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Neutrosophic Vague Topological Spaces

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Abstract: The term topology was introduced by Johann Beredict Listing in the 19th century. Closed sets are fundamental objects in a topological space. In this paper, we use neutrosophic vague sets and topological spaces and we construct and develop a new concept namely "neutrosophic vague topological spaces". By using the fundamental definition and necessary example we have defined the neutrosophic vague topological spaces and have also discussed some of its properties. Also we have defined the neutrosophic vague continuity and neutrosophic vague compact space in neutrosophic vague topological spaces and their properties are deliberated.

Keywords: Neutrosophic vague set, neutrosophic vague topology, neutrosophic vague topological spaces, neutrosophic vague continuity.

1. Introduction:

Zadeh [19] in 1965 introduced and defined the fuzzy set which deals with the degree of membership/truth. Topology has become a powerful instrument of mathematical research. Topology is the modern version of geometry. It is commonly defined as the study of shapes and topological spaces. The topology is an area of mathematics, which is concerned with the properties of space that are preserved under continuous deformation including stretching and bending, but not tearing and gluing which include properties such as connectedness, continuity and boundary. The term topology was introduced by Johann Beredict Listing in the 19th century. Closed sets are fundamental objects in a topological space. In 1970, Levine [11] initiated the study of generalized closed sets.

The theory of fuzzy topology was introduced by Chang [8] in 1967; several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. Atanassov [7] in1986 introduced the degree of non-membership/falsehood (F) and defined the intuitionistic fuzzy set as a generalization of fuzzy sets. Coker [9] in 1997 introduced the intuitionistic fuzzy topological spaces. As an extension of fuzzy set theory in 1993, the theory of vague sets was first proposed by Gau and Buehre[10]. Then, Smarandache [15] introduced the degree of indeterminacy/neutrality (I) as independent component in 1998 and defined the neutrosophic set. Various methods were proposed by Smarandache.et.al [13, 16, 17, 18] and Abdel-Basset.et.al [1, 2, 3] for neutrosophic sets.

Salama and Alblowi [12] in 2012 used this neutrosophic set and introduced neutrosophic topological spaces. Shawkat Alkhazaleh [14] in 2015 introduced the concept of neutrosophic vague set as a combination of neutrosophic set and vague set. Neutrosophic vague theory is an effective tool to process incomplete, indeterminate and inconsistent information. Al-Quran and Hassan [4, 5, 6] in 2017 introduced and gave more application on neutrosophic vague soft.

In this paper we define the notion of neutrosophic vague topological spaces and their properties are obtained. The purpose of this paper is to extend the classical topological spaces to neutrosophic vague topological spaces. Also we have defined the neutrosophic vague continuity and neutrosophic vague compact spaces which give the added advantage in neutrosophic vague topological spaces.

2. Preliminaries

Definition 2.1:[14] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV}=\left\{\left\langle x;\hat{T}_{A_{NV}}(x);\hat{I}_{A_{NV}}(x);\hat{F}_{A_{NV}}(x)\right\rangle;x\in X\right\}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x)=\left[T^{-},T^{+}\right],\hat{I}_{A_{NV}}(x)=\left[I^{-},I^{+}\right],\hat{F}_{A_{NV}}(x)=\left[F^{-},F^{+}\right]$$

Where,

1) $T^{+}=1-F^{-}$
2) $F^{+}=1-T^{-}$ and
3) $^{-}0\leq T^{-}+I^{-}+F^{-}\leq 2^{+}$.

Definition 2.2:[14] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i\in U$, $\hat{T}_{A_{NV}}(u_i)\leq\hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i)\geq\hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i)\geq\hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV}\subseteq B_{NV}$, where $1\leq i\leq n$.

Definition 2.3:[14] The complement of NVS A_{NV} is denoted by A^{c}_{NV} and is defined by

$$\hat{T}^{c}_{A_{NV}}(x)=\left[1-T^{+},1-T^{-}\right],\hat{I}^{c}_{A_{NV}}(x)=\left[1-I^{+},1-I^{-}\right],\hat{F}^{c}_{A_{NV}}(x)=\left[1-F^{+},1-F^{-}\right].$$

Definition 2.4:[14] Let A_{NV} be NVS of the universe U where $\forall u_i\in U$, $\hat{T}_{A_{NV}}(x)=[1,1]$; $\hat{I}_{A_{NV}}(x)=[0,0]$; $\hat{F}_{A_{NV}}(x)=[0,0]$. Then A_{NV} is called unit NVS (1_{NV} in short), where $1\leq i\leq n$.

Definition 2.5:[14] Let A_{NV} be NVS of the universe U where $\forall u_i\in U$, $\hat{T}_{A_{NV}}(x)=[0,0]$; $\hat{I}_{A_{NV}}(x)=[1,1]$; $\hat{F}_{A_{NV}}(x)=[1,1]$. Then A_{NV} is called zero NVS (0_{NV} in short), where $1\leq i\leq n$.

Definition 2.6:[14] The union of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x) = [\max(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}), \max(T^+_{A_{NV_x}}, T^+_{B_{NV_x}})]$$

$$\hat{I}_{C_{NV}}(x) = [\min(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}), \min(I^+_{A_{NV_x}}, I^+_{B_{NV_x}})]$$

$$\hat{F}_{C_{NV}}(x) = [\min(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}), \min(F^+_{A_{NV_x}}, F^+_{B_{NV_x}})].$$

Definition 2.7:[14] The intersection of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x) = [\min(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}), \min(T^+_{A_{NV_x}}, T^+_{B_{NV_x}})]$$

$$\hat{I}_{C_{NV}}(x) = [\max(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}), \max(I^+_{A_{NV_x}}, I^+_{B_{NV_x}})]$$

$$\hat{F}_{C_{NV}}(x) = [\max(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}), \max(F^+_{A_{NV_x}}, F^+_{B_{NV_x}})].$$

Definition 2.8:[14] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i) = \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) = \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) = \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} and B_{NV}, are called equal, where $1 \le i \le n$.

Definition 2.9: Let $\{A_{i_{NV}} : i \in J\}$ be an arbitrary family of NVSs. Then

$$\cup A_{i_{NV}} = \left\{ \left\langle x; \left(\max_{i \in J}\left(T^-_{A_{i_{NV}}}\right), \max_{i \in J}\left(T^+_{A_{i_{NV}}}\right)\right), \left(\min_{i \in J}\left(I^-_{A_{i_{NV}}}\right), \min_{i \in J}\left(I^+_{A_{i_{NV}}}\right)\right), \left(\min_{i \in J}\left(F^-_{A_{i_{NV}}}\right), \min_{i \in J}\left(F^+_{A_{i_{NV}}}\right)\right) \right\rangle; x \in X \right\}$$

$$\cap A_{i_{NV}} = \left\{ \left\langle x; \left(\min_{i \in J}\left(T^-_{A_{i_{NV}}}\right), \min_{i \in J}\left(T^+_{A_{i_{NV}}}\right)\right), \left(\max_{i \in J}\left(I^-_{A_{i_{NV}}}\right), \max_{i \in J}\left(I^+_{A_{i_{NV}}}\right)\right), \left(\max_{i \in J}\left(F^-_{A_{i_{NV}}}\right), \max_{i \in J}\left(F^+_{A_{i_{NV}}}\right)\right) \right\rangle; x \in X \right\}$$

Corollary 2.10: Let A_{NV}, B_{NV} and C_{NV} be NVSs. Then

a) $A_{NV} \subseteq B_{NV}$ and $C_{NV} \subseteq D_{NV} \Rightarrow A_{NV} \cup C_{NV} \subseteq B_{NV} \cup D_{NV}$ and $A_{NV} \cap C_{NV} \subseteq B_{NV} \cap D_{NV}$

b) $A_{NV} \subseteq B_{NV}$ and $A_{NV} \subseteq C_{NV} \Rightarrow A_{NV} \subseteq B_{NV} \cap C_{NV}$

c) $A_{NV} \subseteq C_{NV}$ and $B_{NV} \subseteq C_{NV} \Rightarrow A_{NV} \cup B_{NV} \subseteq C_{NV}$

d) $A_{NV} \subseteq B_{NV}$ and $B_{NV} \subseteq C_{NV} \Rightarrow A_{NV} \subseteq C_{NV}$

e) $\overline{(A_{NV} \cup B_{NV})} = \overline{A_{NV}} \cap \overline{B_{NV}}$

f) $\overline{(A_{NV} \cap B_{NV})} = \overline{A_{NV}} \cup \overline{B_{NV}}$

g) $A_{NV} \subseteq B_{NV} \Rightarrow \overline{B_{NV}} \subseteq \overline{A_{NV}}$

h) $\overline{\left(\overline{A_{NV}}\right)} = A_{NV}$

i) $\bar{1}_{NV} = 0_{NV}$

j) $\bar{0}_{NV} = 1_{NV}$

Corollary 2.11: Let A_{NV}, B_{NV}, C_{NV} and $A_{i_{NV}} (i \in J)$ be NVSs. Then

a) $A_{i_{NV}} \subseteq B_{NV}$ foreach $i \in J \Rightarrow \cup A_{i_{NV}} \subseteq B_{NV}$

b) $B_{NV} \subseteq A_{i_{NV}}$ foreach $i \in J \Rightarrow B_{NV} \subseteq \cap A_{i_{NV}}$

c) $\overline{\cup A_{i_{NV}}} = \cap \overline{A_{i_{NV}}}$ and $\overline{\cap A_{i_{NV}}} = \cup \overline{A_{i_{NV}}}$

3. Neutrosophic Vague Topological Space:

Definition 3.1: A neutrosophic vague topology (NVT) on X_{NV} is a family τ_{NV} of neutrosophic vague sets (NVS) in X_{NV} satisfying the following axioms:

- $0_{NV}, 1_{NV} \in \tau_{NV}$
- $G_1 \cap G_2 \in \tau_{NV}$ for any $G_1, G_2 \in \tau_{NV}$
- $\cup G_i \in \tau_{NV}, \forall \{G_i : i \in J\} \subseteq \tau_{NV}$

In this case the pair (X_{NV}, τ_{NV}) is called neutrosophic vague topological space (NVTS) and any NVS in τ_{NV} is known as neutrosophic vague open set (NVOS) in X_{NV}.

The complement A_{NV}^{c} of NVOS in NVTS (X_{NV}, τ_{NV}) is called neutrosophic vague closed set (NVCS) in X_{NV}.

Example 3.2: Let $X_{NV} = \{e, f, g\}$ and

$$A_{NV} = \left\{ x, \frac{e}{\langle[0.1,0.5];[0.6,0.8];[0.5,0.9]\rangle}, \frac{f}{\langle[0.2,0.3];[0.4,0.5];[0.7,0.8]\rangle}, \frac{g}{\langle[0.2,0.6];[0.7,0.9];[0.4,0.8]\rangle} \right\},$$

$$B_{NV} = \left\{ x, \frac{e}{\langle[0.2,0.4];[0.3,0.7];[0.6,0.8]\rangle}, \frac{f}{\langle[0.5,0.8];[0.2,0.6];[0.2,0.5]\rangle}, \frac{g}{\langle[0.1,0.3];[0.1,0.7];[0.7,0.9]\rangle} \right\},$$

$$C_{NV} = \left\{ x, \frac{e}{\langle[0.2,0.5];[0.3,0.7];[0.5,0.8]\rangle}, \frac{f}{\langle[0.5,0.8];[0.2,0.5];[0.2,0.5]\rangle}, \frac{g}{\langle[0.2,0.6];[0.1,0.7];[0.4,0.8]\rangle} \right\},$$

$$D_{NV} = \left\{ x, \frac{e}{\langle[0.1,0.4];[0.6,0.8];[0.6,0.9]\rangle}, \frac{f}{\langle[0.2,0.3];[0.4,0.6];[0.7,0.8]\rangle}, \frac{g}{\langle[0.1,0.3];[0.7,0.9];[0.7,0.9]\rangle} \right\}.$$

Then the family $\tau_{NV} = \{0_{NV}, A_{NV}, B_{NV}, C_{NV}, D_{NV}, 1_{NV}\}$ of NVSs in X_{NV} is NVT on X_{NV}.

Definition 3.3: Let (X_{NV}, τ_{NV}) be NVTS and $A_{NV} = \left\{\left\langle x, \left[\hat{T}_A, \hat{I}_A, \hat{F}_A\right]\right\rangle\right\}$ be NVS in X_{NV}. Then the neutrosophic vague interior and neutrosophic vague closure are defined by

- $NV\,\text{int}(A_{NV}) = \cup\{G_{NV} / G_{NV}$ is a NVOS in X_{NV} and $G_{NV} \subseteq A_{NV}\}$,
- $NVcl(A_{NV}) = \cap\{K_{NV} / K_{NV}$ is a NVCS in X_{NV} and $A_{NV} \subseteq K_{NV}\}$.

Note that for any NVS A_{NV} in (X_{NV}, τ_{NV}), we have $NVcl\left(A_{NV}^{c}\right) = \left(NV\,\text{int}\left(A_{NV}\right)\right)^{c}$ and $NV\,\text{int}\left(A_{NV}^{c}\right) = \left(NVcl\left(A_{NV}\right)\right)^{c}$.

It can be also shown that $NVcl(A_{NV})$ is NVCS and $NV\,\text{int}(A_{NV})$ is NVOS in X_{NV}.

a) A_{NV} is NVCS in X_{NV} if and only if $NVcl(A_{NV}) = A_{NV}$.

b) A_{NV} is NVOS in X_{NV} if and only if $NV\,\text{int}(A_{NV}) = A_{NV}$.

Example 3.4: Let $X_{NV} = \{e, f\}$ and let $\tau_{NV} = \{0_{NV}, G_1, G_2, 1_{NV}\}$ be NVT on X, where

$$G_1 = \left\{ x, \frac{e}{\langle[0.2,0.4];[0.7,0.9];[0.6,0.8]\rangle}, \frac{f}{\langle[0.3,0.5];[0.6,0.8];[0.5,0.7]\rangle} \right\} \text{ and}$$

$$G_2 = \left\{ x, \frac{e}{\langle[0.4,0.9];[0.1,0.3];[0.1,0.4]\rangle}, \frac{f}{\langle[0.5,0.7];[0.2,0.6];[0.3,0.5]\rangle} \right\}.$$

If $A_{NV} = \left\{ x, \frac{e}{\langle[0.3,0.5];[0.4,0.7];[0.5,0.7]\rangle}, \frac{f}{\langle[0.4,0.6];[0.5,0.8];[0.4,0.6]\rangle} \right\}$ then

$$NV\operatorname{int}(A_{NV}) = G_1 = \left\{ x, \frac{e}{\langle[0.2,0.4];[0.7,0.9];[0.6,0.8]\rangle}, \frac{f}{\langle[0.3,0.5];[0.6,0.8];[0.5,0.7]\rangle} \right\} \text{and}$$

$$NVcl(A_{NV}) = G_1^c = \left\{ x, \frac{e}{\langle[0.8,0.6];[0.1,0.3];[0.2,0.4]\rangle}, \frac{f}{\langle[0.5,0.7];[0.2,0.4];[0.3,0.5]\rangle} \right\}.$$

Proposition 3.5: Let A_{NV} be any NVS in X_{NV}. Then

i) $NV\operatorname{int}(1_{NV} - A_{NV}) = 1_{NV} - (NVcl(A_{NV}))$ and

ii) $NVcl(1_{NV} - A_{NV}) = 1_{NV} - (NV\operatorname{int}(A_{NV}))$

Proof: (i) By definition $NVcl(A_{NV}) = \cap\{K_{NV} / K_{NV}$ is a NVCS in X_{NV} and $A_{NV} \subseteq K_{NV}\}$.

$$\begin{aligned} 1_{NV} - (NVcl(A_{NV})) &= 1_{NV} - \cap\{K_{NV} / K_{NV} \text{ is a NVCS in } X_{NV} \text{ and } A_{NV} \subseteq K_{NV}\} \\ &= \cup\{1_{NV} - K_{NV} / K_{NV} \text{ is a NVCS in } X_{NV} \text{ and } A_{NV} \subseteq K_{NV}\} \\ &= \cup\{G_{NV} / G_{NV} \text{ is an NVOS in } X_{NV} \text{ and } G_{NV} \subseteq 1_{NV} - A_{NV}\} \\ &= NV\operatorname{int}(1_{NV} - A_{NV}) \end{aligned}$$

(ii) The proof is similar to (i).

Proposition 3.6: Let (X_{NV}, τ_{NV}) be a NVTS and A_{NV}, B_{NV} be NVSs in X_{NV}. Then the following properties hold:

a) $NV\operatorname{int}(A_{NV}) \subseteq A_{NV}$,

a′) $A_{NV} \subseteq NVcl(A_{NV})$

b) $A_{NV} \subseteq B_{NV} \Rightarrow NV\operatorname{int}(A_{NV}) \subseteq NV\operatorname{int}(B_{NV})$,

b′) $A_{NV} \subseteq B_{NV} \Rightarrow NVcl(A_{NV}) \subseteq NVcl(B_{NV})$

c) $NV\operatorname{int}(NV\operatorname{int}(A_{NV})) = NV\operatorname{int}(A_{NV})$,

c′) $NVcl(NVcl(A_{NV})) = NVcl(A_{NV})$

d) $NV\operatorname{int}(A_{NV} \cap B_{NV}) = NV\operatorname{int}(A_{NV}) \cap NV\operatorname{int}(B_{NV})$,

d′) $NVcl(A_{NV} \cup B_{NV}) = NVcl(A_{NV}) \cup NVcl(B_{NV})$

e) $NV\operatorname{int}(1_{NV}) = 1_{NV}$,

e′) $NVcl(0_{NV}) = 0_{NV}$

Proof: (a), (b) and (e) are obvious, (c) follows from (a)

d) From $NV\operatorname{int}(A_{NV} \cap B_{NV}) \subseteq NV\operatorname{int}(A_{NV})$ and $NV\operatorname{int}(A_{NV} \cap B_{NV}) \subseteq NV\operatorname{int}(B_{NV})$ we obtain $NV\operatorname{int}(A_{NV} \cap B_{NV}) \subseteq NV\operatorname{int}(A_{NV}) \cap NV\operatorname{int}(B_{NV})$.

On the other hand, from the facts $NV\operatorname{int}(A_{NV}) \subseteq A_{NV}$ and $NV\operatorname{int}(B_{NV}) \subseteq B_{NV}$ $\Rightarrow NV\operatorname{int}(A_{NV}) \cap NV\operatorname{int}(B_{NV}) \subseteq A_{NV} \cap B_{NV}$ and $NV\operatorname{int}(A_{NV}) \cap NV\operatorname{int}(B_{NV}) \in \tau_{NV}$ we see that $NV\operatorname{int}(A_{NV}) \cap NV\operatorname{int}(B_{NV}) \subseteq NV\operatorname{int}(A_{NV} \cap B_{NV})$, for which we obtain the required result.

(a′)–(e′) They can be easily deduced from (a)–(e).

Definition 3.7: A NVS $A_{NV} = \left\langle x, \left[\hat{T}_A, \hat{I}_A, \hat{F}_A\right]\right\rangle$ in NVTS (X_{NV}, τ_{NV}) is said to be

i) *Neutrosophic Vague semi closed set* (NVSCS) if $NV\operatorname{int}(NVcl(A_{NV})) \subseteq A_{NV}$,

ii) *Neutrosophic Vague semi open set* (NVSOS) if $A_{NV} \subseteq NVcl(NV\operatorname{int}(A_{NV}))$,

iii) *Neutrosophic Vague pre- closed set* (NVPCS) if $NVcl(NV\operatorname{int}(A_{NV})) \subseteq A_{NV}$,

iv) *Neutrosophic Vague pre-open set* (NVPOS) if $A_{NV} \subseteq NV\operatorname{int}(NVcl(A_{NV}))$,

v) *Neutrosophic Vague* α *-closed set* (NV α CS) if $NVcl(NV\operatorname{int}(NVcl(A_{NV}))) \subseteq A_{NV}$,

vi) *Neutrosophic Vague* α *-open set* (NV α OS) if $A_{NV} \subseteq NV\operatorname{int}(NVcl(NV\operatorname{int}(A_{NV})))$,

vii) *Neutrosophic Vague semi pre- closed set* (NVSPCS) if $NV\operatorname{int}(NVcl(NV\operatorname{int}(A_{NV}))) \subseteq A_{NV}$,

viii) *Neutrosophic Vague semi pre-open set* (NVSPOS) if $A_{NV} \subseteq NVcl(NV\operatorname{int}(NVcl(A_{NV})))$,

ix) *Neutrosophic Vague regular open set* (NVROS) if $A_{NV} = NV\operatorname{int}(NVcl(A_{NV}))$,

x) *Neutrosophic Vague regular closed set* (NVRCS) if $A_{NV} = NVcl(NV\operatorname{int}(A_{NV}))$.

4. Neutrosophic Vague continuity:

Definition 4.1: We define the image and preimage of NVSs. Let X_{NV} and Y_{NV} be two nonempty sets and $f : X_{NV} \to Y_{NV}$ be a function, then the following statements hold:

a) If $B_{NV} = \left\{ \left\langle x; \hat{T}_B(x); \hat{I}_B(x); \hat{F}_B(x) \right\rangle; x \in X \right\}$ is a NVS in Y_{NV}, then the preimage of B_{NV} under *f*, denoted by $f^{-1}(B_{NV})$, is the NVS in X_{NV} defined by

$$f^{-1}(B_{NV}) = \left\{ \left\langle x; f^{-1}\left(\hat{T}_B\right)(x); f^{-1}\left(\hat{I}_B\right)(x); f^{-1}\left(\hat{F}_B\right)(x) \right\rangle; x \in X_{NV} \right\}.$$

b) If $A_{NV} = \left\{ \left\langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x) \right\rangle; x \in X_{NV} \right\}$ is a NVS in X_{NV}, then the image of A_{NV} under *f*, denoted by $f(A_{NV})$, is the NVS in Y_{NV} defined by

$$f(A_{NV}) = \left\{ \left\langle y; f_{\sup}\left(\hat{T}_A\right)(y); f_{\inf}\left(\hat{I}_A\right)(y); f_{\inf}\left(\hat{F}_A\right)(y) \right\rangle; y \in Y_{NV} \right\}$$

where,

$$f_{\sup}\left(\hat{T}_A\right)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \hat{T}_A(x), & \text{if } f^{-1}(y) \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

$$f_{\inf}\left(\hat{I}_A\right)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \hat{I}_A(x), & \text{if } f^{-1}(y) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

$$f_{\inf}\left(\hat{F}_A\right)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \hat{F}_A(x), & \text{if } f^{-1}(y) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

for each $y \in Y_{NV}$.

Corollary 4.2: Let $A_{NV}, A_{i_{NV}}\ (i \in J)$ be NVSs in X_{NV}, $B_{NV}, B_{j_{NV}}\ (j \in K)$ be NVSs in Y_{NV} and $f : X_{NV} \to Y_{NV}$ a function. Then

a) $A_{1_{NV}} \subseteq A_{2_{NV}} \Rightarrow f\left(A_{1_{NV}}\right) \subseteq f\left(A_{2_{NV}}\right)$, $B_{1_{NV}} \subseteq B_{2_{NV}} \Rightarrow f^{-1}\left(B_{1_{NV}}\right) \subseteq f^{-1}\left(B_{2_{NV}}\right)$,

b) $A_{NV} \subseteq f^{-1}\left(f\left(A_{NV}\right)\right)$ (If f is injective, then $A_{NV} = f^{-1}\left(f\left(A_{NV}\right)\right)$),

c) $f\left(f^{-1}\left(B_{NV}\right)\right) \subseteq B_{NV}$ (If f is surjective, then $f\left(f^{-1}\left(B_{NV}\right)\right) = B_{NV}$),

d) $f^{-1}\left(\cup B_{j_{NV}}\right) = \cup f^{-1}\left(B_{j_{NV}}\right)$,

e) $f^{-1}\left(\cap B_{j_{NV}}\right) = \cap f^{-1}\left(B_{j_{NV}}\right)$,

f) $f\left(\cup A_{i_{NV}}\right)=\cup f\left(A_{i_{NV}}\right)$,

g) $f\left(\cap A_{i_{NV}}\right)\subseteq \cap f\left(A_{i_{NV}}\right)$ (If f is injective, then $f\left(\cap A_{i_{NV}}\right)=\cap f\left(A_{i_{NV}}\right)$),

h) $f^{-1}\left(1_{NV}\right)=1_{NV}$,

i) $f^{-1}\left(0_{NV}\right)=0_{NV}$,

j) $f\left(1_{NV}\right)=1_{NV}$, if f is surjective,

k) $f\left(0_{NV}\right)=0_{NV}$,

l) $\overline{f\left(A_{NV}\right)}\subseteq f\left(\overline{A_{NV}}\right)$, if f is surjective,

m) $f^{-1}\left(\overline{B_{NV}}\right)=\overline{f^{-1}\left(B_{NV}\right)}$.

Definition 4.3: Let (X_{NV},τ_{NV}) and $\left(Y_{NV},\sigma_{NV}\right)$ be two NVTSs and let $f:\left(X_{NV},\tau_{NV}\right)\rightarrow\left(Y_{NV},\sigma_{NV}\right)$ be a function. Then f is said to be neutrosophic vague continuous mapping iff the preimage of each neutrosophic vague closed set is in Y_{NV} is neutrosophic vague closed set in X_{NV}.

Definition 4.4: Let (X_{NV},τ_{NV}) and $\left(Y_{NV},\sigma_{NV}\right)$ be two NVTSs and let $f:\left(X_{NV},\tau_{NV}\right)\rightarrow\left(Y_{NV},\sigma_{NV}\right)$ be a function. Then f is said to be neutrosophic vague open mapping iff the image of each neutrosophic vague open set is in X_{NV} is neutrosophic vague open set in Y_{NV}.

5. Neutrosophic Vague Compact Space:

Definition 5.1: Let $\left(X_{NV},\tau_{NV}\right)$ be NVTS.

i) If a family $\left\{\left\langle x,T_{A_i},I_{A_i},F_{A_i}\right\rangle : i\in J\right\}$ of NVOSs in X satisfies the condition $\bigcup\left\{\left\langle x,T_{A_i},I_{A_i},F_{A_i}\right\rangle : i\in J\right\}=1_{NV}$, then it is called neutrosophic vague open cover of X. A finite subfamily of neutrosophic vague open cover $\left\{\left\langle x,T_{A_i},I_{A_i},F_{A_i}\right\rangle : i\in J\right\}$ of X, which is also a neutrosophic vague cover of X, is called a neutrosophic vague finite subcover of $\left\{\left\langle x, T_{A_i}, I_{A_i}, F_{A_i}\right\rangle : i \in J\right\}$.

ii) A family $\left\{\left\langle x, T_{B_i}, I_{B_i}, F_{B_i}\right\rangle : i \in J\right\}$ of NVCSs in X satisfies the finite intersection property iff every finite subfamily $\left\{\left\langle x, T_{B_i}, I_{B_i}, F_{B_i}\right\rangle : i = 1, 2, ..., n\right\}$ of the family satisfies the condition $\bigcap_{i=1}^{n}\left\{\left\langle x, T_{B_i}, I_{B_i}, F_{B_i}\right\rangle\right\} \neq 0_{NV}$.

Definition 5.2: A NVTS (X_{NV}, τ_{NV}) is called neutrosophic vague compact iff every neutrosophic vague open cover of X has a neutrosophic vague finite subcover.

Corollary 5.3: A NVTS (X_{NV}, τ_{NV}) is neutrosophic vague compact iff every family $\left\{\left\langle x, T_{B_i}, I_{B_i}, F_{B_i}\right\rangle : i \in J\right\}$ of NVCSs in X having the FIP has a nonempty intersection.

Corollary 5.4: Let (X_{NV}, τ_{NV}), (Y_{NV}, σ_{NV}) be NVTSs and $f : (X_{NV}, \tau_{NV}) \to (Y_{NV}, \sigma_{NV})$ a neutrosophic vague continuous surjection. If (X_{NV}, τ_{NV}) is neutrosophic vague compact, then so is (Y_{NV}, σ_{NV}).

Definition 5.5: Let (X_{NV}, τ_{NV}) be NVTS and A_{NV} a NVS in X.

i) If a family $\left\{\left\langle x, T_{A_i}, I_{A_i}, F_{A_i}\right\rangle : i \in J\right\}$ of NVOSs in X satisfies the condition $A_{NV} \subseteq \bigcup\left\{\left\langle x, T_{A_i}, I_{A_i}, F_{A_i}\right\rangle : i \in J\right\}$, then it is called neutrosophic vague open cover of A_{NV}. A finite subfamily of neutrosophic vague open cover $\left\{\left\langle x, T_{A_i}, I_{A_i}, F_{A_i}\right\rangle : i \in J\right\}$ of A_{NV}, which is also a neutrosophic vague cover of A_{NV}, is called a neutrosophic vague finite subcover of $\left\{\left\langle x, T_{A_i}, I_{A_i}, F_{A_i}\right\rangle : i \in J\right\}$.

ii) A NVS in a NVTS (X_{NV}, τ_{NV}) is called neutrosophic vague compact iff every neutrosophic vague cover A_{NV} of has a neutrosophic vague finite subcover.

Corollary 5.6: Let $\left(X_{NV},\tau_{NV}\right)$, $\left(Y_{NV},\sigma_{NV}\right)$ be NVTSs and $f:\left(X_{NV},\tau_{NV}\right)\rightarrow\left(Y_{NV},\sigma_{NV}\right)$ a neutrosophic vague continuous function. If A_{NV} is neutrosophic vague compact in $\left(X_{NV},\tau_{NV}\right)$, then so if $f\left(A_{NV}\right)$ in $\left(Y_{NV},\sigma_{NV}\right)$.

Conclusion: Thus we have given the definition for neutrosophic vague topological spaces and suitable examples are also given. Along with those definition neutrosophic vague continuity and neutrosophic vague compact spaces where also discussed. Further, we can compare with all the neutrosophic vague sets and neutrosophic vague continuous functions in neutrosophic vague topological spaces.

References

1. Abdel-Basset, M.; Mohamed, R.; Zaied, A. E. N. H.; Smarandache, F. A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics. *Symmetry*, 2019; 11(7), 903.
2. Abdel-Basset, M.; Nabeeh, N. A.; El-Ghareeb, H. A.; Aboelfetouh, A. Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. *Enterprise Information Systems*, 2019; 1-21.
3. Abdel-Basset, M.; Chang, V.; Gamal, A. Evaluation of the green supply chain management practices: A novel neutrosophic approach. *Computers in Industry*, 2019; 108,pp. 210-220.
4. Al-Quran, A.; Hassan, N. Neutrosophic vague soft set and its applications. *Malaysian Journal of Mathematical Sciences* , 2017; 11(2), pp. 141-163.
5. Al-Quran, A.; Hassan, N. Neutrosophic vague soft multiset for decision under uncertainty. *Songklanakarin Journal of Science and Technology*, 2018; 40(2), pp. 290-305.
6. Al-Quran, A.; Hassan, N. Neutrosophic vague soft expert set theory. *Journal of Intelligent and Fuzzy Systems*, 2016; 30(6), pp. 3691-3702.
7. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 1986; 20, pp. 87–96.
8. Chang, C.L. Fuzzy topological spaces, *J Math.Anal.Appl*. 1968; 24, pp. 182–190.
9. Coker, D. An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and Systems*, 1997; 88, pp. 81–89.
10. Gau, W.L.; Buehrer, D.J. Vague sets. *IEEE Trans. Systems Man and Cybernet*, 1993; 23(2), pp. 610–614.
11. Levine, N. Generalized closed sets in topological spaces. *Rend. Circ. Mat. Palermo*., 1970; 19, pp. 89–96.
12. Salama, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. *IOSR Journal of Mathematics*, 2012; 3(4), pp. 31-35.
13. Nabeeh, N. A.; Smarandache, F.; Abdel-Basset, M.; El-Ghareeb, H. A.; Aboelfetouh, A. An Integrated Neutrosophic-TOPSIS Approach and Its Application to Personnel Selection: A New Trend in Brain Processing and Analysis. *IEEE Access*, 2019; 7, pp. 29734-29744.
14. Shawkat Alkhazaleh. Neutrosophic vague set theory. *Critical Review*. 2015; 10, pp. 29–39.
15. Smarandache, F. A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth: American Research Press, 1998.

16. Smarandache, F.; Abdel-Basset, M.; Saleh, Abduallah Gamal, M. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, 2019; *77*, pp. 438-452.
17. Smarandache, F.; Abdel-Baset, M.; Victor Chang; Abduallah Gamal. An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field. *Computers in Industry*, 2019; 106, pp. 94-110.
18. Smarandache, F.; Abdel-Basset, M.; Manogaran, G.; Gamal, A. A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, 2019; *43*(2),pp. 38.
19. Zadeh, L.A. Fuzzy Sets. Information and Control, 1965; 8,pp. 338–353.

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