NSS

Neutrosophic Soft Multi-Set Theory and Its Decision Making

Irfan Deli [1], Said Broumi[2] and Mumtaz Ali[3]

[1]Muallim Rıfat Faculty of Education, Kilis 7 Aralık University, 79000 Kilis, Turkey. E-mail:**irfandeli@kilis.edu.tr**

[2]Faculty of Lettres and Humanities, Hay El Baraka Ben M'sik Casablanca B.P. 7951, Hassan II University Mohammedia-Casablanca , Morocco. E-mail:broumisaid78@gmail.com

[3]Department of Mathematics, Quaid-i-Azam University, Islamabad, 44000,Pakistan. E-mail:mumtazali770@yahoo.com

Abstract. In this study, we introduce the concept of neutrosophic soft multi-set theory and study their properties and operations. Then, we give a decision making methods for neutrosophic soft multi-set theory. Finally, an application of this method in decision making problems is presented.

Keywords: Soft set, neutrosophic set, neutrosophic refined set, neutrosophic soft multi-set, decision making.

1. Introduction

In 1999, a Russian researcher Molodtsov [23] initiated the concept of soft set theory as a general mathematical tool for dealing with uncertainty and vagueness. The theory is in fact a set-valued map which is used to describe the universe of discourse based on some parameters which is free from the parameterization inadequacy syndrome of fuzzy set theory [31], rough set theory [25], and so on. After Molodtsov's work several researchers were studied on soft set theory with applications (i.e [13,14,21]). Then, Alkhazaleh et al [3] presented the definition of soft multiset as a generalization of soft set and its basic operation such as complement, union, and intersection. Also, [6,7,22,24] are studied on soft multiset. Later on, in [2] Alkazaleh and Salleh introduced fuzzy soft set multisets, a more general concept, which is a combination of fuzzy set and soft multisets and studied its properties and gave an application of this concept in decision making problem. Then, Alhazaymeh and Hassan [1] introduce the concept of vague soft multisets which is an extension of soft sets and presented application of this concept in decision making problem. These concepts cannot deal with indeterminant and inconsistent information.

In 1995, Smarandache [26,30] founded a theory is called neutrosophic theory and neutrosophic sets has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in real world. The theory is a powerful tool which generalizes the concept of the classical set, fuzzy set [31], interval-valued fuzzy set [29], intuitionistic fuzzy set [4], interval-valued intuitionistic fuzzy set [5], and so on.

Recently, Maji [20] proposed a hybrid structure is called neutrosophic soft set which is a combination of neutrosophic set [26] and soft sets [23] and defined several operations on neutrosophic soft sets and made a theoretical study on the theory of neutrosophic soft sets. After the introduction of neutrosophic soft set, many scholars have done a lot of good researches in this filed [8,9,11,18,19,27,28]. In recently, Deli [16] defined the notion of interval-valued neutrosophic soft set and interval-valued neutrosophic soft set operations to make more functional. After the introduction of interval-valued neutrosophic soft set Broumi et al. [10] examined relations of interval-valued neutrosophic soft set. Many interesting applications of neutrosophic set theory have been combined with soft sets in [12,17]. But until now, there have been no study on neutrosophic soft multisets. In this paper our main objective is to study the concept of neutrosophic soft multisets which is a combination of neutrosophic multi(refined) [15] set and soft multisets [3]. The paper is structured as follows. In Section 2, we first recall the necessary background material on neutrosophic sets and soft set. The concept of neutrosophic soft multisets and some of their properties are presented in Section 3. In Section 4, we present algorithm for neutrosophic soft multisets. In section 5 an application of neutrosophic soft multisets in decision making is presented. Finally we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of, neutrosophic set, soft set and neutrosophic soft sets. For more details, the reader could refer to [15,20,23,26,30].

Definition 2.1.[26] Let U be a universe of discourse then the neutrosophic set A is an object having the form

$$A = \{< x: \mu_{A(x),}\ \nu_{A(x)},\ \omega_{A(x)}>, x \in U\}$$

where the functions μ, ν, ω : $U \rightarrow]^{-}0,1^{+}[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^{-}0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^{+}.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0,1^{+}[$. So instead of $]^{-}0,1^{+}[$ we need to take the interval [0,1] for technical applications cause $]^{-}0,1^{+}[$ will be difficult to apply in the real world applications such as in scientific and engineering problems.

For two NS,

$$_{NS}= \{<x, \mu_A(x), \nu_A(x)\ , \omega_A(x)> \mid x \in X\ \}$$

and

$$B_{NS}= \{<x, \mu_B(x), \nu_B(x)\ , \omega_B(x)> \mid x \in X\ \}$$

Set- theoretic operations;

1. The subset; $_{NS} \subseteq B_{NS}$ if and only if

$\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$ and $\omega_A(x) \geq \omega_B(x)$.

2. $_{NS} = B_{NS}$ if and only if,

$\mu_A(x) = \mu_B(x)$,$\nu_A(x) = \nu_B(x)$ and $\omega_A(x) = \omega_B(x)$

for any x ∈ X.

3. The complement of $_{NS}$ is denoted by $^{o}_{NS}$ and is defined by

$$^{o}_{NS} = \{<x, \ \omega_A(x), 1 - \nu_A(x), \mu_A(x) \mid x \in X\}$$

4. The intersection

$$A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}, \max\{\omega_A(x), \omega_B(x)\}>: x \in X\}$$

5. The union

$$A \cup B = \{<x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}, \min\{\omega_A(x), \omega_B(x)\}>: x \in X\}$$

Definition 2.2 [23] Let U be an initial universe set and E be a set of parameters. Let P (U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K: A → P(U).

For an illustration, let us consider the following example.

Example 2.3. Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_{10}}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_4}, where e_1, e_2, . . ., e_4 stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively. In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, says Mrs X, may be defined like this:

A= {e_1,e_2,e_3,e_4 };

K(e_1) = {h_1, h_3, h_7}, K(e_2) = {h_2 }, K(e_3) = {h_{10}}, K(e_4) = U

Definition 2.4[20] Let **U** be an initial universe set and **A** ⊂ **E** be a set of parameters. Let NS (U) denotes the set of all neutrosophic subsets of **U**. The collection $(\mathbf{F}, \mathbf{A})$ is termed to be the neutrosophic soft set over **U**, where **F** is a mapping given by $\mathbf{F}: \mathbf{A} \rightarrow \mathbf{NS}(\mathbf{U})$.

Example 2.5 [20] Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E = {beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by $U = \{h_1, \ h_2, \ldots, h_5\}$ and the set of parameters $A = \{e_1, e_2, \ e_3, e_4\}$,where e_1 stands for the parameter `beautiful', $e_2$ stands for the parameter `wooden', e_3 stands for the parameter `costly' and the parameter $e_4$stands for `moderate'. Then the neutrosophic soft set (F, A) is defined as follows:

$$(F,A)=\left\{\begin{array}{l}\left(e_1,\left\{\frac{h_1}{(0.5,0.6,0.3)},\frac{h_2}{(0.4,0.7,0.6)},\frac{h_3}{(0.6,0.2,0.3)},\frac{h_4}{(0.7,0.3,0.2)},\frac{h_5}{(0.8,0.2,0.3)}\right\}\right),\\ \left(e_2,\left\{\frac{h_1}{(0.6,0.3,0.5)},\frac{h_2}{(0.7,0.4,0.3)},\frac{h_3}{(0.8,0.1,0.2)},\frac{h_4}{(0.7,0.1,0.3)},\frac{h_5}{(0.8,0.3,0.6)}\right\}\right),\\ \left(e_3,\left\{\frac{h_1}{(0.7,0.4,0.3)},\frac{h_2}{(0.6,0.7,0.2)},\frac{h_3}{(0.7,0.2,0.5)},\frac{h_4}{(0.5,0.2,0.6)},\frac{h_5}{(0.7,0.3,0.4)}\right\}\right),\\ \left(e_4,\left\{\frac{h_1}{(0.8,0.6,0.4)},\frac{h_2}{(0.7,0.9,0.6)},\frac{h_3}{(0.7,0.6,0.4)},\frac{h_4}{(0.7,0.8,0.6)},\frac{h_5}{(0.9,0.5,0.7)}\right\}\right)\end{array}\right\}$$

3-Neutrosophic Soft Multi-Set Theory

In this section, we introduce the definition of a neutrosophic soft multi-set(Nsm-set) and its basic operations such as complement, union and intersection with examples. Some of it is quoted from [1,2,3, 6,7,22,24].

Obviously, some definitions and examples are an extension of soft multi-set [3] and fuzzy soft multi-sets [2].

Definition 3.1. Let $\{U_i : i \in I\}$be a collection of universes such that $\bigcap_{i\in I} U_i = \Phi$, $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters, $U=\prod_{i\in I} NSM(U_i)$ where $NSM(U_i)$ denotes the set of all NSM-subsets of U_i and $E=\prod_{i\in I} E_{U_i}$ and $\subseteq E$. Then, N_A is a neutrosophic soft multi-set (Nsm-set) over U, where N_A is a mapping given by N_A: A→ U.

Thus, a Nsm-set N_A over U can be represented by the set of ordered pairs.

$N_A=\{(x_1, N_A(x_1)) : x_1 \in \quad \subseteq E\}$.

To illustrate this let us consider the following example:

Example 3.2 Suppose that Mr. X has a budget to buy a house, a car and rent a venue to hold a wedding celebration. Let us consider a Nsm-set $N_A$which describes "houses," "cars," and "hotels" that Mr.X is considering for accommodation purchase, transportation-purchase, and a venue to hold a wedding celebration, respectively.

Assume that $U_1=\{u_1, u_2, u_3, u_4\}$, $U_2=\{c_1, c_2, c_3, c_4\}$ and $U_3=\{h_1, h_2, h_3\}$ are three universal set and

$E_1=\{x_1{}^{U_1} = \text{expensiv}, x_2{}^{U_1} = \text{cheap}, x_3{}^{U_1} = \quad \text{wooden}\}$,
$E_2=\{x_1{}^{U_2} = \text{expensive}, x_2{}^{U_2} =$
$\text{in green surroundings}, x_3{}^{U_2} = \text{sporty}\}$ and
$E_3=\{x_1{}^{U_3}=\text{expensive}, x_2{}^{U_3} = \text{majestic}, x_3{}^{U_3} = \text{in Kuala Lumpur}\}$

Three parameter sets that is a collection of sets of decision parameters related to the above
universes.

Let $U=\prod_1^3 NSM(U_i)$ and $E=\prod_1^3 E_{U_i}$ and $\subseteq E$ such that

$A=\{x_1 = \{x_1{}^{U_1}, x_1{}^{U_2}, x_1{}^{U_3}\}, x_2 = \{x_2{}^{U_1}, x_2{}^{U_2}, x_2{}^{U_3}\}\}$

and

$$N_A(x_1)=\left\{\begin{array}{l}\left\{\frac{u_1}{(.5,.3,.4)},\frac{u_2}{(.2,.4,.4)},\frac{u_3}{(.3,.3,.5)},\frac{u_4}{(.7,.8,.4)}\right\},\\ \left\{\frac{c_1}{(.7,.1,.5)},\frac{c_2}{(.2,.5,.7)},\frac{c_3}{(.7,.8,.0)},\frac{c_4}{(.0,.0,.0)}\right\},\\ \left\{\frac{h_1}{(.0,.0,.0)},\frac{h_2}{(1,1,.0)},\frac{h_3}{(.9,.2,.5)}\right\}\end{array}\right\}$$

$$N_A(x_2)=\left\{\begin{array}{l}\left\{\frac{u_1}{(.1,.5,.3)},\frac{u_2}{(.1,.8,.9)},\frac{u_3}{(.0,.0,1)},\frac{u_4}{(.2,.8,.5)}\right\},\\ \left\{\frac{c_1}{(.5,.5,.5)},\frac{c_2}{(.5,.3,.7)},\frac{c_3}{(.5,.4,.3)},\frac{c_4}{(.1,1,1)}\right\},\\ \left\{\frac{h_1}{(1,.2,.5)},\frac{h_2}{(1,1,1)},\frac{h_3}{(.1,.8,.6)}\right\}\end{array}\right.$$

Then a Nsm-set N_A is written by

$$\begin{array}{l}N_A =\\ \left\{\left(x_1,\left(\left\{\frac{u_1}{(.5,.3,.4)},\frac{u_2}{(.2,.4,.4)},\frac{u_3}{(.3,.3,0.5)},\frac{u_4}{(.7,.8,.4)}\right\},\right.\right.\right.\\ \left\{\frac{c_1}{(.7,.1,.5)},\frac{c_2}{(.2,.5,.7)},\frac{c_3}{(.7,.8,.0)},\frac{c_4}{(.0,.0,.0)}\right\},\\ \left.\left.\left\{\frac{h_1}{(.0,.0,.0)},\frac{h_2}{(1,1,.0)},\frac{h_3}{(.9,.2,.5)}\right\}\right)\right),\\ \left(x_2,\left(\left\{\frac{u_1}{(.1,.5,.3)},\frac{u_2}{(.1,.8,.9)},\frac{u_3}{(.0,.0,1)},\frac{u_4}{(.2,.8,.5)}\right\},\right.\right.\\ \left.\left.\left.\left\{\frac{c_1}{(.5,.5,.5)},\frac{c_2}{(.5,.3,.7)},\frac{c_3}{(.5,.4,.3)},\frac{c_4}{(.1,1,1)}\right\},\left\{\frac{h_1}{(1,.2,.5)},\frac{h_2}{(1,1,1)},\frac{h_3}{(.1,.8,.6)}\right\}\right)\right)\right\}\end{array}$$

Definition 3.3. Let N_A be a Nsm-set. Then, a pair $({x_i}^{U_j}, N_A({x_i}^{U_j}))$ is called an U_i-Nsm-set part, ${x_i}^{U_j} \in x_k$ and $N_A({x_i}^{U_j}) \subseteq N_A(x_i)$ such that $x_k \in \{x_1, x_2, \dots, x_n\}$, $i \in \{1,2,\dots,m\}$ and $j \in \{1,2,\dots,r\}$.

Example 3.4. Consider Example 3.2. Then,

$$\begin{aligned}({x_i}^{U_1}, N_A({x_i}^{U_1})) = \Big\{ & \Big({x_1}^{U_1}, \Big\{\frac{u_1}{(0.5,0.3,0.4)}, \frac{u_2}{(0.2,0.4,0.4)}, \frac{u_3}{(0.3,0.3,0.5)}, \frac{u_4}{(0.7,0.8,0.4)}\Big\}\Big), \\ & \Big({x_2}^{U_1}, \Big\{\frac{u_1}{(0.1,0.5,0.3)}, \frac{u_2}{(0.1,0.8,0.9)}, \frac{u_3}{(0.0,0.0,1.0)}, \frac{u_4}{(0.2,0.8,0.5)}\Big\}\Big)\Big\}\end{aligned}$$

is a U_1-Nsm-set part of N_A.

Definition 3.5. Let N_A and N_B be a Nsm-sets. Then, N_A is NSMS-subset of N_B, denoted by $N_A \sqsubseteq N_B$ if and only if $N_A({x_I}^{U_j})$ is a neut – rosophic subset of $N_B({x_i}^{U_j})$ for all ${x_I}^{U_j} \in x_k$ *such that* $x_k \in \{x_1, x_2, \dots, x_n\}$, $i \in \{1,2,\dots,m\}$ and $j \in \{1,2,\dots,r\}$.

Example3.4. Let

$A=\{x_1 = \{{x_1}^{U_1}, {x_1}^{U_2}, {x_1}^{U_3}\},\ x_2 = \{{x_2}^{U_1}, {x_2}^{U_2}, {x_2}^{U_3}\}\}$ and

$B=\{x_1 = \{{x_1}^{U_1}, {x_1}^{U_2}, {x_1}^{U_3}\}, x_2 = \{{x_2}^{U_1}, {x_2}^{U_2}, {x_2}^{U_3}\}$
$x_3 = \{{x_3}^{U_1}, {x_3}^{U_2}, {x_3}^{U_3}\}\}$

Clearly $A \subseteq B$. Let N_A and N_B be two Nsm-set over the same U such that

$$\begin{aligned}N_A = \Big\{ & \Big(x_1, \Big(\Big\{\frac{u_1}{(0.5,0.3,0.4)}, \frac{u_2}{(0.2,0.4,0.4)}, \frac{u_3}{(0.3,0.3,0.5)}, \frac{u_4}{(0.7,0.8,0.4)}\Big\}, \\ & \Big\{\frac{c_1}{(0.7,0.1,0.5)}, \frac{c_2}{(0.2,05,0.7)}, \frac{c_3}{(0.7,0.8,0.0)}, \frac{c_4}{(0.0,0.0,0.0)}\Big\} \\ & \Big\{\frac{h_1}{(0.0,0.0,0.0)}, \frac{h_2}{(1.0,1.0,0.0)}, \frac{h_3}{(0.9,0.2,0.5)}\Big\}\Big)\Big), \\ & \Big(x_2, \Big(\Big\{\frac{u_1}{(0.1,0.5,0.3)}, \frac{u_2}{(0.1,0.8,0.9)}, \frac{u_3}{(0.0,0.0,1.0)}, \frac{u_4}{(0.2,0.8,0.5)}\Big\}, \\ & \Big\{\frac{c_1}{(0.5,0.5,0.5)}, \frac{c_2}{(0.5,0.3,0.7)}, \frac{c_3}{(0.5,0.4,0.3)}, \frac{c_4}{(0.1,1.0,1.0)}\Big\}, \\ & \Big\{\frac{h_1}{(1.0,0.2,0.5)}, \frac{h_2}{(1.0,1.0,1.0)}, \frac{h_3}{(0.1,0.8,0.6)}\Big\}\Big)\Big)\Big\}\end{aligned}$$

$$\begin{aligned}N_B = \Big\{ & \Big(x_1, \Big(\Big\{\frac{u_1}{(0.6,0.1,0.2)}, \frac{u_2}{(0.3,0.3,0.3)}, \frac{u_3}{(0.7,0.2,0.4)}, \frac{u_4}{(0.8,0.6,0.3)}\Big\}, \\ & \Big\{\frac{c_1}{(0.9,0.1,0.4)}, \frac{c_2}{(0.3,0.7,0.6)}, \frac{c_3}{(0.8,0.4,0.0)}, \frac{c_4}{(1.0,0.0,0.0)}\Big\}, \\ & \Big\{\frac{h_1}{(1.0,0.0,0.0)}, \frac{h_2}{(0.9,0.7,0.0)}, \frac{h_3}{(1.0,0.0,0.0)}\Big\}\Big)\Big), \\ & \Big(x_2, \Big(\Big\{\frac{u_1}{(0.8,0.3,0.2)}, \frac{u_2}{(0.7,0.6,0.4)}, \frac{u_3}{(0.8,0.0,0.7)}, \frac{u_4}{(0.5,0.6,0.3)}\Big\}, \\ & \Big\{\frac{c_1}{(0.6,0.4,0.3)}, \frac{c_2}{(0.7,0.2,0.6)}, \frac{c_3}{(0.6,0.1,0.2)}, \frac{c_4}{(1.0,0.3,0.1)}\Big\} \\ & , \Big\{\frac{h_1}{(1.0,0.0,0.0)}, \frac{h_2}{(1.0,0.0,0.1)}, \frac{h_3}{(0.8,0.3,0.4)}\Big\}\Big)\Big)\Big\}, \\ & \Big(x_3, \Big(\Big\{\frac{u_1}{(0.5,0.6,0.4)}, \frac{u_2}{(0.2,0.7,0.5)}, \frac{u_3}{(0.3,0.9,0.3)}, \frac{u_4}{(0.2,0.8,0.7)}\Big\}, \\ & \Big\{\frac{c_1}{(0.8,0.3,0.5)}, \frac{c_2}{(0.8,0.3,0.1)}, \frac{c_3}{(0.3,0.5,0.6)}, \frac{c_4}{(0.9,0.3,0.2)}\Big\}, \\ & \Big\{\frac{h_1}{(0.3,0.8,0.6)}, \frac{h_2}{(0.0,1.0,0.2)}, \frac{h_3}{(0.3,0.6,0.5)}\Big\}\Big)\Big)\Big\}\end{aligned}$$

Then, we have $N_A \subseteq N_B$.

Definition 3.6. Let N_A and N_B are two Nsm-sets. Then, $N_A = N_B$, if and only if $N_A \subseteq N_B$ and $N_B \subseteq N_A$.

Definition 3.7. Let N_A be a Nsm-set. Then, the complement of N_A, denoted by N_A^c, is defined by

$N_A^c = \{(x, N_A^o(x)) : x \in \subseteq E\}$

where $N_A^o(x)$ is a NM complement.

Example3.4.

$$\begin{aligned}N_A^o(x) = \Big\{ & \Big(x_1, \Big(\Big\{\frac{u_1}{(0.4,0.7,0.5)}, \frac{u_2}{(0.4,0.6,0.2)}, \frac{u_3}{(0.5,0.7,0.3)}, \frac{u_4}{(0.4,0.2,0.7)}\Big\}, \\ & \Big\{\frac{c_1}{(0.5,0.9,0.7)}, \frac{c_2}{(0.7,0.5,0.2)}, \frac{c_3}{(0.0,0.2,0.7)}, \frac{c_4}{(0.0,1.0,0.0)}\Big\}, \\ & \Big\{\frac{h_1}{(0.0,1.0,0.0)}, \frac{h_2}{(0.0,9.0,1.0)}, \frac{h_3}{(0.5,0.8,0.9)}\Big\}\Big)\Big), \\ & \Big(x_2, \Big(\Big\{\frac{u_1}{(0.3,0.5,0.1)}, \frac{u_2}{(0.9,0.2,0.1)}, \frac{u_3}{(1.0,1.0,0.0)}, \frac{u_4}{(0.5,0.2,0.2)}\Big\}, \\ & \Big\{\frac{c_1}{(0.5,0.5,0.5)}, \frac{c_2}{(0.7,0.7,0.5)}, \frac{c_3}{(0.3,0.6,0.5)}, \frac{c_4}{(1.0,0.0,0.1)}\Big\}, \\ & \Big\{\frac{h_1}{(0.5,0.8,1.0)}, \frac{h_2}{(1.0,0.0,1.0)}, \frac{h_3}{(0.6,0.2,0.1)}\Big\}\Big)\Big)\Big\}\end{aligned}$$

Definition 3.8. A Nsm-set N_A over U is called a null Nsm-set, denoted by $N_{A_\emptyset}$ if all of the Nsm-set parts of N_A equals $\emptyset$.

Example3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the "at-

tractiveness of stone houses", "cars" and "hotels". Let
A=$\{x_1=\{x_1^{U_1},x_1^{U_2},x_1^{U_3}\}, x_2=\{x_2^{U_1},x_2^{U_2},x_2^{U_3}\}\}$.
The Nsm-set N_A is the collection of approximations as below:

$$N_{A_\emptyset}=\Big\{\Big(x_1,\Big(\Big\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\Big\},\Big\{\frac{c_1}{(0.0,1.0,1.0)},\frac{c_2}{(0.0,1.0,1.0)},\frac{c_3}{(0.0,1.0,1.0)},\frac{c_4}{(0.0,1.0,1.0)}\Big\},\Big\{\frac{h_1}{(0.0,1.0,1.0)},\frac{h_2}{(0.0,1.0,1.0)},\frac{h_3}{(0.0,1.0,1.0)}\Big\}\Big)\Big),\Big(x_2,\Big(\Big\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\Big\},\Big\{\frac{c_1}{(0.0,1.0,1.0)},\frac{c_2}{(0.0,1.0,1.0)},\frac{c_3}{(0.0,1.0,1.0)},\frac{c_4}{(0.0,1.0,1.0)}\Big\},\Big\{\frac{h_1}{(0.0,1.0,1.0)},\frac{h_2}{(0.0,1.0,1.0)},\frac{h_3}{(0.0,1.0,1.0)}\Big\}\Big)\Big)\Big\}$$

Then, $N_{A_\emptyset}$ is a null Nsm-set.

Definition 3.8. A Nsm-set N_A over U is called a semi-null Nsm-set, denoted by$N_{A_{\approx\emptyset}}$if at least all the Nsm-set parts of $N_{A_{\approx\emptyset}}$equals $\emptyset$.

Example3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the "attractiveness of stone houses", "cars" and "hotels". Let
A=$\{x_1=\{x_1^{U_1},x_1^{U_2},x_1^{U_3}\}, x_2=\{x_2^{U_1},x_2^{U_2},x_2^{U_3}\}\}$.
The Nsm-set N_A is the collection of approximations as below:

$$N_{A_{\approx\emptyset}}=\Big\{\Big(x_1,\Big(\Big\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\Big\},\Big\{\frac{c_1}{(0.5,0.9,0.7)},\frac{c_2}{(0.7,0.5,0.2)},\frac{c_3}{(0.0,0.2,0.7)},\frac{c_4}{(0.0,1.0,0.0)}\Big\},\Big\{\frac{h_1}{(0.0,1.0,0.0)},\frac{h_2}{(0.0,9.0,1.0)},\frac{h_3}{(0.5,0.8,0.9)}\Big\}\Big)\Big),\Big(x_2,\Big(\Big\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\Big\},\Big\{\frac{c_1}{(0.5,0.5,0.5)},\frac{c_2}{(0.7,0.7,0.5)},\frac{c_3}{(0.3,0.6,0.5)},\frac{c_4}{(1.0,0.0,0.1)}\Big\},\Big\{\frac{h_1}{(0.5,0.8,1.0)},\frac{h_2}{(1.0,0.0,1.0)},\frac{h_3}{(0.6,0.2,0.1)}\Big\}\Big)\Big)\Big\}$$

Then $N_{A_{\approx\emptyset}}$ is a semi null Nsm-set

Definition 3.8. A Nsm-set N_A over U is called a semi-absolute Nsm-set, denoted by $N_{A\approx U_i}$ if $N_A(x_i^{U_j})=U_i$ for at least one $x_k\in\{x_1,x_2,\dots,x_n\}$, $i\in\{1,2,\dots,m\}$ and $j\in\{1,2,\dots,r\}$.

Example3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the "attractiveness of stone houses", "cars" and "hotels". Let
A= $\{x_1=\{x_1^{U_1},x_1^{U_2},x_1^{U_3}\}, x_2=\{x_2^{U_1},x_2^{U_2},x_2^{U_3}\}\}$.
The Nsm-set N_A is the collection of approximations as below:

$$N_{A\approx U_i}=\Big\{\Big(x_1,\Big(\Big\{\frac{u_1}{(1.0,0.0,0.0)},\frac{u_2}{(1.0,0.0,0.0)},\frac{u_3}{(1.0,0.0,0.0)},\frac{u_4}{(1.0,0.0,0.0)}\Big\},\Big\{\frac{c_1}{(0.5,0.9,0.7)},\frac{c_2}{(0.7,0.5,0.2)},\frac{c_3}{(0.0,0.2,0.7)},\frac{c_4}{(0.0,1.0,0.0)}\Big\},\Big\{\frac{h_1}{(0.0,1.0,0.0)},\frac{h_2}{(0.0,9.0,1.0)},\frac{h_3}{(0.5,0.8,0.9)}\Big\}\Big)\Big),\Big(x_2,\Big(\Big\{\frac{u_1}{(1.0,0.0,0.0)},\frac{u_2}{(1.0,0.0,0.0)},\frac{u_3}{(1.0,0.0,0.0)},\frac{u_4}{(1.0,0.0,0.0)}\Big\},\Big\{\frac{c_1}{(0.5,0.5,0.5)},\frac{c_2}{(0.7,0.7,0.5)},\frac{c_3}{(0.3,0.6,0.5)},\frac{c_4}{(1.0,0.0,0.1)}\Big\},\Big\{\frac{h_1}{(0.5,0.8,1.0)},\frac{h_2}{(1.0,0.0,1.0)},\frac{h_3}{(0.6,0.2,0.1)}\Big\}\Big)\Big)\Big\}$$

Then, $N_{A\approx U_i}$is a semi-absolute Nsm-set.

Definition 3.8.A Nsm-set N_A over U is called an absolute Nsm-set, denoted by N_{AU_i} if $N_A(x_i^{U_j})=U_i$ for all i.

Example 3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the "attractiveness of stone houses", "cars" and "hotels". Let
A= $\{x_1=\{x_1^{U_1},x_1^{U_2},x_1^{U_3}\}, x_2=\{x_2^{U_1},x_2^{U_2},x_2^{U_3}\}\}$.
The Nsm-set N_A is the collection of approximations as below:

$$_{AU_i}=\{(x_1,(\{\frac{u_1}{(1,.0,.0)},\frac{u_2}{(1,.0,.0)},\frac{u_3}{(1,.0,.0)},\frac{u_4}{(1,.0,.0)}\},$$
$$\{\frac{c_1}{(1,.0,.0)},\frac{c_2}{(1,.0,.0)},\frac{c_3}{(1,.0,.0)},\frac{c_4}{(1,.0,.0)}\},$$
$$\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(1,.0,.0)}\})),$$
$$(x_2,(\{\frac{u_1}{(1,.0,.0)},\frac{u_2}{(1,.0,.0)},\frac{u_3}{(1,.0,.0)},\frac{u_4}{(1,.0,.0)}\},$$
$$\{\frac{c_1}{(1,.0,.0)},\frac{c_2}{(1,.0,.0)},\frac{c_3}{(1,.0,.0)},\frac{c_4}{(1,.0,.0)}\},$$
$$\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(1,.0,.0)}\}))\}$$

Then, $_{AU_i}$ is an absolute Nsm-set.

Proposition 3.15. Let $_A, N_B$ and N_C are three Nsm-sets. Then

i. $(N_A^c)^c=N_A$
ii. $(_{A\approx\emptyset})^c=N_{A\approx U_i}$
iii. $(_{A\emptyset})^c=N_{AU_i}$
iv. $(_{A\approx U_i})^c=N_{A\approx\emptyset}$
v. $(_{AU_i})^c=N_{A\emptyset}$

Proof: The proof is straightforward

Definition 3.8. Let N_A and N_B are two Nsm-sets. Then, union of $_A$ and N_B denoted by $N_A \sqcup N_B$, is defined by
$N_A \sqcup N_B=\{(x_i, N_A(x_i)\cup N_B(x_i)): x_i \in E\}$
where $\cup$ *is a NS union*, $i\in\{1,2,\ldots,m\}$ and $j\in\{1,2,\ldots,r\}$.

Example 3.10.

Let
$A=\{x_1=\{x_1^{U_1},x_1^{U_2},x_1^{U_3}\},\ x_2=\{x_2^{U_1},x_2^{U_2},x_2^{U_3}\}\}$
and
$B=\{x_1=\{x_1^{U_1},x_1^{U_2},x_1^{U_3}\}, x_2=\{x_2^{U_1},x_2^{U_2},x_2^{U_3}\}$
$x_3=\{x_3^{U_1},x_3^{U_2},x_3^{U_3}\}\}$

$$N_A=\{(x_1,(\{\frac{u_1}{(.5,.3,.4)},\frac{u_2}{(.2,.4,.4)},\frac{u_3}{(.3,.3,.5)},\frac{u_4}{(.7,.8,.4)}\},$$
$$\{\frac{c_1}{(.7,.1,.5)},\frac{c_2}{(.2,.5,.7)},\frac{c_3}{(.7,.8,.0)},\frac{c_4}{(.0,.0,.0)}\},$$
$$\{\frac{h_1}{(.0,.0,.0)},\frac{h_2}{(1,1,.0)},\frac{h_3}{(.9,.2,.5)}\})),$$
$$(x_2,(\{\frac{u_1}{(.1,.5,.3)},\frac{u_2}{(.1,.8,.9)},\frac{u_3}{(.0,.0,1)},\frac{u_4}{(.2,.8,.5)}\},$$
$$\{\frac{c_1}{(.5,.5,.5)},\frac{c_2}{(.5,.3,.7)},\frac{c_3}{(.5,.4,.3)},\frac{c_4}{(.1,1,1)}\},$$
$$\{\frac{h_1}{(1,.2,.5)},\frac{h_2}{(1,1,1)},\frac{h_3}{(.1,.8,.6)}\}))\},$$

$$N_B=\{(x_1,(\{\frac{u_1}{(.3,.7,.2)},\frac{u_2}{(.4,.3,.8)},\frac{u_3}{(.6,.5,.4)},\frac{u_4}{(.6,.7,.4)}\},$$
$$\{\frac{c_1}{(.5,.6,.8)},\frac{c_2}{(.5,.7,.8)},\frac{c_3}{(.3,.5,.6)},\frac{c_4}{(1,.0,.0)}\},$$
$$\{\frac{h_1}{(1,.0,.1)},\frac{h_2}{(.5,.6,.3)},\frac{h_3}{(1,.0,.0)}\})),$$
$$(x_2,(\{\frac{u_1}{(.7,.3,.5)},\frac{u_2}{(.6,.7,.8)},\frac{u_3}{(.6,.8,.6)},\frac{u_4}{(.6,.7,.3)}\},$$
$$\{\frac{c_1}{(.4,.3,.2)},\frac{c_2}{(.5,.6,.7)},\frac{c_3}{(.9,.1,.3)},\frac{c_4}{(1,.2,.1)}\},$$
$$\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.0,.1)},\frac{h_3}{(.4,.2,.3)}\})),$$
$$(x_3,(\{\frac{u_1}{(.6,.3,.6)},\frac{u_2}{(.3,.2,.6)},\frac{u_3}{(.6,.7,.5)},\frac{u_4}{(.3,.7,.6)}\},$$
$$\{\frac{c_1}{(.7,.5,.3)},\frac{c_2}{(.6,.7,.2)},\frac{c_3}{(.5,.4,.5)},\frac{c_4}{(.3,.6,.5)}\},$$
$$\{\frac{h_1}{(.3,.5,.6)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(.3,.2,.7)}\}))\}$$

$$N_A \sqcup N_B=\Big\{\Big(x_1,\Big(\{\frac{u_1}{(0.5,0.3,0.4)},\frac{u_2}{(0.4,0.3,0.4)},\frac{u_3}{(0.6,0.3,0.4)},\frac{u_4}{(0.7,0.7,0.4)}\},$$
$$\{\frac{c_1}{(0.7,0.1,0.5)},\frac{c_2}{(0.5,0.5,0.7)},\frac{c_3}{(0.7,0.3,0.0)},\frac{c_4}{(1.0,0.0,0.0)}\},$$
$$\{\frac{h_1}{(1.0,0.0,0.0)},\frac{h_2}{(1.0,0.1,0.0)},\frac{h_3}{(0.9,0.2,0.5)}\}\Big)\Big),$$
$$\Big(x_2,\Big(\{\frac{u_1}{(0.7,0.3,0.5)},\frac{u_2}{(0.6,0.7,0.8)},\frac{u_3}{(0.6,0.0,0.6)},\frac{u_4}{(0.6,0.7,0.3)}\},$$
$$\{\frac{c_1}{(0.5,0.3,0.2)},\frac{c_2}{(0.5,0.3,0.7)},\frac{c_3}{(0.9,0.1,0.3)},\frac{c_4}{(1.0,0.2,0.1)}\}$$
$$\{\frac{h_1}{(1.0,0.0,0.0)},\frac{h_2}{(1.0,0.0,0.1)},\frac{h_3}{(0.4,0.2,0.3)}\}\Big)\Big)\Big\},$$
$$\Big(x_3,\Big(\{\frac{u_1}{(0.6,0.3,0.6)},\frac{u_2}{(0.3,0.2,0.6)},\frac{u_3}{(0.6,0.7,0.5)},\frac{u_4}{(0.3,0.7,0.6)}\},$$
$$\{\frac{c_1}{(0.7,0.5,0.3)},\frac{c_2}{(0.6,0.7,0.2)},\frac{c_3}{(0.5,0.4,0.5)},\frac{c_4}{(0.3,0.6,0.5)}\},$$
$$\{\frac{h_1}{(0.3,0.5,0.6)},\frac{h_2}{(1.0,0.0,0.0)},\frac{h_3}{(0.3,0.2,0.7)}\}\Big)$$

Proposition 3.15. Let $_A, N_B$ and N_C are three Nsm-sets. Then

i. $N_A \sqcup (N_B \sqcup N_C) = (N_A \sqcup N_B) \sqcup N_C$
ii. $N_A \sqcup N_A = N_A$
iii. $N_A \sqcup N_{A\emptyset} = N_A$
iv. $N_A \sqcup N_{B\emptyset} = N_A$

Proof: The proof is straightforward

Definition 3.8. Let N_A and N_B are two Nsm-sets. Then, intersection of N_A and N_B, denoted by $N_A \sqcap N_B$, is defined by

$$N_A \sqcap N_B = \{(x_i, N_A(x_i) \cap N_B(x_i)) : x_i \in E\}$$

where $\cap$ *is a NS intersection*,
$i \in \{1,2, \ldots ,m\}$ and $j \in \{1,2, \ldots ,r\}$.

Example 3.10.

$$N_A=\{(x_1, (\{\frac{u_1}{(.5,.3,.4)}, \frac{u_2}{(.2,.4,.4)}, \frac{u_3}{(.3,.3,.5)}, \frac{u_4}{(.7,.8,.4)}\},$$
$$\{\frac{c_1}{(.7,.1,.5)}, \frac{c_2}{(.2,.5,.7)}, \frac{c_3}{(.7,.8,.0)}, \frac{c_4}{(.0,.0,.0)}\},$$
$$\{\frac{h_1}{(.0,.0,.0)}, \frac{h_2}{(1,.1,.0)}, \frac{h_3}{(.9,.2,.5)}\})),$$
$$(x_2, (\{\frac{u_1}{(.1,.5,.3)}, \frac{u_2}{(.1,.8,.9)}, \frac{u_3}{(.0,.0,1)}, \frac{u_4}{(.2,.8,.5)}\},$$
$$\{\frac{c_1}{(.5,.5,5)}, \frac{c_2}{(.5,.3,.7)}, \frac{c_3}{(.5,.4,.3)}, \frac{c_4}{(.1,1,1)}\},$$
$$\{\frac{h_1}{(1,.2,.5)}, \frac{h_2}{(1,1,1)}, \frac{h_3}{(.1,.8,.6)}\})),$$

$$N_B=\{(x_1, (\{\frac{u_1}{(.3,.7,.2)}, \frac{u_2}{(.4,.3,.8)}, \frac{u_3}{(.6,.5,.4)}, \frac{u_4}{(.6,.7,.4)}\},$$
$$\{\frac{c_1}{(.5,.6,.8)}, \frac{c_2}{(.5,.7,.8)}, \frac{c_3}{(.3,.5,.6)}, \frac{c_4}{(1,.0,.0)}\},$$
$$\{\frac{h_1}{(1,.0,.1)}, \frac{h_2}{(.5,.6,.3)}, \frac{h_3}{(1,.0,.0)}\})),$$
$$(x_2, (\{\frac{u_1}{(.7,.3,.5)}, \frac{u_2}{(.6,.7,.8)}, \frac{u_3}{(.6,.8,.6)}, \frac{u_4}{(.6,.7,.3)}\},$$
$$\{\frac{c_1}{(.4,.3,.2)}, \frac{c_2}{(.5,.6,.7)}, \frac{c_3}{(.9,.1,.3)}, \frac{c_4}{(1,.2,.1)}\},$$
$$\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.0,..1)}, \frac{h_3}{(.4,.2,.3)}\})),$$
$$(x_3, (\{\frac{u_1}{(.6,.3,.6)}, \frac{u_2}{(.3,.2,.6)}, \frac{u_3}{(.6,.7,.5)}, \frac{u_4}{(.3,.7,.6)}\},$$
$$\{\frac{c_1}{(.7,.5,.3)}, \frac{c_2}{(.6,.7,.2)}, \frac{c_3}{(.5,.4,.5)}, \frac{c_4}{(.3,.6,.5)}\},$$
$$\{\frac{h_1}{(.3,.5,.6)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.3,.2,.7)}\}))\}$$

$N_A \sqcap N_B =$
$$(x_1, (\{\frac{u_1}{(.3,.7,.4)}, \frac{u_2}{(.2,.4,.8)}, \frac{u_3}{(.3,.5,.5)}, \frac{u_4}{(.6,.8,.4)}\},$$
$$\{\frac{c_1}{(.5,.6,.8)}, \frac{c_2}{(.2,.7,.8)}, \frac{c_3}{(.3,.8,.6)}, \frac{c_4}{(1,.0,.0)}\},$$
$$\{\frac{h_1}{(.0,.0,.1)}, \frac{h_2}{(.5,1,.3)}, \frac{h_3}{(.9,.2,.5)}\})),$$
$$(x_2, (\{\frac{u_1}{(.1,.5,.5)}, \frac{u_2}{(.1,.8,.9)}, \frac{u_3}{(.0,.8,.6)}, \frac{u_4}{(.2,.7,.5)}\},$$
$$\{\frac{c_1}{(.4,.5,.5)}, \frac{c_2}{(.5,.6,.7)}, \frac{c_3}{(.5,.4,.3)}, \frac{c_4}{(.1,1,1)}\},$$
$$\{\frac{h_1}{(1,.2,.5)}, \frac{h_2}{(1,1,1)}, \frac{h_3}{(.1,.8,.6)}\})),$$

Proposition 3.15. Let $_A, N_B$ and N_C are three Nsm-sets. Then

i. $N_A \sqcap (N_B \sqcap N_C) = (N_A \sqcap N_B) \sqcap N_C$
ii. $N_A \sqcap N_A = N_A$
iii. $N_A \sqcap N_{A\emptyset} = N_A$
iv. $N_A \sqcap N_{B\emptyset} = N_A$

Proof: The proof is straightforward.

4. NS-multi-set Decision Making

In this section we recall the algorithm designed for solving a neutrosophic soft set and based on algorithm proposed by Alkazaleh and Saleh [20] for solving fuzzy soft multisets based decision making problem, we propose a new algorithm to solve neutrosophic soft multiset(NS-mset) based decision-making problem.
Now the algorithm for most appropriate selection of an object will be as follows.

4-1 Algorithm (Maji's algorithm using scores)

Maji [20] used the following algorithm to solve a decision-making problem.

(1) input the neutrosophic Soft Set (F, A).
(2) input P, the choice parameters of Mrs. X which is a subset of A.
(3) consider the NSS (F, P) and write it in tabular form.
(4) compute the comparison matrix of the NSS (F, P).
(5) compute the score S_i, for all i using S_i=T_i+I_i-F_i
(6) find S_k = max$_{x_i}$ S_i
(7) if k has more than one value then any one of bi may be chosen.

4.2 NS-multiset Theoretic Approch to Decision–Making Problem

In this section, we construct a Ns-mutiset decision making method by the following algorithm;

(1) Input the neutrosophic soft multiset (H, C) which is introduced by making any operations between (F, A) and (G, B).
(2) Apply MA to the first neutrosophic soft multi-set part in (H, C) to get the decision S_{k_1}.
(3) Redefine the neutrosophic soft multiset (H, C) by keeping all values in each row where $S_{k_1}$is maximum and replacing the values in the other rows by zero, to get $(H, C)_1$.
(4) Apply MA to the second neutrosophic soft multiset part in$(H, C)_1$to get the decision S_{k_2}.
(5) Redefine the neutrosophic soft set$(H, C)_1$ by keeping the first and second parts and apply the method in step (c) to the third part.
(6) Apply MA to the third neutrosophic soft multiset part in $(H, C)_2$ to get the decision S_{k_3}.
(7) The decision is $(S_{k_1}, S_{k_2}, S_{k_3})$.

5-Application in a Decision Making Problem

Assume that $U_1=\{u_1, u_2, u_3, u_4\}$, $U_2=\{c_1, c_2, c_3, c_4\}$ and $U_3=\{h_1, h_2, h_3\}$ be the sets of es” ,”cars”, and “hotels” , respectively and $\{E_1, E_2, E_3\}$be a collection of sets of decision parameters related to the above universe, where

$E_1=\{x_1{}^{U_1} = \text{expensive}, x_2{}^{U_1} = \text{cheap}, x_3{}^{U_1} = \text{wooden}\}$,

$E_2=\{x_1{}^{U_2} = \text{expensive}, x_2{}^{U_2} = \text{in green surroundings}, x_3{}^{U_2} = \text{sporty}\}$
and

$E_3=\{x_1{}^{U_3}=\text{expensive}, x_2{}^{U_3} = \text{majestic}, x_3{}^{U_3} = \text{in Kuala Lumpur}\}$

Let $A=\{x_1 = \{x_1{}^{U_1}, x_1{}^{U_2}, x_1{}^{U_3}\}, x_2 = \{x_2{}^{U_1}, x_2{}^{U_2}, x_2{}^{U_3}\}, x_4 = \{x_3{}^{U_1}, x_2{}^{U_2}, x_1{}^{U_3}\}\}$
and
$B=\{x_1 = \{x_1{}^{U_1}, x_1{}^{U_2}, x_1{}^{U_3}\}, x_2 = \{x_2{}^{U_1}, x_2{}^{U_2}, x_2{}^{U_3}\}, x_3 = \{x_3{}^{U_1}, x_3{}^{U_2}, x_3{}^{U_3}\}\}$

Suppose that a person wants to choose objects from the set of given objects with respect to the sets of choices parameters. Let there be two observation N_A and N_B by two expert Y_1 and Y_2, respectively.

$$N_A=\{(x_1, (\left\{\frac{u}{(.5,.3,.4)}, \frac{u_2}{(.2,.4,.4)}, \frac{u_3}{(.3,.3,.5)}, \frac{u_4}{(.7,.8,.4)}\right\},$$
$$\left\{\frac{c}{(.7,.1,.5)}, \frac{c_2}{(.2,.5,.7)}, \frac{c_3}{(.7,.8,.0)}, \frac{c_4}{(.0,.0,.0)}\right\},$$
$$\left\{\frac{h}{(.0,.0,.0)}, \frac{h_2}{(1,.1,.0)}, \frac{h_3}{(.9,.2,.5)}\right\})),$$
$$(x_2, (\left\{\frac{u}{(.1,.5,.3)}, \frac{u_2}{(.1,.8,.9)}, \frac{u_3}{(.0,.1,1)}, \frac{u_4}{(.2,.8,.5)}\right\},$$
$$\left\{\frac{c}{(.5,.5,5)}, \frac{c_2}{(.5,.3,.7)}, \frac{c_3}{(.5,.4,.3)}, \frac{c_4}{(.1,1,1)}\right\},$$
$$\left\{\frac{h}{(1,.2,.5)}, \frac{h_2}{(1,1,1)}, \frac{h_3}{(.1,.8,.6)}\right\})),$$
$$(x_4, (\left\{\frac{u}{(.2,.5,.6)}, \frac{u_2}{(.6,.2,.3)}, \frac{u_3}{(.8,.7,.6)}, \frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c}{(.3,.5,.7)}, \frac{c_2}{(.3,.6,.2)}, \frac{c_3}{(.8,.5,.3)}, \frac{c_4}{(.3,.5,.5)}\right\},$$
$$\left\{\frac{h}{(.2,.6,.5)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.5,.2,.3)}\right\}))\}$$

$$N_B=\{(x_1, (\left\{\frac{u}{(.3,.7,.2)}, \frac{u_2}{(.4,.3,.8)}, \frac{u_3}{(.6,.5,.4)}, \frac{u_4}{(.6,.7,.4)}\right\},$$
$$\left\{\frac{c}{(.5,.6,.8)}, \frac{c_2}{(.5,.7,.8)}, \frac{c_3}{(.3,.5,.6)}, \frac{c_4}{(1,.0,.0)}\right\},$$
$$\left\{\frac{h}{(1,.0,.1)}, \frac{h_2}{(.5,.6,.3)}, \frac{h_3}{(1,.0,.0)}\right\})),$$
$$(x_2, (\left\{\frac{u}{(.7,.3,.5)}, \frac{u_2}{(.6,.7,.8)}, \frac{u_3}{(.6,.8,.6)}, \frac{u_4}{(.6,.7,.3)}\right\},$$
$$\left\{\frac{c}{(.4,.3,.2)}, \frac{c_2}{(.5,.6,.7)}, \frac{c_3}{(.9,.1,.3)}, \frac{c_4}{(1,.2,.1)}\right\},$$
$$\left\{\frac{h}{(1,.0,.0)}, \frac{h_2}{(1,.0,..1)}, \frac{h_3}{(.4,.2,.3)}\right\})),$$
$$(x_3, (\left\{\frac{u}{(.6,.3,.6)}, \frac{u_2}{(.3,.2,.6)}, \frac{u_3}{(.6,.7,.5)}, \frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c}{(.7,.5,.3)}, \frac{c_2}{(.6,.7,.2)}, \frac{c_3}{(.5,.4,.5)}, \frac{c_4}{(.3,.6,.5)}\right\},$$
$$\left\{\frac{h}{(.3,.5,.6)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.3,.2,.7)}\right\}))\}$$

$$N_A \sqcup N_B=$$
$$\{(x_1, (\left\{\frac{u_1}{(.5,.3,.4)}, \frac{u_2}{(.4,.3,.4)}, \frac{u_3}{(.6,.3,.4)}, \frac{u_4}{(.7,.7,.4)}\right\},$$
$$\left\{\frac{c_1}{(.7,.1,.5)}, \frac{c_2}{(.5,.5,.7)}, \frac{c_3}{(.7,.3,.0)}, \frac{c_4}{(1,.0,.0)}\right\},$$
$$\left\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.1,.0)}, \frac{h_3}{(.9,.2,.5)}\right\})),$$
$$(x_2, (\left\{\frac{u_1}{(.7,.3,.5)}, \frac{u_2}{(.6,.7,.8)}, \frac{u_3}{(.6,.0,.6)}, \frac{u_4}{(.6,.7,.3)}\right\},$$
$$\left\{\frac{c_1}{(.5,.3,.2)}, \frac{c_2}{(.5,.3,.7)}, \frac{c_3}{(.9,.1,.3)}, \frac{c_4}{(1,.2,.1)}\right\},$$
$$\left\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.0,..1)}, \frac{h_3}{(.4,.2,.3)}\right\}))$$

$$(x_3,(\left\{\frac{u_1}{(.6,.3,.6)},\frac{u_2}{(.3,.2,.6)},\frac{u_3}{(.6,.7,.5)},\frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c_1}{(.7,.5,.3)},\frac{c_2}{(.6,.7,.2)},\frac{c_3}{(.5,.4,.5)},\frac{c_4}{(.3,.6,.5)}\right\},$$
$$\left\{\frac{h_1}{(.3,.5,.6)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(.3,.2,.7)}\right\})),$$
$$(x_4,(\left\{\frac{u_1}{(.2,.5,.6)},\frac{u_2}{(.6,.2,.3)},\frac{u_3}{(.8,.7,.6)},\frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c_1}{(.3,.5,.7)},\frac{c_2}{(.3,.6,.2)},\frac{c_3}{(.8,.5,.3)},\frac{c_4}{(.3,.5,.5)}\right\},$$
$$\left\{\frac{h_1}{(.2,.6,.5)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(.5,.2,.3)}\right\}))\}$$

Now we apply MA to the first neutrosophic soft multiset part in (H,D) to take the decision from the availability set U_1. The tabular representation of the first resultant neutrosophic soft multiset part will be as in Table 1. The comparison table for the first resultant neutrosophic soft multiset part will be as in Table 2.
Next we compute the row-sum, column-sum, and the score for each u_i as shown in Table 3.
From Table 3, it is clear that the maximum score is 6, scored by u_3.

Table 1 :Tabular representation: U_1- neutrosophic soft multiset part of (H, D).

U_1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
u_1	(.5 ,.3 ,.4)	(.7 ,.3 ,.5)	(.6 ,.3 ,.6)	(.2 ,.5 ,.6)
u_2	(.4 ,.3 ,.4)	(.6 ,.7 ,.8)	(.3 ,.2 ,.6)	(.6 ,.2 ,.3)
u_3	(.6 ,.3 ,.4)	(.6 ,.0 ,.6)	(.6 ,.7 ,.5)	(.8 ,.7 ,.6)
u_4	(.7 ,.7 ,.4)	(.6 ,.7 ,.3)	(.3 ,.7 ,.6)	(.3 ,.7 ,.6)

Table 2 :Comparison table: U_1- neutrosophic soft multiset part of (H, D).

U_1	u_1	u_2	u_3	u_4
u_1	4	3	1	1
u_2	2	4	1	1
u_3	3	3	4	3
u_4	3	2	1	4

Table 3 :Score table: U_1- neutrosophic soft multiset part of (H, D).

	Row sum	Column sum	Score
u_1	9	12	-3
u_2	8	12	-4
u_3	13	7	**6**
u_4	10	9	1

Now we redefine the neutrosophic soft multiset (H, D) by keeping all values in each row where u_3 is maximum and replacing the values in the other rows by zero (1, 0 , 0):

$$(H,D)_1=\{(x_1,(\left\{\frac{u_1}{(.5,.3,.4)},\frac{u_2}{(.4,.3,.4)},\frac{u_3}{(.6,.3,.4)},\frac{u_4}{(.7,.7,.4)}\right\},$$
$$\left\{\frac{c_1}{(.7,.1,.5)},\frac{c_2}{(.5,.5,.7)},\frac{c_3}{(.7,.3,.0)},\frac{c_4}{(1,.0,.0)}\right\},$$
$$\left\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.1,.0)},\frac{h_3}{(.9,.2,.5)}\right\})),$$
$$(x_2,(\left\{\frac{u_1}{(.7,.3,.5)},\frac{u_2}{(.6,.7,.8)},\frac{u_3}{(.6,.0,.6)},\frac{u_4}{(.6,.7,.3)}\right\},$$
$$\left\{\frac{c_1}{(1,.0,.0)},\frac{c_2}{(1,.0,.0)},\frac{c_3}{(1,.0,.0)},\frac{c_4}{(1,.0,.0)}\right\},$$
$$\left\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(1,.0,.0)}\right\})),$$
$$(x_3,(\left\{\frac{u_1}{(.6,.3,.6)},\frac{u_2}{(.3,.2,.6)},\frac{u_3}{(.6,.7,.5)},\frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c_1}{(.7,.5,.3)},\frac{c_2}{(.6,.7,.2)},\frac{c_3}{(.5,.4,.5)},\frac{c_4}{(.3,.6,.5)}\right\},$$
$$\left\{\frac{h_1}{(.3,.5,.6)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(.3,.2,.7)}\right\})),$$
$$(x_4,(\left\{\frac{u_1}{(.2,.5,.6)},\frac{u_2}{(.6,.2,.3)},\frac{u_3}{(.8,.7,.6)},\frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c_1}{(.3,.5,.7)},\frac{c_2}{(.3,.6,.2)},\frac{c_3}{(.8,.5,.3)},\frac{c_4}{(.3,.5,.5)}\right\},$$
$$\left\{\frac{h_1}{(.2,.6,.5)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(.5,.2,.3)}\right\}))\}$$

U_2	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
c_1	(.7 ,.1 ,.5)	(1 ,.0 ,.0)	(.7 ,.5 ,.3)	(.3 ,.5 ,.67)
c_2	(.5 ,.5 ,.7)	(1 ,.0 ,.0)	(.6 ,.7 ,.2)	(.3 ,.6 ,.2)
c_3	(.7 ,.3 ,.0)	(1 ,.0 ,.0)	(.5 ,.4 ,.5)	(.8 ,.5 ,.3)
c_4	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(.3 ,.6 ,.5)	(.3 ,.5 ,.5)

Table 4 :Tabular representation: U_2- neutrosophic soft multiset part of $(H, D)_1$.

Table 5 :Comparison table: U_2- neutrosophic soft multiset part of $(H, D)_1$

U_2	c_1	c_2	c_3	c_4
c_1	4	2	2	2
c_2	4	4	3	3
c_3	3	3	4	4
c_4	2	2	3	4

Table 6 :Score table: U_2- neutrosophic soft multiset part of $(H, D)_1$

	Row sum	Column sum	Score
c_1	10	13	-3
c_2	14	11	**3**
c_3	14	12	2
c_4	11	13	-2

Now we apply MA to the second neutrosophic soft multiset part in $(H, D)_1$ to take the decision from the availability set U_2. The tabular representation of the first resultant neutrosophic soft multiset part will be as in Table 4.

The comparison table for the first resultant neutrosophic soft multiset part will be as in

Table 5.

Next we compute the row-sum, column-sum, and the score for each u_i as shown in Table 3.

From Table 6, it is clear that the maximum score is 3, scored by c_2.

Now we redefine the neutrosophic soft multiset $(H, D)_2$by keeping all values in each row where c_2 is maximum and replacing the values in the other rows by zero (1, 0 , 0):

$$(H, D)_2=\{(x_1, (\left\{\frac{u_1}{(.5,.3,.4)}, \frac{u_2}{(.4,.3,.4)}, \frac{u_3}{(.6,.3,.4)}, \frac{u_4}{(.7,.7,.4)}\right\},$$
$$\left\{\frac{c_1}{(.7,.1,.5)}, \frac{c_2}{(.5,.5,.7)}, \frac{c_3}{(.7,.3,.0)}, \frac{c_4}{(1,.0,.0)}\right\},$$
$$\left\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.1,.0)}, \frac{h_3}{(.9,.2,.5)}\right\})),$$
$$(x_2, (\left\{\frac{u_1}{(.7,.3,.5)}, \frac{u_2}{(.6,.7,.8)}, \frac{u_3}{(.6,.0,.6)}, \frac{u_4}{(.6,.7,.3)}\right\},$$
$$\left\{\frac{c_1}{(1,.0,.0)}, \frac{c_2}{(1,.0,.0)}, \frac{c_3}{(1,.0,.0)}, \frac{c_4}{(1,.0,.0)}\right\},$$
$$\left\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(1,.0,.0)}\right\}))$$
$$(x_3, (\left\{\frac{u_1}{(.6,.3,.6)}, \frac{u_2}{(.3,.2,.6)}, \frac{u_3}{(.6,.7,.5)}, \frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c_1}{(.7,.5,.3)}, \frac{c_2}{(.6,.7,.2)}, \frac{c_3}{(.5,.4,.5)}, \frac{c_4}{(.3,.6,.5)}\right\},$$
$$\left\{\frac{h_1}{(.3,.5,.6)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.3,.2,.7)}\right\})),$$
$$(x_4, (\left\{\frac{u_1}{(.2,.5,.6)}, \frac{u_2}{(.6,.2,.3)}, \frac{u_3}{(.8,.7,.6)}, \frac{u_4}{(.3,.7,.6)}\right\},$$
$$\left\{\frac{c_1}{(.3,.5,.7)}, \frac{c_2}{(.3,.6,.2)}, \frac{c_3}{(.8,.5,.3)}, \frac{c_4}{(.3,.5,.5)}\right\},$$
$$\left\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(1,.0,.0)}\right\}))\}$$

U_3	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
h_1	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(.3 ,.5 ,.6)	(1 ,.0 ,.0)
h_2	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(1 ,.0 ,.0)
h_3	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(.3 ,.2 ,.7)	(1 ,.0 ,.0)

Table 7: Tabular representation: U_3- neutrosophic soft multiset part of $(H, D)_2$.

Table 8 :Comparison table: U_3- neutrosophic soft multiset part of $(H, D)_2$

U_3	h_1	h_2	h_3
h_1	3	3	4
h_2	4	3	4
h_3	3	3	3

Table 9 :Score table: U_3- neutrosophic soft multiset part of $(H, D)_2$

	Row sum	Column sum	Score
h_1	10	10	0
h_2	11	9	**2**
h_3	9	11	-2

Now we apply MA to the third neutrosophic soft multiset part in $(H, D)_2$ to take the decision from the availability set U_3. The tabular representation of the first resultant neutrosophic soft multiset part will be as in Table 7. The comparison table for the first resultant neutrosophic soft multiset part will be as in Table 8. Next we compute the row-sum, column-sum, and the score for each u_i as shown in Table 3. From Table 9, it is clear that the maximum score is 2, scored by h_2. Then from the above results the decision for Mr.X is **(u_3 , c_2, h_2).**

6. Conclusion

In this work, we present neutrosophic soft multi-set theory and study their properties and operations. Then, we give a decision making methods. An application of this method in decsion making problem is shown.

References

[1] K. Alhazaymeh and N. Hassan, Vague soft multisets, International Journal of Pure and Applied Mathematics, 93(4) (2014) 511-523.

[2] S. Alkhazaleh and A. R. Salleh, Fuzzy soft Multiset theory, Abstarct and applied Analysis (2012) doi:10.1155/2012/350603.

[3] S. Alkhazaleh, A. R. Salleh, N. Hassan, Soft Multisets Theory, Applied Mathematical Sciences, 5(72) (2011) 3561 – 3573.

[4] K. Atanassov, Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1986) 87–96.

[5] K. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31 (1989) 343-349.

[6] K. V. Babitha, S. J. John, On soft multi sets, Annals of Fuzzy Mathematics and Informatics 5(1) (2013) 35-44.

[7] H. M. Balami and A. M. Ibrahim, Soft Multiset and its Application in Information System, International Journal of scientific research and management 1(9) (2013) 471-482.

[8] S. Broumi and F. Smarandache, Intuitionistic Neutrosophic Soft Set, Journal of Information and Computing Science, 8(2) (2013) 130-140.

[9] S. Broumi, Generalized Neutrosophic Soft Set, International Journal of Computer Science, Engineering and Information Technology, 3(2) (2013) 17-30.

[10] S. Broumi, I. Deli, and F. Smarandache, Relations on Interval Valued Neutrosophic Soft Sets, Journal of New Results in Science, 5 (2014) 1-20

[11] S. Broumi, F. Smarandache, More on Intuitionistic Neutrosophic Soft Sets, Computer Science and Information Technology 1(4) (2013) 257-268.

[12] S. Broumi, I. Deli, F. Smarandache, Neutrosophic Parametrized Soft Set theory and its decision making problem, International Frontier Science Letters, 1 (1) (2014) 01-11.

[13] N. Çağman, I. Deli, Product of FP-Soft Sets and its Applications, Hacettepe Journal of Mathematics and Statistics, 41/3 (2012) 365 - 374.

[14] N. Çağman, I. Deli, Means of FP-Soft Sets and its Applications, Hacettepe Journal of Mathematics and Statistics, 41/5 (2012) 615–625.

[15] I. Deli and S. Broumi,Neutrosophic refined sets and its application in medical diagnosis (2014) (submmited).

[16] I. Deli, .Interval-valued neutrosophic soft sets and its decision making http://arxiv.org/abs/1402.3130

[17] I. Deli, Y. Toktas and S. Broumi, Neutrosophic Parameterized Soft Relations and Their Applications, Neutrosophic Sets and Systems, 4 (2014) 25-34.

[18] İ. Deli and S. Broumi, Neutrosophic soft relations and some properties, Annals of Fuzzy Mathematics and Informatics x(x) (201x) xx-xx.

[19] F. Karaaslan, Neutrosophic soft set with applications in decision making. http://arxiv.org/abs/1405.7964V2.

[20] P. K. Maji, Neutrosophic Soft Set, **A**nnals of **F**uzzy **M**athematics and **I**nformatics, 5(1) (2013) 157-168.

[21] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9(3) (2001) 589-602.

[22] P. Majumdar, Soft multisets, J. Math. Comput. Sci. 2(6) (2012) 1700-1711.

[23] D. A. Molodtsov, Soft Set Theory First Result, Computers and Mathematics with Applications, 37 (1999) 19-31.

[24] T. J. Neog, D. K. Sut, On Soft Multisets Theory, International Journal of Advanced Computer and Mathematical Sciences 3(3) (2012) 295-304.

[25] Z. Pawlak, Rough sets, Int. J. Comput. Inform. Sci., 11 (1982) 341-356.

[26] F. Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, (1998).

[27] R. Şahin and A. Kücük, Generalised Neutrosophic Soft Set and its Integration to Decision Making Problem, Appl. Math. Inf. Sci. 8(6) (2014) 2751-2759

[28] R. Şahin and A. Küçük, On Similarity and Entropy of Neutrosophic Soft Sets, Journal of Intelligent and Fuzzy Systems, DOI: 10.3233/IFS-141211.

[29] I. B. Turksen, Interval valued fuzzy sets based on normal forms, Fuzzy Sets and Systems, 20 (1986) 191–210.

[30] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, Interval Neutrosophic Sets and Logic: Theory and Applications in Computing, Hexis, Phoenix, AZ, (2005).

[31] L. A. Zadeh, Fuzzy sets. Information and Control, 8 (1965) 338-353. P.K. Maji, R. Biswas, A.R. Roy, Intuitionistic fuzzy soft sets, J. Fuzzy Math. 9, 677–692 (2001).

Received: June 26, 2014. Accepted: August 15, 2014.