\mathbf{3} + \varepsilon_{21})x_{21} + (\mathbf{4} + \varepsilon_{22})x_{22} + (\ \mathbf{5} + \varepsilon_{23})x_{23} + (\mathbf{6} + \varepsilon_{24})x_{24} + (\mathbf{2} + \varepsilon_{31})x_{31} + (\mathbf{1} + \varepsilon_{32})x_{32} + (\mathbf{1} + \varepsilon_{33})x_{33} + (\mathbf{2} + \varepsilon_{34})x_{34} + (\mathbf{4} + \varepsilon_{41})x_{41} + (\mathbf{3} + \varepsilon_{42})x_{42} + (\mathbf{5} + \varepsilon_{43})x_{43} + (\mathbf{6} + \varepsilon_{44})x_{44}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

Since the number of works is equal to the number of machines, the issue is a standard assignment issue, and the optimal solution can be obtained using several methods, including the Hungarian method in this research.

This method was named after the scientist who created it, a mathematician D.Konig. Its principle depends on finding the total opportunity-cost matrix, references [16, 17].

Explanation of the method based on what was stated in the reference [18]:

This method is based on a mathematical property discovered by the scientist D.Konig,

If the cost is non-negative values, then subtracting or adding a fixed number of elements of any row or any column in the standard allocation cost matrix does not affect the optimal assignment, and specifically does not affect the optimal values x_{ij} .

The algorithm begins by identifying the smallest element in each row, and subtracting it from all the elements of the row , or by selecting the smallest element in each column and subtracting it from all the elements of that column, we get a new cost matrix that includes at least one element equal to zero in each row or column. We do the assignment process using cells with a cost equal to zero. If possible, we have obtained the optimal allocation. For this assignment, the cost elements (c_{ij}) are non-negative, so the minimum value of the objective function cannot be $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$ is less than zero.

We will use the above to find the optimal assignment for the problem in Example 2 based on the following information:

Taking the indeterminacy $\varepsilon_{ij} = \varepsilon \in [0\ ,5]$, the problem becomes:

Example 3:

We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table:

Business / The machines	N_1	N_2	N_3	N_4

M_1	$[10, 15]$	$[9, 14]$	$[8, 13]$	$[7, 12]$
M_2	$[3, 8]$	$[4, 9]$	$[5, 10]$	$[6, 11]$
M_3	$[2, 7]$	$[1, 6]$	$[1, 6]$	$[2, 7]$
M_4	$[4, 9]$	$[3, 8]$	$[5, 10]$	$[6, 11]$

Table No. (3) Table of Example data

To formulate the linear mathematical model:
We assume:

$$x_{ij} = \begin{cases} 1 & \text{if job } j \text{ was given to machine } i \\ 0 & \text{otherwise} \end{cases} \quad ; i, j = 1, 2, 3, 4$$

Using the problem data, we get the following objective function:

$$Z \in \{[10, 15]x_{11} + [9, 14]x_{12} + [8, 13]x_{13} + [7, 12]x_{14} + [3, 8]x_{21} + [4, 9]x_{22} + [5, 10]x_{23} + [6, 11]x_{24} + [2, 7]x_{31} + [1, 6]x_{32} + [1, 6]x_{33} + [2, 7]x_{34} + [4, 9]x_{41} + [3, 8]x_{42} + [5, 10]x_{43} + [6, 11]x_{44}\}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:
Find the minimum value of the function:

$$Z \in \{[10, 15]x_{11} + [9, 14]x_{12} + [8, 13]x_{13} + [7, 12]x_{14} + [3, 8]x_{21} + [4, 9]x_{22} + [5, 10]x_{23} + [6, 11]x_{24} + [2, 7]x_{31} + [1, 6]x_{32} + [1, 6]x_{33} + [2, 7]x_{34} + [4, 9]x_{41} + [3, 8]x_{42} + [5, 10]x_{43} + [6, 11]x_{44}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

Solution using the Hungarian method:
From the table Number 3 we find the following:

Business / The machines	N_1	N_2	N_3	N_4
M_1	$[10, 15]$	$[9 , 14]$	$[8 , 13]$	$[7 , 12]$
M_2	$[3 , 8]$	$[4 , 9]$	$[5 , 10]$	$[6 , 11]$
M_3	$[2 , 7]$	$[1, 6]$	$[1, 6]$	$[2 , 7]$
M_4	$[4 , 9]$	$[3 , 8]$	$[5 , 10]$	$[6 , 11]$

To form the opportunity cost matrix for the rows, we do the following:
We form the opportunity cost matrix for the rows as follows:

In the first row, the lowest cost is $[7 , 12]$ we subtract it from all the elements of the first row.

In the second row, the lowest cost is $[3 , 8]$, we subtract from all elements of the second row.

In the third row, the lowest cost is $[1, 6]$, we subtract it from all elements of the third row.

In the fourth row, the lowest cost is $[3 , 8]$, we subtract from all elements of the fourth row.

We get the opportunity cost matrix for the following rows:

Business / The machines	N_1	N_2	N_3	N_4
M_1	3	2	1	0
M_2	0	1	2	3
M_3	1	0	0	1
M_4	1	0	2	3

Table No. (4) Table of Total opportunity cost matrix table

We try to make the assignment using cells with cost equal to zero we find:
Dedicate the machine M_1 to get the job N_4done.

Dedicate the machine M_2 to get the job N_1done.

Dedicate the machine M_3 to get the job N_3 done.

Dedicate the machine M_4 to get the job N_2 done.

Thus, we have obtained the optimal assignment and the minimum cost:

$$\begin{aligned} Z \in \{&[10, 15] \times 0 + [9 , 14] \times 0 + [8 , 13] \times 0 + [7 , 12] \times 1 + [3 , 8] \times 1 + [4 , 9] \times 0 + [5 , 10] \times 0 \\ &+ [6 , 11] \times 0 + [2 , 7] \times 0 + [1, 6] \times 0 + [1 , 6] \times 1 + [2 , 7] \times 0 + [4 , 9] \times 0 \\ &+ [3 , 8] \times 1 + [5 , 10] \times 0 + [6 , 11] \times 0\} \end{aligned}$$

$$Z \in [7 , 12] + [3 , 8] + [1 , 6] + [3 , 8] = [14 , 34]$$

That is, the optimal allocation is:
Dedicate the machine M_1 to get the job N_4 done.

Dedicate the machine $M_2$to get the job N_1done.

Dedicate the machine M_3 to get the job N_3 done.

Dedicate the machine M_4 to get the job N_2 done.

The cost:

$$Z \in [14\ ,34].$$

The Hungarian method is summarized based on what was stated in the reference [17]:

1- We determine the smallest element in each row and subtract it from the rest of the elements of that row. Thus, we get a new matrix that is the opportunity cost matrix for the rows.

2- We determine the smallest element in each column of the opportunity cost matrix for the rows and Subtract it from the elements of that column. Thus, we get the total opportunity cost matrix.

3- We draw as few horizontal and vertical straight lines as possible to pass through all zero elements of the total opportunity cost matrix.

4- If the number of the straight lines drawn passing through the zero elements is equal to the number of rows (columns). Then we say that we have reached the optimal assignment**.**

5- If the number of straight lines passing through the zero elements is less than the number of rows (Columns).Then we move on to the next step.

6- We choose the lesser element from the elements that no straight line passed through and subtract it from all the elements that no straight line. Then we add it to all the elements that lie at the intersection of two lines. The elements that the straight lines passed through remain the same without any change. We get a new matrix that we call it the modified total opportunity cost matrix.

7- We draw vertical and horizontal straight lines passing through all the zero elements in the modified total opportunity cost matrix. If the number of straight lines drawn passing through the zero elements is equal to the number of rows (columns). Then we have reached the optimal assignment solution.

8- If the number of the lines is not equal to the number of rows (columns). We go back to step (1), we Repeat the previous steps until reaching the optimal assignment that makes the total opportunity cost equal to zero.

Example 4:

We have three machines $M_1\ , M_2, M_3$ and three works $N_1,\ N_2, N_3$ and each work is done completely using any of the three machines and in return each machine can perform any of the three works as well. What is required is to allocate these mechanisms to the existing works so that we get the optimal assignment , i.e. the assignment that gives us here the minimum total cost, bearing in mind that the costs of completing these works vary according to the different mechanisms implemented for these works, and this cost is related to the performance of each work and is shown in the following table:

Business / The machines	N_1	N_2	N_3
M_1	$[20, 23]$	$[27, 30]$	$[30, 33]$

M_2	$[10,13]$	$[18,21]$	$[16,19]$
M_3	$[14,17]$	$[16,19]$	$[12,15]$

Table No. (5) Table of allocation cost neutrosophic values example data

Mathematical model:

Find the minimum value of the function:

$$Z \in \{[\mathbf{20,23}]x_{11} + [\mathbf{27,30}]x_{12} + [\mathbf{30,33}]x_{13} + [\mathbf{10,13}]x_{21} + [\mathbf{18,21}]x_{22} + [\mathbf{16,19}]x_{23} + [\mathbf{14,17}]x_{31} + [\mathbf{16,19}]x_{32} + [\mathbf{12,15}]x_{33}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} = 1$$
$$x_{21} + x_{22} + x_{23} = 1$$
$$x_{31} + x_{32} + x_{33} = 1$$
$$x_{11} + x_{21} + x_{31} = 1$$
$$x_{12} + x_{22} + x_{32} = 1$$
$$x_{13} + x_{23} + x_{33} = 1$$

Where x_{ij} it is either equal to zero or one.

Finding the optimal assignment using the Hungarian method:

We take Table No. (5)

Business / The machines	N_1	N_2	N_3
M_1	$[20,23]$	$[27,30]$	$[30,33]$
M_2	$[10,13]$	$[18,21]$	$[16,19]$
M_3	$[14,17]$	$[16,19]$	$[12,15]$

1.

In the first row, the lowest cost is $[20,23]$, which we subtract from all the elements of the first row.

In the second row, the least cost is $[10,13]$ and we subtract it from all the elements of the second row.

In the third row, the lowest cost is $[12,15]$, which we subtract from all the elements of the third row.

We get the opportunity cost table for the following rows:

Business / The machines	N_1	N_2	N_3
M_1	**0**	**7**	**10**
M_2	**0**	**8**	**6**
M_3	**2**	**4**	**0**

Table No. (6) Table of opportunity cost matrix for lines

2.

In the first column, the lowest cost is $\mathbf{0}$ we subtract it from all the items in the first column.

In the second column, the lowest cost is $\mathbf{4}$ we subtract it from all the items in the second column.

In the third column, the lowest cost is 4 we subtract it from all the items in the third column.

We get the table:

Business / The machines	N_1	N_2	N_3
M_1	0	3	10
M_2	0	4	6
M_3	2	0	0

Table No. (7) Table of total opportunity cost matrix

3. We draw as few horizontal and vertical straight lines as possible to pass through all zero elements of the total opportunity cost matrix.
4. If the number of straight lines drawn passing through the zero elements is equal to the number of rows (columns), then we say that we have reached the optimal assignment.
5. If the number of straight lines passing through the zero elements is less than the number of rows or columns, then we move on to the third step.

Business / The machines	N_1	N_2	N_3
M_1	0	3	10
M_2	0	4	6
M_3	2	0	0

Table No. (7) Total Opportunity Cost Matrix

We Note that the number of lines is less than the number of rows (columns). So we go to (6).

6.

a. We choose the lowest element through which no straight line has passed. Smallest element is (3).

b. We subtract it from the rest of the elements through which none of the lines drawn are passed.

c. We add it to all the elements at the intersection of two straight lines drawn.

d. Elements through which straight lines pass remain unchanged.

e. We draw vertical and horizontal straight lines passing through all zero elements of the adjusted total opportunity cost matrix, and we get:

Business / The machines	N_1	N_2	N_3
M_1	0	0	7
M_2	0	1	3
M_3	5	0	0

Table No. (8) Modified Total Opportunity Cost Matrix

7. If the number of drawn lines is equal to the number of rows (columns), then we have reached the optimal assignment.

   From Table No. (8), we note that the number of drawn lines is equal to the number of rows, meaning that we have obtained the optimal assignment, which is as follows:

   In the third column, we have zero only in the cell that is $\boldsymbol{M_3N_3}$, so the third machine is used to perform the third work. $\boldsymbol{M_3} \rightarrow \boldsymbol{N_3}$

   We delete the third row and the third column, we get the following table:

Business / The machines	$\boldsymbol{N_1}$	$\boldsymbol{N_2}$
$\boldsymbol{M_1}$	**0**	**0**
$\boldsymbol{M_2}$	**0**	**1**

In the same way. The first machine is used to perform the second work, $\boldsymbol{M_1} \rightarrow \boldsymbol{N_2}$.

The second machine used to perform the first work, $\boldsymbol{M_2} \rightarrow \boldsymbol{N_1}$.

The minimum total cost is:

$$Z \in \{[\mathbf{20}, \mathbf{23}] \times 0 + [\mathbf{27}, \mathbf{30}] \times 1 + [\mathbf{30}, \mathbf{33}] \times 0 + [\mathbf{10}, \mathbf{13}] \times 1 + [\mathbf{18}, \mathbf{21}] \times 0 + [\mathbf{16}, \mathbf{19}] \times 0 + [\mathbf{14}, \mathbf{17}] \times 0 + [\mathbf{16}, \mathbf{19}] \times 0 + [\mathbf{12}, \mathbf{15}] \times 1\}$$

$$Z \in [\mathbf{27}, \mathbf{30}] + [\mathbf{10}, \mathbf{13}] + [\mathbf{12}, \mathbf{15}] = [\mathbf{49}, \mathbf{58}]$$

That is, the optimal assignment is:

Machine $\boldsymbol{M_1}$ is assigned to do $\boldsymbol{N_2}$work.

Machine $\boldsymbol{M_2}$ is assigned to perform $\boldsymbol{N_1}$work.

Machine $\boldsymbol{M_3}$ is assigned to perform $\boldsymbol{N_3}$work.

The cost:

$$Z \in [\mathbf{49}\ , \mathbf{58}]$$

Important notes:

When we study the issues of optimal assignment, we come across the following:

1. **There is two types of assignment issues according to the objective function:**

The first type:

It is required to obtain a minimum value for the objective function, knowing that the cost of completing any work by a machine is a known value, and therefore the total cost is as small as possible.

The second type:

What is required is to obtain a maximum value for the objective function, and here it is known that we have the profit accruing from the completion of any work, by a machine, and therefore the total cost is the greatest possible.

In this type, we transform matter to the first type according to the following steps:

a. Multiply the elements of the cost matrix by the value (-1).
b. If some elements of the matrix are negative, we add enough positive numbers to the corresponding rows and columns so that all elements become non-negative.
c. Then the issue becomes a matter of assignment and we want to make the objective function smaller and all elements of the cost matrix are non-negative, so we can apply the Hungarian method.

2. **There are two types of customization issues according to the number of businesses and the number of machines**:

   In this research, we studied the standard assignment issues. It should be noted that there are non-standard assignment issues. In these issues, the number of works is not equal to the number of machines , and here we convert them into standard issues by adding a fictitious work or a fictitious machine and make the cost equal to zero So that the objective function does not change, then we build the mathematical model as it is in the standard models.

Conclusion and results:

Due to the importance of the issue of assignment in our practical life, it received great attention from scholars and researchers, as this issue was addressed on the basis that it is a transfer issue, and special methods were followed to solve transfer issues to find the optimal assignment ,but we find that many references deal with these issues according to the Hungarian method that It was found to be addressed. And through our study of these issues, we find that the use of the Hungarian method greatly helps to find the optimal assignment with less effort than other methods. On the other hand according to the results obtained when using the data Neutrosophic values , in issues of assignment and in all practical matters that are affected by the conditions surrounding the work environment so that decision makers can make appropriate decisions for all circumstances that achieve companies and institutions the greatest profit and the lowest cost.

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