# On Single Valued Neutrosophic Signed Digraph and its Applications

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**Abstract:** The development of the theory of the single valued neutrosophic (SVN) digraph is done in this paper. Also this paper introduces the concept of SVN signed digraph. Some basic terminologies and operations of SVN digraphs and SVN signed digraphs have been defined. Finally classification problem in a signed network system is solved with the help of SVN signed digraphs.

**Keywords:** SVN set, SVN digraph, SVN signed digraph, Classification Problem.

## 1 Introduction

Uncertainty is something that we cannot be sure about. It is a common phenomenon of our daily existence, because our world is full of uncertainties. There are many situations and complex physical processes, where we encounter uncertainties of different types and often face many problems due to it. Therefore it is natural for us to understand and try to model these uncertain situations prevailing in those physical processes. From centuries, the Science, whether Physics or Biology, or in Philosophy, i.e. every domain of knowledge has strived to understand the manifestations and features of uncertainty. Perhaps that is the main reason behind the development of Probability theory and Stochastic techniques which started in early eighteenth century, which has the ability to model uncertainties arising due to randomness. But the traditional view of Science, especially Mathematics was to worship certainty and to avoid uncertainty by all possible means. Therefore the classical mathematics failed to model many complex physical phenomena such as complex chemical processes or biological systems where uncertainty was unavoidable. Again probabilistic techniques cannot also model all kinds of uncertain situations. Natural language processing is an example of such problem where the above method fails. Thus the need for a fundamentally different approach to study such problems, where uncertainty plays a key role, was felt and that stimulated new developments in Mathematics.

Recently a new theory has been introduced and which is known as neutrosophic logic and sets. The term neutro-sophy means knowledge of neutral thought and this neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. Neutrosophic logic was introduced by Florentin Smarandache in 1995. It is a logic in which each proposition is estimated to have a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F). A Neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies between, the non-standard unit interval. Unlike in intuitionistic fuzzy sets, where the incorporated uncertainty is dependent of the degree of belongingness and degree of non belongingness, here the uncertainty present, i.e. indeterminacy factor, is independent of truth and falsity values. In 2005, Wang et. Al. introduced an instance of neutrosophic set known as single valued neutrosophic sets which were motivated from the practical point of view and that can be used in real scientific and engineering applications. The single valued neutrosophic set is a generalization of classical set, fuzzy set, intuitionistic fuzzy set and paraconsistent sets etc.

The recently proposed notion of neutrosophic sets is a general formal framework for studying uncertainties arising due to indeterminacy factors. From the philosophical point of view, it has been shown that a neutrosophic set generalizes a classical set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set etc. Also single valued neutrosophic (SVN) set can be used in modeling real scientific and engineering problems. The SVN set is a generalization of classical fuzzy set [54], intuitionstic fuzzy set [7] etc. Therefore the study of neutrosophic sets and its properties have a considerable significance in the the sense of applications as well as in understanding the fundamentals of uncertainty [See [2, 3, 4, 5, 8, 10, 15, 16, 28, 29, 32, 33, 35, 36, 37, 38, 39, 40, 55]]. This new topic is very sophisticated and only a handful of papers have been published till date but it has immense possibilities which are to be explored.

Graphs and Digraphs play an important role to solve many pratical problems in algebra, analysis, geometry etc. A couple of researchers are continuously engaged in research on fuzzy graph theory, fuzzy digraph theory, intuitionstic fuzzy graphs, soft digraphs [17, 22, 23, 24, 49]. However Neutrosophic graphs, SVN graphs concept have been defined by Samarandache and Broumi et al. in their papers [12, 45]. We have defined the SVN digraphs in our previous paper [50]

In this paper we have developed the notion of SVN digraphs. Some preliminaries regarding SVN sets, graph theory etc. are discussed in Section 2. In section 3, we have defined the some terminologies regarding SVN digraph with examples. We have discussed SVN signed digraphs for the first time in Section 4. In section 5, we have solved a real life networking problem by using SVN signed digraph. Section 6 concludes the paper.

## 2 Preliminaries

Neutrosophic sets play an important role in decision making under uncertain environment of Mathematics. Most of the preliminary ideas regarding Neutrosophic sets and its possible applications can be easily found in any standard reference say [30, 43, 45, 46]. However we will discuss some definitions and terminologies regarding neutrosophic sets which will be used in the rest of the paper. Also we have added some new definitions and results on SVN digraphs in this section.

**Definition 1** *[30] Let $X$ be a universal set. A neutrosophic set $A$ on $X$ is characterized by a truth membership function $t_A$, an indeterminacy function $i_A$ and a falsity function $f_A$, where $t_A, i_A, f_A : X \to [0,1]$, are functions and $\forall\, x \in X$, $x = x(t_A(x), i_A(x), f_A(x)) \in A$ is a single valued neutrosophic element of $A$. A single valued neutrosophic set (SVNS) $A$ over a finite universe $X = \{x_1, x_2, \ldots, x_n\}$ is represented as below:*

$$A = \sum_{i=1}^{n} \frac{x_i}{\langle t_A(x_i), i_A(x_i), f_A(x_i)\rangle}$$

**Definition 2** *[1] Let $A = \{\langle x; t_A(x); i_A(x); f_A(x)\rangle; x \in X\}$ be a single-valued neutrosophic set of the set X . For $\alpha \in [0,1]$, the $\alpha$-cut of $A$ is the crisp set $A_\alpha$*

$$A_\alpha = \{x \in X : \text{either}(t_A(x); i_A(x)) \geq \alpha \text{ or } f_A(x) < 1-\alpha\}.$$

*Let $B = \{\langle (x,y); t_B(x,y); i_B(x,y); f_B(x,y)\rangle\}$ be a neutrosophic set on $E \subseteq X \times X$ . For $\alpha \in [0,1]$, the $\alpha$-cut is the crisp set $B_\alpha$ defined by,*

$$B_\alpha = \{(x,y) \in E : \text{either } (t_B(x,y); i_B(x,y) \geq \alpha) \text{ or } f_B(x,y) \leq 1-\alpha\}.$$

**Definition 3** *[30] Suppose $N(X)$ be the collection of all SVN sets on $X$ and $A, B \in N(X)$. A similarity measure between two SVN sets $A$ and $B$ is a function $S : N(X) \times N(X) \to [0,1]$ which satisfies the following condition:*

(i) $0 \leq S(A,B) \leq 1$,

(ii) $S(A,B) = 1$ *if and only if* $A = B$.

(iii) $S(A,B) = S(B,A)$

(iv) *If $A \subseteq B \subseteq C$, then $S(A,C) \leq S(A,B)$ and $S(A,C) \leq S(B,C)$ for all $A, B, C \in N(X)$.*

Note that here (i)-(iii) are essential for any similarity measure and (iv) is a desirable property although not mandatory.

**Definition 4** *The entropy of SVNS $A$ is defined as a function $E : N(X) \to [0,1]$ which satisfies the following axioms:*

(i) *$E(A) = 0$ if $A$ is a crisp set.*

(ii) *$E(A) = 1$ if $(t_A(x), i_A(x), f_A(x)) = (0.5, 0.5, 0.5)\ \forall x \in X$.*

(iii) *$E(A) \geq E(B)$ if $A$ is more uncertain than $B$ i.e. $t_A(x) + f_A(x) \leq t_B(x) + f_B(x)$ and $|i_A(x) - i_{A^c}(x)| \leq |i_B(x) - i_{B^c}(x)|\ \forall\ x \in X\ \ A, B \in X$.*

(iv) *$E(A) = E(A^c)\ \forall A \in N(X)$, where $N(X)$ is the collection of all SVNS over $X$.*

**Example 5** *An entropy measure of an element $x_1$ of a SVNS $A$ can be calculated as follows:*

$$E_1(x_1) = 1 - (t_A(x_1) + f_A(x_1)) \times |i_A(x_1) - i_{A^c}(x_1)|.$$

Graph theory are widely used in different areas of neutrosophic theory. Many authors have used different types of graphs in neutrosophic theory. Consider a SVN set $V_D = \{(v_i, \langle t_{V_D}(v_i), i_{V_D}(v_i), f_{V_D}(v_i)\rangle), i = 1, \ldots, n\}$ over a finite universal set $X$.

**Definition 6** *[50] A SVN digraph $D$ is of the form $D = (V_D, A_D)$ where,*

(i) *$V_D = \{v_1, v_2, v_3, \ldots, v_n\}$ and the functions $t_{V_D} : V_D \to [0,1]$, $i_{V_D} : V_D \to [0,1]$, $f_{V_D} : V_D \to [0,1]$ denote the truth-membership function, a indeterminacy-membership function and falsity-membership function of the element $v_i \in V_D$ respectively such that $0 \leq t_{V_D}(v_i) + i_{V_D}(v_i) + f_{V_D}(v_i) \leq 3$, $\forall v_i \in V_D, i = 1, 2, \ldots, n$.*

(ii) *$A_D = \{(v_i, v_j); (v_i, v_j) \in V_D \times V_D\}$ provided $0 < E(v_i) - E(v_j) \leq 0.5$ and the functions $t_{A_D} : A_D \to [0,1], i_{A_D} : A_D \to [0,1], f_{A_D} : A_D \to [0,1]$ are defined by*

$$t_{A_D}(\{v_i, v_j\}) \leq min[t_{V_D}(v_i), t_{V_D}(v_j)],$$
$$i_{A_D}(\{v_i, v_j\}) \geq max[i_{V_D}(v_i), i_{V_D}(v_j)],$$
$$f_{A_D}(\{v_i, v_j\}) \geq max[f_{V_D}(v_i), f_{V_D}(v_j)]$$

*where $t_{A_D}, i_{A_D}, f_{A_D}$ denotes the truth-membership function, a indeterminacy-membership function and falsity-membership function of the arc $(v_i, v_j) \in A_D$ respectively where $0 \leq t_{A_D}(v_i, v_j) + i_{A_D}(v_i, v_j) + f_{A_D}(v_i, v_j) \leq 3, \forall (v_i, v_j) \in A_D,\ i, j \in \{1, 2, \ldots n\}$.*

*We call $V_D$ as the vertex set of $D$, $A_D$ as the arc set of $D$ where $E(v)$ is the entropy of the vertex $v$. Please note that if $E(v_i) = E(v_j)$, then $\{(v_i, v_j), (v_j, v_i)\} \in A_D$. Since for a vertex $v \in V_D$ of a SVN digraph $D$ we have $E(v) = E(v)$, thus every vertex of a SVN digraph $D$ contains a loop $(v, v)$ at $v$. On the other hand, if $E(v_i) - E(v_j) > 0.5$, we define that there exists no arc between the vertices $v_i$ and $v_j$.*

**Example 7** *Consider the SVN digraph $D_0 = (V_{D_0}, A_{D_0})$ in Figure 1 with vertex set $V_{D_0} = \{v_1, v_2, v_3\}$ and arc set $A_{D_0} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3)\}$ with one loop at each vertex as follows:*

$$\begin{bmatrix} & v_1 & v_2 & v_3 \\ t_{V_D} & 0.4 & 0.4 & 0.5 \\ i_{V_D} & 0.1 & 0.3 & 0.2 \\ f_{V_D} & 0.2 & 0.1 & 0.5 \\ E & 0.52 & 0.8 & 0.4 \end{bmatrix},$$

$$\begin{bmatrix} & (v_2, v_1) & (v_2, v_3) & (v_1, v_3) \\ t_{A_D} & 0.3 & 0.2 & 0.4 \\ i_{A_D} & 0.4 & 0.3 & 0.3 \\ f_{A_D} & 0.2 & 0.6 & 0.4 \end{bmatrix}.$$

*It is clear that the $D_0$ is a SVN digraph.*

**Definition 8** *Suppose $D = (V_D, A_D)$ and $H = (V_H, A_H)$ be two SVN digraphs with $|V_D| = |V_H|$ corresponding to the SVNS $V_D$ and $V_H$ over an universal set $X$. Then the cartesian product of two SVN digraphs $D$ and $H$ is defined as a SVN digraph $C = (V_C, A_C)$ in which the following holds:*

(i) $V_C = V_D \times V_H$,

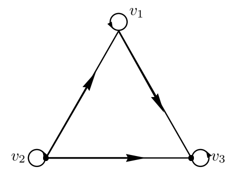


Figure 1: The SVN Digraph $D_0$

(ii) $t_{V_C}(v_1,v_2) \leq min(t_{V_D}(v_1), t_{V_H}(v_2))$; $i_{V_C}(v_1,v_2) \leq min(i_{V_D}(v_1), i_{V_H}(v_2))$; $f_{V_C}(v_1,v_2) \geq max(f_{V_D}(v_1), f_{V_H}(v_2)); \forall\ (v_1,v_2) \in V_D \cap V_H$ *and ,*

(iii) $A_C = \{\big((v_i,v_j),(v_k,v_l)\big); \big((v_i,v_j),(v_k,v_l)\big) \in V_C \times V_C\}$ *provided* $0 < E(v_i,v_j) - E(v_k,v_l) \leq 0.5$.

**Definition 9** *The degree and the total degree of a vertex $v_i$ of a SVN digraph $D = (V, A)$ are denoted by*

$$\begin{aligned} d_D(v_i) &= (d_t(v_i), d_i(v_i), d_f(v_i)) \\ &= \big(\sum_{j,i\neq j} t_A(v_i,v_j), \sum_{j,i\neq j} i_A(v_i,v_j), \sum_{j,i\neq j} f_A(v_i,v_j)\big), \\ Td_D(v_i) &= \big(\sum_{j,i\neq j} t_A(v_i,v_j) + t_V(v_i), \sum_{j,i\neq j} i_A(v_i,v_j) + \\ &\quad i_V(v_i), \sum_{j,i\neq j} f_A(v_i,v_j) + f_V(v_i)\big). \end{aligned}$$

**Example 10** *The degree and total degree of the vertex $v_2$ of the digraph $D_0$ in Example 7 are $d_D(v_2) = (0.5, 0.7, 0.8)$ and $Td_D(v_2) = (0.9, 1, 0.9)$.*

**Definition 11** *A SVN digraph $D = (V_D, A_D)$ is called a k-regular SVN digraph if $d_D(v_i) = (k, k, k)\ \forall v_i \in V_D$.*

**Definition 12** *A SVN digraph $D = (V_D, A_D)$ is called a totally regular SVN digraph of degree $(k_1, k_2, k_3)$ if $Td_D(v_i) = (k_1, k_2, k_3)\ \forall v_i \in V_D$.*

It is quite clear that the concept of a regular SVN digraph and totally regular SVN digraph are completely different. We have seen that the arc set $A_D$ in $D$ forms a SVN set [50]. Now we consider the concept of degree and total degree of an arc of a SVN digraph in the next definition.

**Definition 13** *The degree and the total degree of an arc $(u, v)$ of a SVN digraph are denoted by $d_D(u, v) = (d_t(u, v), d_i(u, v), d_f(u, v))$ and $Td_D(u, v) = (Td_t(u, v), Td_i(u, v), Td_f(u, v))$, respectively and are defined as follows:*

$$\begin{aligned} d_D(u,v) &= d_D(u) + d_D(v) - \frac{1}{2}(t_A(u,v), i_A(u,v), f_A(u,v)), \\ Td_D(u,v) &= d_D(u,v) + (t_A(u,v), i_A(u,v), f_A(u,v)) \end{aligned}$$

**Example 14** *Consider the SVN digraph $D_0$ in Figure 1. Here the degree and total degree of the vertices $\{v_1, v_2, v_3\}$ of $D_0$ as follows:*

$$\begin{aligned} d_D(v_1) &= (0.4, 0.3, 0.4), Td_D(v_1) = (0.8, 0.4, 0.6), \\ d_D(v_2) &= (0.5, 0.7, 0.8), Td_D(v_2) = (0.9, 1, 0.9), \\ d_D(v_3) &= (0, 0, 0), Td_D(v_3) = (0.5, 0.2, 0.5). \end{aligned}$$

*Now we calculate the degree and total degree of each arc of $A_{D_0}$ of $D_0$ as follows:*

$$\begin{aligned} d_D(v_2,v_1) &= (0.85, 0.8, 1.1), Td_D(v_2,v_1) = (0.6, 0.6, 0.1), \\ d_D(v_2,v_3) &= (0.4, 0.55, 0.5), Td_D(v_2,v_3) = (0.3, 0.4, 0.2), \\ d_D(v_1,v_3) &= (0.2, 0.15, 0.2), Td_D(v_1,v_3) = (0, 0, 0). \end{aligned}$$

**Definition 15** *The maximum degree of a SVN digraph $D = (V_D, A_D)$ is defined as $\Delta(D) = (\Delta_t(D), \Delta_i(D), \Delta_f(D))$ where*

$$\begin{aligned} \Delta_t(D) &= max\{d_t(v) : v \in V_D\}, \\ \Delta_i(D) &= max\{d_i(v) : v \in V_D\}, \\ \Delta_f(D) &= max\{d_f(v) : v \in V_D\}, \end{aligned}$$

**Definition 16** *The minimum degree of a SVN digraph $D = (V_D, A_D)$ is defined as $\delta(D) = (\delta_t(D), \delta_i(D), \delta_f(D))$ where*

$$\begin{aligned} \delta_t(D) &= min\{d_t(v) : v \in V_D\}, \\ \delta_i(D) &= min\{d_i(v) : v \in V_D\}, \\ \delta_f(D) &= min\{d_f(v) : v \in V_D\}, \end{aligned}$$

**Example 17** *For the SVN digraph $D_0$ in Figure 1, we have $\Delta(D) = (0.5, 0.7, 0.8)$ and $\delta(D) = (0, 0, 0, )$.*

**Definition 18** *Suppose $D = (V_D, A_D)$ be a SVN digraph corresponding to a SVN set $V_D$. Then $D$ is said to be*

(i) *arc regular SVN digraph if every arc in $D$ has the same degree $(k_1, k_2, k_3)$.*

(ii) *equally arc regular SVN digraph if $k_1 = k_2 = k_3$.*

(iii) *totally arc regular SVN digraph if every arc in $D$ has the same total degree $(k_1, k_2, k_3)$.*

It is also quite clear that the above three concepts are completely different to each other.

# 3 SVN Signed Digraph

In this paper, we will define the SVN signed digraph for the first time.

**Definition 19** *Suppose $D = (V_D, A_D)$ be a SVN digraph over a single valued neutrosophic set $V_D$. A signing of a SVN digraph $D$ is an assignment of a sign ($+$ or $-$) to each arc of the digraph; the sign of arc $(v, w)$ is denoted $sgn(v, w)$. The result of a signing of $D$ is called a SVN signed digraph.*

*However to assign the sign of the arcs, we will follow some rules. For this, we will consider the $\alpha$-level subdigraph $D_1$ of a SVN digraph $D$. Then we will assign $+$ sign only to those arcs of $D$ which are also the arcs of $D_1$. For the rest of arcs of $D$, we will assign $-$ sign.*

**Example 20** *Consider the SVN digraph $D_1 = (V_{D_1}, A_{D_1})$ in Figure 2 with vertex set $V_{D_1} = \{v_1, v_2, v_3, v_4\}$ and arc set $A_{D_1} = \{(v_2, v_1), (v_1, v_3), (v_2, v_3),$ $(v_2, v_4), (v_3, v_4), (v_4, v_1)\}$ with one loop at each vertex as follows:*

$$\begin{bmatrix} & v_1 & v_2 & v_3 & v_4 \\ t_{V_{D_1}} & 0.4 & 0.4 & 0.5 & 0.2 \\ i_{V_{D_1}} & 0.1 & 0.3 & 0.2 & 0.5 \\ f_{V_{D_1}} & 0.2 & 0.1 & 0.5 & 0.3 \\ E & 0.52 & 0.8 & 0.4 & 1 \end{bmatrix},$$

$$\begin{bmatrix} & (v_2, v_1) & (v_2, v_3) & (v_4, v_1) & (v_1, v_3) & (v_4, v_2) \\ t_{A_{D_1}} & 0.3 & 0.2 & 0.1 & 0.4 & 0.2 \\ i_{A_{D_1}} & 0.4 & 0.3 & 0.5 & 0.3 & 0.5 \\ f_{A_{D_1}} & 0.2 & 0.6 & 0.3 & 0.4 & 0.4 \end{bmatrix}.$$

*We take $\alpha = 0.5$. In this case, the vertices $\{v_1, v_2, v_4\}$ of $D_1$ are $\alpha$-level vertices and the arcs $\{(v_2, v_1), (v_4, v_1), (v_4, v_2)\}$ are the $\alpha$-level arcs. Thus we will assign the sign as follows to the arcs of $D_1$*

$$\begin{aligned} sgn(v_2, v_1) = sgn(v_4, v_1) = sgn(v_4, v_2) = + \\ sgn(v_1, v_1) = sgn(v_2, v_2) = sgn(v_4, v_4) = +, \\ sgn(v_2, v_3) = sgn(v_1, v_3) = sgn(v_3, v_3) = - \end{aligned}$$

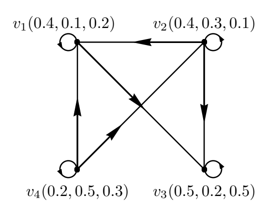


Figure 2: The SVN Digraph $D_1$

**Remark 21** *Throughout this paper, we have taken the value of $\alpha$ is 0.5. However, for different values of $\alpha$ we will get different signed SVN digraphs. Also, by $K_n$, we denote the complete SVN digraph of $n$-vertices.*

**Definition 22** *The sets of positive and negative arcs of a SVN signed digraph $D$ are respectively denoted by $D^+$ and $D^-$. Thus $D = D^+ \cup D^-$.*

**Definition 23** *A SVN signed digraph is said to be homogeneous if all of its arcs have either positive sign or negative sign, otherwise heterogeneous.*

**Definition 24** *The sign of a SVN signed digraph is defined as the product of signs of its arcs.A SVN signed digraph is said to be positive (negative) if its sign is positive (negative) i.e., it contains an even (odd) number of negative arcs. A signed digraph is said to be all-positive (respectively, all negative) if all its arcs are positive (negative).*

**Example 25** *It is clear that the sign of the SVN digraph $D_1$ in Example 20 is negative. It is clear that the SVN digraph $D_1$ is neither all positive nor all negative.*

**Definition 26** *A SVN signed digraph is said to be cycle balanced if each of its cycles is positive, otherwise non cycle balanced.*

**Definition 27** *A SVN signed digraph is symmetric if $(u, v) \in D^+$ $(or D^-)$ then $(v, u) \in D^+$ $(or D^-)$ where $u, v \in V_D$.*

**Definition 28** *The adjacency matrix of a SVN signed digraph $D$ is the square matrix $M = (a_{ij})$ whose $(i, j)$ entry $a_{ij}$ is $+1$ if arc $(v_i, v_j)$ in $D$ has a $+$ sign, $-1$ if arc $(v_i, v_j)$ in $D$ has a $-$ sign, and $0$ if arc $(v_i, v_j)$ is not in $D$.*

**Example 29** *The adjacency matrix $M$ of the SVN signed digraph $D_1$ in Figure 2 is as following:*

$$M = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

**Definition 30** *The characteristic polynomial $\delta(t) = |tI - M|$ of the adjacency matrix $M$ of a SVN signed digraph $D$ is called the characteristic polynomial of $D$ and it is denoted by $p(t)$. The eigenvalues of $M$ are called the spectral of the digraph $D$.*

**Example 31** *The characteristic polynomial $\delta(t)$ of the SVN signed digraph $D_1$ in Figure 2 is $\delta(t) = (t-1)^3(t+1)$ and spectral values are $1, 1, 1, -1$.*

**Definition 32** *Suppose $D = (V_D, A_D)$ be a SVN signed digraph over a single valued neutrosophic set $V_D = \{v_1, v_2, \ldots, v_n\}$. Consider the complement SVN digraph $D^C$ corresponding to the complement SVN set $V_D^C$. The digraph $V_D^C$ with a signing of arcs is called the signed complement of the SVN signed digraph $V_D$.*

Here we choose the same value of $\alpha$ as of $D$ and also consider the $\alpha$-arcs and $\alpha$-vertices. According to the $\alpha$-arcs and $\alpha$-vertices we assign signs to the arcs of $V_D^c$.

**Example 33** *Consider the SVN complement digraph $D^c$ of the SVN digraph $D_1$ in Figure 2.*

$$\begin{aligned} sgn(v_2, v_1) &= sgn(v_4, v_1) = sgn(v_4, v_2) = -, \\ sgn(v_1, v_1) &= sgn(v_2, v_2) = sgn(v_4, v_4) = +, \\ sgn(v_2, v_3) &= sgn(v_1, v_3) = sgn(v_3, v_3) = +. \end{aligned}$$

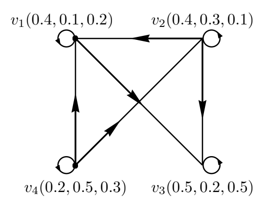


Figure 3: The SVN Digraph $D^C$

# 4 Some important results of a SVN Signed Digraph

In this section we will discuss some results regarding SVN signed Neutrosophic digraphs. Like wise a SVN digraph $D$, we define the terminologies of a SVN signed digraph. However, the *order* of a SVN signed digraph $D$, denoted by $|D|$, is the number of vertices of $D$. The *size* of a SVN signed digraph $D$, is the number of arcs of $D$ i.e. $|A_D|$.

**Theorem 34** *A SVN (signed) digraph $D \neq K_n$ of order $\geq 3$ is always acyclic.*

**Proof 35** *Suppose there exist a cyclic SVN (signed) digraph $D = (V_D, A_D)$ has vertex set $V_D = \{v_1, v_2, v_3, \ldots, v_n\}$. Without loss of generality, let $D$ has a cycle of length $k$, where $k \geq 3$ say $\langle v_1, v_2, \ldots, v_k\rangle$. Then we have $E(v_1) > E(v_2) > \ldots,\ E(v_k) > E(v_1)$- which is impossible. Hence $D$ does not have a cycle of length $k$.*

**Corollary 36** *Any asymmetric SVN signed digraph of order $\geq 3$ is not balanced.*

**Theorem 37** *Any asymmetric SVN (signed) digraph $H$ of order $\geq 3$ is not strongly connected.*

**Proof 38** *Since there does not exists any SVN (signed) digraph with a cycle of length $\geq 3$, hence the results follows.*

**Theorem 39** *In any complete symmetric SVN digraph $D = (V_D, A_D)$, where $V_D = \{v_1, v_2, \ldots, v_n\}$,*

$$\begin{aligned} \sum d_D(v_i) &= (d_t(v_i), d_i(v_i), d_f(v_i)) \\ &= \Big( \sum_{j, i\neq j} t_A(v_i, v_j),\ \sum_{j, i\neq j} i_A(v_i, v_j),\ \sum_{j, i\neq j} f_A(v_i, v_j) \Big), \end{aligned}$$

$\forall v_i \in V_D$.

**Proof 40** *For any symmetric complete SVN digraph $D = (V_D, A_D)$, where $V_D = \{v_1, v_2, \dots, v_n\}$, we have,* $\sum d_D(v_i) = (\sum d_t(v_i), \sum d_i(v_i), \sum d_f(v_i))$
$= (d_t(v_1), d_i(v_1), d_f(v_1)) + \dots + (d_t(v_n), d_i(v_n), d_f(v_n))$
$= \big( \sum_{j,1\neq j} t_A(v_1, v_j), \sum_{j,1\neq j} i_A(v_1, v_j), \sum_{j,1\neq j} f_A(v_1, v_j)\big) + \dots +$
$\big( \sum_{j,n\neq j} t_A(v_n, v_j), \sum_{j,n\neq j} i_A(v_n, v_j), \sum_{j,n\neq j} f_A(v_n, v_j)\big)$
$= 2\big( \sum_{j,i\neq j} t_A(v_i, v_j), \sum_{j,i\neq j} i_A(v_i, v_j), \sum_{j,i\neq j} f_A(v_i, v_j)\big)$.

However the converse of the Theorem 39 is not true for a asymmetric and incomplete SVN digraph which can be followed from the Example 10. The SVN digraph $D_0$ is asymmetric as well as incomplete. Clearly the Theorem 39 does not hold.

**Theorem 41** *Suppose $D = (V_D, A_D)$ be a SVN symmetric digraph which has a cycle $C$ on p-vertices, say $\{v_1, v_2, \dots, v_p\}$. Then,*
$\sum d_D(v_i)$

$$= \sum d_D(v_i, v_j) + \frac{3}{2}\big( \sum_{j,i\neq j} t_A(v_i, v_j), \sum_{j,i\neq j} i_A(v_i, v_j), \sum_{j,i\neq j} f_A(v_i, v_j)\big)$$

*where $(v_i, v_j) \in C, i \neq j$.*

**Proof 42** *We have,* $\sum d_D(v_i, v_j) = d_D(v_1, v_2) + \dots + d_D(v_p, v_1)$
$= d_D(v_1) + d_D(v_2) - \frac{1}{2}(t_A(v_1, v_2), i_A(v_1, v_2), f_A(v_1, v_2)) + \dots +$
$d_D(v_p) + d_D(v_1) - \frac{1}{2}(t_A(v_p, v_1), i_A(v_p, v_1), f_A(v_p, v_1))$,
$= 2\sum_{v_i\in C} d_D(v_i) -$
$\frac{1}{2}\big\{ \sum_{j,i\neq j} t_A(v_i, v_j), \sum_{j,i\neq j} i_A(v_i, v_j), \sum_{j,i\neq j} f_A(v_i, v_j)\big\}$
$= \sum_{v_i\in C} d_D(v_i) +$,
$2\big\{ \sum_{j,i\neq j} t_A(v_i, v_j), \sum_{j,i\neq j} i_A(v_i, v_j), \sum_{j,i\neq j} f_A(v_i, v_j)\big\}$
$-\frac{1}{2}\big( \sum_{j,i\neq j} t_A(v_i, v_j), \sum_{j,i\neq j} i_A(v_i, v_j), \sum_{j,i\neq j} f_A(v_i, v_j)\big)$,
$= \sum d_D(v_i, v_j) +$,
$\frac{3}{2}\big( \sum_{j,i\neq j} t_A(v_i, v_j), \sum_{j,i\neq j} i_A(v_i, v_j), \sum_{j,i\neq j} f_A(v_i, v_j)\big)$.

**Theorem 43** *The maximum value of the degree of any vertex in a complete SVN digraph $D$ with $n$ vertices is $(n-1, n-1, n-1)$.*

**Proof 44** *Suppose $D = (V_D, A_D)$ be a complete SVN digraph. Then the maximum truth-membership value given to an arc is 1 and the number of arcs incident on a vertex can be at most $n-1$. Hence the maximum truth-membership degree of any vertex in a complete SVN-digraph with $n$ vertices is $n-1$. Similar argument can be done for indeterminacy-membership degree and falsity-membership degree of any vertex. Hence the result follows.*

The following remarks are quite natural for a SVN signed digraph:

**Remark 45** (i) *A single valued neutrosophic signed digraph is a single valued neutrosophic positive signed digraph if every even length cycles having all negative signed arcs.*

(ii) *Odd length cycle having all negative signed arcs is always a negative signed digraph.*

(iii) *An odd length single valued signed neutrosophic cycle is balanced if and only if it contains at least one positive arcs or odd number of positive arcs.*

# 5 Applications of a SVN Signed digraph

The applications of SVN sets in solving real life problems under uncertainty has been shown by many authors. In this section we have shown the application of our SVN signed digraphs in solving two problems namely a classification problem and a decision making problem.

## 5.1 Classification problem

Consider the SVN set $V(D) = \{v_1, v_2, v_3.v_4\}$ in Example 20 and the corresponding SVN signed digraph $D = (V_D, A_D)$ in Figure 2. To draw SVN signed digraph, we have taken $\alpha = 0.5$. Based on this $\alpha$, we find that the vertices $\{v_1, v_2, v_4\}$ of $D_1$ as $\alpha$-level vertices and the arcs $\{(v_2, v_1), (v_4, v_1), (v_4, v_2)\}$ as the $\alpha$-level arcs. Then we assign the signs to the arcs of $D$ as follows:

$$sgn(v_2, v_1) = sgn(v_4, v_1) = sgn(v_4, v_2) = +,$$
$$sgn(v_1, v_1) = sgn(v_2, v_2) = sgn(v_4, v_4) = +,$$
$$sgn(v_2, v_3) = sgn(v_1, v_3) = sgn(v_3, v_3) = -.$$

Hence, we can form a partition of two sets namely $P, Q$, where $P = \{v_1, v_2, v_4\}$ and $Q = \{v_3\}$ from the elements of a SVN set $V(D)$. The partition is done on the basis of signing of the $\alpha$-level vertices. Thus by drawing SVN signed digraph of a SVN set, we can get a 2-point classification of a SVN set.

## 5.2 Algorithm for 2-point classification of a SVN set

One can attempt for 2-point classification of a SVN set by using the following algorithm:

(i) Consider a SVN set $V(D)$.

(ii) Draw a SVN digraph $D = (V(D), A(D))$, where $V(D), A(D)$ are the vertex set and arc set of $D$ respectively.

(iii) Choose the value of $\alpha$ and find out $\alpha$ level vertices of $D$. The choice of the value of the $\alpha$ is completely depend on the programmer.

(iv) Assign the positive sign with the $\alpha$ level vertices, arcs and negative sign to rest of the vertices, arcs of $D$. In that case $D$ turns into a SVN signed digraph.

(v) Finally consider two sets $P, Q$ s.t $P$ consists the positive vertices and $Q$ contains the negative vertices. Hence a partition of the SVN sets $V(D)$ is done consisting of two sets $P$ and $Q$ respectively.

### 5.3 A Decision Making Problem

Suppose $A, B, C$ be three nations willing to explore the possibility trade between them. Considering various situations in there countries like, political stability, case of doing business, human resource, trade laws etc. Each country was assigned grades of positive factors, indeterminacy and negative factors as follows:

$$A(0.4, 0.3, 0.2), B(0.4, 0.1, 0.2), C(0.5, 0.2, 0.4).$$

In these way, we can characterize the three country $A, B, C$ respectively. We must to find the possibility of trade between them. For this, we consider $A, B, C$ as the three vertices $v_1, v_2, v_3$ respectively as a vertex set $V_{D_4}$ of a proposed SVN digraph $D_4 = (V_{D_4}, A_{D_4})$. Now we draw the SVN digraph $D_4$ as follows:

$$\begin{array}{c|ccc} & v_1 & v_2 & v_3 \\ \hline t_{V_D} & 0.4 & 0.4 & 0.5 \\ i_{V_D} & 0.3 & 0.1 & 0.2 \\ f_{V_D} & 0.2 & 0.2 & 0.4 \\ E & 0.76 & 0.52 & 0.46 \end{array}, \quad \begin{array}{c|ccc} & (v_1, v_2) & (v_1, v_3) & (v_2, v_3) \\ \hline t_{A_D} & 0.3 & 0.2 & 0.4 \\ i_{A_D} & 0.4 & 0.3 & 0.3 \\ f_{A_D} & 0.2 & 0.6 & 0.4 \end{array}.$$

Here, we have seen that $A_{D_4} = \{(v_1, v_2), (v_1, v_3), (v_2, v_3)\}$. So we can say that, there is a good transport communication between the country pair $(A, B), (A, C), (B, C)$

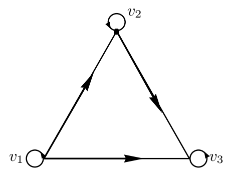


Figure 4: The SVN Digraph $D_4$

respectively. Now consider $\alpha = 0.3$. Here, the vertices $\{v_1, v_2\}$ of $D_4$ are $\alpha$-level vertices and the arcs $\{(v_1, v_2)\}$ is the only $\alpha$-level arcs. Thus we will assign the sign as follows to the arcs of $D_4$

$$\begin{aligned} sgn(v_1, v_2) &= sgn(v_1, v_1) = sgn(v_2, v_2) = +, \\ sgn(v_2, v_3) &= sgn(v_1, v_3) = sgn(v_3, v_3) = - \end{aligned}$$

From this SVN signed digraph $D_4$ we can conclude that both $A$ and $B$ have a common enemy $C$. Hence although there is a good communication between two country $(A, C)$ and $(B, C)$, it is not possible to do business between them due to their political situation. Hence a cyclic triple SVN signed digraph $D_4$ with one positive arcs can evaluate the real networks.

# 6 Conclusion

F. Smarandache introduced the neutrosophic set theory in his paper [43] as a generalization of fuzzy intuitionistic set theory. After that many researchers have developed the neutrosophic set theory, SVN theory, neutrosophic graph theory etc. and have applied those theories in solving many practical problems ([1, 6, 10, 12, 13, 14, 19, 20, 21, 26, 31, 41, 42, 48, 50, 51, 52, 53] etc.). We have developed earlier SVN digraph theory corresponding to a SVN set in our paper [50]. In this paper we have further developed the SVN digraph theory and introduced the notion of SVN signed digraphs and studied some of its important properties and applied it in a decision making problem. In future, one may study the decision making problems using SVN signed digraphs. The study of deeper properties of SVN signed digraphs and solution of more real life problems will be done in our subsequent papers.

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