On Neutrosophic Crisp Semi Alpha Closed Sets

Riad K. Al-Hamido[1], Qays Hatem Imran[2], Karem A. Alghurabi[3] and Taleb Gharibah[4]

[1,4]Department of Mathematics, College of Science, Al-Baath University, Homs, Syria.
E-mail: riad-hamido1983@hotmail.com
E-mail: taleb.gharibah@gmail.com
[2]Department of Mathematics, College of Education for Pure Science, Al Muthanna University, Samawah, Iraq.
E-mail: qays.imran@mu.edu.iq
[3]Department of Mathematics, College of Education for Pure Science, Babylon University, Hilla, Iraq.
E-mail: kareemalghurabi@yahoo.com

Abstract. In this paper, we presented another concept of neutrosophic crisp generalized closed sets called neutrosophic crisp semi-α-closed sets and studied their fundamental properties in neutrosophic crisp topological spaces. We also present neutrosophic crisp semi-α-closure and neutrosophic crisp semi-α-interior and study some of their fundamental properties.
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1. Introduction

The concept of "neutrosophic set" was first given by F. Smarandache [4,5]. A. A. Salama and S. A. Alblowi [1] presented the concept of neutrosophic topological space (briefly NTS). Q. H. Imran, F. Smarandache, R. K. Al-Hamido and R. Dhavaseelan [6] presented the idea of neutrosophic semi-α-open sets in neutrosophic topological spaces. In 2014, A. A. Salama, F. Smarandache and V. Kroumov [2] presented the concept of neutrosophic crisp topological space (briefly NCTS). The objective of this paper is to present the concept of neutrosophic crisp semi-α-closed sets and study their fundamental properties in neutrosophic crisp topological spaces. We also present neutrosophic crisp semi-α-closure and neutrosophic crisp semi-α-interior and obtain some of its properties.

2. Preliminaries

Throughout this paper, $(\mathcal{U},T)$ (or simply $\mathcal{U}$) always mean a neutrosophic crisp topological space. The complement of a neutrosophic crisp open set (briefly NC-OS) is called a neutrosophic crisp closed set (briefly NC-CS) in $(\mathcal{U},T)$. For a neutrosophic crisp set $\mathcal{A}$ in a neutrosophic crisp topological space $(\mathcal{U},T)$, $NCcl(\mathcal{A})$, $NCint(\mathcal{A})$ and $\mathcal{A}^c$ denote the neutrosophic crisp closure of $\mathcal{A}$, the neutrosophic crisp interior of $\mathcal{A}$ and the neutrosophic crisp complement of $\mathcal{A}$, respectively.

Definition 2.1:
A neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U},T)$ is said to be:
(i) A neutrosophic crisp pre-open set (briefly NCP-OS) [3] if $\mathcal{A} \subseteq NCint(NCcl(\mathcal{A}))$. The complement of a NCP-OS is called a neutrosophic crisp pre-closed set (briefly NCP-CS) in $(\mathcal{U},T)$. The family of all NCP-OS (resp. NCP-CS) of $\mathcal{U}$ is denoted by NCPO$(\mathcal{U})$ (resp. NCPC$(\mathcal{U})$).
(ii) A neutrosophic crisp semi-open set (briefly NCS-OS) [3] if $\mathcal{A} \subseteq NCcl(NCint(\mathcal{A}))$. The complement of a NCS-OS is called a neutrosophic crisp semi-closed set (briefly NCS-CS) in $(\mathcal{U},T)$. The family of all NCS-OS (resp. NCS-CS) of $\mathcal{U}$ is denoted by NCSO$(\mathcal{U})$ (resp. NCSC$(\mathcal{U})$).
(iii) A neutrosophic crisp α-open set (briefly NCα-OS) [3] if $\mathcal{A} \subseteq NCint(NCcl(NCint(\mathcal{A})))$. The complement of a NCα-OS is called a neutrosophic crisp α-closed set (briefly NCα-CS) in $(\mathcal{U},T)$. The family of all NCα-OS (resp. NCα-CS) of $\mathcal{U}$ is denoted by NCαO$(\mathcal{U})$ (resp. NCαC$(\mathcal{U})$).

Definition 2.2:
(i) The neutrosophic crisp pre-interior of a neutrosophic crisp set $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U},T)$ is the union of all NCP-OS contained in $\mathcal{A}$ and is denoted by $PNCint(\mathcal{A})$[3].

(ii) The neutrosophic crisp semi-interior of a neutrosophic crisp set $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$ is the union of all NCS-OS contained in $\mathcal{A}$ and is denoted by $SNCint(\mathcal{A})$[3].
(iii) The neutrosophic crisp α-interior of a neutrosophic crisp set $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$ is the union of all NCα-OS contained in $\mathcal{A}$ and is denoted by $\alpha NCint(\mathcal{A})$[3].

Definition 2.3:
(i) The neutrosophic crisp pre-closure of a neutrosophic crisp set $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$ is the intersection of all NCP-CS that contain $\mathcal{A}$ and is denoted by $PNCcl(\mathcal{A})$[3].
(ii) The neutrosophic crisp semi-closure of a neutrosophic crisp set $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$ is the intersection of all NCS-CS that contain $\mathcal{A}$ and is denoted by $SNCcl(\mathcal{A})$[3].
(iii) The neutrosophic crisp α-closure of a neutrosophic crisp set $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$ is the intersection of all NCα-CS that contain $\mathcal{A}$ and is denoted by $\alpha NCcl(\mathcal{A})$[3].

Proposition 2.4 [7]:
In a neutrosophic crisp topological space $(\mathcal{U}, T)$, the following statements hold, and the equality of each statement are not true:
(i) Every NC-CS (resp. NC-OS) is a NCα-CS (resp. NCα-OS).
(ii) Every NCα-CS (resp. NCα-OS) is a NCS-CS (resp. NCS-OS).
(iii) Every NCα-CS (resp. NCα-OS) is a NCP-CS (resp. NCP-OS).

Proposition 2.5 [7]:
A neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$ is a NCα-CS (resp. NCα-OS) iff $\mathcal{A}$ is a NCS-CS (resp. NCS-OS) and NCP-CS (resp. NCP-OS).

Theorem 2.6 [7]:
For any neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$, $\mathcal{A} \in$ NCαO$(\mathcal{U})$ iff there exists a NC-OS $\mathcal{H}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq NCint(NCcl(\mathcal{H}))$.

Proposition 2.7 [7]:
The union of any family of NCα-OS is a NCα-OS.

Proposition 2.8:
(i) If $\mathcal{K}$ is a NC-OS, then $SNCcl(\mathcal{K}) = NCint(NCcl(\mathcal{K}))$.
(ii) If $\mathcal{A}$ is a neutrosophic crisp subset of a neutrosophic crisp topological space $(\mathcal{U}, T)$, then $SNCint\big(NCcl(\mathcal{A})\big) = NCcl(NCint(NCcl(\mathcal{A})))$.
Proof: This follows directly from the definition (2.1) and proposition (2.4).

3. Neutrosophic Crisp Semi-α-Closed Sets

In this section, we present and study the neutrosophic crisp semi-α-closed sets and some of its properties.

Definition 3.1:
A neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$ is called neutrosophic crisp semi-α-closed set (briefly NCSα-CS) if there exists a NCα-CS $\mathcal{H}$ in $\mathcal{U}$ such that $NCint(\mathcal{H}) \subseteq \mathcal{A} \subseteq \mathcal{H}$ or equivalently if $NCint(\alpha NCcl(\mathcal{A})) \subseteq \mathcal{A}$. The family of all NCSα-CS of $\mathcal{U}$ is denoted by NCSαC$(\mathcal{U})$.

Definition 3.2:
A neutrosophic crisp set $\mathcal{A}$ is called a neutrosophic crisp semi-α-open set (briefly NCSα-OS) if and only if its complement $\mathcal{A}^c$ is a NCSα-CS or equivalently if there exists a NCα-OS $\mathcal{H}$ in $\mathcal{U}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq NCcl(\mathcal{H})$. The family of all NCSα-OS of $\mathcal{U}$ is denoted by NCSαO$(\mathcal{U})$.

Proposition 3.3:
It is evident by definitions that in a neutrosophic crisp topological space $(\mathcal{U}, T)$, the following hold:
(i) Every NC-CS (resp. NC-OS) is a NCSα-CS (resp. NCSα-OS).
(ii) Every NCα-CS (resp. NCα-OS) is a NCSα-CS (resp. NCSα-OS).
The converse of Proposition (3.3) need not be true as shown by the following example.

Example 3.4:
Let $\mathcal{U} = \{p, q, r, s\}, \mathcal{A} = \langle\{p\}, \{q, s\}, \{r\}\rangle, \mathcal{B} = \langle\{p\}, \{q\}, \{r\}\rangle$.Then $T = \{\emptyset_N, \mathcal{A}, \mathcal{B}, \mathcal{U}_N\}$ is a neutrosophic crisp topology on $\mathcal{U}$.

(i) Let $\mathcal{H} = \langle\{p\},\{q,r,s\},\emptyset\rangle$, $\mathcal{A} \subseteq \mathcal{H} \subseteq NCcl(\mathcal{A}) = \mathcal{U}_N$, the neutrosophic crisp set $\mathcal{H}$ is a NCSα-OS but not NC-OS. It is clear that $\mathcal{H}^c = \langle\{q,r,s\},\{p\},\mathcal{U}\rangle$ is a NCSα-CS but not NC-CS.
(ii) Let $\mathcal{K} = \langle\emptyset,\{q,r,s\},\{r,s\}\rangle$ and so $\mathcal{K} \nsubseteq NCint(NCcl(NCint(\mathcal{K})))$, the neutrosophic crisp set $\mathcal{K}$ is a NCSα-OS but not NCα-OS. It is clear that $\mathcal{K}^c = \langle\mathcal{U},\{p\},\{p,q\}\rangle$ is a NCSα-CS but not NCα-CS.

Remark 3.5:
The concepts of NCSα-CS (resp. NCSα-OS) and NCP-CS (resp. NCP-OS) are independent, as the following examples show.

Example 3.6:
Let $\mathcal{U} = \{p,q,r,s\}, \mathcal{A} = \langle\{p\},\{q\},\{r\}\rangle, \mathcal{B} = \langle\{r\},\{q\},\{s\}\rangle, \mathcal{C} = \langle\{p,r\},\{q\},\emptyset\rangle, \mathcal{D} = \langle\emptyset,\{q\},\{r,s\}\rangle$.
Then $T = \{\emptyset_N, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{U}_N\}$ is a neutrosophic crisp topology on $\mathcal{U}$. Let $\mathcal{H} = \langle\{r,s\},\{p,q\},\{s\}\rangle$, $\mathcal{B} \subseteq \mathcal{H} \subseteq NCcl(\mathcal{B}) = \langle\{r,s\},\{q\},\emptyset\rangle$, the neutrosophic crisp set $\mathcal{H}$ is a NCSα-OS but not NCP-OS. It is clear that $\mathcal{H}^c = \langle\{s\},\{p,q\},\{r,s\}\rangle$ is a NCSα-CS but not NCP-CS.

Example 3.7:
Let $\mathcal{U} = \{p,q,r,s\}, \mathcal{A}_1 = \langle\{p\},\{q\},\{r\}\rangle, \mathcal{A}_2 = \langle\{p\},\{q,s\},\{r\}\rangle$. Then $T = \{\emptyset_N, \mathcal{A}_1, \mathcal{A}_2, \mathcal{U}_N\}$ is a neutrosophic crisp topology on $\mathcal{U}$. If $\mathcal{A}_3 = \langle\{p,q\},\{r\},\{s\}\rangle$, then $\mathcal{A}_3$ is a NCP-OS but not NCSα-OS. It is clear that ${\mathcal{A}_3}^c = \langle\{s\},\{r\},\{p,q\}\rangle$ is a NCP-CS but not NCSα-CS.

Remark 3.8:
(i) If every NC-OS is a NC-CS and every nowhere neutrosophic crisp dense set is NC-CS in any neutrosophic crisp topological space $(\mathcal{U},T)$, then every NCSα-CS (resp. NCSα-OS) is a NC-CS (resp. NC-OS).
(ii) If every NC-OS is a NC-CS in any neutrosophic crisp topological space $(\mathcal{U},T)$, then every NCSα-CS (resp. NCSα-OS) is a NCα-CS (resp. NCα-OS).

Remark 3.9:
(i) It is clear that every NCS-CS (resp. NCS-OS) and NCP-CS (resp. NCP-OS) of any neutrosophic crisp topological space $(\mathcal{U},T)$ is a NCSα-CS (resp. NCSα-OS) (by Proposition (2.5) and Proposition (3.3) (ii)).
(ii) A NCSα-CS (resp. NCSα-OS) in any neutrosophic crisp topological space $(\mathcal{U},T)$ is a NCP-CS (resp. NCP-OS) if every NC-OS of $\mathcal{U}$ is a NC-CS (from Proposition (2.4) (iii) and Remark (3.8) (ii)).

Theorem 3.10:
For any neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U},T)$. The following properties are equivalent:
(i) $\mathcal{A} \in$ NCSαO$(\mathcal{U})$.
(ii) There exists a NC-OS, say $\mathcal{H}$, such that $\mathcal{H} \subseteq \mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H})))$.
(iii) $\mathcal{A} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$.
Proof:
$(i) \Longrightarrow (ii)$ Let $\mathcal{A} \in$ NCSαO$(\mathcal{U})$. Then, there exists $\mathcal{K} \in$ NCαO$(\mathcal{U})$, such that $\mathcal{K} \subseteq \mathcal{A} \subseteq NCcl(\mathcal{K})$. Hence there exists $\mathcal{H}$ NC-OS such that $\mathcal{H} \subseteq \mathcal{K} \subseteq NCint(NCcl(\mathcal{H}))$(by Theorem (2.6)). Therefore, $NCcl(\mathcal{H}) \subseteq NCcl(\mathcal{K}) \subseteq NCcl(NCint(NCcl(\mathcal{H})))$, implies that $NCcl(\mathcal{K}) \subseteq NCcl(NCint(NCcl(\mathcal{H})))$.
Then $\mathcal{H} \subseteq \mathcal{K} \subseteq \mathcal{A} \subseteq NCcl(\mathcal{K}) \subseteq NCcl(NCint(NCcl(\mathcal{H})))$. Hence, $\mathcal{H} \subseteq \mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H})))$, for some $\mathcal{H}$ NC-OS.
$(ii) \Longrightarrow (iii)$ Suppose that there exists a NC-OS $\mathcal{H}$ such that $\mathcal{H} \subseteq \mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H})))$. We know that $NCint(\mathcal{A}) \subseteq \mathcal{A}$. On the other hand, $\mathcal{H} \subseteq NCint(\mathcal{A})$ (since $NCint(\mathcal{A})$ is the largest NC-OS contained in $\mathcal{A}$). Hence $NCcl(\mathcal{H}) \subseteq NCcl(NCint(\mathcal{A}))$, then $NCint(NCcl(\mathcal{H})) \subseteq NCint(NCcl(NCint(\mathcal{A})))$,
therefore $NCcl(NCint(NCcl(\mathcal{H}))) \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$. But $\mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H})))$ (by hypothesis). Hence $\mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H}))) \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$,
then $\mathcal{A} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$.
$(iii) \Longrightarrow (i)$ Let $\mathcal{A} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$. To prove $\mathcal{A} \in$ NCSαO$(\mathcal{U})$, let $\mathcal{K} = NCint(\mathcal{A})$; we know that $NCint(\mathcal{A}) \subseteq \mathcal{A}$. To prove $\mathcal{A} \subseteq NCcl(NCint(\mathcal{A}))$.
Since $NCint(NCcl(NCint(\mathcal{A}))) \subseteq NCcl(NCint(\mathcal{A}))$.
Hence, $NCcl(NCint(NCcl(NCint(\mathcal{A})))) \subseteq NCcl(NCcl(NCint(\mathcal{A}))) = NCcl(NCint(\mathcal{A}))$.
But $\mathcal{A} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ (by hypothesis). Hence, $\mathcal{A} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$
$\subseteq NCcl(NCint(\mathcal{A})) \Longrightarrow \mathcal{A} \subseteq NCcl(NCint(\mathcal{A}))$. Hence, there exists an NC-OS say $\mathcal{K}$, such that $\mathcal{K} \subseteq \mathcal{A} \subseteq NCcl(\mathcal{A})$. On the other hand, $\mathcal{K}$ is a NCα-OS (since $\mathcal{K}$ is a NC-OS). Hence $\mathcal{A} \in$ NCSαO$(\mathcal{U})$.

Corollary 3.11:
For any neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$, the following properties are equivalent:
(i) $\mathcal{A} \in$ NCSαC$(\mathcal{U})$.
(ii) There exists a NC-CS $\mathcal{F}$ such that $NCint(NCcl(NCint(\mathcal{F}))) \subseteq \mathcal{A} \subseteq \mathcal{F}$.
(iii) $NCint(NCcl(NCint(NCcl(\mathcal{A})))) \subseteq \mathcal{A}$.
Proof:
$(i) \Longrightarrow (ii)$ Let $\mathcal{A} \in$ NCSαC$(\mathcal{U})$, then $\mathcal{A}^c \in$ NCSαO$(\mathcal{U})$. Hence there is $\mathcal{H}$ NC-OS such that $\mathcal{H} \subseteq \mathcal{A}^c \subseteq NCcl(NCint(NCcl(\mathcal{H})))$ (by Theorem (3.10)). Hence $(NCcl(NCint(NCcl(\mathcal{H}))))^c \subseteq \mathcal{A}^{cc} \subseteq \mathcal{H}^c$, i.e., $NCint(NCcl(NCint(\mathcal{H}^c))) \subseteq \mathcal{A} \subseteq \mathcal{H}^c$. Let $\mathcal{H}^c = \mathcal{F}$, where $\mathcal{F}$ is a NC-CS in $\mathcal{U}$.
Then $NCint(NCcl(NCint(\mathcal{F}))) \subseteq \mathcal{A} \subseteq \mathcal{F}$, for some $\mathcal{F}$ NC-CS.
$(ii) \Longrightarrow (iii)$ Suppose that there exists $\mathcal{F}$ NC-CS such that $NCint\left(NCcl\left(NCint(\mathcal{F})\right)\right) \subseteq \mathcal{A} \subseteq \mathcal{F}$, but $NCcl(\mathcal{A})$ is the smallest NC-CS containing $\mathcal{A}$. Then $NCcl(\mathcal{A}) \subseteq \mathcal{F}$, and therefore: $NCint\left(NCcl(\mathcal{A})\right) \subseteq NCint(\mathcal{F})$
$\Longrightarrow NCcl\left(NCint\left(NCcl(\mathcal{A})\right)\right) \subseteq NCcl\left(NCint(\mathcal{F})\right) \Longrightarrow NCint(NCcl(NCint(NCcl(\mathcal{A})))) \subseteq NCint(NCcl(NCint(\mathcal{F}))) \subseteq \mathcal{A} \Longrightarrow NCint(NCcl(NCint(NCcl(\mathcal{A})))) \subseteq \mathcal{A}$.
$(iii) \Longrightarrow (i)$ Let $NCint(NCcl(NCint(NCcl(\mathcal{A})))) \subseteq \mathcal{A}$. To prove $\mathcal{A} \in$ NCSαC$(\mathcal{U})$, i.e., to prove $\mathcal{A}^c \in$ NCSαO$(\mathcal{U})$. Then $\mathcal{A}^c \subseteq (NCint(NCcl(NCint(NCcl(\mathcal{A})))))^c = NCcl(NCint(NCcl(NCint(\mathcal{A}^c))))$, but $(NCint(NCcl(NCint(NCcl(\mathcal{A})))))^c = NCcl(NCint(NCcl(NCint(\mathcal{A}^c))))$.
Hence $\mathcal{A}^c \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}^c))))$, and therefore $\mathcal{A}^c \in$ NCSαO$(\mathcal{U})$, i.e., $\mathcal{A} \in$ NCSαC$(\mathcal{U})$.

Theorem 3.12:
The union of any family of NCSα-OS is a NCSα-OS.
Proof: Let $\{\mathcal{A}_\lambda\}_{\lambda\in\Lambda}$ be a family of NCSα-OS. To prove $\bigcup_{\lambda\in\Lambda}\mathcal{A}_\lambda$ is a NCSα-OS. Since $\mathcal{A}_\lambda \in$ NCSαO$(\mathcal{U})$. Then there is a NCα-OS $\mathcal{B}_\lambda$ such that $\mathcal{B}_\lambda \subseteq \mathcal{A}_\lambda \subseteq NCcl(\mathcal{B}_\lambda)$, $\forall \lambda \in \Lambda$. Hence $\bigcup_{\lambda\in\Lambda}\mathcal{B}_\lambda \subseteq \bigcup_{\lambda\in\Lambda}\mathcal{A}_\lambda \subseteq \bigcup_{\lambda\in\Lambda} NCcl(\mathcal{B}_\lambda) \subseteq NCcl(\bigcup_{\lambda\in\Lambda}\mathcal{B}_\lambda)$. But $\bigcup_{\lambda\in\Lambda}\mathcal{B}_\lambda \in$ NCαO$(\mathcal{U})$ (by Proposition (2.7)). Hence $\bigcup_{\lambda\in\Lambda}\mathcal{A}_\lambda \in$ NCSαO$(\mathcal{U})$.

Corollary 3.13:
The intersection of any family of NCSα-CS is a NCSα-CS.
Proof: This follows directly from Theorem (3.12).

Remark 3.14:
The following diagram shows the relations among the different types of weakly neutrosophic crisp closed sets that were studied in this section:

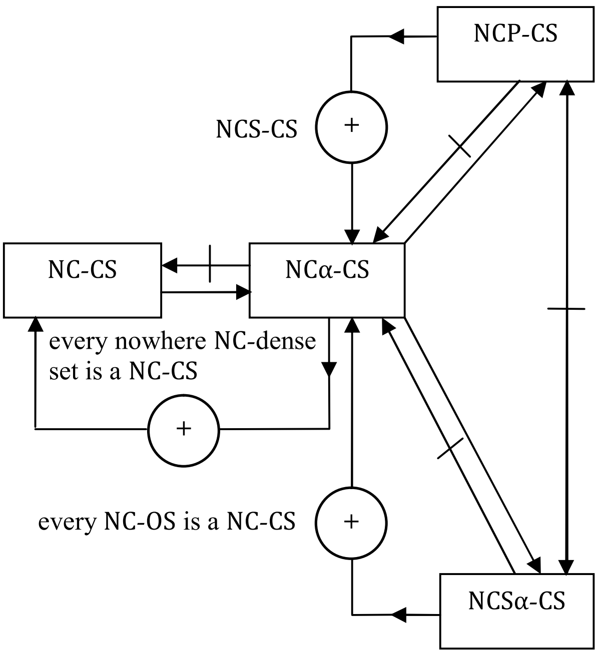

Diagram (3.1)

4. Neutrosophic Crisp Semi-α-Closure and Neutrosophic Crisp Semi-α-Interior

We present neutrosophic crisp semi-α-closure and neutrosophic crisp semi-α-interior and obtain some of their properties in this section.

Definition 4.1:
The intersection of all NCSα-CS in a neutrosophic crisp topological space $(\mathcal{U},T)$ containing $\mathcal{A}$ is called neutrosophic crisp semi-α-closure of $\mathcal{A}$ and is denoted by $S\alpha NCcl(\mathcal{A})$, $S\alpha NCcl(\mathcal{A}) = \bigcap\{\mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NCSα-CS}.

Definition 4.2:
The union of all NCSα-OS in a neutrosophic crisp topological space $(\mathcal{U},T)$ contained in $\mathcal{A}$ is called neutrosophic crisp semi-α-interior of $\mathcal{A}$ and is denoted by $S\alpha NCint(\mathcal{A})$, $S\alpha NCint(\mathcal{A}) = \bigcup\{\mathcal{B}: \mathcal{B} \subseteq \mathcal{A}, \mathcal{B}$ is a NCSα-OS}.

Proposition 4.3:
Let $\mathcal{A}$ be any neutrosophic crisp set in a neutrosophic crisp topological space $(\mathcal{U},T)$, the following properties are true:
(i) $S\alpha NCcl(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a NCSα-CS.
(ii) $S\alpha NCint(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a NCSα-OS.
(iii) $S\alpha NCcl(\mathcal{A})$ is the smallest NCSα-CS containing $\mathcal{A}$.
(iv) $S\alpha NCint(\mathcal{A})$ is the largest NCSα-OS contained in $\mathcal{A}$.
Proof: (i), (ii), (iii) and (iv) are obvious.

Proposition 4.4:
Let $\mathcal{A}$ be any neutrosophic crisp set in a neutrosophic crisp topological space $(\mathcal{U},T)$, the following properties hold:
(i) $S\alpha NCint(\mathcal{U}_N - \mathcal{A}) = \mathcal{U}_N - (S\alpha NCcl(\mathcal{A}))$,
(ii) $S\alpha NCcl(\mathcal{U}_N - \mathcal{A}) = \mathcal{U}_N - (S\alpha NCint(\mathcal{A}))$.
Proof: (i) By definition (2.3), $S\alpha NCcl(\mathcal{A}) = \bigcap\{\mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NCSα-CS}
$\mathcal{U}_N - (S\alpha NCcl(\mathcal{A})) = \mathcal{U}_N - \bigcap\{\mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NCSα-CS}
$= \bigcup\{\mathcal{U}_N - \mathcal{B}: \mathcal{A} \subseteq \mathcal{B}, \mathcal{B}$ is a NCSα-CS}
$= \bigcup\{\mathcal{H}: \mathcal{H} \subseteq \mathcal{U}_N - \mathcal{A}, \mathcal{H}$ is a NCSα-OS}
$= S\alpha NCint(\mathcal{U}_N - \mathcal{A})$.
(ii) The proof is similar to (i).

Theorem 4.5:
Let $\mathcal{A}$ and $\mathcal{B}$ be two neutrosophic crisp sets in a neutrosophic crisp topological space $(\mathcal{U},T)$. The following properties hold:
(i) $S\alpha NCcl(\emptyset_N) = \emptyset_N$, $S\alpha NCcl(\mathcal{U}_N) = \mathcal{U}_N$.
(ii) $\mathcal{A} \subseteq S\alpha NCcl(\mathcal{A})$.
(iii) $\mathcal{A} \subseteq \mathcal{B} \Longrightarrow S\alpha NCcl(\mathcal{A}) \subseteq S\alpha NCcl(\mathcal{B})$.
(iv) $S\alpha NCcl(\mathcal{A}\bigcap\mathcal{B}) \subseteq S\alpha NCcl(\mathcal{A})\bigcap S\alpha NCcl(\mathcal{B})$.
(v) $S\alpha NCcl(\mathcal{A})\bigcup S\alpha NCcl(\mathcal{B}) \subseteq S\alpha NCcl(\mathcal{A}\bigcup\mathcal{B})$.
(vi) $S\alpha NCcl(S\alpha NCcl(\mathcal{A})) = S\alpha NCcl(\mathcal{A})$.
Proof: (i) and (ii) are evident.
(iii) By (ii), $\mathcal{B} \subseteq S\alpha NCcl(\mathcal{B})$. Since $\mathcal{A} \subseteq \mathcal{B}$, we have $\mathcal{A} \subseteq S\alpha NCcl(\mathcal{B})$. But $S\alpha NCcl(\mathcal{B})$ is a NCSα-CS. Thus $S\alpha NCcl(\mathcal{B})$ is a NCSα-CS containing $\mathcal{A}$.
Since $S\alpha NCcl(\mathcal{A})$ is the smallest NCSα-CS containing $\mathcal{A}$, we have $S\alpha NCcl(\mathcal{A}) \subseteq S\alpha NCcl(\mathcal{B})$. Hence, $\mathcal{A} \subseteq \mathcal{B} \Longrightarrow S\alpha NCcl(\mathcal{A}) \subseteq S\alpha NCcl(\mathcal{B})$.
(iv) We know that $\mathcal{A}\bigcap\mathcal{B} \subseteq \mathcal{A}$ and $\mathcal{A}\bigcap\mathcal{B} \subseteq \mathcal{B}$. Therefore, by (iii), $S\alpha NCcl(\mathcal{A}\bigcap\mathcal{B}) \subseteq S\alpha NCcl(\mathcal{A})$ and $S\alpha NCcl(\mathcal{A}\bigcap\mathcal{B}) \subseteq S\alpha NCcl(\mathcal{B})$. Hence $S\alpha NCcl(\mathcal{A}\bigcap\mathcal{B}) \subseteq S\alpha NCcl(\mathcal{A})\bigcap S\alpha NCcl(\mathcal{B})$.
(v) Since $\mathcal{A} \subseteq \mathcal{A}\bigcup\mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A}\bigcup\mathcal{B}$, it follows from part (iii) that $S\alpha NCcl(\mathcal{A}) \subseteq S\alpha NCcl(\mathcal{A}\bigcup\mathcal{B})$ and $S\alpha NCcl(\mathcal{B}) \subseteq S\alpha NCcl(\mathcal{A}\bigcup\mathcal{B})$. Hence $S\alpha NCcl(\mathcal{A})\bigcup S\alpha NCcl(\mathcal{B}) \subseteq S\alpha NCcl(\mathcal{A}\bigcup\mathcal{B})$.
(vi) Since $S\alpha NCcl(\mathcal{A})$ is a NCSα-CS, we have by Proposition (4.3)(i), $S\alpha NCcl(S\alpha NCcl(\mathcal{A})) = S\alpha NCcl(\mathcal{A})$.

Theorem 4.6:
Let $\mathcal{A}$ and $\mathcal{B}$ be two neutrosophic crisp sets in a neutrosophic crisp topological space $(\mathcal{U},T)$. The following properties hold:
(i) $S\alpha NCint(\emptyset_N) = \emptyset_N$, $S\alpha NCint(\mathcal{U}_N) = \mathcal{U}_N$.
(ii) $S\alpha NCint(\mathcal{A}) \subseteq \mathcal{A}$.

(iii) $\mathcal{A} \subseteq \mathcal{B} \Longrightarrow S\alpha NCint(\mathcal{A}) \subseteq S\alpha NCint(\mathcal{B})$.
(iv) $S\alpha NCint(\mathcal{A} \cap \mathcal{B}) \subseteq S\alpha NCint(\mathcal{A}) \cap S\alpha NCint(\mathcal{B})$.
(v) $S\alpha NCint(\mathcal{A}) \cup S\alpha NCint(\mathcal{B}) \subseteq S\alpha NCint(\mathcal{A} \cup \mathcal{B})$.
(vi) $S\alpha NCint(S\alpha NCint(\mathcal{A})) = S\alpha NCint(\mathcal{A})$.
Proof: (i), (ii), (iii), (iv), (v) and (vi) are obvious.

Proposition 4.7:
For any neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U}, T)$, then:
(i) $NCint(\mathcal{A}) \subseteq \alpha NCint(\mathcal{A}) \subseteq S\alpha NCint(\mathcal{A}) \subseteq S\alpha NCcl(\mathcal{A}) \subseteq \alpha NCcl(\mathcal{A}) \subseteq NCcl(\mathcal{A})$.
(ii) $NCint(S\alpha NCint(\mathcal{A})) = S\alpha NCint(NCint(\mathcal{A})) = NCint(\mathcal{A})$.
(iii) $\alpha NCint(S\alpha NCint(\mathcal{A})) = S\alpha NCint(\alpha NCint(\mathcal{A})) = \alpha NCint(\mathcal{A})$.
(iv) $NCcl(S\alpha NCcl(\mathcal{A})) = S\alpha NCcl(NCcl(\mathcal{A})) = NCcl(\mathcal{A})$.
(v) $\alpha NCcl(S\alpha NCcl(\mathcal{A})) = S\alpha NCcl(\alpha NCcl(\mathcal{A})) = \alpha NCcl(\mathcal{A})$.
(vi) $S\alpha NCcl(\mathcal{A}) = \mathcal{A} \cup NCint(NCcl(NCint(NCcl(\mathcal{A}))))$.
(vii) $S\alpha NCint(\mathcal{A}) = \mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$.
(viii) $NCint(NCcl(\mathcal{A})) \subseteq S\alpha NCint(S\alpha NCcl(\mathcal{A}))$.
Proof: We shall prove only (ii), (iii), (iv), (vii) and (viii).
(ii) To prove $NCint(S\alpha NCint(\mathcal{A})) = S\alpha NCint(NCint(\mathcal{A})) = NCint(\mathcal{A})$, we know that $NCint(\mathcal{A})$ is a NC-OS. It follows that $NCint(\mathcal{A})$ is a NCSα-OS. Hence $NCint(\mathcal{A}) = S\alpha NCint(NCint(\mathcal{A}))$ (by Proposition (4.3)).
Therefore: $NCint(\mathcal{A}) = S\alpha NCint(NCint(\mathcal{A}))$……..(1)
Since $NCint(\mathcal{A}) \subseteq S\alpha NCint(\mathcal{A}) \Longrightarrow NCint(NCint(\mathcal{A})) \subseteq NCint(S\alpha NCint(\mathcal{A})) \Longrightarrow NCint(\mathcal{A}) \subseteq NCint(S\alpha NCint(\mathcal{A}))$. Also, $S\alpha NCint(\mathcal{A}) \subseteq \mathcal{A} \Longrightarrow NCint(S\alpha NCint(\mathcal{A})) \subseteq NCint(\mathcal{A})$.
Hence: $NCint(\mathcal{A}) = NCint(S\alpha NCint(\mathcal{A}))$…………………..(2)
Therefore by (1) and (2), we get $NCint(S\alpha NCint(\mathcal{A})) = S\alpha NCint(NCint(\mathcal{A})) = NCint(\mathcal{A})$.
(iii) Now we prove $\alpha NCint(S\alpha NCint(\mathcal{A})) = S\alpha NCint(\alpha NCint(\mathcal{A})) = \alpha NCint(\mathcal{A})$.
Since $\alpha NCint(\mathcal{A})$ is NCα-OS, therefore $\alpha NCint(\mathcal{A})$ is NCSα-OS. Therefore by Proposition (4.3):
$\alpha NCint(\mathcal{A}) = S\alpha NCint(\alpha NCint(\mathcal{A}))$………..(1)
Now, to prove $\alpha NCint(\mathcal{A}) = \alpha NCint(S\alpha NCint(\mathcal{A}))$, we have $\alpha NCint(\mathcal{A}) \subseteq S\alpha NCint(\mathcal{A}) \Longrightarrow \alpha NCint(\alpha NCint(\mathcal{A})) \subseteq \alpha NCint(S\alpha NCint(\mathcal{A})) \Longrightarrow \alpha NCint(\mathcal{A}) \subseteq \alpha NCint(S\alpha NCint(\mathcal{A}))$.
Also, $S\alpha NCint(\mathcal{A}) \subseteq \mathcal{A} \Longrightarrow \alpha NCint(S\alpha NCint(\mathcal{A})) \subseteq \alpha NCint(\mathcal{A})$.
Hence: $\alpha NCint(\mathcal{A}) = \alpha NCint(S\alpha NCint(\mathcal{A}))$………………...(2)
Therefore by (1) and (2), we get $\alpha NCint(S\alpha NCint(\mathcal{A})) = S\alpha NCint(\alpha NCint(\mathcal{A})) = \alpha NCint(\mathcal{A})$.
(iv) To prove $NCcl(S\alpha NCcl(\mathcal{A})) = S\alpha NCcl(NCcl(\mathcal{A})) = NCcl(\mathcal{A})$. We know that $NCcl(\mathcal{A})$ is a NC-CS, so it is NCSα-CS. Hence by proposition (4.3), we have: $NCcl(\mathcal{A}) = S\alpha NCcl(NCcl(\mathcal{A}))$….......(1)
To prove $\boldsymbol{NCcl(\mathcal{A}) = NCcl(S\alpha NCcl(\mathcal{A}))}$, we have $\boldsymbol{S\alpha NCcl(\mathcal{A}) \subseteq NCcl(\mathcal{A})}$ (by part (i)).
Then $\boldsymbol{NCcl(S\alpha NCcl(\mathcal{A})) \subseteq NCcl(NCcl(\mathcal{A})) = NCcl(\mathcal{A}) \Longrightarrow NCcl(S\alpha NCcl(\mathcal{A})) \subseteq NCcl(\mathcal{A})}$.
Since $\boldsymbol{\mathcal{A} \subseteq S\alpha NCcl(\mathcal{A}) \subseteq NCcl(S\alpha NCcl(\mathcal{A}))}$, then $\boldsymbol{\mathcal{A} \subseteq NCcl(S\alpha NCcl(\mathcal{A}))}$.
Hence, $NCcl(\mathcal{A}) \subseteq NCcl\left(NCcl(S\alpha NCcl(\mathcal{A}))\right) = NCcl(S\alpha NCcl(\mathcal{A})) \Longrightarrow NCcl(\mathcal{A}) \subseteq NCcl(S\alpha NCcl(\mathcal{A}))$ and therefore: $NCcl(\mathcal{A}) = NCcl(S\alpha NCcl(\mathcal{A}))$……...(2)
Now, by (1) and (2), we get that $NCcl(S\alpha NCcl(\mathcal{A})) = S\alpha NCcl(NCcl(\mathcal{A}))$. Hence $NCcl(S\alpha NCcl(\mathcal{A})) = S\alpha NCcl(NCcl(\mathcal{A})) = NCcl(\mathcal{A})$.
(vii) To prove $S\alpha NCint(\mathcal{A}) = \mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$, since $S\alpha NCint(\mathcal{A}) \in$ NCSαO$(\mathcal{U}) \Longrightarrow S\alpha NCint(\mathcal{A}) \subseteq NCcl(NCint(NCcl(NCint(S\alpha NCint(\mathcal{A}))))) = NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ (by part (ii)). Hence, $S\alpha NCint(\mathcal{A}) \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$, also $S\alpha NCint(\mathcal{A}) \subseteq \mathcal{A}$. Then:
$S\alpha NCint(\mathcal{A}) \subseteq \mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$..(1)
To prove $\mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ is a NCSα-OS contained in $\mathcal{A}$.
It is clear that $\mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A})))) \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ and also it is clear that $NCint(\mathcal{A}) \subseteq NCcl(NCint(\mathcal{A})) \Longrightarrow NCint(NCint(\mathcal{A})) \subseteq NCint(NCcl(NCint(\mathcal{A}))) \Longrightarrow NCint(\mathcal{A}) \subseteq NCint(NCcl(NCint(\mathcal{A}))) \Longrightarrow NCcl(NCint(\mathcal{A})) \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ and $NCint(\mathcal{A}) \subseteq NCcl(NCint(\mathcal{A})) \Longrightarrow NCint(\mathcal{A}) \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ and $NCint(\mathcal{A}) \subseteq \mathcal{A} \Longrightarrow NCint(\mathcal{A}) \subseteq \mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$. We get $NCint(\mathcal{A}) \subseteq \mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A})))) \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$. Hence $\mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ is a NCSα-OS (by Proposition (4.3)). Also, $\mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ is contained in $\mathcal{A}$.
Then $\mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A})))) \subseteq S\alpha NCint(\mathcal{A})$ (since $S\alpha NCint(\mathcal{A})$ is the largest NCSα - OS contained in $\mathcal{A}$). Hence: $\mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A})))) \subseteq S\alpha NCint(\mathcal{A})$...............(2)
By (1) and (2), we get that $S\alpha NCint(\mathcal{A}) = \mathcal{A} \cap NCcl(NCint(NCcl(NCint(\mathcal{A}))))$.
(viii) To prove that $NCint(NCcl(\mathcal{A})) \subseteq S\alpha NCint(S\alpha NCcl(\mathcal{A}))$, we know that $S\alpha NCcl(\mathcal{A})$ is a NCSα-CS, therefore $NCint(NCcl(NCint(NCcl(S\alpha NCcl(\mathcal{A}))))) \subseteq S\alpha NCcl(\mathcal{A})$ (by Corollary (3.11)). Hence $NCint(NCcl(\mathcal{A})) \subseteq NCint(NCcl(NCint(NCcl(\mathcal{A})))) \subseteq S\alpha NCcl(\mathcal{A})$ (by part (iv)). Therefore,

$S\alpha NCint\left(NCint(NCcl(\mathcal{A}))\right) \subseteq S\alpha NCint(S\alpha NCcl(\mathcal{A})) \Rightarrow NCint(NCcl(\mathcal{A})) \subseteq S\alpha NCint(S\alpha NCcl(\mathcal{A}))$ (by (ii)).

Theorem 4.8:
For any neutrosophic crisp subset $\mathcal{A}$ of a neutrosophic crisp topological space $(\mathcal{U},T)$. The following properties are equivalent:
(i) $\mathcal{A} \in$ NCSαO$(\mathcal{U})$.
(ii) $\mathcal{H} \subseteq \mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H})))$, for some NC-OS $\mathcal{H}$.
(iii) $\mathcal{H} \subseteq \mathcal{A} \subseteq SNCint(NCcl(\mathcal{H}))$, for some NC-OS $\mathcal{H}$.
(iv) $\mathcal{A} \subseteq SNCint(NCcl(NCint(\mathcal{A})))$.
Proof:
$(i) \Longrightarrow (ii)$ Let $\mathcal{A} \in$ NCSαO$(\mathcal{U})$, then $\mathcal{A} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ and $NCint(\mathcal{A}) \subseteq \mathcal{A}$. Hence $\mathcal{H} \subseteq \mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H})))$, where $\mathcal{H} = NCint(\mathcal{A})$.
$(ii) \Longrightarrow (iii)$ Suppose $\mathcal{H} \subseteq \mathcal{A} \subseteq NCcl(NCint(NCcl(\mathcal{H})))$, for some NC-OS $\mathcal{H}$. But $SNCint(NCcl(\mathcal{H})) = NCcl(NCint(NCcl(\mathcal{H})))$ (by Proposition (2.8)). Then $\mathcal{H} \subseteq \mathcal{A} \subseteq SNCint(NCcl(\mathcal{H}))$, for some NC-OS $\mathcal{H}$.
$(iii) \Longrightarrow (iv)$ Suppose that $\mathcal{H} \subseteq \mathcal{A} \subseteq SNCint(NCcl(\mathcal{H}))$, for some NC-OS $\mathcal{H}$. Since $\mathcal{H}$ is a NC-OS contained in $\mathcal{A}$. Then $\mathcal{H} \subseteq NCint(\mathcal{A}) \Longrightarrow NCcl(\mathcal{H}) \subseteq NCcl(NCint(\mathcal{A}))$
$\Longrightarrow SNCint(NCcl(\mathcal{H})) \subseteq SNCint(NCcl(NCint(\mathcal{A})))$. But $\mathcal{A} \subseteq SNCint(NCcl(\mathcal{H}))$ (by hypothesis), then $\mathcal{A} \subseteq SNCint(NCcl(NCint(\mathcal{A})))$.
$(iv) \Longrightarrow (i)$ Let $\mathcal{A} \subseteq SNCint(NCcl(NCint(\mathcal{A})))$.
But $SNCint(NCcl(NCint(\mathcal{A}))) = NCcl(NCint(NCcl(NCint(\mathcal{A}))))$ (by Proposition (2.8)).
Hence, $\mathcal{A} \subseteq NCcl(NCint(NCcl(NCint(\mathcal{A})))) \Longrightarrow \mathcal{A} \in$ NCSαO$(\mathcal{U})$.

Corollary 4.9:
For any neutrosophic crisp subset $\mathcal{B}$ of a neutrosophic crisp topological space $(\mathcal{U},T)$, the following properties are equivalent:
(i) $\mathcal{B} \in$ NCSαC$(\mathcal{U})$.
(ii) $NCint(NCcl(NCint(\mathcal{F}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ NC-CS.
(iii) $SNCcl(NCint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ NC-CS.
(iv) $SNCcl(NCint(NCcl(\mathcal{B}))) \subseteq \mathcal{B}$.
Proof:
$(i) \Longrightarrow (ii)$ Let $\mathcal{B} \in$ NCSαC$(\mathcal{U}) \Longrightarrow NCint(NCcl(NCint(NCcl(\mathcal{B})))) \subseteq \mathcal{B}$ (by Corollary(3.11))
and $\mathcal{B} \subseteq NCcl(\mathcal{B})$. Hence we obtain $NCint(NCcl(NCint(NCcl(\mathcal{B})))) \subseteq \mathcal{B} \subseteq NCcl(\mathcal{B})$.
Therefore, $NCint(NCcl(NCint(\mathcal{F}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$, where $\mathcal{F} = NCcl(\mathcal{B})$.
$(ii) \Longrightarrow (iii)$ Let $NCint(NCcl(NCint(\mathcal{F}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ NC-CS. But $NCint(NCcl(NCint(\mathcal{F}))) = SNCcl(NCint(\mathcal{F}))$ (by Proposition (2.8)). Hence $SNCcl(NCint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ NC-CS.
$(iii) \Longrightarrow (iv)$ Let $SNCcl(NCint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ NC-CS. Since $\mathcal{B} \subseteq \mathcal{F}$ (by hypothesis), then we have $NCcl(\mathcal{B}) \subseteq \mathcal{F} \Longrightarrow NCint(NCcl(\mathcal{B}) \subseteq NCint(\mathcal{F}) \Longrightarrow SNCcl(NCint(NCcl(\mathcal{B}))) \subseteq SNCcl(NCint(\mathcal{F})) \subseteq \mathcal{B} \Longrightarrow SNCcl(NCint(NCcl(\mathcal{B}))) \subseteq \mathcal{B}$.
$(iv) \Longrightarrow (i)$ Let $SNCcl(NCint(NCcl(\mathcal{B}))) \subseteq \mathcal{B}$.
But $SNCcl(NCint(NCcl(\mathcal{B}))) = NCint(NCcl(NCint(NCcl(\mathcal{B}))))$ (by Proposition (2.8)).
Hence, $NCint(NCcl(NCint(NCcl(\mathcal{B})))) \subseteq \mathcal{B} \Longrightarrow \mathcal{B} \in$ NCSαC$(\mathcal{U})$.

5. Conclusion

In this work, we have the new concept of neutrosophic crisp closed sets called neutrosophic crisp semi-α-closed sets and studied their fundamental properties in neutrosophic crisp topological spaces. The neutrosophic crisp semi-α-closed sets can obtain to derive a new decomposition of neutrosophic crisp continuity.

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