Neutrosophic $b^*g\alpha$-Closed Sets

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Abstract: This article introduces the concept of neutrosophic $b^*g\alpha$-closed sets, neutrosophic $b^*g\alpha$-border of a set, neutrosophic $b^*g\alpha$-frontier of a set in neutrosophic topological spaces and the properties of these sets are discussed. The connection between neutrosophic $b^*g\alpha$-border of a set and neutrosophic $b^*g\alpha$-frontier of a set in neutrosophic topological spaces are established.

Keywords: Neutrosophic $g\alpha$-closed sets, Neutrosophic $^*g\alpha$-closed sets, Neutrosophic $b^*g\alpha$-closed sets, Neutrosophic $b^*g\alpha$-border, Neutrosophic $b^*g\alpha$-frontier.

1 Introduction

Neutrosophic set initially proposed by Smarandache[8, 9] which is a generalization of Atanassov's[11] intuitionistic fuzzy sets and Zadeh's[12] fuzzy sets. Also it considers truth-membership function, indeterminacy-membership function and falsity-membership function. Since fuzzy sets and intuitionistic fuzzy sets fails to deal with indeterminacy-membership functions, Smarandache introduced the neutrosophic concept in various fields, including probability, algebra, control theory, topology, etc. Later Alblowi et al.,[20] introduced neutrosophic set based concepts in the neutrosophic field. These effective concepts has been applied by many researchers in the last two decades to propose many concepts in topology. Salama and Alblowi[3] proposed a new concept in neutrosophic topological spaces and it provides a brief idea about neutrosophic topology, which is a generalization of Coker's[6] intuitionistic fuzzy topology and Chang's[5] fuzzy topology.

Salama et al.,[4, 1, 2] introduced the generalization of neutrosophic sets, neutrosophic crisp sets and the neutrosophic closed sets in the field of neutrosophic topological spaces. Some neutrosophic continuous functions were introduced by Salama et al.,[2] as an initial continuous functions in neutrosophic topology. Further several researchers have defined some closed sets in neutrosophic topology, namely neutrosophic α-closed sets[10], neutrosophic αg-closed sets[7], neutrosophic b-closed sets[15], neutrosophic ω-closed sets[19], generalized neutrosophic closed sets[18] and neutrosophic $\alpha\psi$-closed sets[13] in neutrosophic topological spaces. Recently Iswarya and Bageerathi[16] proposed a new concept of neutrosophic frontier operator and neutrosophic semi-frontier operator in neutrosophic topological spaces, which provides the relationship between the operators of neutrosophic interior and neutrosophic closure. Vigneshwaran and Saranya[14] defined a new closed set as $b^*g\alpha$-closed sets in topological spaces, and it has been applied to define some topological functions as continuous functions, irresolute functions and homeomorphic functions with some separable axioms.

In this article, the notion of neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces are introduced and investigated their properties and the relation with other existing properties. The concept of neutrosophic $b^*g\alpha$-interior, neutrosophic $b^*g\alpha$-closure, neutrosophic $b^*g\alpha$-border and neutrosophic $b^*g\alpha$-frontier are introduced and discussed their properties. The connection between neutrosophic $b^*g\alpha$-border and neutrosophic $b^*g\alpha$-frontier in neutrosophic topological spaces are established with their related properties.

2 Preliminaries

In this section, we recall some of basic definitions which was already defined by various authors.

Definition 2.1. [3] Let X be a non empty fixed set. A neutrosophic set E is an object having the form $E = \{< x, mv(E(x)), iv(E(x)), nmv(E(x)) > \ \forall\ x \in X\}$, where $mv(E(x))$ represents the degree of membership, $iv(E(x))$ represents the degree of indeterminacy and $nmv(E(x))$ represents the degree of non-membership functions of each element $x \in X$ to the set E.

Remark 2.2. [3] A neutrosophic set $E = \{< x, mv(E(x)), iv(E(x)), nmv(E(x)) > \ \forall\ x \in X\}$ can be identified to an ordered triple $< mv(E), iv(E), nmv(E) >$ in $\left]^{-}0, 1^{+}\right[$ on X.

Definition 2.3. [3] Let E and F be two neutrosophic sets of the form, $E = \{< x, mv(E(x)), iv(E(x)), nmv(E(x)) > \ \forall\ x \in X\}$ and $F = \{< x, mv(F(x)), iv(F(x)), nmv(F(x)) > \ \forall\ x \in X\}$. Then,

i) $E \subseteq F$ if and only if $mv(E(x)) \leq mv(F(x))$, $iv(E(x)) \leq iv(F(x))$ and $nmv(E(x)) \geq nmv(F(x))$ $\forall\ x \in X$,

ii) $E = F$ if and only if $E \subseteq F$ and $F \subseteq E$,

iii) $\overline{E} = \{< x, nmv(E(x)), 1 - iv(E(x)), mv(E(x)) > \ \forall\ x \in X\}$,

iv) $E \cup F = \{x, max[mv(E(x)), mv(F(x))], min[iv(E(x)), iv(F(x))], min[nmv(E(x)), nmv(F(x))]$ $\forall\ x \in X\}$,

v) $E \cap F = \{x, min[mv(E(x)), mv(F(x))], max[iv(E(x)), iv(F(x))], max[nmv(E(x)), nmv(F(x))]$ $\forall\ x \in X\}$.

Definition 2.4. [3] A neutrosophic topology on a non empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

i) $0_N, 1_N \in \tau$,

ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

iii) $\cup G_i \in \tau\ \forall\ \{G_i : i \in J\} \subseteq \tau$.

Then the pair (X, τ) or simply X is called a neutrosophic topological space.

Definition 2.5. [10] A neutrosophic set E in a neutrosophic topological space (X, τ) is called

i) a neutrosophic semiopen set (briefly NSOS) if $E \subseteq Ncl(Nint(E))$.

ii) a neutrosophic α-open set (briefly NαOS) if $E \subseteq Nint(Ncl(Nint(E)))$.

iii) a neutrosophic preopen set (briefly NPOS) if $E \subseteq Nint(Ncl(E))$.

iv) a neutrosophic regular open set (briefly NROS) if $E = Nint(Ncl(E))$.

v) a neutrosophic semipreopen or β-open set (briefly NβOS) if $E \subseteq Ncl(Nint(Ncl(E)))$.

A neutrosophic set E is called neutrosophic semiclosed (resp. neutrosophic α-closed, neutrosophic preclosed, neutrosophic regular closed and neutrosophic β-closed) (briefly NSCS, NαCS, NPCS, NRCS and NβCS) if the complement of E is a neutrosophic semiopen (resp. neutrosophic α-open, neutrosophic preopen, neutrosophic regular open and neutrosophic β-open).

Definition 2.6. [15] Let E be a subset of a neutrosophic topological space (X, τ). Then E is called a neutrosophic b(N_b.In brief)-closed set if $[Ncl(Nint(E))] \cup [Nint(Ncl(E))] \subseteq E$.

Definition 2.7. [17] Let E be a neutrosophic set in a neutrosophic topological space (X, τ). Then,

i) $Nint(E) = \bigcup\{F|\ F$ is a neutrosophic open set in (X, τ) and $F \subseteq E\}$ is called the neutrosophic interior of E;

ii) $Ncl(E) = \bigcap\{F|\ F$ is a neutrosophic closed set in (X, τ) and $F \supseteq E\}$ is called the neutrosophic closure of E.

3 Neutrosophic $b^*g\alpha$-closed sets

In this section, the new concept of neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces was defined and studied.

Definition 3.1. Let E be a subset of a neutrosophic topological space (X, τ). Then E is called

i) a neutrosophic $g\alpha$-open set($N_{g\alpha}OS$) if $V \subseteq N_\alpha int(E)$ whenever $V \subseteq E$ and V is a neutrosophic α-closed set in (X, τ).

ii) a neutrosophic $g\alpha$-closed set($N_{g\alpha}CS$) if $N_\alpha cl(E) \subseteq V$ whenever $E \subseteq V$ and V is a neutrosophic α-open set in (X, τ).

iii) a neutrosophic $^*g\alpha$-open set($N_{^*g\alpha}OS$) if $V \subseteq Nint(E)$ whenever $V \subseteq E$ and V is a neutrosophic $g\alpha$-closed set in (X, τ).

iv) a neutrosophic $^*g\alpha$-closed set($N_{^*g\alpha}CS$) if $Ncl(E) \subseteq V$ whenever $E \subseteq V$ and V is a neutrosophic $g\alpha$-open set in (X, τ).

Definition 3.2. Let E be a subset of a neutrosophic topological space (X, τ). Then E is called

i) a neutrosophic $b^*g\alpha$-open set($N_{b^*g\alpha}OS$) if $V \subseteq N_b int(E)$ whenever $V \subseteq E$ and V is a neutrosophic $^*g\alpha$-closed set in (X, τ).

ii) a neutrosophic $b^*g\alpha$-closed set($N_{b^*g\alpha}CS$) if $N_bcl(E) \subseteq V$ whenever $E \subseteq V$ and V is a neutrosophic $^*g\alpha$-open set in (X, τ).

Example 3.3. Let $X = \{p, q, r\}$ and the neutrosophic sets L and M are defined as,

$$L = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{2/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{1/5}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\},$$

$$M = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{1/5}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\}.$$

Then the neutrosophic topology $\tau = \{0_N, L, M, 1_N\}$, which are neutrosophic open sets in the neutrosophic topological space (X, τ).

If $N = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{4/5}, \tfrac{q}{4/5}, \tfrac{r}{9/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\}$ and

$$E = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{3/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\}.$$

Then the complement of L, M, N and E are,

$$\overline{L} = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{4/5}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{2/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\},$$

$$\overline{M} = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{4/5}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\},$$

$\overline{N} = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{1/5}, \tfrac{q}{1/5}, \tfrac{r}{1/10}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\}$ and

$$\overline{E} = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\}.$$

Hence N is a neutrosophic $^*g\alpha$-open set, $\overline{N}$ is a neutrosophic $^*g\alpha$-closed set, E is a neutrosophic $b^*g\alpha$-closed set, $\overline{E}$ is a neutrosophic $b^*g\alpha$-open set of a neutrosophic topological space (X, τ). Since $N_bcl(E)$ = $\{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{1/5}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\}$, which is contained in N. That is $N_bcl(E) \subseteq N$.

Definition 3.4. Let E be a subset of a neutrosophic topological space (X, τ). Then $N_{b^*g\alpha}$-$int(E) = \bigcup\{F : F$ is neutrosophic $b^*g\alpha$-open set and $F \subset E\}$. The complement of $N_{b^*g\alpha}$-$int(E)$ is $N_{b^*g\alpha}$-$cl(E)$.

Remark 3.5. Let A be a subset of a neutrosophic topological space (X, τ), then $N_{b^*g\alpha}$-$int(A)$ is $N_{b^*g\alpha}$-open in (X, τ).

Theorem 3.6. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since E is neutrosophic closed, $Ncl(E) = E$. But $N_bcl(E) \subseteq Ncl(E) = E$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed set.*

The converse of the above theorem need not be true. It can be seen by the following example.

Example 3.7. Let $X = \{p, q, r\}$ and the neutrosophic sets L and M are defined as,

$L = \{< x, \left(\frac{p}{1/2}, \frac{q}{2/5}, \frac{r}{1/2}\right), \left(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{1/5}\right), \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\right) > \ \forall\ x \in X\}$,

$M = \{< x, \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\right), \left(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{1/5}\right), \left(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\right) > \ \forall\ x \in X\}$.

Then the neutrosophic topology $\tau = \{0_N, L, M, 1_N\}$ and the complement of neutrosophic sets L and M are defined as,

$\overline{L} = \{< x, \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\right), \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{4/5}\right), \left(\frac{p}{1/2}, \frac{q}{2/5}, \frac{r}{1/2}\right) > \ \forall\ x \in X\}$,

$\overline{M} = \{< x, \left(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\right), \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{4/5}\right), \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\right) > \ \forall\ x \in X\}$.

If $E = \{< x, \left(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\right), \left(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{3/10}\right), \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\right) > \ \forall\ x \in X\}$.

Then E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic closed set of a neutrosophic topological space (X, τ). Since $Ncl(E) = \overline{M}$ which is not equal to the neutrosophic set E.

Theorem 3.8. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic pre-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since E is neutrosophic pre-closed, $N_pcl(E)$ = E. But $N_bcl(E) \subseteq N_pcl(E) = E$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed set.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.9. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic pre-closed set of a neutrosophic topological space (X, τ). Since $Ncl(Nint(E)) = \overline{M}$ which is not contained in the neutrosophic set E.

Theorem 3.10. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic α-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since E is neutrosophic α-closed, $N_\alpha cl(E) = E$. But $N_bcl(E) \subseteq N_\alpha cl(E) = E$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed set.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.11. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic α-closed set of a neutrosophic topological space (X, τ). Since $Ncl(Nint(Ncl(E))) = \overline{M}$ which is not contained in the neutrosophic set E.

Theorem 3.12. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic $g\alpha$-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since every neutrosophic $^*g\alpha$-open set is neutrosophic α-open, V is neutrosophic α-open. Since E is neutrosophic $g\alpha$-closed in X, $N_\alpha cl(E) \subseteq V$. But $N_bcl(E) \subseteq N_\alpha cl(E) \subseteq V$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.13. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic $g\alpha$-closed set of a neutrosophic topological space (X,τ). Since N_α-open set $F = \{< x, \big(\frac{p}{5/10}, \frac{q}{5/10}, \frac{r}{5/10}\big), \big(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{3/10}\big), \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\big) > \ \forall\ x \in X\}$.

Theorem 3.14. *In the neutrosophic topological space (X,τ), if a subset E is a neutrosophic $^*g\alpha$-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since every neutrosophic $^*g\alpha$-open set is neutrosophic $g\alpha$-open, V is neutrosophic $g\alpha$-open. Since E is neutrosophic $^*g\alpha$-closed in X, $Ncl(E) \subseteq V$. But $N_bcl(E) \subseteq Ncl(E) \subseteq V$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.15. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic $^*g\alpha$-closed set of a neutrosophic topological space (X,τ). Since $N_{g\alpha}$-open set $G = \{< x, \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\big), \big(\frac{p}{3/10}, \frac{q}{2/5}, \frac{r}{3/10}\big), \big(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\big) > \ \forall\ x \in X\}$.

Theorem 3.16. *The union of any two neutrosophic $b^*g\alpha$-closed sets in (X,τ) is also a neutrosophic $b^*g\alpha$-closed set in (X,τ).*

Proof. *Let E and F be two neutrosophic $b^*g\alpha$-closed sets in (X,τ). Let V be a neutrosophic $^*g\alpha$-open set in X such that $E \subseteq V$ and $F \subseteq V$. Then we have, $E \cup F \subseteq V$. Since E and F are neutrosophic $b^*g\alpha$-closed sets in (X,τ), which implies $N_bcl(E) \subseteq V$ and $N_bcl(F) \subseteq V$. Now, $N_bcl(E \cup F) = N_bcl(E) \cup N_bcl(F) \subseteq V$. Thus, we have $N_bcl(E \cup F) \subseteq V$ whenever $E \cup F \subseteq V$, V is neutrosophic $^*g\alpha$-open set in (X,τ) which implies $E \cup F$ is a neutrosophic $b^*g\alpha$-closed set in (X,τ).*

Theorem 3.17. *Let E be a neutrosophic $b^*g\alpha$-closed subset of (X,τ). If $E \subseteq F \subseteq N_bcl(E)$, then F is also a neutrosophic $b^*g\alpha$-closed subset of (X,τ).*

Proof. *Let $F \subseteq V$, where V is neutrosophic $^*g\alpha$-open in (X,τ). Then $E \subseteq F$ implies $E \subseteq V$. Since E is neutrosophic $b^*g\alpha$-closed, $N_bcl(E) \subseteq V$. Also $F \subseteq N_bcl(E)$ implies $N_bcl(F) \subseteq N_bcl(E)$. Thus, $N_bcl(F) \subseteq V$ and so F is neutrosophic $b^*g\alpha$-closed.*

Theorem 3.18. *Let E be a neutrosophic $b^*g\alpha$-closed set in (X,τ). Then $N_bcl(E) - E$ has no non-empty neutrosophic $^*g\alpha$-closed set.*

Proof. *Let E be a neutrosophic $b^*g\alpha$-closed set in (X,τ), and F be a neutrosophic $^*g\alpha$-closed subset of $N_bcl(E) - E$. That is, $F \subseteq N_bcl(E) - E$, which implies that, $F \subseteq N_bcl(E) \cap \overline{E}$. That is $F \subseteq N_bcl(E)$ and $F \subseteq \overline{E}$, which implies $E \subseteq \overline{F}$, where $\overline{F}$ is a neutrosophic $^*g\alpha$-open set. Since E is neutrosophic $b^*g\alpha$-closed, $N_bcl(E) \subseteq \overline{F}$. That is $F \subseteq \overline{N_bcl(E)}$. Thus $F \subseteq N_bcl(E) \cap \overline{N_bcl(E)}$. Therefore $F = \phi$.*

4 Neutrosophic $b^*g\alpha$-Border

Definition 4.1. For any subset E of X, the neutrosophic $b^*g\alpha$-border of E is defined by

$$N_{b^*g\alpha}[Bd(E)] = E \setminus N_{b^*g\alpha}\text{-}int(E).$$

Theorem 4.2. *In the neutrosophic topological space (X, τ), for any subset E of X, the following statements are hold.*

i) $N_{b^*g\alpha}[Bd(\phi)] = N_{b^*g\alpha}[Bd(X)] = \phi$

ii) $E = N_{b^*g\alpha}\text{-}int(E) \cup N_{b^*g\alpha}[Bd(E)]$

iii) $N_{b^*g\alpha}\text{-}int(E) \cap N_{b^*g\alpha}[Bd(E)] = \phi$

iv) $N_{b^*g\alpha}\text{-}int(E) = E \setminus N_{b^*g\alpha}[Bd(E)]$

v) $N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)]) = \phi$

vi) *E is $N_{b^*g\alpha}$-open iff* $N_{b^*g\alpha}[Bd(E)] = \phi$

vii) $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}\text{-}int(E))] = \phi$

viii) $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}[Bd(E)])] = N_{b^*g\alpha}[Bd(E)]$

ix) $N_{b^*g\alpha}[Bd(E)] = E \cap N_{b^*g\alpha}\text{-}cl(X \setminus E)$

Proof. *Statements i) to iv) are obvious by the definition of neutrosophic $b^*g\alpha$-border of E. If possible, let $x \in N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)])$. Then $x \in N_{b^*g\alpha}[Bd(E)]$, since $N_{b^*g\alpha}[Bd(E)] \subseteq E$, $x \in N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)]) \subseteq N_{b^*g\alpha}\text{-}int(E)$. Therefore $x \in N_{b^*g\alpha}\text{-}int(E) \cap N_{b^*g\alpha}[Bd(E)]$, which is the contradiction to iii). Hence v) is proved. E is neutrosophic $b^*g\alpha$-open iff $N_{b^*g\alpha}\text{-}int(E) = E$. But $N_{b^*g\alpha}[Bd(E)] = E \setminus N_{b^*g\alpha}\text{-}Int(E)$ implies $N_{b^*g\alpha}[Bd(E)] = \phi$. This proves vi) & vii). When $E = N_{b^*g\alpha}[Bd(E)]$, then the definition of neutrosophic $b^*g\alpha$-border of E becomes $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}[Bd(E)])] = N_{b^*g\alpha}[Bd(E)] \setminus N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)])$. By using vii), we get the proof of viii). Now, $N_{b^*g\alpha}[Bd(E)] = E \setminus N_{b^*g\alpha}\text{-}int(E) = E \cap (X \setminus N_{b^*g\alpha}\text{-}int(E)) = E \cap N_{b^*g\alpha}\text{-}cl(X \setminus E)$.*

5 Neutrosophic $b^*g\alpha$-Frontier

Definition 5.1. For any subset E of X, the neutrosophic $b^*g\alpha$-frontier of E is defined by

$$N_{b^*g\alpha}[Fr(E)] = N_{b^*g\alpha}\text{-}cl(E) \setminus N_{b^*g\alpha}\text{-}int(E).$$

Theorem 5.2. *In the Neutrosophic topological space (X, τ), for any subset E of X, the following statements will be hold.*

i) $N_{b^*g\alpha}[Fr(\phi)] = N_{b^*g\alpha}[Fr(X)] = \phi$

ii) $N_{b^*g\alpha}\text{-}int(E) \cap N_{b^*g\alpha}[Fr(E)] = \phi$

iii) $N_{b^*g\alpha}[Fr(E)] \subseteq N_{b^*g\alpha}\text{-}cl(E)$

iv) $N_{b^*g\alpha}\text{-}int(E) \cup N_{b^*g\alpha}[Fr(E)] = N_{b^*g\alpha}\text{-}cl(E)$

v) $N_{b^*g\alpha}\text{-}int(E) = E \setminus N_{b^*g\alpha}[Fr(E)]$

vi) *If* E *is* $N_{b^*g\alpha}$*-closed, then* $E = N_{b^*g\alpha}\text{-}int(E) \cup N_{b^*g\alpha}[Fr(E)]$

vii) $Fr(E) = Fr(N_{b^*g\alpha}[Fr(E)]$

viii) *If* E *is* $N_{b^*g\alpha}$*-open, then* $E \cap N_{b^*g\alpha}[Fr(E)] = \phi$

ix) $X = N_{b^*g\alpha}\text{-}cl(E) \cup N_{b^*g\alpha}\text{-}cl(X \setminus E)$

x) *If* E *is* $N_{b^*g\alpha}$*-open, then* $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}int(E))] \subseteq N_{b^*g\alpha}[Fr(E)]$

xi) *If* E *is* $N_{b^*g\alpha}$*-closed, then* $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}cl(E))] \subseteq N_{b^*g\alpha}[Fr(E)]$

xii) *If* E *is* $N_{b^*g\alpha}$*-open iff* $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}int(E))] \cap N_{b^*g\alpha}\text{-}int(E) = \phi$

Proof. *Statements i) to vii) are true by the definition of neutrosophic* $b^*g\alpha$*-frontier of* E*. By remark (3.5), If* E *is neutrosophic* $b^*g\alpha$*-open,* $E = N_{b^*g\alpha}\text{-}int(E)$ *and by statement - ii),* $E \cap N_{b^*g\alpha}[Fr(E)] = \phi$*. Hence viii) is proved. statement ix) is obvious. Since* $N_{b^*g\alpha}\text{-}int(E)$ *is* $N_{b^*g\alpha}$*-open, then* $N_{b^*g\alpha}\text{-}int(E) = E$*, which implies* $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}int(E))] \subseteq N_{b^*g\alpha}[Fr(E)]$*. Similarly, xi) can be proved. By remark(3.5) and by statement-ii), xii) is straight forward.*

6 Connection between Neutrosophic $b^*g\alpha$-Frontier and Neutrosophic $b^*g\alpha$-Border

Theorem 6.1. *In the neutrosophic topological space* (X, τ)*, for any subset* E *of* X*, the following statements will be hold.*

i) $N_{b^*g\alpha}[Bd(E)] \setminus N_{b^*g\alpha}[Fr(E)] = \phi$

ii) $N_{b^*g\alpha}[Bd(E)] \subseteq N_{b^*g\alpha}[Fr(E)]$

iii) $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}[Bd(E)])] = N_{b^*g\alpha}[Bd(E)]$

iv) $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}[Fr(E)])] = N_{b^*g\alpha}[Fr(E)]$

v) *If* E *is neutrosophic* $b^*g\alpha$*- open, then* $N_{b^*g\alpha}[Fr(E)] \cup N_{b^*g\alpha}[Bd(E)] = N_{b^*g\alpha}[Fr(E)]$

vi) $N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)] = N_{b^*g\alpha}[Bd(E)]$

vii) $\overline{N_{b^*g\alpha}[Fr(E)]} \cup \overline{N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Bd(E)]}$

viii) $\overline{N_{b^*g\alpha}[Fr(E)]} \cap \overline{N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Fr(E)]}$

Proof. *Statement i) to iv) are obvious by the definitions of Neutrosophic* $b^*g\alpha$*-Frontier and Neutrosophic* $b^*g\alpha$*-border of a set. Since* E *is Neutrosophic* $b^*g\alpha$*- open, then we have a statement from Neutrosophic* $b^*g\alpha$*-border of a set,* $N_{b^*g\alpha}[Bd(E)] = \phi$*, which implies* $N_{b^*g\alpha}[Fr(E)] \cup \phi = N_{b^*g\alpha}[Fr(E)]$*.Hence v) is proved. We know from statement - ii),* $N_{b^*g\alpha}[Bd(E)] \subseteq N_{b^*g\alpha}[Fr(E)]$ *which implies* $N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)] = N_{b^*g\alpha}[Bd(E)]$*. It gives the proof of vi). By the above statement,* $\overline{N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)]}$*, and by using De Morgan's law,* $\overline{N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Fr(E)]} \cup \overline{N_{b^*g\alpha}[Bd(E)]}$*, it gives the proof of vii). Similarly we can prove the statement viii).*

7 Conclusion

This article defined neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces and discussed some of their properties. Also neutrosophic $b^*g\alpha$-interior, neutrosophic $b^*g\alpha$-closure, neutrosophic $b^*g\alpha$-border and neutrosophic $b^*g\alpha$-frontier of a set were introduced and discussed their properties. The connection between neutrosophic $b^*g\alpha$-border of a set and neutrosophic $b^*g\alpha$-frontier of a set in neutrosophic topological spaces were established. This set can be used to derive few more new functions of neutrosophic $b^*g\alpha$-continuous and neutrosophic $b^*g\alpha$-homeomorphisms in neutrosophic topological spaces. In addition to this, it can be extended in the field of contra neutrosophic functions.

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