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Characterization of γ-Single Valued Neutrosophic Rings and Ideals

Muhammad Shazib Hameed[1,*], Zaheer Ahmad[2] and Shahbaz Ali[3]

[1]Department of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan 64200, Punjab, Pakistan. Email: shazib.hameed@kfueit.edu.pk

[2]Department of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan 64200, Punjab, Pakistan. Email: zaheeir@gmail.com

[3]Department of Mathematics, The Islamia University of Bahawalpur, Rahim Yar Khan Campus 64200, Punjab, Pakistan. Email: shahbaz.math@gmail.com

[*]Correspondence: Muhammad Shazib Hameed, shazib.hameed@kfueit.edu.pk

Abstract. In this paper, we investigate the notion of γ-single valued neutrosophic subrings and ideals. Also, several properties related to the algebraic structure of rings and ideals are discussed. Moreover, many characterizations are proposed on γ-single valued neutrosophic subrings and ideals.

Keywords: γ-single valued neutrosophic subrings; γ-single valued neutrosophic normal subrings; γ-single valued neutrosophic ideals.

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1. Introduction

Generally, the inconvenience of previously established strategies and designs is overcome by recently established fuzzy algebraic structures. Routine mathematics cannot always be used because of unclear and missing knowledge in certain regular structures. Various methodologies were seen as alternative groups to deal with these issues and avoid vulnerabilities, like probability, rough set, anda fuzzy set hypothesis. Unfortunately, each of these alternate mathematics has a side and inconveniences such as the majority of words like real, beautiful, famous that are not clearly observed or indeed vague. Henceforth, the rules for such terms vary from person to person.

Zadeh [1], proposed the idea of the fuzzy set which is focussed on the possibility of the support highlight doling out an enrollment grade in $[0, 1]$ to deal with such sort of vague and questionable data. Taking into account the possibility of enrolment and non-investment, Atanassov [2, 3] proposed an intuitionistic fuzzy set which is an augmentation of a fuzzy set. As an extension of intuitionistic fuzzy set, Smarandache's [4, 5] introduced neutrosophic logic and sets. A neutrosophic set is based on three degrees: the level of participation, indeterminacy, and non-enrollment degree. The notion of a soft set is introduced in [6] by Molodtsov. Several operations were added by Ali et al. in soft set in [7]. In [8]- [10], Yager has executed the idea of the Pythagorean fuzzy set. Peng et al. presented several findings in [11, 12] on the measurements of the Pythagorean fuzzy and soft sets. Moreover, several new models have been investigated in [13]- [16].

In 1971, the concept of a fuzzy subgroup was proposed by Rosenfeld [17] and the investigation of fuzzy subgroups began. Later on, many algebraic structures; like groups, rings, fields, graphs, and modules, etc. have been developed in [18]- [38]. In this piece of work, we investigate the notion of γ-single valued neutrosophic rings, ideals, and sum and product of γ-single valued neutrosophic ideals. The proposed work is the generalization of many existing algebraic structures on fuzzy set, intuitionistic fuzzy set, (α, β)-intuitionistic fuzzy set etc.

The paper is structured as follows: we provide some basic concepts relating to γ-single valued neutrosophic rings and ideals in Section 3. We give an overview of the sum and product of γ-single valued neutrosophic ideals, also suggested several suggested several characterizations in Section 4.

2. Preliminaries

In this section neutrosophic subrings, neutrosophic normal subrings, and neutrosophic ideals are defined.

Definition 2.1. [18] A single valued neutrosophic set U on the universe of discourse R is defined as:

$$U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\},$$

where $i, t, f : R \to [0, 1]$ and $0 \leq i_U(u) + t_U(u) + f_U(u) \leq 3$. Here, $i_U(u), t_U(u)$ and $f_U(u)$ are called membership function, hesitancy function and non-membership function respectively.

Definition 2.2. [18] Let U & V be two SVNS on R. Then

(1) $U \subseteq V, \Leftrightarrow U(u) \leq V(u)$. i.e. $i_U(u) \leq i_V(u),\ t_U(u) \leq t_V(u)$ and $f_U(u) \geq f_V(u)$. Also $U = V \Leftrightarrow U \subseteq V$ and $V \subseteq U$.

(2) $W = U \cup V$ such that $W(u) = U(u) \vee V(u)$ where $U(u) \vee V(u) = (i_U(u) \vee i_V(u), t_U(u) \vee t_V(u), f_U(u) \wedge f_V(u))$, for each $u \in R$. i.e. $i_W(u) = \max\{i_U(u), i_V(u)\}, t_W(u) = \max\{t_U(u), t_V(u)\}$ and $f_W(u) = \min\{f_U(u), f_V(u)\}$.

(3) $W = U \cap V$ such that $W(u) = U(u) \wedge V(u)$ where
$U(u) \wedge V(u) = (i_U(u) \wedge i_V(u),\ t_U(u) \wedge t_V(u),\ f_U(u) \vee f_V(u))$, for each $u \in R$.
i.e. $i_W(u) = \min\{i_U(u), i_V(u)\},\ t_W(u) = \min\{t_U(u), t_V(u)\}$ and
$f_W(u) = \max\{f_U(u), f_V(u)\}$.

(4) $U^c(u) = (f_U(u), 1 - t_U(u), i_U(u))$, for each $u \in R$. Here $(U^c)^c = U$.

Definition 2.3. [39] A single valued neutrosophic set $(SVNS)$ $U = (i_U, t_U, f_U)$ of a ring R is said to be an single valued neutrosophic subring $(SVNSR)$ if

(1) $i_U(u - v) \geq \wedge\{i_U(u),\ i_U(v)\}$.
(2) $t_U(u - v) \geq \wedge\{t_U(u),\ t_U\}$.
(3) $f_U(u - v) \leq \vee\{f_U(u),\ f_U(v)\}$.
(4) $i_U(uv) \geq \wedge\{i_U(u),\ i_U(v)\}$.
(5) $t_U(uv) \geq \wedge\{t_U(u),\ t_U(v)\}$.
(6) $f_U(uv) \leq \vee\{f_U(u),\ f_U(v)\},\ \forall\ u, v \in R$.

Definition 2.4. [39] A subset $U = (i_U, t_U, f_U)$ of a ring R is said to be an single valued neutrosophic normal subring $(SVNNSR)$ of R if

(1) $i_U(uv) = i_U(vu)$.
(2) $t_U(uv) = t_U(vu)$.
(3) $f_U(uv) = f_U(vu),\ \forall\ u, v \in R$.

Definition 2.5. [39] A single valued neutrosophic set $U = (i_U, t_U, f_U)$ a ring R is said to be an single valued neutrosophic left ideal $(SVNLI)$ if

(1) $i_U(u - v) \geq \wedge\{i_U(u),\ i_U(v)\}$.
(2) $i_U(uv) \geq i_U(v)$.
(3) $t_U(u - v) \geq \wedge\{t_U(u),\ t_U(v)\}$.
(4) $t_U(uv) \geq t_U(v)$.
(5) $f_U(u - v) \leq \vee\{f_U(u),\ f_U(v)\}$.
(6) $f_U(uv) \leq f_U(v),\ \forall\ u, v \in R$.

Definition 2.6. [39] A single valued neutrosophic set $U = (i_U, t_U, f_U)$ a ring R is said to be an single valued neutrosophic right ideal $(SVNRI)$ if

(1) $i_U(u - v) \geq \wedge\{i_U(u),\ i_U(v)\}$.
(2) $i_U(uv) \geq i_U(u)$.
(3) $t_U(u - v) \geq \wedge\{t_U(u),\ t_U(v)\}$.
(4) $t_U(uv) \geq t_U(u)$.
(5) $f_U(u - v) \leq \vee\{f_U(u),\ f_U(v)\}$.

(6) $f_U(uv) \leq f_U(u), \ \forall \ u, v \in R$.

Definition 2.7. [39] A single valued neutrosophic set $U = (i_U, t_U, f_U)$ a ring R is said to be an single valued neutrosophic ideal ($SVNI$) if

(1) $i_U(u - v) \geq \wedge\{i_U(u), \ i_U(v)\}$.
(2) $t_U(u - v) \geq \wedge\{t_U(u), \ t_U(v)\}$.
(3) $f_U(u - v) \leq \vee\{f_U(u), \ f_U(v)\}$.
(4) $i_U(uv) \geq \vee\{i_U(u), \ i_U(v)\}$.
(5) $t_U(uv) \geq \vee\{t_U(u), \ t_U(v)\}$.
(6) $f_U(uv) \leq \wedge\{f_U(u), \ f_U(v)\}, \ \forall \ u, v \in R$.

3. γ-Single Valued Neutrosophic Subrings and Ideals

This section discusses some basic concepts and results related to γ-single valued neutrosophic subrings and ideals.

Definition 3.1. If U be a single valued neutrosophic subset of ring R then γ-single valued neutrosophic subset U is described as,

$$U^\gamma = \Big\{\langle u, i^\gamma(u), t^\gamma(u), f^\gamma(u)\rangle \mid i^\gamma(u) = \wedge\{i_U(u), \ \gamma\}, \ t^\gamma(u) = \wedge\{t_U(u), \gamma\}, \ f^\gamma(u) = \vee\{f_U(u), \gamma\}, \ u \in R\Big\},$$

where $\gamma \in [0, 1]$.

Definition 3.2. Let U & V be two γ-SVNS on R. Then

(1) $U^\gamma \subseteq V^\gamma, \Leftrightarrow U^\gamma(u) \leq V^\gamma(u)$. i.e. $i_{U^\gamma}(u) \leq i_{V^\gamma}(u), \ t_{U^\gamma}(u) \leq t_{V^\gamma}(u)$ and $f_{U^\gamma}(u) \geq f_{V^\gamma}(u)$. Also $U^\gamma = V^\gamma \Leftrightarrow U^\gamma \subseteq V^\gamma$ and $V^\gamma \subseteq U^\gamma$.
(2) $W^\gamma = U^\gamma \cup V^\gamma$ such that $W^\gamma(u) = U^\gamma(u) \vee V^\gamma(u)$ where $U^\gamma(u) \vee V^\gamma(u) = (i_{U^\gamma}(u) \vee i_{V^\gamma}(u), t_{U^\gamma}(u) \vee t_{V^\gamma}(u), f_{U^\gamma}(u) \wedge f_{V^\gamma}(u))$, for each $u \in R$. i.e. $i_{W^\gamma}(u) = \max\{i_{U^\gamma}(u), i_{V^\gamma}(u)\}, \ t_{W^\gamma}(u) = \max\{t_{U^\gamma}(u), t_{V^\gamma}(u)\}$ and $f_{W^\gamma}(u) = \min\{f_{U^\gamma}(u), f_{V^\gamma}(u)\}$.
(3) $W^\gamma = U^\gamma \cap V^\gamma$ such that $W^\gamma(u) = U^\gamma(u) \wedge V^\gamma(u)$ where $U^\gamma(u) \wedge V^\gamma(u) = (i_{U^\gamma}(u) \wedge i_{V^\gamma}(u), t_{U^\gamma}(u) \wedge t_{V^\gamma}(u), f_{U^\gamma}(u) \vee f_{V^\gamma}(u))$, for each $u \in R$. i.e. $i_{W^\gamma}(u) = \min\{i_{U^\gamma}(u), i_{V^\gamma}(u)\}, \ t_{W^\gamma}(u) = \min\{t_{U^\gamma}(u), t_{V^\gamma}(u)\}$ and $f_{W^\gamma}(u) = \max\{f_{U^\gamma}(u), f_{V^\gamma}(u)\}$.
(4) $U^{\gamma c}(u) = (f_{U^\gamma}(u), 1 - t_{U^\gamma}(u), i_{U^\gamma}(u))$, for each $u \in R$. Here $(U^{\gamma c})^c = U^\gamma$.

Definition 3.3. A γ-single valued neutrosophic set (γ-$SVNS$) $U^\gamma = (i_U^\gamma, t_U^\gamma, f_U^\gamma)$ of a ring R is said to be an γ-single valued neutrosophic subring (γ-$SVNSR$) if

(1) $i_U^\gamma(u - v) \geq \wedge\{i_U^\gamma(u), \ i_U^\gamma(v)\}$.
(2) $t_U^\gamma(u - v) \geq \wedge\{t_U^\gamma(u), \ t_U^\gamma(v)\}$.

(3) $f_U^{\gamma}(u-v) \leq \vee\{f_U^{\gamma}(u),\ f_U^{\gamma}(v)\}$.
(4) $i_U^{\gamma}(uv) \geq \wedge\{i_U^{\gamma}(u),\ i_U^{\gamma}(v)\}$.
(5) $t_U^{\gamma}(uv) \geq \wedge\{t_U^{\gamma}(u),\ t_U^{\gamma}(v)\}$.
(6) $f_U^{\gamma}(uv) \leq \vee\{f_U^{\gamma}(u),\ f_U^{\gamma}(v)\},\ \forall\ u,v \in R$.

Example 3.4. Let us consider the ring $(Z_2, +_2, *_2)$ where $Z_2 = \{0,1\}$.
Let we define $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in Z_2\}$ such that
$i_U(0) = 0.8,\ i_U(1) = 0.4,\ t_U(0) = 0.4,\ t_U(1) = 0.3$ and $f_U(0) = 0.3,\ f_U(1) = 0.6$.
Consider $\gamma = 0.5$, then $U^{\gamma} = \{\langle u, i_U^{\gamma}(u), t_U^{\gamma}(u), f_U^{\gamma}(u)\rangle \mid u \in Z_2\}$ where
$i_U^{\gamma}(0) = 0.5,\ i_U^{\gamma}(1) = 0.4,\ t_U^{\gamma}(0) = 0.4,\ t_U^{\gamma}(1) = 0.3$ and $f_U^{\gamma}(0) = 0.5,\ f_U^{\gamma}(1) = 0.6$,
$\Rightarrow SVNS\ U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in Z_2\}$ is an 0.5-$SVNSR$ of Z_2.

Proposition 3.5. *If U and V be two γ-single-valued neutrosophic subset of ring R then $(U \cap V)^{\gamma}$=$U^{\gamma} \cap V^{\gamma}$.*

Proof. Assume that U and V are two γ-single-valued neutrosophic subset of ring R.

$$(U\cap V)^{\gamma}(u) = \Big\{ \min\{\min\{i_U(u), i_V(u)\}, \gamma\}, \min\{\min\{t_U(u),\ t_V(u)\}, \gamma\}, \max\{\max\{f_U(u), f_V(u)\}, \gamma\} \Big\}$$
$$= \Big\{ \min\{\min\{i_U(u), \gamma\}, \min\{i_V(u), \gamma\}\}, \min\{\min\{t_U(u), \gamma\}, \min\{t_V(u), \gamma\}\}, \max\{\max\{f_U(u), \gamma\}, \max\{f_V(u), \gamma\}\} \Big\}$$
$$= \Big\{ \min(\{i_U^{\gamma}(u)\}, \{i_V^{\gamma}(u)\}), \min(\{t_U^{\gamma}(u)\}, \{t_V^{\gamma}(u)\}), \max(\{f_U^{\gamma}(u)\}, \{f_V^{\gamma}(u)\}) \Big\} = U^{\gamma}(u) \cap V^{\gamma}(u),\ \forall\, u \in R.$$

□

Theorem 3.6. *Let U and V be two γ-$SVNSRs$ of a ring R. Then $U \cap V$ is also an γ-$SVNSR$ of R.*

Proof. Let $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ and $V = \{\langle u, i_V(u), t_V(u), f_V(u)\rangle \mid u \in R\}$ be any two γ-$SVNRs$ of a ring R.

$$\Rightarrow\ U^{\gamma} = \{\langle u, i_U^{\gamma}(u), t_U^{\gamma}(u), f_U^{\gamma}(u)\rangle \mid u \in R\}\ and\ V^{\gamma} = \{\langle u, i_V^{\gamma}(u), t_V^{\gamma}(u), f_V^{\gamma}(u)\rangle \mid u \in R\}.$$

Then by using Proposition 3.5

$$(U \cap V)^{\gamma} = U^{\gamma} \cap V^{\gamma} = \{\langle u, (i_U^{\gamma} \wedge i_V^{\gamma})(u), (t_U^{\gamma} \wedge t_V^{\gamma})(u), (f_U^{\gamma} \vee f_V^{\gamma})(u)\rangle \mid u \in R\}.$$

Now for any $u, v \in R$, we have
(i) $(i_U^{\gamma} \wedge i_V^{\gamma})(u-v) = \wedge\{i_U^{\gamma}(u-v), i_V^{\gamma}(u-v)\}$
$\geq \wedge\{\wedge\{i_U^{\gamma}(u), i_U^{\gamma}(v)\}, \wedge\{i_V^{\gamma}(u), i_V^{\gamma}(v)\}\}$
$= \wedge\{\wedge\{i_U^{\gamma}(u), i_V^{\gamma}(u)\}, \wedge\{i_U^{\gamma}(v), i_U^{\gamma}(v)\}\}$
$= \wedge\{(i_U^{\gamma} \wedge i_V^{\gamma})(u), (i_U^{\gamma} \wedge i_V^{\gamma})(v)\}$.
(ii) $(i_U^{\gamma} \wedge i_V^{\gamma})(uv) = \wedge\{i_U^{\gamma}(uv), i_V^{\gamma}(uv)\}$
$\geq \wedge\{\wedge\{i_U^{\gamma}(u), i_U^{\gamma}(v)\}, \wedge\{i_V^{\gamma}(u), i_V^{\gamma}(v)\}\}$

$= \wedge\{\wedge\{i_U^\gamma(u), i_V^\gamma(u)\}, \wedge\{i_U^\gamma(v), i_U^\gamma(v)\}\}$
$= \wedge\{(i_U^\gamma \wedge i_V^\gamma)(u), (i_U^\gamma \wedge i_V^\gamma)(v)\}.$
(iii) $(t_U^\gamma \wedge t_V^\gamma)(u-v) = \wedge\{t_U^\gamma(u-v), t_V^\gamma(u-v)\}$
$\geq \wedge\{\wedge\{t_U^\gamma(u), t_U^\gamma(v)\}, \wedge\{t_V^\gamma(u), t_V^\gamma(v)\}\}$
$= \wedge\{\wedge\{t_U^\gamma(u), t_V^\gamma(u)\}, \wedge\{t_U^\gamma(v), t_U^\gamma(v)\}\}$
$= \wedge\{(t_U^\gamma \wedge t_V^\gamma)(u), (t_U^\gamma \wedge t_V^\gamma)(v)\}.$
(iv) $(t_U^\gamma \wedge t_V^\gamma)(uv) = \wedge\{t_U^\gamma(uv), t_V^\gamma(uv)\}$
$\geq \wedge\{\wedge\{t_U^\gamma(u), t_U^\gamma(v)\}, \wedge\{T_B^\gamma(u), t_V^\gamma(v)\}\}$
$= \wedge\{\wedge\{t_U^\gamma(u), T_B^\gamma(u)\}, \wedge\{t_U^\gamma(v), t_U^\gamma(v)\}\}$
$= \wedge\{(t_U^\gamma \wedge t_V^\gamma)(u), (t_U^\gamma \wedge t_V^\gamma)(v)\}.$
(v) $(f_U^\gamma \vee f_V^\gamma)(u-v) = \vee\{f_U^\gamma(u-v), f_V^\gamma(u-v)\}$
$\leq \vee\{\vee\{f_U^\gamma(u), f_U^\gamma(v)\}, \vee\{f_V^\gamma(u), f_V^\gamma(v)\}\}$
$= \vee\{\vee\{f_U^\gamma(u), f_V^\gamma(u)\}, \vee\{f_U^\gamma(v), f_U^\gamma(v)\}\}$
$= \vee\{(f_U^\gamma \vee f_V^\gamma)(u), (f_U^\gamma \vee f_V^\gamma)(v)\}.$
(vi) $(f_U^\gamma \vee f_V^\gamma)(uv) = \vee\{f_U^\gamma(uv), f_V^\gamma(uv)\}$
$\leq \vee\{\vee\{f_U^\gamma(u), f_U^\gamma(v)\}, \vee\{f_V^\gamma(u), f_V^\gamma(v)\}\}$
$= \vee\{\vee\{f_U^\gamma(u), f_V^\gamma(u)\}, \vee\{f_U^\gamma(v), f_U^\gamma(v)\}\}$
$= \vee\{(f_U^\gamma \vee f_V^\gamma)(u), (f_U^\gamma \vee f_V^\gamma)(v)\}.$
Therefore $(U \cap V)$ is an γ-$SVNSR$ of R. □

Remark 3.7. However, the union of two γ-$SVNSRs$ is not an γ-$SVNSR$. For example, consider the set $R = \{0, a, b, a+b\}$, where $a+a = 0 = b+b$ and $a+b = b+a$ and $u.v = 0$ for every $u, v \in R$. Then $(R, +, .)$ is a ring.
Let $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ and $V = \{\langle u, i_V(u), t_V(u), f_V(u)\rangle \mid u \in R\}$, where
$i_U(0) = 0.8,\ i_U(a) = 0.5,\ i_U(b) = 0.4 = i_U(a+b).$
$t_U(0) = 0.7,\ t_U(a) = 0.3,\ t_U(b) = 0.2 = t_U(a+b).$
$f_U(0) = 0.4,\ f_U(a) = 0.7,\ f_U(b) = 0.8 = f_U(a+b).$
$i_V(0) = 0.6,\ i_V(a) = 0.1,\ i_V(b) = 0.5, i_V(a+b) = 0.1.$
$t_V(0) = 0.7,\ t_V(a) = 0.1,\ t_V(b) = 0.3, t_V(a+b) = 0.1.$
$f_V(0) = 0.1,\ f_V(a) = 0.2,\ f_V(b) = 0.2, f_V(a+b) = 0.2.$
Consider $\gamma = 0.6$ then $U^\gamma = \{\langle u, i_U^\gamma(u), t_U^\gamma(u), f_U^\gamma(u)\rangle \mid u \in R\}$ and
$V^\gamma = \{\langle u, i_V^\gamma(u), t_V^\gamma(u), f_V^\gamma(u)\rangle \mid u \in R\}$, where
$i_U^\gamma(0) = 0.6,\ i_U^\gamma(a) = 0.5,\ i_U^\gamma(b) = 0.4 = i_U^\gamma(a+b).$
$t_U^\gamma(0) = 0.6,\ t_U^\gamma(a) = 0.3,\ t_U^\gamma(b) = 0.2 = t_U^\gamma(a+b).$
$f_U^\gamma(0) = 0.6,\ f_U^\gamma(a) = 0.7,\ f_U^\gamma(b) = 0.8 = f_U^\gamma(a+b).$
$i_V^\gamma(0) = 0.6,\ i_V^\gamma(a) = 0.1,\ i_V^\gamma(b) = 0.5, i_V^\gamma(a+b) = 0.1.$

$t_V^\gamma(0) = 0.6,\ t_V^\gamma(a) = 0.1,\ t_V^\gamma(b) = 0.3, t_V^\gamma(a+b) = 0.1.$
$f_V^\gamma(0) = 0.6,\ f_V^\gamma(a) = 0.6,\ f_V^\gamma(b) = 0.6, f_V^\gamma(a+b) = 0.6.$
Then U and V are γ-$SVNSRs$ of R. Now
$(U \cup V)^\gamma == \{\langle u, (i_U \vee i_V)^\gamma)(u), (t_U \vee t_V)^\gamma(u), (f_U^\gamma \wedge f_V)^\gamma(u), (u)\rangle \mid u \in R\},$
Here $(i_U \vee i_V)^\gamma(0) = 0.8,\ (i_U \vee i_V)^\gamma(a) = 0.5,\ (i_U \vee i_V)^\gamma(b) = 0.5,\ (i_U \vee i_V)^\gamma(a+b) = 0.4;$
$(t_U \vee t_V)^\gamma(0) = 0.7,\ (t_U \vee t_V)^\gamma(a) = 0.3,\ (t_U \vee t_V)^\gamma(b) = 0.3,\ (t_U \vee t_V)^\gamma(a+b) = 0.2;$
$(f_U^\gamma \wedge f_V)^\gamma(0) = 0.1,\ (f_U^\gamma \wedge f_V)^\gamma(a) = 0.2,\ (f_U^\gamma \wedge f_V)^\gamma(b) = 0.2,\ (f_U^\gamma \wedge f_V)^\gamma(a+b) = 0.2.$
Now
$(i_U \vee i_V)^\gamma(a+b) = 0.4 < \wedge\{((i_U^\gamma \vee i_V)^\gamma(a), (i_U \vee i_V)^\gamma)(b))\} = 0.5$
Therefore $(U \cup V)^\gamma$ is not an γ-$SVNSR$ of R.

Definition 3.8. A $U^\gamma = (i_U^\gamma, t_U^\gamma, f_U^\gamma)$ of a ring R is said to be an γ-single valued neutrosophic normal subring γ-$SVNNSR$ of R if

(1) $i_U^\gamma(uv) = i_U^\gamma(vu)$.
(2) $t_U^\gamma(uv) = t_U^\gamma(vu)$.
(3) $f_U^\gamma(uv) = f_U^\gamma(vu),\ \forall\ u, v \in R$.

Definition 3.9. A γ-single valued neutrosophic set $U^\gamma = (i_U^\gamma, t_U^\gamma, f_U^\gamma)$ a ring R is said to be an γ-single valued neutrosophic left ideal (γ-$SVNLI$) if

(1) $i_U^\gamma(u - v) \geq \wedge\{i_U^\gamma(u),\ i_U^\gamma(v)\}$.
(2) $i_U^\gamma(uv) \geq i_U^\gamma(v)$.
(3) $t_U^\gamma(u - v) \geq \wedge\{t_U^\gamma(u),\ t_U^\gamma(v)\}$.
(4) $t_U^\gamma(uv) \geq t_U^\gamma(v)$.
(5) $f_U^\gamma(u - v) \leq \vee\{f_U^\gamma(u),\ f_U^\gamma(v)\}$.
(6) $f_U^\gamma(uv) \leq f_U^\gamma(v),\ \forall\ u, v \in R$.

Definition 3.10. A γ-single valued neutrosophic set $U^\gamma = (i_U^\gamma, t_U^\gamma, f_U^\gamma)$ a ring R is said to be an γ-single valued neutrosophic right ideal (γ-$SVNRI$) if

(1) $i_U^\gamma(u - v) \geq \wedge\{i_U^\gamma(u),\ i_U^\gamma(v)\}$.
(2) $i_U^\gamma(uv) \geq i_U^\gamma(u)$.
(3) $t_U^\gamma(u - v) \geq \wedge\{t_U^\gamma(u),\ t_U^\gamma(v)\}$.
(4) $t_U^\gamma(uv) \geq t_U^\gamma(u)$.
(5) $f_U^\gamma(u - v) \leq \vee\{f_U^\gamma(u),\ f_U^\gamma(v)\}$.
(6) $f_U^\gamma(uv) \leq f_U^\gamma(u),\ \forall\ u, v \in R$.

Definition 3.11. A γ-single valued neutrosophic set $U^\gamma = (i_U^\gamma, t_U^\gamma, f_U^\gamma)$ a ring R is said to be an γ-single valued neutrosophic ideal (γ-SVNI) if

(1) $i_U^\gamma(u - v) \geq \wedge\{i_U^\gamma(u),\ i_U^\gamma(v)\}$.

(2) $t_U^\gamma(u-v) \geq \wedge\{t_U^\gamma(u),\ t_U^\gamma(v)\}$.
(3) $f_U^\gamma(u-v) \leq \vee\{f_U^\gamma(u),\ f_U^\gamma(v)\}$.
(4) $i_U^\gamma(uv) \geq \vee\{i_U^\gamma(u),\ i_U^\gamma(v)\}$.
(5) $t_U^\gamma(xv) \geq \vee\{t_U^\gamma(u),\ t_U^\gamma(v)\}$.
(6) $f_U^\gamma(uv) \leq \wedge\{f_U^\gamma(u),\ f_U^\gamma(v)\},\ \forall\ u, v \in R$.

Example 3.12. Let us consider a ring $(Z_4, +_4, \times_4)$ where $Z_4 = \{0, 1, 2, 3\}$ and
Consider $U = \{\langle i_U, t_U, f_U\rangle \mid u \in Z_4\}$ be a single valued neutrosophic subset of Z_4, where
$i_U(0) = 0.4,\ i_U(1) = 0.3 = i_U(3),\ i_U(2) = 0.5.$
$t_U(0) = 0.3,\ t_U(1) = 0.2 = t_U(3),\ t_U(2) = 0.6.$ and
$f_U(0) = 0.2,\ f_U(1) = 0.7 = f_U(3),\ f_U(2) = 0.6.$
Suppose $\gamma = 0.5$ then $U^\gamma = \{\langle i_U^\gamma, t_U^\gamma, f_U^\gamma\rangle \mid u \in Z_4\}$ be an γ-single valued neutrosophic subset of Z_4, where
$i_U^\gamma(0) = 0.4,\ i_U^\gamma(1) = 0.3 = i_U^\gamma(3),\ i_U^\gamma(2) = 0.5.$
$t_U^\gamma(0) = 0.3,\ t_U^\gamma(1) = 0.2 = t_U^\gamma(3),\ t_U^\gamma(2) = 0.5.$ and
$f_U^\gamma(0) = 0.5,\ f_U^\gamma(1) = 0.7 = f_U^\gamma(3),\ f_U^\gamma(2) = 0.6.$
$\Rightarrow U$ is an γ-$SVNI$ of Z_4.

Theorem 3.13. *If $U^\gamma = \{\langle i_U^\gamma, t_U^\gamma, f_U^\gamma\rangle \mid u \in R\}$ is a γ-$SVNI$ of a ring R, then*
$i_U^\gamma(0) \geq i_U^\gamma(u),\ t_U^\gamma(0) \geq t_U^\gamma(u),\ f_U^\gamma(0) \leq f_U^\gamma(u)$
and $i_U^\gamma(-u) = i_U^\gamma(u),\ t_U^\gamma(-u) = t_U^\gamma(u),\ f_U^\gamma(-u) = F_U^\gamma(u),\ \forall\ u \in R.$

Proof. Let $i_U^\gamma(0) = i_U^\gamma(u-u) \geq \wedge\{i_U^\gamma(u), i_U^\gamma(u)\} = i_U^\gamma(u)$.
$t_U^\gamma(0) = t_U^\gamma(u-u) \geq \wedge\{t_U^\gamma(u), t_U^\gamma(u)\} = t_U^\gamma(u)$.
Similarly $f_U^\gamma(0) = f_U^\gamma(u-u) \leq \vee\{f_U^\gamma(u), f_U^\gamma(u)\} = f_U^\gamma(u)$.
Next $i_U^\gamma(-x) = i_U^\gamma(0-u) \geq \wedge\{i_U^\gamma(0), i_U^\gamma(u)\} = i_U^\gamma(u)$.
Also $i_U^\gamma(u) = i_U^\gamma\{0-(-u)\} \geq \wedge\{i_U^\gamma(0), i_U^\gamma(-u)\} = i_U^\gamma(-u)$.
Therefore $i_U^\gamma(-u) = i_U^\gamma(u)$.
So $t_U^\gamma(-u) = t_U^\gamma(0-u) \geq \wedge\{t_U^\gamma(0), t_U^\gamma(u)\} = t_U^\gamma(u)$.
Also $t_U^\gamma(u) = t_U^\gamma\{0-(-u)\} \geq \wedge\{t_U^\gamma(0), t_U^\gamma(-u)\} = t_U^\gamma(-u)$.
Therefore $t_U^\gamma(-u) = t_U^\gamma(u)$.
Finally $f_U^\gamma(-u) = f_U^\gamma(0-u) \leq \vee\{f_U^\gamma(0), f_U^\gamma(u)\} = f_U^\gamma(u)$.
Also $f_U^\gamma(u) = f_U^\gamma\{0-(-u)\} \geq \vee\{f_U^\gamma(-u), f_U^\gamma(0)\} = f_U^\gamma(-u)$.
Therefore $f_U^\gamma(-u) = f_U^\gamma(u)$. □

Remark 3.14. Every γ-$SVNI$ of a ring R is an γ-$SVNSR$ of R. However the converse is not true.

For example, let $(R, +, .)$ be the ring of real numbers.

Define, $U = \{\langle u, i_U(u), t_U(u), i_U(u)\rangle | \ u \in R\}$ such that

$i_U(u) = 0.5$ if u is rational, $t_U(u) = 0.8$ if u is rational, $f_U(u) = 0.1$ if u is rational.

$i_U(u) = 0.4$ if u is irrational, $t_U(u) = 0.3$ if u is irrational, $f_U(u) == 0.7$ if u is irrational.

Consider $\gamma = 0.6$, now define $U^\gamma = \{\langle u, i_U^\gamma(u), t_U^\gamma(u), i_U^\gamma(u)\rangle | \ u \in R\}$ then

$i_U^\gamma(u) = 0.5$ if u is rational, $t_U^\gamma(u) = 0.6$ if u is rational, $f_U^\gamma(u) = 0.6$ if u is rational.

$i_U^\gamma(u) = 0.4$ if u is irrational, $t_U^\gamma(u) = 0.3$ if u is irrational, $f_U^\gamma(u) = 0.7$ if u is irrational.

Then U is an γ-$SVNSR$ of R.

But U is not an γ-$SVNI$ of R, since $i_U^\gamma(2\sqrt{2}) = 0.4 < \vee\{i_U^\gamma(2), i_U^\gamma(\sqrt{2})\}$.

Theorem 3.15. *Let U and V be two γ-$SVNIs$ of a ring R. Then $U \cap V$ is also a γ-$SVNI$ of R.*

Proof. Let $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ and $V = \{\langle u, i_V(u), t_V(u), f_V(u)\rangle \mid u \in R\}$ be any two γ-$SVNIs$ of a ring R. Then,

$U^\gamma = \{\langle u, i_U^\gamma(u), t_U^\gamma(u), f_U^\gamma(u)\rangle \mid u \in R\}$ and $V^\gamma = \{\langle u, i_V^\gamma(u), t_V^\gamma(u), f_V^\gamma(u)\rangle \mid u \in R\}$, then by using Proposition 3.5

$(U \cap V)^\gamma = U^\gamma \cap V^\gamma = \{\langle u, (i_U^\gamma \wedge i_V^\gamma)(u), (t_U^\gamma \wedge t_V^\gamma)(u), (f_U^\gamma \vee f_V^\gamma)(u)\rangle \mid u \in R\}$.

Now for any $u, v \in R$, we have

(i) $(i_U^\gamma \wedge i_V^\gamma)(u - v) = \wedge\{i_U^\gamma(u - v), i_V^\gamma(u - v)\}$

$\geq \wedge\{\wedge\{i_U^\gamma(u), i_U^\gamma(v)\}, \wedge\{i_V^\gamma(u), i_V^\gamma(v)\}\}$

$= \wedge\{\wedge\{i_U^\gamma(u), i_V^\gamma(u)\}, \wedge\{i_U^\gamma(v), i_U^\gamma(v)\}\}$

$= \wedge\{(i_U^\gamma \wedge i_V^\gamma)(u), (i_U^\gamma \wedge i_V^\gamma)(v)\}$.

(ii) $(i_U^\gamma \wedge i_V^\gamma)(uv) = \wedge\{i_U^\gamma(uv), i_V^\gamma(xv)\}$

$\geq \wedge\{\vee\{i_U^\gamma(u), i_U^\gamma(v)\}, \vee\{i_V^\gamma(u), i_V^\gamma(v)\}\}$

$\geq \vee\{\wedge\{i_U^\gamma(u), i_V^\gamma(u)\}, \wedge\{i_U^\gamma(v), i_U^\gamma(v)\}\}$

$= \vee\{(i_U^\gamma \wedge i_V^\gamma)(x), (i_U^\gamma \wedge i_V^\gamma)(v)\}$.

(iii) $(t_U^\gamma \wedge t_V^\gamma)(u - v) = \wedge\{t_U^\gamma(u - v), t_V^\gamma(u - v)\}$

$\geq \wedge\{\wedge\{t_U^\gamma(u), t_U^\gamma(v)\}, \wedge\{t_V^\gamma(u), t_V^\gamma(v)\}\}$

$= \wedge\{\wedge\{t_U^\gamma(u), t_V^\gamma(u)\}, \wedge\{t_U^\gamma(v), t_U^\gamma(v)\}\}$

$= \wedge\{(t_U^\gamma \wedge t_V^\gamma)(u), (t_U^\gamma \wedge t_V^\gamma)(v)\}$.

(iv) $(t_U^\gamma \wedge t_V^\gamma)(uv) = \wedge\{t_U^\gamma(uv), t_V^\gamma(uv)\}$

$\geq \wedge\{\vee\{t_U^\gamma(u), t_U^\gamma(v)\}, \vee\{t_V^\gamma(u), t_V^\gamma(v)\}\}$

$\geq \vee\{\wedge\{t_U^\gamma(u), t_V^\gamma(u)\}, \wedge\{t_U^\gamma(v), t_U^\gamma(v)\}\}$

$= \vee\{(t_U^\gamma \wedge t_V^\gamma)(u), (t_U^\gamma \wedge t_V^\gamma)(v)\}$.

(v) $(f_U^\gamma \vee f_V^\gamma)(u - v) = \vee\{f_U^\gamma(u - v), f_V^\gamma(u - v)\}$

$\leq \vee\{\vee\{f_U^\gamma(u), f_U^\gamma(v)\}, \vee\{f_V^\gamma(u), f_V^\gamma(v)\}\}$

$= \vee\{\vee\{f_U^\gamma(u), f_V^\gamma(u)\}, \vee\{f_U^\gamma(v), f_U^\gamma(v)\}\}$

$= \vee\{(f_U^\gamma \vee f_V^\gamma)(u), (f_U^\gamma \vee f_V^\gamma)(v)\}$.

(vi) $(f_U^\gamma \vee f_V^\gamma)(uv) = \vee\{f_U^\gamma(uv), f_V^\gamma(uv)\}$

$\leq \vee\{\wedge\{f_U^\gamma(u), f_U^\gamma(v)\}, \wedge\{f_V^\gamma(u), f_V^\gamma(v)\}\}$

$\leq \wedge\{\vee\{f_U^\gamma(u), f_V^\gamma(u)\}, \vee\{f_U^\gamma(v), f_U^\gamma(v)\}\}$

$= \wedge\{(f_U^\gamma \vee f_V^\gamma)(u), (f_U^\gamma \vee f_V^\gamma)(v)\}$.

Therefore $U \cap V$ is an γ-SVNI of R. □

Remark 3.16. Union of two γ-$SVNIs$ of R need not to be γ-$SVNI$ of R.

Remark 3.17. If U is an γ-$SVNSR$ and V is an γ-$SVNI$ of a ring R then $U \cap V$ is an γ-$SVNSR$ of R but not an γ-$SVNI$ of R. For example, consider the ring $(R, +, .)$ of real numbers and define,

$U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ such that

$i_U(u) = 0.7$ if u is rational, $t_U(u) = 0.6$ if u is rational, $f_U(u) = 0.1$ if u is rational.

$i_U(u) = 0.2$ if u is irrational, $i_U(u) = 0.1$ if u is irrational, $f_U(u) = 0.8$ if u is irrational.

Also define $V = \{\langle u, i_V(u), t_V(u), f_V(u)\rangle | u \in R\}$ such that

$i_V(u) = 0.5$, $t_V(u) = 0.4$ and $f_V(u) = 0.6 \;\forall\; u \in R$. Consider $\gamma = 0.5$ then

$i_U^\gamma(u) = 0.5$ if u is rational, $t_U^\gamma(u) = 0.5$ if u is rational, $f_U^\gamma(u) = 0.5$ if u is rational.

$i_U^\gamma(u) = 0.2$ if u is irrational, $i_U^\gamma(u) = 0.1$ if u is irrational, $f_U^\gamma(u) = 0.8$ if u is irrational.

Then $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ is an γ-$SVNSR$ of R.

Similarly, $i_V^\gamma(u) = 0.5$, $t_V^\gamma(u) = 0.4$ and $i_V^\gamma(u) = 0.6 \;\forall\; u \in R$.

Then $V = \{\langle u, i_V(u), t_V(u), f_V(u)\rangle | u \in R\}$ is an γ-$SVNI$ of R.

Then by using Proposition 3.5

$(U \cap V)^\gamma = U^\gamma \cap V^\gamma = \{\langle u, (i_U^\gamma \wedge i_V^\gamma)(u), (t_U^\gamma \wedge t_V^\gamma)(u), (f_U^\gamma \vee f_V^\gamma)(u)\rangle \mid u \in R\}$ is not an γ-$SVNI$ of R, because $(i_U^\gamma \wedge i_V^\gamma)(2\sqrt{2}) < \vee\{(i_U^\gamma \wedge i_V^\gamma)(2), (i_U^\gamma \wedge i_V^\gamma)(\sqrt{2})\}$.

4. **Sum and Product of γ-Single Valued Neutrosophic Ideal (γ-SVNI)**

In this section, we elaborate some fundamental principles and results related to the sum and product of the γ-single valued neutrosophic ideal.

Definition 4.1. Let U and V be two γ-$SVNIs$ of a ring R then their sum $(U+V)^\gamma$ is defined as $(U+V)^\gamma = \{\langle u, (i_U^\gamma + i_V^\gamma)(u), (t_U^\gamma + t_V^\gamma)(u), (f_U^\gamma + f_V^\gamma)(u)\rangle \mid u \in R\}$, where

$(i_U^\gamma + i_V^\gamma)(u) = \sup\limits_{u=a+b}\{\wedge\ \{i_U^\gamma(a), i_U^\gamma(b)\}\}$,

$(t_U^\gamma + t_V^\gamma)(u) = \sup\limits_{u=a+b}\{\wedge\ \{t_U^\gamma(a), t_U^\gamma(b)\}\}$, and

$(f_U^\gamma + f_V^\gamma)(u) = \inf\limits_{u=a+b}\{\vee\ \{f_U^\gamma(a), f_U^\gamma(b)\}\}$.

Definition 4.2. Let U and V be two γ-$SVNIs$ of a ring R then their product $(UV)^\gamma$ is defined as $(UV)^\gamma = \{\langle u, (i_U^\gamma i_V^\gamma)(u), (t_U^\gamma t_V^\gamma)(u), (f_U^\gamma f_V^\gamma)(u)\rangle \mid u \in R\}$, where

$(i_U^\gamma i_V^\gamma)(u) = \sup\limits_{\substack{u=\sum a_i b_i \\ i<\infty}} \{\wedge \ \{\wedge\{i_U^\gamma(a_i), i_U^\gamma(b_i)\}\}\}$,
$(t_U^\gamma t_V^\gamma)(u) = \sup\limits_{\substack{u=\sum a_i b_i \\ i<\infty}} \{\wedge \ \{\wedge\{t_U^\gamma(a_i), t_U^\gamma(b_i)\}\}\}$, and
$(f_U^\gamma f_V^\gamma)(u) = \inf\limits_{\substack{u=\sum a_i b_i \\ i<\infty}} \{\vee \ \{\vee\{f_U^\gamma(a_i), f_U^\gamma(b_i)\}\}\}$.

Theorem 4.3. *If U and V are two γ-$SVNIs$ of a ring R, then $U+V$ is also an γ-$SVNI$ of R.*

Proof. Let $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ and $V = \{\langle u, i_V(u), t_V(u), f_V(u) \mid u \in R\rangle\}$ be two γ-$SVNIs$ of a ring R, so $U^\gamma = \{\langle u, i_U^\gamma(u), t_U^\gamma(u), f_U^\gamma(u)\rangle \mid u \in R\}$ and
$V^\gamma = \{\langle u, i_V^\gamma(u), t_V^\gamma(u), f_V^\gamma(u) \mid u \in R\rangle\}$, then their sum $(U+V)^\gamma$ is given by
$(U+V)^\gamma = \{\langle u, (i_U^\gamma + i_V^\gamma)(u), (t_U^\gamma + t_V^\gamma)(u), (f_U^\gamma + f_V^\gamma)(u)\rangle \mid u \in R\}$.
Let $u, v \in R$ and let $\wedge\{(i_U^\gamma i_V^\gamma)(u),\ (i_U^\gamma i_V^\gamma)(v)\} = l$. Then for any $\epsilon > 0$,
$l - \epsilon < (i_U^\gamma + i_V^\gamma)(u) = \sup\limits_{u=a+b} \{\wedge \ \{i_U^\gamma(a), i_U^\gamma(b)\}\}$,
$l - \epsilon < (i_U^\gamma + i_V^\gamma)(v) = \sup\limits_{v=c+d} \{\wedge \ \{i_U^\gamma(c), i_U^\gamma(d)\}\}$.
So there exist representations $u = a+b,\ v = c+d$, where $a, b, c, d \in R$ such that
$l - \epsilon < \wedge\{i_U^\gamma(a), i_V^\gamma(b)\}$ and $l - \epsilon < \wedge\{i_U^\gamma(c), i_U^\gamma(d)\}$.
$\Rightarrow\ l - \epsilon < i_U^\gamma(a), i_V^\gamma(b)$ and $l - \epsilon < i_U^\gamma(c), i_U^\gamma(d)$.
$\Rightarrow\ l - \epsilon < \wedge\{i_U^\gamma(a), i_U^\gamma(c)\} \leq i_U^\gamma(a+c)$ and $l - \epsilon < \wedge\{i_V^\gamma(b), i_V^\gamma(d)\} \leq i_V^\gamma(b+d)$.
Thus we get $u + v = (a+b) + (c+d) = (a+c) + (b+d)$ such that
$l - \epsilon < \wedge\{i_U^\gamma(a+c), i_V^\gamma(b+d\}$.
$\Rightarrow\ l - \epsilon < \sup\limits_{u+v=(a+c)+(b+d)} \{\wedge i_U^\gamma(a+c), i_V^\gamma(b+d)\} = (i_U^\gamma + i_V^\gamma)(u+v)$.
Since ϵ is arbitrary, it follows that,
$(i_U^\gamma + i_V^\gamma)(u+v) \geq l = \wedge\{(i_U^\gamma + i_V^\gamma)(u), (i_U^\gamma + i_V^\gamma)(v)\}$.
Next, let $m = \vee\{(i_U^\gamma + i_V^\gamma)(u), (i_U^\gamma + i_V^\gamma)(v)\} = (i_U^\gamma + i_V^\gamma)(u)$ (say) and $\epsilon > 0$.
Then $m - \epsilon < (i_U^\gamma + i_V^\gamma)(u) = \sup\limits_{u=a+b} \{\wedge i_U^\gamma(a), i_V^\gamma(b)\}$.
So there exists a representation $u = a + b$ such that
$m - \epsilon < \wedge\{i_U^\gamma(a), i_V^\gamma(b)\}$.
$\Rightarrow\ m - \epsilon < i_U^\gamma(a), i_V^\gamma(b)$.
$m - \epsilon < \vee\{i_U^\gamma(a), i_U^\gamma(c+d)\} = i_U^\gamma(a(c+d))$, where $v = c+d$,
and $m - \epsilon < \vee\{i_V^\gamma(b), i_V^\gamma(c+d)\} = i_V^\gamma(b(c+d))$.
$\Rightarrow\ m - \epsilon < \wedge\{i_U^\gamma(a(c+d)), i_V^\gamma(b(c+d))\}$.
So we get, $uv = (a+b)(c+d) = a(c+d) + b(c+d)$, such that
$m - \epsilon < \wedge\{i_U^\gamma(a(c+d)), i_V^\gamma(b(c+d))\}$.
$\Rightarrow\ m - \epsilon < \sup\limits_{uv=a(c+d)+b(c+d)} \{\wedge\{i_U^\gamma(a(c+d)), i_V^\gamma(b(c+d))\}\} = (i_U^\gamma + i_V^\gamma)(uv)$.
Since ϵ is arbitrary,

$(i_U^\gamma + i_V^\gamma)(uv) \geq m = \vee\{(i_U^\gamma + i_V^\gamma)(u), (i_U^\gamma + i_V^\gamma)(v)\}$.
Similarly we can show that
$(t_U^\gamma + t_V^\gamma)(uv) \geq s = \vee\{(t_U^\gamma + t_V^\gamma)(u), (t_U^\gamma + t_V^\gamma)(v)\}$.
Next let $\vee\{(f_U^\gamma + f_V^\gamma)(u), (f_U^\gamma + f_V^\gamma)(v)\} = n$ and $\epsilon > 0$.
Then $n + \epsilon > (f_U^\gamma + f_V^\gamma)(u) = \inf\limits_{u=a+b}\{\vee\{f_U^\gamma(a), f_U^\gamma(b)\}\}$,
and $n + \epsilon > (f_U^\gamma + f_V^\gamma)(v) = \inf\limits_{v=c+d}\{\vee\{f_U^\gamma(c), f_U^\gamma(d)\}\}$.
So, there exist representations $u = a + b$ and $v = c + d$, for some $a, b, c, d \in R$ such that
$n + \epsilon > \vee\{f_U^\gamma(a), f_V^\gamma(b)\}$ and $n + \epsilon > \vee\{f_U^\gamma(c), f_V^\gamma(d)\}$.
$\Rightarrow n + \epsilon > f_U^\gamma(a), f_V^\gamma(b)$ and $n + \epsilon > f_U^\gamma(c), f_V^\gamma(d)$.
$\Rightarrow n + \epsilon > \vee\{f_U^\gamma(a), f_U^\gamma(c)\} = f_U^\gamma(a + c)$, and $n + \epsilon > \vee\{f_U^\gamma(b), f_U^\gamma(d)\} \geq f_U^\gamma(b + d)$.
Thus we get, $u + v = (a + b) + (c + d) = (a + c) + (b + d)$, such that
$n + \epsilon > \vee\{f_U^\gamma(a + c), f_V^\gamma(b + d)\}$.
$\Rightarrow\ n + \epsilon < \inf\limits_{u+v=(a+c)+(b+d)}\{\vee\{f_U^\gamma(a + c), f_V^\gamma(b + d)\}\} = (f_U^\gamma + f_V^\gamma)(u + v)$.
Since ϵ is arbitrary,
$(f_U^\gamma + f_V^\gamma)(u + v) \leq n = \vee\{(f_U^\gamma + f_V^\gamma)(u), (f_U^\gamma + f_V^\gamma)(v)\}$.
Finally, if $w = \wedge\{(f_U^\gamma + f_V^\gamma)(u), (f_U^\gamma + f_V^\gamma)(v)\} = (f_U^\gamma + f_V^\gamma)(u)$ (say), and $\epsilon > 0$,
then $w + \epsilon > (f_U^\gamma + f_V^\gamma)(u) = \inf\limits_{u=a+b}\{\vee f_U^\gamma(a), f_U^\gamma(b)\}$.
So there exists a representation $u = a + b$ such that $w + \epsilon > \vee\{f_U^\gamma(a), f_V^\gamma(b)\}$.
$\Rightarrow w + \epsilon > f_U^\gamma(a)$ and $w + \epsilon > f_V^\gamma(b)$.
$\Rightarrow w + \epsilon > \wedge\{f_U^\gamma(a), f_U^\gamma(c + d)\} = f_U^\gamma(a(c + d))$, and
$w + \epsilon > \wedge\{f_V^\gamma(b), f_V^\gamma(c + d)\} = f_V^\gamma(b(c + d))$, where $v = c + d$.
So, we get $uv = (a + b)(c + d) = a(c + d) + b(c + d)$ such that
$w + \epsilon > \vee\{f_U^\gamma(a(c + d)), f_V^\gamma(b(c + d))\}$.
$\Rightarrow w + \epsilon > \inf\limits_{uv=a(c+d)+b(c+d)}\{\vee(f_U^\gamma(a(c + d)), f_V^\gamma(b(c + d)))\} = (f_U^\gamma + f_V^\gamma)(uv)$.
Since ϵ is arbitrary,
$(f_U^\gamma + f_V^\gamma)(uv) \leq w = \wedge\{(f_U^\gamma + f_V^\gamma)(u), (f_U^\gamma + f_V^\gamma)(v)\}$.
Hence $U + V$ is an γ-$SVNI$ of R. □

Theorem 4.4. *If U and V are two γ-$SVNIs$ of a ring R, then UV is also an γ-$SVNI$ of R.*

Proof. Let $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ and $V = \{\langle u, i_V(u), t_V(u), f_V(u) \mid u \in R\rangle\}$ be two γ-$SVNIs$ of a ring R, so
$U^\gamma = \{\langle u, i_U^\gamma(u), t_U^\gamma(u), f_U^\gamma(u)\rangle \mid u \in R\}$ and $V^\gamma = \{\langle u, i_V^\gamma(u), t_V^\gamma(u), f_V^\gamma(u) \mid u \in R\rangle\}$.
Then $(UV)^\gamma = \{\langle u, (i_U^\gamma i_V^\gamma)(u), (t_U^\gamma t_V^\gamma)(u), (f_U^\gamma f_V^\gamma)(u)\rangle \mid u \in R\}$.
Let $u, v \in R$ and let $\wedge\{(i_U^\gamma i_V^\gamma)(u), (i_U^\gamma i_V^\gamma)(v)\} = \varsigma$.
Then for any $\epsilon > 0$,

$\varsigma - \epsilon < (i^{\gamma}_U i^{\gamma}_V)(u) = \sup\limits_{\substack{u=\sum a_i b_i \\ i<\infty}} \{\wedge\ \{\wedge\{i^{\gamma}_U(a_i), i^{\gamma}_U(b_i)\}\}\}$, and

$\varsigma - \epsilon < (i^{\gamma}_U i^{\gamma}_V)(v) = \sup\limits_{\substack{u=\sum m_i n_i \\ i<\infty}} \{\wedge\ \{\wedge\{i^{\gamma}_U(m_i), i^{\gamma}_U(n_i)\}\}\}$.

So we get representations $u = \sum\limits_{i<\infty} a_i b_i$ and $v = \sum\limits_{i<\infty} m_i n_i$ such that

$\varsigma - \epsilon < \{\wedge\ \{\wedge\{i^{\gamma}_U(a_i), i^{\gamma}_U(b_i)\}\}\}$, and $\varsigma - \epsilon < \{\wedge\ \{\wedge\{i^{\gamma}_U(m_i), i^{\gamma}_U(n_i)\}\}\}$,

$\Rightarrow\ \varsigma - \epsilon < \wedge\{i^{\gamma}_U(a_i), i^{\gamma}_U(b_i)\}$, and $\varsigma - \epsilon < \wedge\{i^{\gamma}_U(m_i), i^{\gamma}_U(n_i)\}\ \forall\ i$,

$\Rightarrow\ \varsigma - \epsilon < i^{\gamma}_U(a_i), i^{\gamma}_U(b_i)$, and $\varsigma - \epsilon < i^{\gamma}_U(m_i), i^{\gamma}_U(n_i)\ \forall\ i$,

$\Rightarrow\ \varsigma - \epsilon < \wedge\{i^{\gamma}_U(a_i), i^{\gamma}_U(b_i)\} \leq i^{\gamma}_U(a_i + m_i)$, and $\varsigma - \epsilon < \wedge\{i^{\gamma}_U(m_i), i^{\gamma}_U(n_i)\} \leq i^{\gamma}_V(b_i + n_i)\ \forall\ i$.

Thus, we get $u + v = \sum\limits_{i<\infty} (a_i b_i + m_i n_i)$, where $a_i, b_i, m_i, n_i \in R$, such that

$\varsigma - \epsilon < \{\wedge\{i^{\gamma}_U(a_i + m_i), i^{\gamma}_V(b_i + n_i)\}\},\ \forall\ i$,

$\Rightarrow\ \varsigma - \epsilon < \bigwedge\limits_i \{\wedge\{i^{\gamma}_U(a_i + m_i), i^{\gamma}_V(b_i + n_i)\}\}$,

$\varsigma - \epsilon < \sup\limits_{\substack{u=\sum(a_i b_i + m_i n_i) \\ i<\infty}} \{\bigwedge\limits_i \{\wedge\{i^{\gamma}_U(a_i + m_i), i^{\gamma}_U(b_i + n_i)\}\}\} = (i^{\gamma}_U i^{\gamma}_V)(u + v)$.

Since ϵ is arbitrary, so we have,

$(i^{\gamma}_U i^{\gamma}_V)(u + v) \geq \varsigma = \wedge\{(i^{\gamma}_U i^{\gamma}_V)(u), (i^{\gamma}_U i^{\gamma}_V)(v)\}$.

Next let $g = \vee\{(i^{\gamma}_U i^{\gamma}_V)(u), (i^{\gamma}_U i^{\gamma}_V)(v)\} = (i^{\gamma}_U i^{\gamma}_V)(u)$ (say) and let $\epsilon > 0$, then

$g - \epsilon < (i^{\gamma}_U i^{\gamma}_V)(u) = \sup\limits_{\substack{u=\sum a_i b_i \\ i<\infty}} \{\bigwedge\limits_i \{\wedge\{i^{\gamma}_U(a_i), \{i^{\gamma}_V b_i)\}\}\}$.

So there exists a representation $u = \sum\limits_{i<\infty} a_i b_i$ such that

$g - \epsilon < \bigwedge\limits_i \{\wedge\{i^{\gamma}_U(a_i), \{i^{\gamma}_V b_i)\}\}\ \Rightarrow \wedge\{i^{\gamma}_U(a_i), \{i^{\gamma}_V b_i\},\ \forall\ i$.

$\Rightarrow\ g - \epsilon <\ i^{\gamma}_U(a_i), i^{\gamma}_V(b_i),\ \forall\ i$.

If $v = \sum\limits_{i<\infty} m_i n_i$ then

$g - \epsilon < \vee\{i^{\gamma}_U(a_i), i^{\gamma}_U(m_i)\} = i^{\gamma}_U(a_i m_i)\ \forall\ i$,

and $g - \epsilon < \vee\{i^{\gamma}_V(b_i), i^{\gamma}_V(n_i)\} = i^{\gamma}_V(b_i n_i),\ \forall\ i$.

Thus, we get $uv = \sum\limits_{i<\infty} (a_i b_i)(m_i n_i) = \sum\limits_{i<\infty} (a_i m_i)(b_i n_i)$

such that $g - \epsilon < \wedge\{i^{\gamma}_U(a_i m_i), i^{\gamma}_V(b_i n_i)\},\ \forall\ i$.

$\Rightarrow\ g - \epsilon < \bigwedge\limits_i \{\wedge\{i^{\gamma}_U(a_i m_i), i^{\gamma}_V(b_i n_i)\}\}$.

$\Rightarrow\ g - \epsilon < \sup\limits_{\substack{uv=\sum(a_i m_i)(b_i n_i) \\ i<\infty}} \{\wedge\{i^{\gamma}_U(a_i m_i), i^{\gamma}_V(b_i n_i)\}\} = (i^{\gamma}_U i^{\gamma}_V)(uv)$.

Since ϵ is arbitrary

$(i^{\gamma}_U i^{\gamma}_V)(uv) \geq g = \vee\{(i^{\gamma}_U i^{\gamma}_V)(u), (i^{\gamma}_U i^{\gamma}_V)(v)\}$.

Similarly, we can show that

$(t^{\gamma}_U t^{\gamma}_V)(u + v) \geq j = \wedge\{(t^{\gamma}_U t^{\gamma}_V)(u), (t^{\gamma}_U i^{\gamma}_V)(v)\}$.

$(t^{\gamma}_U t^{\gamma}_V)(uv) \geq \delta = \vee\{(t^{\gamma}_U t^{\gamma}_V)(u), (t^{\gamma}_U t^{\gamma}_V)(v)\}$.

Next, let $l = \vee\{(f^{\gamma}_U f^{\gamma}_V)(u), (f^{\gamma}_U f^{\gamma}_V)(v)\}$ and $\epsilon > 0$, then

$\Rightarrow\ l+\epsilon > (f_U^{\gamma} f_V^{\gamma})(u) = \inf_{\substack{u=\sum a_i b_i \\ i<\infty}} \{\vee\{\bigvee_i \{f_U^{\gamma}(a_i), f_V^{\gamma}(b_i)\}\}\}$,

$\Rightarrow\ l+\epsilon > (f_U^{\gamma} f_V^{\gamma})(v) = \inf_{\substack{u=\sum m_i n_i \\ i<\infty}} \{\vee\{\bigvee_i \{f_U^{\gamma}(m_i), f_V^{\gamma}(n_i)\}\}\}$.

So, we get representations $u = \sum_{i<\infty} a_i b_i$ and $v = \sum_{i<\infty} m_i n_i$, where $a_i, b_i, m_i, n_i \in R$, such that

$l+\epsilon > \bigvee_i \{\vee\{f_U^{\gamma}(a_i), f_V^{\gamma}(b_i)\}$ and $l+\epsilon > \bigvee_i \{\vee\{f_U^{\gamma}(m_i), f_V^{\gamma}(n_i)\}\}$.

$\Rightarrow\ l+\epsilon > \vee\{f_U^{\gamma}(a_i), f_V^{\gamma}(b_i)\}$ and $l+\epsilon > \vee\{f_U^{\gamma}(m_i), f_V^{\gamma}(n_i)\},\ \forall\ i$.

$\Rightarrow\ l+\epsilon > f_U^{\gamma}(a_i), f_V^{\gamma}(b_i)$ and $l+\epsilon > f_U^{\gamma}(m_i), f_V^{\gamma}(n_i),\ \forall\ i$.

$\Rightarrow\ l+\epsilon > \vee\{f_U^{\gamma}(a_i), f_V^{\gamma}(m_i)\} \geq f_U^{\gamma}(a_i+m_i)$ and $l+\epsilon > \vee\{f_U^{\gamma}(b_i), f_V^{\gamma}(n_i)\} \geq f_U^{\gamma}(b_i+n_i),\ \forall\ i$.

Thus, we get $u+v = \sum_{i<\infty}(a_i b_i + m_i n_i)$, where $a_i, b_i, m_i, n_i \in R$, such that

$l+\epsilon > \vee\{f_U^{\gamma}(a_i+m_i), f_V^{\gamma}(b_i+n_i)\},\ \forall\ i$.

$\Rightarrow\ l+\epsilon > \bigvee_i \{\vee\{f_U^{\gamma}(a_i+m_i), f_V^{\gamma}(b_i+n_i)\}\}$,

$l+\epsilon > \sup_{\substack{u=\sum(a_i b_i + m_i n_i) \\ i<\infty}} \{\bigvee_i \{\vee\{f_U^{\gamma}(a_i+m_i), f_U^{\gamma}(b_i+n_i)\}\}\} = (f_U^{\gamma} f_V^{\gamma})(u+v)$.

Since ϵ is arbitrary, so we have,

$(f_U^{\gamma} f_V^{\gamma})(u+v) \leq o = \vee\{(f_U^{\gamma} f_V^{\gamma})(u), (f_U^{\gamma} f_V^{\gamma})(v)\}$.

Finally, let $o = \wedge\{(f_U^{\gamma} f_V^{\gamma})(u), (f_U^{\gamma} f_V^{\gamma})(v)\} = (f_U^{\gamma} f_V^{\gamma})(u)$ (say) and let $\epsilon > 0$, then

$o+\epsilon > (f_U^{\gamma} f_V^{\gamma})(u) = \inf_{\substack{u=\sum a_i b_i \\ i<\infty}} \{\bigvee_i \{\vee\{f_U^{\gamma}(a_i), \{f_V^{\gamma} b_i)\}\}\}$.

So there exists a representation $u = \sum_{i<\infty} a_i b_i$ such that

$o+\epsilon > \bigvee_i \{\vee\{f_U^{\gamma}(a_i), \{f_V^{\gamma} b_i)\}\}\ \Rightarrow \vee\{f_U^{\gamma}(a_i), \{f_V^{\gamma} b_i\},\ \forall\ i$.

$\Rightarrow\ r+\epsilon >\ f_U^{\gamma}(a_i), f_V^{\gamma}(b_i),\ \forall\ i$.

If $v = \sum_{i<\infty} m_i n_i$ then

$o+\epsilon > \vee\{f_U^{\gamma}(a_i), f_U^{\gamma}(m_i)\} \geq f_U^{\gamma}(a_i m_i)\ \forall\ i$,

and $o+\epsilon > \vee\{f_V^{\gamma}(b_i), f_V^{\gamma}(n_i)\} \geq f_V^{\gamma}(b_i n_i),\ \forall\ i$.

Thus, we get $uv = \sum_{i<\infty}(a_i b_i)(m_i n_i) = \sum_{i<\infty}(a_i m_i)(b_i n_i)$

such that $o+\epsilon > \vee\{f_U^{\gamma}(a_i m_i), f_V^{\gamma}(b_i n_i)\},\ \forall\ i$.

$\Rightarrow\ o+\epsilon > \bigvee_i \{\vee\{f_U^{\gamma}(a_i m_i), f_V^{\gamma}(b_i n_i)\}\}$.

$\Rightarrow\ o+\epsilon > \inf_{\substack{uv=\sum(a_i m_i)(b_i n_i) \\ i<\infty}} \{\vee\{f_U^{\gamma}(a_i m_i), f_V^{\gamma}(b_i n_i)\}\} = (f_U^{\gamma} f_V^{\gamma})(uv)$.

Since ϵ is arbitrary

$(f_U^{\gamma} f_V^{\gamma})(uv) \leq o = \wedge\{(f_U^{\gamma} f_V^{\gamma})(u), (f_U^{\gamma} f_V^{\gamma})(v)\}$.

Hence UV is an γ-$SVNI$ of R. □

Remark 4.5. According to the definition given by Atanassov [1] the sum and product of two γ-$SVNIs$ of a ring R is not necessarily an γ-$SVNI$ of R as shown by the following example: Consider the ring $R = \{0, a, b, a+b\}$ where $a+a = 0 = b+b, a+b = b+a$ and $uv = 0$

$\forall\ u,\ v \in R$. We define,
$i^{\gamma}_{U}(0) = 0.9 = i^{\gamma}_{U}(a), i^{\gamma}_{U}(b) = 0.4 = i^{\gamma}_{U}(a+b)$;
$t^{\gamma}_{U}(0) = 0.9 = t^{\gamma}_{U}(a), t^{\gamma}_{U}(b) = 0.4 = t^{\gamma}_{U}(a+b)$;
$f^{\gamma}_{U}(0) = 0.1 = f^{\gamma}_{U}(a), f^{\gamma}_{U}(b) = 0.4 = f^{\gamma}_{U}(a+b)$.
And $i^{\gamma}_{V}(0) = 0.7, i^{\gamma}_{V}(a) = 0.3 = i^{\gamma}_{V}(a+b), i^{\gamma}_{V}(b) = 0.5$;
$i^{\gamma}_{V}(0) = 0.7, i^{\gamma}_{V}(a) = 0.3 = i^{\gamma}_{V}(a+b), i^{\gamma}_{V}(b) = 0.5$;
$f^{\gamma}_{V}(0) = 0.2, f^{\gamma}_{V}(a) = 0.6 = f^{\gamma}_{V}(a+b), f^{\gamma}_{V}(b) = 0.5$.
Then $U = \{\langle u, i_U(u), t_U(u), f_U(u)\rangle \mid u \in R\}$ and $V = \{\langle u, i_V(u), t_V(u), f_V(u)\rangle \mid u \in R\}$ are γ-$SVNIs$ of R. According to Atanassov [1],
$(U+V)^{\gamma} = \{\langle u, i^{\gamma}_{U}(u)+i^{\gamma}_{V}(u)-i^{\gamma}_{U}(u)i^{\gamma}_{V}(u), t^{\gamma}_{U}(u)+t^{\gamma}_{V}(u)-t^{\gamma}_{U}(u)t^{\gamma}_{V}(u), f^{\gamma}_{U}(u)f^{\gamma}_{U}(u)\rangle \mid u \in R\}$.
And $(UV)^{\gamma} = \{\langle u, i^{\gamma}_{U}(u)i^{\gamma}_{V}(u), t^{\gamma}_{U}(u)t^{\gamma}_{U}(u), f^{\gamma}_{U}(u)+f^{\gamma}_{V}(u)-f^{\gamma}_{U}(u)f^{\gamma}_{U}(u)\rangle \mid uinR\}$.
Now $i^{\gamma}_{U}(a-b)+i^{\gamma}_{V}(a-b)-i^{\gamma}_{U}(a-b)i^{\gamma}_{V}(a-b) = 0.4+0.3-0.12 = 0.58$,
$i^{\gamma}_{U}(a)+i^{\gamma}_{V}(a)-i^{\gamma}_{U}(a)i^{\gamma}_{V}(a) = 0.9+0.3-0.27 = 0.93$,
and $i^{\gamma}_{U}(b)+i^{\gamma}_{V}(b)-i^{\gamma}_{U}(b)i^{\gamma}_{V}(b) = 0.4+0.5-0.2 = 0.7$.
Therefore,
$i^{\gamma}_{U}(a-b)+i^{\gamma}_{V}(a-b)-i^{\gamma}_{U}(a-b)i^{\gamma}_{V}(a-b) < \wedge\{i^{\gamma}_{U}(a)+i^{\gamma}_{V}(a)-i^{\gamma}_{U}(a)i^{\gamma}_{V}(a), i^{\gamma}_{U}(b)+i^{\gamma}_{V}(b)-i^{\gamma}_{U}(b)i^{\gamma}_{V}(b)\}$.
Hence $U+V$ is not an γ-$SVNI$ of R. Again for the product, we see that

$$f^{\gamma}_{U}(a-b)+f^{\gamma}_{V}(a-b)-f^{\gamma}_{U}(a-b)f^{\gamma}_{V}(a-b) = 0.76,$$
$$f^{\gamma}_{U}(a)+f^{\gamma}_{V}(a)-f^{\gamma}_{U}(a)f^{\gamma}_{V}(a) = 0.64,$$
$$and\ f^{\gamma}_{U}(b)+f^{\gamma}_{V}(b)-f^{\gamma}_{U}(b)f^{\gamma}_{V}(b) = 0.7.$$

Therefore

$$f^{\gamma}_{U}(a-b)+f^{\gamma}_{V}(a-b)-f^{\gamma}_{U}(a-b)f^{\gamma}_{V}(a-b) > \vee\{f^{\gamma}_{U}(a)+f^{\gamma}_{V}(a)-f^{\gamma}_{U}(a)f^{\gamma}_{V}(a), f^{\gamma}_{U}(b)+f^{\gamma}_{V}(b)-f^{\gamma}_{U}(b)f^{\gamma}_{V}(b)\}.$$

Hence UV is not an γ-$SVNI$ of R.

5. Conclusions

A γ-single valued neutrosophic set is a type of SVNS that can be used to tackle real-world challenges for research and engineering. In this work, we introduce the notion of γ-single valued neutrosophic subrings, γ-single valued neutrosophic ideals also the sum and product of γ-single valued neutrosophic ideals. On γ-single valued neutrosophic subrings and ideals, a variety of characterizations have been proposed. Therefore, it is important for researchers to examine γ-single valued neutrosophic subrings and ideals and their characteristics in applications and to understand the basics of uncertainty. We agreed to include the concept of a γ-SVNSR & γ-SVNI in research also examine its key feature. As a consequence of this research, various principles are to be applied to achieve some adequate research value results of γ-SVNSR &

γ-SVNI. In further work, researchers can extend this idea in topological spaces, modules, and fields.

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