Neutrosophic Fuzzy X-Sub algebra of Near-Subtraction Semigroups

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Abstract: Neutrosophy fuzzy set is the extended research version of the fuzzy set that deals with imprecise and indeterminate data Neutrosophic deals with the membership, non-membership and indeterminacy function. Neutrosophy have achieved in various fields such as medical diagnosis, decision making problems, image processing etc.,The motivation of the present article is to extend the concept of Neutrosophic fuzzy X-subalgebra in near-subtraction semigroups. We will discuss along with some fundamentals and their algebraic Properties.

Keywords: Near subtraction Semigroup, Fuzzy Sub algebra , Fuzzy X-sub algebra, Neutrosophic Fuzzy Sub algebra, Neutrosophic Fuzzy X-sub algebra

1. Introduction

The Theory of Fuzzy subsets, fuzzy logic found in the research area of L.A. Zadeh[15]. The theory of Intuitionistic fuzzy set is the extension of the fuzzy set that deals with truth and false membership data. From the extension version, the term Neutrosophy was identified in the F. Smarandache [13]. Neutrosophy is a new concept in philosophy. Neutrosophic deals with the membership, non-membership and indeterminacy function. Neutrosophy have achieved in various fields such as medical diagnosis, decision making problems, image processing etc., Neutrosophy became the motivation of our manuscript.

Our present manuscript describes the Neutrosophic Fuzzy X-sub algebra (NFX-SA) of Near-Subtraction Semigroup and has conceptualized some basic algebraic properties.

The results obtained are entirely more beneficial to the researchers. Our aim of this manuscript is given as follows:

(i)To examine the some basic properties and fundamentals.

(ii) Also expand the Intersection, Quotient of the Set.

(iii) We also describe the Complement of the set.

2. Preliminaries

2.1 Definition[8]

Consider X to be define as a non empty along with the operations '-' and '•' is said to be a ***right near-subtraction semigroups*** if for p,q and r in X.

(i)With respect to '-' it defines as a subtraction algebra

(ii) With respect to '•' it defines as a semigroup

(iii)Right Distributive Law follows.

2.2Definition[9]

A fuzzy set μ in X is defined to be a ***fuzzy X-sub algebra*** of X if

(i) $\mu(p-q) \geq \min\{\mu(p), \mu(q)\}$

(ii) $\mu(pq) \geq \mu(q)$

(iii) $\mu(pq) \geq \mu(p)$ for each $p, q \in X$

(i)and (ii) gives μ is called ***fuzzy left X-Sub algebra*** of X and conditions (i) and (iii) gives μ is a ***fuzzy right X-sub algebra*** of X .

2.3Definition[9]

A ***Intuitionistic Fuzzy*** (IF) set $\upsilon = (\mu_\upsilon, \lambda_\upsilon)$ of X is said to be IF X-Sub algebra of X if

(i) $\mu_\upsilon(p-q) \geq \min\{\mu_\upsilon(p), \mu_\upsilon(q)\}$

$\lambda_\upsilon(p-q) \leq \max\{\lambda_\upsilon(p), \lambda_\upsilon(q)\}$

(ii) $\mu_\upsilon(pq) \geq \mu_\upsilon(p)$

$\lambda_\upsilon(pq) \leq \lambda_\upsilon(p)$

(iii) $\mu_\upsilon(pq) \geq \mu_\upsilon(q)$

$\lambda_\upsilon(pq) \leq \lambda_\upsilon(q)$for each p,q$\epsilon$X

Conditions that satisfy equation (i) and (ii) is called IF right X-sub algebra of X and the conditions that satisfies equation (i) and(iii) is called IF left X-sub algebra of X .

2.4Definition[8]

A ***Neutrosophic Fuzzy*** Set S defines on the universe of discourse X defined by a truth membership $T_S(p)$, indeterminacy function $I_S(p)$, and a false membership function $F_S(p)$as $S = \{< p, T_S(p), I_S(p), F_S(p) >/p$ in X}.Here, T_S, I_S, F_S:X→[0,1] and 0≤ $T_S(p) + I_S(p) + F_S(p)$ ≤ 3.

2.5Definition[8]

Consider a Neutrosophic fuzzy set V in X is defined to be ***Neutrosphic fuzzy near - subtraction subsemigroup*** of X if for all p,q, in X.

(i) $T_V(p-q) \geq \min\{T_V(p), T_V(q)\}$; $T_V(pq) \geq \min\{T_V(p), T_V(q)\}$

(ii) $I_V(p-q) \leq \max\{I_V(p), I_V(q)\}$;$I_V(pq) \leq \max\{I_V(p), I_V(q)\}$

(iii) $F_V(p-q) \leq \max\{F_V(p), F_V(q)\}$; $F_V(pq) \leq \max\{F_V(p), F_V(q)\}$

3. Neutrosophic Fuzzy X-sub algebra of Near-Subtraction Semigroups

This Section we introduced the basic properties of NFX-SA in Near-Subtraction Semigroup.

3.1Definition

A Neutrosophic fuzzy set $S=(T_S, I_S, F_S)$ in X is said to be **NFX-SA** of X if for each p,q in X.

(i) $T_S(p-q)\geq\min\{T_S(p), T_S(q)\}$;$I_S(p-q)\leq\max\{I_S(p), I_S(q)\}$;$F_S(p-q)\leq\max\{F_S(p), F_S(q)\}$

(ii) $T_S(pq)\geq T_S(p)$;$I_S(pq)\leq I_S(p)$;$F_S(pq)\leq F_S(p)$

(iii)$T_S(pq)\geq T_S(q)$;$I_S(pq)\leq I_S(q)$;$F_S(pq)\leq F_S(q)$

Conditions that satisfies equation (i) and (ii) is called ***Neutrosophic Fuzzy right X-sub algebra*** of X and the conditions that satisfies equation(i) and(iii) is called ***Neutrosophic Fuzzy left X-sub algebra*** of X.

3.2 Example

Define X={0,p,q,r} to be a set defined by binary operations '- ' and '•' is

-	**0**	**p**	**q**	**r**
0	**0**	**0**	**0**	**0**
p	**p**	**0**	**p**	**0**
q	**q**	**q**	**0**	**0**
r	**r**	**q**	**P**	**0**

•	**0**	**p**	**q**	**r**
0	**0**	**0**	**0**	**0**
p	**0**	**p**	**0**	**p**
q	**0**	**q**	**0**	**q**
r	**0**	**r**	**0**	**r**

Let $S:X\to[0,1]$ be a fuzzy subset of X defined by

$T_S(0)=.7$ $T_S(p)=.5$ $T_S(q)=.4$ $T_S(r)=.3$

$I_S(0)=.1$ $I_S(p)=.2$ $I_S(q)=.3$ $I_S(r)=.5$

$F_S(0)=.02$ $F_S(p)=.3$ $F_S(q)=.5$ $F_S(r)=.7$

Hence, S is a NFX-SA of X.

3.3Theorem

If $S=(T_S, I_S, F_S)$ be a NFX-SA of X ,then the set $X_S=\{p \text{ in } X/ T_S(p)= T_S(0); I_S(p)= I_S(0); F_S(p)= F_S(0)\}$ is a X-sub algebra of X.

Proof:

Choose p,q in X_S. Thus $T_S(p)= T_S(0)$; $I_S(p)= I_S(0)$; $F_S(p)= F_S(0)$;$T_S(q)= T_S(0)$; $I_S(q)= I_S(0)$; $F_S(q)= F_S(0)$.

(i)$T_S(p-q)\geq\min\{T_S(p), T_S(q)\}= T_S(0)$

$I_S(p-q)\leq\max\{I_S(p), I_S(q)\}= I_S(0)$.

$F_S(p-q)\leq\max\{F_S(p), F_S(q)\}= F_S(0)$.

So,$p-q\in X_S$. Now

(ii)$T_S(pq)\geq T_S(p)= T_S(0)$.

$I_S(pq)\leq I_S(p)= I_S(0)$.

$F_S(pq)\leq F_S(p)= F_S(0)$.

(iii)$T_S(pq) \geq T_S(q) = T_S(0)$.

$I_S(pq) \leq I_S(q) = I_S(0)$.

$F_S(pq) \leq F_S(q) = F_S(0)$.

So, $pq \in X_S$.

Thus, X_S is a X-sub algebra of X.

3.4 Theorem

The Complement of NFX-SA is again a NFX-SA of X.

Proof:

Assume that $S^c=(T_S^c, I_S^c, F_S^c)$ be the Complement set of the Neutrosophic fuzzy set $S=(T_S , I_S , F_S)$ of X.

Select $p,q,r \in X$.

Then

(i)$T_S^c(p-q)=1- T_S(pq)$

$\leq 1-\min\{ T_S(p), T_S(q)\}$

$=\max\{1- T_S(p),1- T_S(q)\}$

$=\max\{ T_S^c(p), T_S^c(q)\}$

$I_S^c(p-q) \quad =1- I_S(pq)$

$\geq 1-\max\{ I_S(p), I_S(q)\}$

$=\min\{1- I_S(p),1- I_S(q)\}$

$=\min\{ I_S^c(p), I_S^c(q)\}$

$F_S^c(p-q) \quad =1- F_S(pq)$

$\geq 1-\max\{ F_S(p), F_S(q)\}$

$=\min\{1- F_S(p),1- F_S(q)\}$

$=\min\{ F_S^c(p), F_S^c(q)\}$

(ii)$T_S^c(pq)=1- T_S(pq)$

$\leq 1-T_S(p)$

$= T_S^c(p)$

$I_S^c(pq) \quad =1- I_S(pq)$

$\geq 1- I_S(p)$

$=I_S^c(p)$

$F_S^c(pq) \quad =1- F_S(pq)$

$\geq 1- F_S(p)$

$= F_S^c(p)$

(iii)$T_S^c(pq)=1- T_S(pq)$

$\leq 1- T_S(q)$

$= T_S^c(q)$

$I_S^c(pq) \quad =1- I_S(pq)$

$\geq 1-I_S(q)$

$= I_S^c(q)$

$F_S^c(pq) \quad =1- F_S(pq)$

$\geq 1- F_S(q)$

$= F_{S^c}(q)$

Therefore, S^c is a NFX-SA of X.

3.5 Corollary

A Neutrosophic fuzzy set S=(T_S, I_S, F_S) of X is a NFX-SA of X iff (T_S,I_S T_{S^c}), (F_{S^c}, I_S, T_{S^c}), (F_{S^c}, I_S F_S) are NFX-SA's of X.

3.6 Theorem

If S_j ={(T_{S_j}, I_{S_j}, F_{S_j})/j∊ δ}be a family of NFX-SA on X, then the set $\bigcap_{j\epsilon\delta} T_j$,$\bigcup_{j\epsilon\delta} I_{S_j}$ and $\bigcup_{j\epsilon\delta} F_{S_j}$ are also family of NFX-SA of X, where δ defines an index set.

Proof:

Assume that p,q,r in X.

Also $\bigcap_{j\epsilon\delta} T_{S_j}(p)=\inf_{j\epsilon\delta} T_{S_j}(p)$

$\bigcup_{j\epsilon\delta} I_{S_j}(p) = \sup_{j\epsilon\delta} I_{S_j}(p)$;

$\bigcup_{j\epsilon\delta} F_{S_j}(p) = \sup_{j\epsilon\delta} F_{S_j}(p)$.

Also T_{S_j} , I_{S_j} and F_{S_j} be a family of fuzzy X-sub algebra of X.

Now

(i) $\bigcap_{j\epsilon\delta} T_{S_j}(p-q) = \inf_{j\epsilon\delta} T_{S_j}(p-q) \geq \inf_{j\epsilon\delta} \min\{T_{S_j}(p), T_{S_j}(q)\}$

$=\min\{\inf_{j\epsilon\delta} T_{S_j}(p), \inf_{j\epsilon\delta} T_{S_j}(q)\}$

$=\min\{\bigcap_{j\epsilon\delta} T_{S_j}(p), \bigcap_{j\epsilon\delta} T_{S_j}(q)\}$

$\bigcup_{j\epsilon\delta} I_{S_j}(p-q)= \sup_{j\epsilon\delta} I_{S_j}(p-q) \leq \sup_{j\epsilon\delta} \max\{I_{S_j}(p), I_{S_j}(q)\}$

$=\max\{\sup_{j\epsilon\delta} I_{S_j}(p), \sup_{j\epsilon\delta} I_{S_j}(q)\}$

$=\max\{\bigcup_{j\epsilon\delta} I_{S_j}(p), \bigcup_{j\epsilon\delta} I_{S_j}(q)\}$

$\bigcup_{j\epsilon\delta} F_{S_j}(p-q)= \sup_{j\epsilon\delta} F_{S_j}(p-q) \leq \sup_{j\epsilon\delta} \max\{F_{S_j}(p), F_{S_j}(q)\}$

$=\max\{\sup_{j\epsilon\delta} F_{S_j}(p), \sup_{j\epsilon\delta} F_{S_j}(q)\}$

$=\max\{\bigcup_{j\epsilon\delta} F_{S_j}(p), \bigcup_{j\epsilon\delta} F_{S_j}(q)\}$

(ii) $\bigcap_{j\epsilon\delta} T_{S_j}(pq) = \inf_{j\epsilon\delta} T_{S_j}(pq) \geq \inf_{j\epsilon\delta} T_j(p)=\bigcap_{j\epsilon\delta} T_{S_j}(p)$

$\bigcup_{j\epsilon\delta}\ I_{S_j}(pq)=\sup_{j\in\delta} I_{S_j}(pq)\leq \sup_{j\in\delta}\ I_{S_j}(p)=\bigcup_{j\epsilon\delta}\ I_{S_j}\ (p)$

$\bigcup_{j\epsilon\delta}\ F_{S_j}(pq)=\sup_{j\in\delta} F_{S_j}(pq)\leq \sup_{j\in\delta} F_{S_j}(p)=\bigcup_{j\epsilon\delta}\ F_{S_j}\ (p)$

(iii) $\bigcap_{j\epsilon\delta}\ T_{S_j}(pq) = \inf_{j\in\delta} T_{S_j}(pq)\geq \inf_{j\in\delta} T_{S_j}(q)= \bigcap_{j\epsilon\delta}\ T_{S_j}\ (q)$

$\bigcup_{j\epsilon\delta}\ I_{S_j}(pq)=\sup_{j\in\delta} I_{S_j}(pq)\leq \sup_{j\in\delta} I_{S_j}(q)=\bigcup_{j\epsilon\delta}\ I_{S_j}\ (q)$

$\bigcup_{i\epsilon\delta}\ F_{S_j}(pq)=\sup_{j\in\delta} F_{S_j}(pq)\leq \sup_{j\in\delta} F_{S_j}(q)=\bigcup_{j\epsilon\delta}\ F_{S_j}\ (q)$

Hence the Proof.

3.7 Theorem

Consider S as a NFX-SA of X, then the fuzzy set S of X/I, where I is an ideal of X defined by

$T_S{}^{o}(p+I)=\sup_{q\in I}\ T_S(p+q);$

$I_S{}^{o}(p+I)=\inf_{q\in I}\ I_S(p+q);$

$F_S{}^{o}(p+I)=\inf_{q\in I}\ F_S(p+q)$

is a NFX-SA of Quotient near-subtraction Semigroup $X/_I$.

Proof:

Choose l,m in X so that l+I=m+I.
Then m=l+q where q in I.
To prove that S is well-defined.

$T_S{}^{o}(m+I)=\sup_{p\in I}\ T_S(m+p)$

$=\sup_{p\in I}\ T_S(l+q+p)$

$= \sup_{q+p=u\in I}\ T_S(l+u)$

$=\ T_S{}^{o}(l+I)$

$I_S{}^{o}(m+I)=\inf_{p\in I}\ I_S(m+p)$

$=\inf_{p\in I}\ I_S(l+q+p)$

$= \inf_{q+p=u\in I}\ I_S(l+u)$

$=\ I_S{}^{o}(l+I)$

$F_S{}^{o}(m+I)=\inf_{p\in I}\ F_S(m+p)$

$=\inf_{p\in I}\ F_S(l+q+p)$

$=\inf_{q+p=u\in I}\ F_S(l+u)$

$=F_S{}^{o}(l+I)$

Now

(i) $T_S{}^{o}((p+I)-(q+I))\geq\min\{\ T_S{}^{o}(p+I),\ T_S{}^{o}(q+I)\}$

$I_S{}^{o}((p+I)-(q+I))\leq\max\{\ I_S{}^{o}(p+I),\ I_S{}^{o}(q+I)\}$

$F_S{}^{o}((p+I)-(q+I))\leq\max\{\ F_S{}^{o}(p+I),\ F_S{}^{o}(q+I)\}$

Let p+I,q+I in $X/_I$

(ii) $T_S{}^{o}[(p+I)(q+I)]=\ T_S{}^{o}(pq+I)=\sup_{l\in I}\ T_S(pq+l)$

$=\sup_{l=ab\in I}\ T_S(pq+ab)$

$=\sup_{a,b\in I}\ T_S[(p+a)(q+b)]$

$\geq\sup_{a\in I}\ \{T_S(p+a)\}$

$=T_S{}^{o}(p+I)$

$I_S{}^{o}[(p+I)(q+I)]=\ I_S{}^{o}(pq+I)=\inf_{l\in I}\ I_S(pq+l)$

$=\inf_{l=ab\in I}\ I_S(pqr+ab)$

$=\inf_{a,b\in I}I_S[(p+a)(q+b)]$

$\leq\inf_{a,\in I}\ \{I_S(p+a)\}$

$=\ I_S{}^{o}(p+I)$

$F_S{}^{o}[(p+I)(q+I)]=\ F_S{}^{o}(pq+I)=\inf_{l\in I}\ F_S(pq+l)$

$=\inf_{l=ab\in I}\ F_S(pq+ab)$

$=\inf_{a,b\in I}\ F_S[(p+a)(q+b)]$

$\leq \inf_{a,b,c\in I} \{F_S(p+a)\}$

$= F_S{}^{o}(p+I)$

(iii)$T_S{}^{o}[(p+I)(q+I)] = T_S{}^{o}(pq+I) = \sup_{l\in I} T_S(pq+l)$

$= \sup_{l=ab\in I} T_S(pq+ab)$

$= \sup_{a,b\in I} T_S[(p+a)(q+b)]$

$\geq \sup_{a\in I} \{T_S(q+b)\}$

$= T_S{}^{o}(q+I)$

$I_S{}^{o}[(p+I)(q+I)] = I_S{}^{o}(pq+I) \quad = \inf_{l\in I} I_S(pq+l)$

$= \inf_{l=ab\in I} I_S(pqr+ab)$

$= \inf_{a,b\in I} I_S[(p+a)(q+b)]$

$\leq \inf_{a,\in I} \{I_S(q+b)\}$

$= I_S{}^{o}(q+I)$

$F_S{}^{o}[(p+I)(q+I)] = F_S{}^{o}(pq+I) \quad = \inf_{l\in I} F_S(pq+l)$

$= \inf_{l=ab\in I} F_S(pq+ab)$

$= \inf_{a,b\in I} F_S[(p+a)(q+b)]$

$\leq \inf_{a,b,c\in I} \{F_S(q+b)\}$

$= F_S{}^{o}(q+I)$

Hence the Proof.

4. Conclusion

In the present manuscript, we have defined the Intersection, Complement set, Quotient Set of NFX-SA in Near subtraction Semi group. This research work can be extended to other types of ideals and other algebraic structures of Near Subtraction Semi groups.

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