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On The Symbolic 2-Plithogenic Split-Complex Real Square Matrices and Their Algebraic Properties

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Abstract:

The objective of this paper is to study for the first time the concept of square real matrices with symbolic 2-plithogenic split-complex entries. Many of their algebraic properties will be discussed and handled, where we find the formula of computing inverses, exponents, and powers of these matrices by building a ring isomorphism between the ring of split-complex symbolic 2-plithogenic matrices and the direct product of the symbolic 2-plithogenic matrices with itself. Also, we give the interested reader many related examples to clarify the validity of our work.

Keywords: split-complex number, symbolic 2-plithogenic number, split-complex 2-plithogenic matrix.

Introduction

The concept of symbolic 2-plithogenic rings is considered as a generalization of algebraic rings [1], and these rings were used by many authors to generalize classical algebraic structures such as vector space, modules, and functions into novel symbolic 2-plithogenic and 3-plithogenic versions, see [2-9].

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Symbolic 2-plithogenic matrices and other types have been studied in [10], where many results were obtained such as diagonalizations, and eigenvalues.

Symbolic 2-plithogenic split-complex numbers were defined as a combination between split-complex numbers, and symbolic 2-plithogenic numbers, for more details about split-complex structures and their applications, see [11-13]. For similar results about neutrosophic matrices and n-plithogenic matrices check [14-19].

In this paper, we define symbolic 2-plithogenic split-complex matrices, and we present many elementary properties of these matrices, especially those are related to classical matrix theory and applications.

Main Discussion

Definition.

Let $A = (a_{ij})$ be a matrix, it is called symbolic 2-plithogenic split-complex if and only if:

$a_{ij} = \left(a_{ij}{}^{(0)} + a_{ij}{}^{(1)}P_1 + a_{ij}{}^{(2)}P_2\right) + \left(b_{ij}{}^{(0)} + b_{ij}{}^{(1)}P_1 + b_{ij}{}^{(2)}P_2\right)J$, where $a_{ij}{}^{(k)}, b_{ij}{}^{(k)} \in R, J^2 = 1,\ P_i \times P_j = P_{max(i,j)}, P_i{}^2 = P_i$.

Example.

The matrix $A = \begin{pmatrix} 1 + 2P_1 + P_2 + J(1 - P_2) & (P_1 + P_2)J \\ (2 - P_2) + J(3 + P_1 + P_2) & 5 - (P_1 + 4P_2)J \end{pmatrix}$ is a 2×2 symbolic 2-plithogenic split-complex matrix.

$$A = \begin{pmatrix} 1 + 2P_1 + P_2 & 0 \\ 2 - P_2 & 5 \end{pmatrix} + J\begin{pmatrix} 1 - P_2 & P_1 + P_2 \\ 3 + P_1 + P_2 & -(P_1 + 4P_2) \end{pmatrix}$$

Remark.

Any symbolic 2-plithogenic split-complex matrix can be written as follows $A = T + KJ$; T, K are two symbolic 2-plithogenic real matrices.

Also, it can be written as follows:

$A = (A_0 + A_1P_1 + A_2P_2) + J(B_0 + B_1P_1 + B_2P_2)$; A_i, B_i are classical square real matrices.

The matrix presented in the previous example can be written as:

$$\begin{pmatrix} 1 & 0 \\ 2 & 5 \end{pmatrix} + P_1\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} + P_2\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} + J\left[\begin{pmatrix} 1 & 0 \\ 3 & 0 \end{pmatrix} + P_1\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} + P_2\begin{pmatrix} -1 & 1 \\ 1 & -4 \end{pmatrix}\right]$$

Remark.

We denote the ring of all symbolic 2-plithogenic split-complex matrices by $2-SP_{MS}$.

$(2-SP_{MS},+,.)$ is a ring.

Definition.

Let $X=M_0+N_0J, Y=M_1+N_1J;\ M_0,N_0,M_1,N_1\in 2-P_M$, then:

$X+Y=(M_0+M_1)+(N_0+N_1)J.$

$X.Y=(M_0M_1+N_0N_1)+(M_0N_1+N_0M_1)J.$

Example.

Take:

$$X=\begin{pmatrix}1+P_1 & P_2\\ P_1 & -P_1+P_2\end{pmatrix}+J\begin{pmatrix}2+P_1-P_2 & 3P_1\\ P_1-P_2 & 2P_2\end{pmatrix}=M_0+N_0J$$

$$Y=\begin{pmatrix}1+P_1-P_2 & 2-P_2\\ 1+P_2 & 1+P_1\end{pmatrix}+J\begin{pmatrix}2-P_2 & P_1-P_2\\ 1+P_1 & P_2\end{pmatrix}=M_1+N_1J$$

$$X+Y=\begin{pmatrix}2+2P_1-P_2 & 2-P_1+P_2\\ 1+P_1+P_2 & 1+P_2\end{pmatrix}+J\begin{pmatrix}4+P_1-2P_2 & 4P_1-P_2\\ 1+2P_1-P_2 & 3P_2\end{pmatrix}$$

$$M_0M_1=\begin{pmatrix}1+P_1 & P_2\\ P_1 & -P_1+P_2\end{pmatrix}\begin{pmatrix}1+P_1-P_2 & 2-P_2\\ 1+P_2 & 1+P_1\end{pmatrix}=\begin{pmatrix}1+3P_1 & 2+2P_2\\ P_1 & -P_1+2P_2\end{pmatrix}$$

$$N_0N_1=\begin{pmatrix}2+P_1-P_2 & 3P_1\\ P_1-P_2 & 2P_2\end{pmatrix}\begin{pmatrix}2-P_2 & P_1-P_2\\ 1+P_1 & P_2\end{pmatrix}=\begin{pmatrix}4+8P_1-4P_2 & 3P_1\\ 2P_1+2P_2 & P_1+P_2\end{pmatrix}$$

$$M_0N_1=\begin{pmatrix}1+P_1 & P_2\\ P_1 & -P_1+P_2\end{pmatrix}\begin{pmatrix}2-P_2 & P_1-P_2\\ 1+P_1 & P_2\end{pmatrix}=\begin{pmatrix}2+2P_1 & 2P_1-P_2\\ P_2 & P_1-P_2\end{pmatrix}$$

$$N_0M_1=\begin{pmatrix}2+P_1-P_2 & 3P_1\\ P_1-P_2 & 2P_2\end{pmatrix}\begin{pmatrix}1+P_1-P_2 & 2-P_2\\ 1+P_2 & 1+P_1\end{pmatrix}=\begin{pmatrix}1+7P_1-P_2 & 4+5P_1-P_2\\ 2P_1+2P_2 & P_1+2P_2\end{pmatrix}$$

So that:

$$X.Y=\begin{pmatrix}5+11P_1-4P_2 & 2+3P_1+2P_2\\ 3P_1+2P_2 & 3P_2\end{pmatrix}+J\begin{pmatrix}4+9P_1-P_2 & 2+3P_1+2P_2\\ 2P_1+3P_2 & 2P_1+P_2\end{pmatrix}$$

Theorem.

Let $2-SP_{MS}$ be the ring of all $n\times n$ symbolic 2-plithogenic split-complex matrices, let $2-P_M$ be the ring of all $n\times n$ square symbolic 2-plithogenic real matrices, then:

$$f:2-SP_{MS}\to 2-P_M\times 2-P_M$$

Such that:

$f(M+NJ)=(M+N,M-N); M,N \in 2-P_M$ is a ring isomorphism.

Proof.

For $M_0+N_0J=M_1+N_1J$, we have:

$M_0=M_1, N_0=N_1$, thus $(M_0+N_0, M_0-N_0)=(M_1+N_1, M_1-N_1)$,

hence $f(M_0+N_0J)=f(M_1+N_1J)$.

For $X=M_0+N_0J, Y=M_1+N_1J \in 2-SP_{MS}$,

we have:

$f(X+Y)=(M_0+N_0+M_1+N_1, M_0+M_1-N_0-N_1)=f(X)+f(Y)$.

$f(X.Y)=f[(M_0M_1+N_0N_1)+(M_0N_1+N_0M_1)J]=[(M_0+N_0)(M_1+N_1),(M_0-N_0)(M_1-N_1)]=f(X).f(Y)$.

$$f(X)=0 \Leftrightarrow \begin{cases} M_0+N_0=0 \\ M_0-N_0=0 \end{cases} \Leftrightarrow M_0=N_0=0$$

Thus, $ker(f)=\{0\}$.

For any arbitrary element $(M,N) \in 2-P_M \times 2-P_M$, there exists $X=\frac{1}{2}(M+N)+\frac{1}{2}(M-N)J \in 2-P_M$, such that $f(X)=(M,N)$, so that f is a ring isomorphism.

Remark.

The inverse isomorphism is:

$f^{-1}: 2-P_M \times 2-P_M \to 2-SP_{MS}$; $f^{-1}(M,N)=\frac{1}{2}(M+N)+\frac{1}{2}(M-N)J$.

Example.

Consider:

$$X=\begin{pmatrix} 1+P_1+P_2 & 3-P_2 & 1+P_1 \\ P_1-P_2 & 5+P_1 & P_2 \\ P_1+P_2 & 2P_1 & -P_2 \end{pmatrix}+J\begin{pmatrix} 1+P_1-P_2 & 1 & 1 \\ 4+2P_1-P_2 & P_1 & 2P_2 \\ 2P_1-P_2 & P_1 & 5P_1 \end{pmatrix}=M+NJ,$$

then:

$$f(X)=\left(\begin{pmatrix} 2+2P_1 & 4-P_2 & 1+P_1 \\ 4+3P_1-2P_2 & 5+2P_1 & 3P_2 \\ 2P_1 & 3P_1 & 5P_1-P_2 \end{pmatrix},\begin{pmatrix} 2P_2 & 2-P_2 & -1+P_1 \\ -4-P_1 & 5 & -P_2 \\ -P_1+2P_2 & P_1 & -5P_1-P_2 \end{pmatrix}\right)$$

Results from the isomorphism.

Let $X=M+NJ \in 2-SP_M; M=M_0+M_1P_1+M_2P_2, N=N_0+N_1P_1+N_2P_2 \in 2-P_M$,

then:

1). X is invertible if and only if $M+N, M-N$ are invertible, which is equivalent to:

$M_0+N_0, M_0-N_0, (M_0+M_1)+(N_0+N_1), (M_0+M_1)-(N_0+N_1)$,

$(M_0+M_1+M_2)+(N_0+N_1+N_2), (M_0+M_1+M_2)+(N_0+N_1-N_2)$,

Are invertible matrices.

2). If X is inverible, then:

$$X^{-1}=\frac{1}{2}[(M+N)^{-1}+(M-N)^{-1}]+\frac{1}{2}[(M+N)^{-1}-(M-N)^{-1}]J$$

Where:

$$(M+N)^{-1}=(M_0+N_0)^{-1}+[(M_0+M_1+N_0+N_1)^{-1}-(M_0+N_0)^{-1}]P_1 \\ +[(M_0+M_1+M_2+N_0+N_1+N_2)^{-1}-(M_0+M_1+N_0+N_1)^{-1}]P_2$$

$$(M-N)^{-1}=(M_0-N_0)^{-1}+[(M_0+M_1-N_0-N_1)^{-1}-(M_0-N_0)^{-1}]P_1+ \\ [(M_0+M_1+M_2-N_0-N_1-N_2)^{-1}-(M_0+M_1-N_0-N_1)^{-1}]P_2.$$

3). $detX=\frac{1}{2}[det(M+N)+det(M-N)]+\frac{1}{2}[det(M+N)-det(M-N)]J$

$$det(M+N)=det(M_0+N_0)+[det(M_0+M_1+N_0+N_1)-det(M_0+N_0)]P_1+ \\ [det(M_0+M_1+M_2+N_0+N_1+N_2)-det(M_0+M_1+N_0+N_1)]P_2.$$

$$det(M-N)=det(M_0-N_0)+[det(M_0+M_1-N_0-N_1)-det(M_0-N_0)]P_1+ \\ [det(M_0+M_1+M_2-N_0-N_1-N_2)-det(M_0+M_1-N_0-N_1)]P_2.$$

4). $X^n=\frac{1}{2}[(M+N)^n+(M-N)^n]+\frac{1}{2}[(M+N)^n-(M-N)^n]J$

$$(M+N)^n=(M_0+N_0)^n+[(M_0+M_1+N_0+N_1)^n-(M_0+N_0)^n]P_1+[(M_0+M_1+ \\ M_2+N_0+N_1+N_2)^n-(M_0+M_1+N_0+N_1)^n]P_2.$$

$$(M-N)^n=(M_0-N_0)^n+[(M_0+M_1-N_0-N_1)^n-(M_0-N_0)^n]P_1+[(M_0+M_1+ \\ M_2-N_0-N_1-N_2)^n-(M_0+M_1-N_0-N_1)^n]P_2.$$

Example.

Consider the following 2×2 symbolic 2-plithogenic split-complex matrix:

$$X=\begin{pmatrix} 2+\frac{3}{2}P_1+4P_2 & \frac{1}{2} \\ \frac{3}{2}P_1 & \frac{3}{2}+\frac{1}{2}P_1+\frac{1}{2}P_2 \end{pmatrix}+J\begin{pmatrix} 1-\frac{7}{2}P_1+2P_2 & \frac{1}{2}-P_1+P_2 \\ \frac{3}{2}P_1-3P_2 & \frac{1}{2}-\frac{3}{2}P_1-\frac{1}{2}P_2 \end{pmatrix}=M+$$

NJ, where:

$$M=\begin{pmatrix} 2+\frac{3}{2}P_1+4P_2 & \frac{1}{2} \\ \frac{3}{2}P_1 & \frac{3}{2}+\frac{1}{2}P_1+\frac{1}{2}P_2 \end{pmatrix}, N=\begin{pmatrix} 1-\frac{7}{2}P_1+2P_2 & \frac{1}{2}-P_1+P_2 \\ \frac{3}{2}P_1-3P_2 & \frac{1}{2}-\frac{3}{2}P_1-\frac{1}{2}P_2 \end{pmatrix}$$

$$M+N=\begin{pmatrix}3-P_1+6P_2 & 1-P_1+P_2\\ 3P_1-3P_2 & 2-P_1\end{pmatrix}$$

$$M-N=\begin{pmatrix}1+2P_1+2P_2 & P_1-P_2\\ 3P_2 & 1+2P_1+P_2\end{pmatrix}$$

We write:

$M+N=K_0+K_1P_1+K_2P_2, M-N=L_0+L_1P_1+L_2P_2$, where:

$$K_0=\begin{pmatrix}3 & 1\\ 0 & 2\end{pmatrix}, K_1=\begin{pmatrix}-5 & -1\\ 3 & -1\end{pmatrix}, K_2=\begin{pmatrix}6 & 1\\ -3 & 0\end{pmatrix}, K_0+K_1=\begin{pmatrix}-2 & 0\\ 3 & 1\end{pmatrix}, K_0+K_1+K_2=\begin{pmatrix}4 & 1\\ 0 & 1\end{pmatrix}$$

$$L_0=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, L_1=\begin{pmatrix}2 & 1\\ 0 & 2\end{pmatrix}, L_2=\begin{pmatrix}2 & -1\\ 3 & 1\end{pmatrix}, L_0+L_1=\begin{pmatrix}3 & 1\\ 0 & 3\end{pmatrix}, L_0+L_1+L_2=\begin{pmatrix}5 & 0\\ 3 & 4\end{pmatrix}$$

$det(M+N)=det(K_0)+[det(K_0+K_1)-det(K_0)]P_1+[det(K_0+K_1+K_2)-det(K_0+K_1)]P_2=6+(-2-6)P_1+(4+2)P_2=6-8P_1+6P_2.$

$det(M-N)=det(L_0)+[det(L_0+L_1)-det(L_0)]P_1+[det(L_0+L_1+L_2)-det(L_0+L_1)]P_2=1+(9-1)P_1+(20-9)P_2=1+8P_1+11P_2.$

$$detX=\frac{1}{2}[det(M+N)+det(M-N)]+\frac{1}{2}[det(M+N)-det(M-N)]J$$
$$=\frac{1}{2}(7+17P_2)+\frac{1}{2}J(5-16P_1-5P_2)$$
$$=\left(\frac{7}{2}+\frac{17}{2}P_2\right)+J\left(\frac{5}{2}-8P_1-\frac{5}{2}P_2\right)$$

now, let's find the inverse of X:

$$K_0^{-1}=-\frac{1}{6}\begin{pmatrix}2 & -1\\ 0 & 3\end{pmatrix}=\begin{pmatrix}\frac{1}{3} & \frac{1}{6}\\ 0 & \frac{1}{2}\end{pmatrix}, (K_0+K_1)^{-1}=-\frac{1}{2}\begin{pmatrix}1 & 0\\ -3 & -2\end{pmatrix}=\begin{pmatrix}-\frac{1}{2} & 0\\ \frac{3}{2} & 1\end{pmatrix}$$

$$(K_0+K_1+K_2)^{-1}=\frac{1}{4}\begin{pmatrix}1 & -1\\ 0 & 4\end{pmatrix}=\begin{pmatrix}\frac{1}{4} & -\frac{1}{4}\\ 0 & 1\end{pmatrix}$$

$$L_0^{-1}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, (L_0+L_1)^{-1}=\frac{1}{9}\begin{pmatrix}3 & -1\\ 0 & 3\end{pmatrix}=\begin{pmatrix}\frac{1}{3} & -\frac{1}{9}\\ 0 & \frac{1}{3}\end{pmatrix}$$

$$(L_0+L_1+L_2)^{-1}=\frac{1}{20}\begin{pmatrix}4 & 0\\ -3 & 5\end{pmatrix}=\begin{pmatrix}\frac{1}{5} & 0\\ -\frac{3}{20} & \frac{1}{4}\end{pmatrix}$$

$$(M+N)^{-1} = (K_0)^{-1} + [(K_0+K_1)^{-1} - (K_0)^{-1}]P_1 + [(K_0+K_1+K_2)^{-1} - (K_0+K_1)^{-1}]P_2$$

$$= \begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{-5}{6} & \frac{1}{6} \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix} P_1 + \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{3}{2} & 0 \end{pmatrix} P_2$$

$$= \begin{pmatrix} \frac{1}{3} - \frac{5}{6}P_1 + \frac{3}{4}P_2 & \frac{1}{6} + \frac{1}{6}P_1 - \frac{1}{4}P_2 \\ \frac{3}{2}P_1 - \frac{3}{2}P_2 & \frac{1}{2} + \frac{1}{2}P_1 \end{pmatrix}$$

$$(M-N)^{-1} = (L_0)^{-1} + [(L_0+L_1)^{-1} - (L_0)^{-1}]P_1 + [(L_0+L_1+L_2)^{-1} - (L_0+L_1)^{-1}]P_2$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{-2}{3} & \frac{-1}{9} \\ 0 & \frac{-2}{3} \end{pmatrix} P_1 + \begin{pmatrix} -\frac{2}{15} & \frac{1}{9} \\ -\frac{3}{20} & -\frac{1}{12} \end{pmatrix} P_2$$

$$= \begin{pmatrix} 1 - \frac{2}{3}P_1 - \frac{2}{15}P_2 & -\frac{1}{9}P_1 + \frac{1}{9}P_2 \\ -\frac{3}{20}P_2 & 1 - \frac{2}{3}P_1 - \frac{1}{12}P_2 \end{pmatrix}$$

$$X^{-1} = \frac{1}{2}[(M+N)^{-1} + (M-N)^{-1}] + \frac{1}{2}[(M+N)^{-1} - (M-N)^{-1}]J$$

$$= \frac{1}{2}\begin{pmatrix} \frac{4}{3} - \frac{3}{2}P_1 + \frac{37}{60}P_2 & -\frac{1}{6} + \frac{1}{18}P_1 - \frac{5}{36}P_2 \\ \frac{3}{2}P_1 - \frac{33}{20}P_2 & \frac{3}{2} - \frac{1}{6}P_1 - \frac{1}{12}P_2 \end{pmatrix}$$

$$+ \frac{1}{2}J\begin{pmatrix} \frac{-3}{2} - \frac{1}{6}P_1 + \frac{53}{60}P_2 & -\frac{1}{6} + \frac{5}{18}P_1 - \frac{13}{36}P_2 \\ \frac{3}{2}P_1 - \frac{27}{20}P_2 & -\frac{1}{2} + \frac{7}{6}P_1 + \frac{1}{12}P_2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{3} - \frac{3}{4}P_1 + \frac{37}{120}P_2 & -\frac{1}{12} + \frac{1}{36}P_1 - \frac{5}{72}P_2 \\ \frac{3}{4}P_1 - \frac{33}{40}P_2 & \frac{3}{4} - \frac{1}{12}P_1 - \frac{1}{24}P_2 \end{pmatrix}$$

$$+ J\begin{pmatrix} \frac{-1}{3} - \frac{1}{12}P_1 + \frac{53}{120}P_2 & -\frac{1}{12} + \frac{5}{36}P_1 - \frac{13}{72}P_2 \\ \frac{3}{4}P_1 - \frac{27}{40}P_2 & -\frac{1}{4} + \frac{7}{12}P_1 + \frac{1}{24}P_2 \end{pmatrix}$$

As an additional application of the matrix ring isomorphism between $2-SP_M$ and $2-P_M \times 2-P_M$ is to find all possible eigenvalues/vectors for the symbolic 2-plithogenic split-complex matrix.

To find the eigenvalue for a symbolic 2-plithogenic split-complex matrix, we must find all symbolic 2-plithogenic eigen values of $M+N, M-N$.

For the matrix defined in the previous example, we can see:

The eigenvalues of K_0 are $\{3,2\}=Q_0$.

The eigenvalues of K_0+K_1 are $\{-2,1\}=Q_1$.

The eigenvalues of $K_0+K_1+K_2$ are $\{4,1\}=Q_2$.

The eigenvalues of $M+N$ are:

$Q=\{a_0+(a_1-a_0)P_1+(a_2-a_1)P_2; a_i \in Q_i\}=\{3-5P_1+6P_2, 3-5P_1+3P_2, 3-2P_1+3P_2, 3-2P_1, 2-4P_1+6P_2, 2-4P_1+3P_2, 2-P_1+3P_2, 2-P_1\}$.

The eigenvalue of L_0 is $\{1\}=s_0$.

The eigenvalue of L_0+L_1 is $\{3\}=S_1$.

The eigenvalues of $L_0+L_1+L_2$ are $\{5,4\}=S_2$.

The eigenvalues of $M-N$ are:

$S=\{b_0+(b_1-b_0)P_1+(b_2-b_1)P_2; b_i \in S_i\}=\{1+2P_1+2P_2, 1+2P_1+P_2\}$

The eigen values of the duplet $(M+N, M-N)$ are:

$\{(3-5P_1+6P_2, 1+2P_1+2P_2), (3-5P_1+6P_2, 1+2P_1+P_2), (3-5P_1+3P_2, 1+2P_1+P_2), (3-5P_1+3P_2, 1+2P_1+2P_2), (3-2P_1+3P_2, 1+2P_1+2P_2), (3-2P_1+3P_2, 1+2P_1+P_2), (2-P_1, 1+2P_1+2P_2), (2-P_1, 1+2P_1+P_2), (2-4P_1+6P_2, 1+2P_1+2P_2), (2-4P_1+6P_2, 1+2P_1+P_2), (2-4P_1+3P_2, 1+2P_1+2P_2), (2-P_1+3P_2, 1+2P_1+2P_2), (2-P_1+3P_2, 1+2P_1+P_2), (3-2P_1, 1+2P_1+2P_2), (3-2P_1, 1+2P_1+P_2)\}$.

We put:

$T_1=(3-5P_1+6P_2, 1+2P_1+2P_2)$

$f^{-1}(T_1)=\frac{1}{2}(4-3P_1+8P_2)+\frac{1}{2}(2-7P_1+4P_2)J=\left(2-\frac{3}{2}P_1+4P_2\right)+\left(1-\frac{7}{2}P_1+2P_2\right)J$.

$T_2=(3-5P_1+6P_2, 1+2P_1+P_2)$

$f^{-1}(T_2)=\frac{1}{2}(4-3P_1+7P_2)+\frac{1}{2}(2-7P_1+5P_2)J=\left(2-\frac{3}{2}P_1+\frac{7}{2}P_2\right)+\left(1-\frac{7}{2}P_1+\frac{5}{2}P_2\right)J$.

$T_3=(3-2P_1+3P_2, 1+2P_1+2P_2)$

$f^{-1}(T_3)=\frac{1}{2}(4+5P_2)+\frac{1}{2}(2-4P_1+P_2)J=\left(2+\frac{5}{2}P_2\right)+\left(1-2P_1+\frac{1}{2}P_2\right)J.$

$T_4=(3-2P_1+3P_2,1+2P_1+P_2)$

$f^{-1}(T_4)=\frac{1}{2}(4+4P_2)+\frac{1}{2}(2-4P_1+2P_2)J=(2+2P_2)+(1-2P_1+P_2)J.$

$T_5=(3-5P_1+3P_2,1+2P_1+2P_2)$

$f^{-1}(T_5)=\frac{1}{2}(4-3P_1+5P_2)+\frac{1}{2}(2-7P_1+P_2)J=\left(2-\frac{3}{2}P_1+\frac{5}{2}P_2\right)+\left(1-\frac{7}{2}P_1+\frac{1}{2}P_2\right)J.$

$T_6=(3-5P_1+3P_2,1+2P_1+P_2)$

$f^{-1}(T_6)=\frac{1}{2}(4-3P_1+4P_2)+\frac{1}{2}(2-7P_1+2P_2)J=\left(2-\frac{3}{2}P_1+2P_2\right)+\left(1-\frac{7}{2}P_1+P_2\right)J.$

$T_7=(2-P_1,1+2P_1+2P_2)$

$f^{-1}(T_7)=\frac{1}{2}(3+P_1+2P_2)+\frac{1}{2}(1-3P_1-2P_2)J=\left(\frac{3}{2}+\frac{1}{2}P_1+P_2\right)+\left(\frac{1}{2}-\frac{3}{2}P_1-P_2\right)J.$

$T_8=(2-P_1,1+2P_1+P_2)$

$f^{-1}(T_8)=\frac{1}{2}(3+P_1+P_2)+\frac{1}{2}(1-3P_1-P_2)J=\left(\frac{3}{2}+\frac{1}{2}P_1+\frac{1}{2}P_2\right)+\left(\frac{1}{2}-\frac{3}{2}P_1-\frac{1}{2}P_2\right)J.$

$T_9=(2-4P_1+6P_2,1+2P_1+2P_2)$

$f^{-1}(T_9)=\frac{1}{2}(3-2P_1+8P_2)+\frac{1}{2}(1-6P_1+4P_2)J=\left(\frac{3}{2}-P_1+4P_2\right)+\left(\frac{1}{2}-3P_1+2P_2\right)J.$

$T_{10}=(2-4P_1+6P_2,1+2P_1+P_2)$

$f^{-1}(T_{10})=\frac{1}{2}(3-2P_1+7P_2)+\frac{1}{2}(1-6P_1+5P_2)J=\left(\frac{3}{2}-P_1+\frac{7}{2}P_2\right)+\left(\frac{1}{2}-3P_1+\frac{5}{2}P_2\right)J.$

$T_{11}=(2-4P_1+6P_2,1+2P_1+2P_2)$

$f^{-1}(T_{11})=\frac{1}{2}(3-2P_1+5P_2)+\frac{1}{2}(1-6P_1+P_2)J=\left(\frac{3}{2}-P_1+\frac{5}{2}P_2\right)+\left(\frac{1}{2}-3P_1+\frac{1}{2}P_2\right)J.$

$T_{12}=(2-4P_1+3P_2,1+2P_1+P_2)$

$f^{-1}(T_{12})=\frac{1}{2}(3-2P_1+4P_2)+\frac{1}{2}(1-6P_1+2P_2)J=\left(\frac{3}{2}-P_1+2P_2\right)+\left(\frac{1}{2}-3P_1+P_2\right)J.$

$T_{13}=(2-P_1+3P_2,1+2P_1+2P_2)$

$f^{-1}(T_{13}) = \frac{1}{2}(3 + P_1 + 5P_2) + \frac{1}{2}(1 - 3P_1 + P_2)J = \left(\frac{3}{2} + \frac{1}{2}P_1 + \frac{5}{2}P_2\right) + \left(\frac{1}{2} - \frac{3}{2}P_1 + \frac{1}{2}P_2\right)J.$

$T_{14} = (2 - P_1 + 3P_2, 1 + 2P_1 + P_2)$

$f^{-1}(T_{14}) = \frac{1}{2}(3 + P_1 + 4P_2) + \frac{1}{2}(1 - 3P_1 + 2P_2)J = \left(\frac{3}{2} + \frac{1}{2}P_1 + 2P_2\right) + \left(\frac{1}{2} - \frac{3}{2}P_1 + P_2\right)J.$

$T_{15} = (3 - 2P_1, 1 + 2P_1 + 2P_2)$

$f^{-1}(T_{15}) = \frac{1}{2}(4 + 2P_2) + \frac{1}{2}(2 - 4P_1 - 2P_2)J = (2 + P_2) + (1 - 2P_1 - P_2)J.$

$T_{16} = (3 - 2P_1, 1 + 2P_1 + P_2)$

$f^{-1}(T_{16}) = \frac{1}{2}(4 + P_2) + \frac{1}{2}(2 - 4P_1 - P_2)J = \left(2 + \frac{1}{2}P_2\right) + \left(1 - 2P_1 - \frac{1}{2}P_2\right)J.$

Conclusion

In this paper, we studied for the first time the concept of square real matrices with symbolic 2-plithogenic split-complex entries, where we find the formula of computing inverses, exponents, and powers of these matrices by building a ring isomorphism between the ring of split-complex symbolic 2-plithogenic matrices and the direct product of the symbolic 2-plithogenic matrices with itself. Also, we give the interested reader many related examples to clarify the validity of our work.

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