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Neutrosophic ags Continuity And Neutrosophic ags Irresolute Maps

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Abstract. Neutrosophic Continuity functions very first introduced by A.A.Salama et.al.Aim of this present paper is, we introduce and investigate new kind of Neutrosophic continuity is called Neutrosophic α gs Continuity maps in Neutrosophic topological spaces and also discussed about some properties and characterization of Neutrosophic α gs Irresolute Map.

Keywords: Neutrosophic α -closed sets, Neutrosophic semi-closed sets, Neutrosophic α gs-closed sets Neutrosophic α gs Continuity maps, Neutrosophic α gs irresolute maps

1. Introduction

Neutrosophic set theory concepts first initiated by F.Smarandache[11] which is Based on K. Atanassov's intuitionistic[6]fuzzy sets & L.A.Zadeh's [20]fuzzy sets. Also it defined by three parameters truth(T), indeterminacy (I), and falsity(F)-membership function. Smarandache's neutrosophic concept have wide range of real time applications for the fields of [1,2,3,4&5] Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, decision making. Mechanics, Electrical & Electronic, Medicine and Management Science etc,.

A.A.Salama[16] introduced Neutrosophic topological spaces by using Smarandache's Neutrosophic sets. I.Arokiarani.[7] et.al., introduced Neutrosophic α -closed sets.P. Ishwarya, [13]et.al., introduced and studied Neutrosophic semi-open sets in Neutrosophic topological spaces. Neutrosophic continuity functions introduced by A.A.Salama[15]. Neutrosophic α gs-closed set[8] introduced by V.Banu priya&S.Chandrasekar. Aim of this present paper is, we introduce and investigate new kind of Neutrosophic continuity is called Neutrosophic α gs Continuity maps in Neutrosophic topological spaces and also we discussed about properties and characterization Neutrosophic α gs Irresolute Maps

2. Preliminaries

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

Definition 2.1 [11]

Let E be a non-empty fixed set. A Neutrosophic set λ writing the format is

 $\lambda = \{\langle e, \eta_{\lambda}(e), \sigma_{\lambda}(e), \gamma_{\lambda}(e) \rangle : e \in E\}$

Where $\eta_{\lambda}(e)$, $\sigma_{\lambda}(e)$ and $\gamma_{\lambda}(e)$ which represents Neutrosophic topological spaces the degree of membership function, indeterminacy and non-membership function respectively of each element $e \in E$ to the set λ

Remark 2.2 [11]

A Neutrosophic set $\lambda = \{ \langle e, \eta_{\lambda}(e), \sigma_{\lambda}(e) \rangle : e \in E \}$ can be identified to an ordered triple $\langle \eta_{\lambda}, \sigma_{\lambda}, \gamma_{\lambda} \rangle$ in]-0,1+[on E.

Remark 2.3[11]

Neutrosophic set $\lambda = \{\langle e, \eta_{\lambda}(e), \sigma_{\lambda}(e), \gamma_{\lambda}(e) \rangle : e \in E\}$ our convenient we can write $\lambda = \langle e, \eta_{\lambda}, \sigma_{\lambda}, \gamma_{\lambda} \rangle$.

Example 2.4 [11]

we must introduce the Neutrosophic set 0_N and 1_N in E as follows:

0_N may be defined as:

- $(0_1) 0_N = \{ \langle e, 0, 0, 1 \rangle : e \in E \}$
- $(0_2) 0_N = \{ \langle e, 0, 1, 1 \rangle : e \in E \}$
- $(0_3) 0_N = \{ < e, 0, 1, 0 > : e \in E \}$
- $(0_4) 0_N = \{ \langle e, 0, 0, 0 \rangle : e \in E \}$

1_N may be defined as:

- (1_1) $1_N = \{ \langle e, 1, 0, 0 \rangle : e \in E \}$
- (1_2) $1_N = \{ \langle e, 1, 0, 1 \rangle : e \in E \}$
- (13) $1_N = \{ \langle e, 1, 1, 0 \rangle : e \in E \}$
- (1_4) $1_N = \{ \langle e, 1, 1, 1 \rangle : e \in E \}$

Definition 2.5 [11]

Let $\lambda = \langle \eta_{\lambda}, \sigma_{\lambda}, \gamma_{\lambda} \rangle$ be a Neutrosophic set on E, then λ^{C} defined as $\lambda^{C} = \{\langle e, \gamma_{\lambda}(e), 1 - \sigma_{\lambda}(e), \eta_{\lambda}(e) \rangle : e \in E\}$

Definition 2.6 [11]

Let E be a non-empty set, and Neutrosophic sets λ and μ in the form

 $\lambda = \{\langle e, \eta_{\lambda}(e), \sigma\lambda(e), \gamma\lambda(e)\rangle : e \in E\}$ and

 $\mu = \{ \langle e, \eta_{\mu}(e), \sigma_{\mu}(e), \gamma_{\mu}(e) \rangle : e \in E \}.$

Then we consider definition for subsets $(\lambda \subseteq \mu)$.

 $\lambda \subseteq \mu$ defined as: $\lambda \subseteq \mu \Leftrightarrow \eta_{\lambda}(e) \le \eta_{\mu}(e)$, $\sigma_{\lambda}(e) \le \sigma_{\mu}(e)$ and $\gamma_{\lambda}(e) \ge \gamma_{\mu}(e)$ for all $e \in E$

Proposition 2.7 [11]

For any Neutrosophic set λ , then the following condition are holds:

- (i) $0_N \subseteq \lambda$, $0_N \subseteq 0_N$
- (ii) $\lambda \subseteq 1_N$, $1_N \subseteq 1_N$

Definition 2.8 [11]

Let E be a non-empty set, and $\lambda = \langle e, \eta_{\mu}(e), \sigma_{\lambda}(e) \rangle$, $\mu = \langle e, \eta_{\mu}(e), \sigma_{\mu}(e), \gamma_{\mu}(e) \rangle$ be two

Neutrosophic sets. Then

- (i) $\lambda \cap \mu$ defined as $:\lambda \cap \mu = \langle e, \eta_{\lambda}(e) \wedge \eta_{\mu}(e), \sigma_{\lambda}(e) \wedge \sigma_{\mu}(e), \gamma_{\lambda}(e) \vee \gamma_{\mu}(e) \rangle$
- (ii) $\lambda \cup \mu$ defined as : $\lambda \cup \mu = \langle e, \eta_{\lambda}(e) V \eta_{\mu}(e), \sigma_{\lambda}(e) V \sigma_{\mu}(e), \gamma_{\lambda}(e) \Lambda \gamma_{\mu}(e) \rangle$

Proposition 2.9 [11]

For all λ and μ are two Neutrosophic sets then the following condition are true:

- (i) $(\lambda \cap \mu)^C = \lambda^C \cup \mu^C$
- (ii) $(\lambda \cup \mu)^C = \lambda^C \cap \mu^C$.

Definition 2.10 [16]

A Neutrosophic topology is a non-empty set E is a family τ_N of Neutrosophic subsets in E satisfying the following axioms:

- (i) 0_N , $1_N \in \tau_N$,
- (ii) $G_1 \cap G_2 \in \tau_N$ for any G_1 , $G_2 \in \tau_N$,
- (iii) $\cup G_i \in \tau_N$ for any family $\{G_i \mid i \in J\} \subseteq \tau_N$.

the pair (E, τ_N) is called a Neutrosophic topological space.

The element Neutrosophic topological spaces of τ_N are called Neutrosophic open sets.

A Neutrosophic set λ is closed if and only if λ^{C} is Neutrosophic open.

Example 2.11[16]

Let E={e} and

 $A_1 = \{ < e, .6, .6, .5 > : e \in E \}$

 $A_2 = \{ \langle e, .5, .7, .9 \rangle : e \in E \}$

 $A_3 = \{ < e, .6, .7, .5 > : e \in E \}$

 $A_4 = \{ \langle e, .5, .6, .9 \rangle : e \in E \}$

Then the family TN={0N, 1N,A1, A2, A3, A4}is called a Neutrosophic topological space on E.

Definition 2.12[16]

Let (E, τ_N) be Neutrosophic topological spaces and $\lambda = \{ < e, \eta_\lambda(e), \sigma_\lambda(e), \gamma_\lambda(e) > : e \in E \}$ be a Neutrosophic set in E. Then the Neutrosophic closure and Neutrosophic interior of λ are defined by

Neu-cl(λ)= \cap {D:D is a Neutrosophic closed set in E and $\lambda\subseteq$ D}

Neu-int(λ)= \cup {C:C is a Neutrosophic open set in E and C \subseteq λ }.

Definition 2.13

Let (E, τ_N) be a Neutrosophic topological space. Then λ is called

- (i) Neutrosophic regular Closed set [7] (Neu-RCS in short) if λ =Neu-Cl(Neu-Int(λ)),
- (ii) Neutrosophic α -Closed set[7] (Neu- α CS in short) if Neu-Cl(Neu-Int(Neu-Cl(λ))) $\subseteq \lambda$,
- (iii) Neutrosophic semi Closed set [13] (Neu-SCS in short) if Neu-Int(Neu-Cl(λ)) $\subseteq \lambda$,
- (iv) Neutrosophic pre Closed set [18] (Neu-PCS in short) if Neu-Cl(Neu-Int(λ))⊆λ,

Definition 2.14

Let (E, τ_N) be a Neutrosophic topological space. Then λ is called

- (i). Neutrosophic regular open set [7](Neu-ROS in short) if λ =Neu-Int(Neu-Cl(λ)),
- (ii). Neutrosophic α -open set [7](Neu- α OS in short) if $\lambda\subseteq$ Neu-Int(Neu-Cl(Neu-Int(λ))),
- (iii). Neutrosophic semi open set [13](Neu-SOS in short) if $\lambda \subseteq \text{Neu-Cl}(\text{Neu-Int}(\lambda))$,
- (iv). Neutrosophic pre open set [18] (Neu-POS in short) if $\lambda \subseteq \text{Neu-Int}(\text{Neu-Cl}(\lambda))$,

Definition 2.15

Let (E, τ_N) be a Neutrosophic topological space. Then λ is called

(i). Neutrosophic generalized closed set[9] (Neu-GCS in short) if Neu-cl(λ) \subseteq U whenever λ \subseteq U and U is a Neu-

OS in E,

(ii). Neutrosophic generalized semi closed set[17] (Neu-GSCS in short) if Neu-scl(λ) \subseteq U Whenever λ \subseteq U and U

is a Neu-OS in E,

(iii). Neutrosophic α generalized closed set [14](Neu- α GCS in short) if Neu- α cl(λ) \subseteq U whenever λ \subseteq U and U is a

Neu-OS in E,

(iv). Neutrosophic generalized alpha closed set [10] (Neu-G α CS in short) if Neu- α cl(λ) \subseteq U whenever λ \subseteq U and U

is a Neu- α OS in E .

The complements of the above mentioned Neutrosophic closed sets are called their respective Neutrosophic open sets.

Definition 2.16 [8]

Let (E, τ_N) be a Neutrosophic topological space. Then λ is called Neutrosophic α generalized Semi closed set $(Neu-\alpha GSCS \text{ in short})$ if $Neu-\alpha cl(\lambda) \subseteq U$ whenever $\lambda \subseteq U$ and U is a Neu-SOS in E

The complements of Neutrosophic αGS closed sets is called Neutrosophic αGS open sets.

3. Neutrosophic ags-Continuity maps

In this section we Introduce Neutrosophic α -generalized semi continuity maps and study some of its properties.

Definition 3.1.

A maps f:(E₁, τ_N) \rightarrow (E₂, σ_N) is called a Neutrosophic α -generalized semi continuity (Neu- α GS continuity in short) f-1(μ) is a Neu- α GSCS in (E₁, τ_N) for every Neu-CS μ of (E₂, σ_N)

Example 3.2.

Let $E_1=\{a_1,a_2\}$, $E_2=\{b_1,b_2\}$, $U=\langle e_1,(.7,.5,.8),(.5,.5,.4)\rangle$ and $V=\langle e_2,(1,.5,.9),(.2,.5,.3)\rangle$. Then $\tau_N=\{0_N,U,1_N\}$ and $\sigma_N=\{0_N,V,1_N\}$ are Neutrosophic Topologies on E_1 and E_2 respectively.

Define a maps $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Then f is a Neu- α GS continuity maps.

Theorem 3.3.

Every Neu-continuity maps is a Neu- α GS continuity maps.

Proof.

Let $f:(E_1, \tau_N) \to (E_2, \sigma_N)$ be a Neu-continuity maps. Let λ be a Neu-CS in E_2 . Since f is a Neu-continuity maps, $f^1(\lambda)$ is a Neu- α GSCS in E_1 . Hence f is a Neu- α GS continuity maps.

Example 3.4.

Neu- α GS continuity maps is not Neu-continuity maps

Let $E_1=\{a_1, a_2\}$, $E_2=\{b_1, b_2\}$, $U=<e_1$, (.5,.5,.3), (.7,.5,.8)> and $V=<e_2$, (.4,.5,.3), (.8,.5,.9)>. Then $\tau_N=\{0_N,U,1_N\}$ and $\sigma_N=\{0_N,V,1_N\}$ are Neutrosophic sets on E_1 and E_2 respectively. Define a maps $f:(E_1, \tau_N)\to (E_2, \sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Since the Neutrosophic set $\lambda=<y$, (.3,.5,.4), (.9,.5,.8)> is Neu-CS in E_2 , $f^1(\lambda)$ is a Neu- α GSCS but not Neu-CS in E_1 . Therefore f is a Neu- α GS continuity maps but not a Neu-continuity maps. **Theorem 3.5.**

Every Neu- α continuity maps is a Neu- α GS continuity maps.

Proof.

Let $f:(E_1, \tau_N) \to (E_2, \sigma_N)$ be a Neu- α continuity maps. Let λ be a Neu-CS in E_2 . Then by hypothesis $f^{-1}(\lambda)$ is a Neu- α CS in E_1 . Since every Neu- α CS is a Neu- α GSCS, $f^{-1}(\lambda)$ is a Neu- α GSCS in E_1 . Hence f is a Neu- α GS continuity maps.

Example 3.6.

Neu- α GS continuity maps is not Neu- α continuity maps

Let $E_1=\{a_1,a_2\}$, $E_2=\{b_1,b_2\}$, $U=<e_1,(.5,.5,.6)$, (.7,.5,.6)> and $V=<e_2$, (.3,.5,.9), (.5,.5,.7)>. Then $\tau_N=\{0_N,U,1_N\}$ and $\sigma_N=\{0_N,V,1_N\}$ are Neutrosophic Topologies on E_1 and E_2 respectively. Define a maps $f:(E_1,\tau_N)\to(E_2,\sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Since the Neutrosophic set $\lambda=<e_2$, (.9,.5,.3), (.7,.5,.5)> is Neu-CS in E_2 , $f^{-1}(\lambda)$ is a Neu- α GSCS continuity maps.

Remark 3.7.

Neu-G continuity maps and Neu- α GS continuity maps are independent of each other.

Example 3.8.

Neu- α GS continuity maps is not Neu-G continuity maps.

Let E₁={a₁, a₂}, E₂={b₁, b₂}, U=< e₁,(.5,.5, .6), (.8,.5,.4)> and V=< e₂,(.7,.5,.4), (.9,.5, .3)>. Then τ_N ={0_N,U,1_N} and σ_N ={0_N,V,1_N} are Neutrosophic Topologies on E₁ and E₂ respectively. Define a maps f:(E₁, τ_N)→(E₂, σ_N) by f(a₁)=b₁ and f(a₂)=b₂. Then f is Neu- α GS continuity maps but not Neu-G continuity maps. Since λ =< e₁,(.4,.5, .7), (.3,.5, .9)> is Neu-CS in E₂, f¹(λ)=< e₂, (.4,.5, .7), (.7,.5, .3)> is not Neu-GCS in E₁.

Example 3.9.

Neu-G continuity maps is not Neu- α GS continuity maps.

Let $E_1=\{a_1, a_2\}$, $E_2=\{b_1,b_2\}$, $U=\langle e_1,(.6,.5,.4)$, $(.8,.5,.2)\rangle$ and $V=\langle e_2,(.3,.5,.7)$, $(.1,.5,.9)\rangle$. Then $\tau_N=\{0_N,U,1_N\}$ and $\sigma_N=\{0_N,V,1_N\}$ are Neutrosophic Topologies on E_1 and E_2 respectively. Define a maps $f:(E_1, \tau_N) \to (E_2, \sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Then f is Neu-G continuity maps but not a Neu- α GS continuity maps. Since $\lambda=\langle e_2,(.7,.5,.3),(.9,.5,.1)\rangle$ is Neu-CS in E_2 , $f^1(\lambda)=\langle e_1,(.7,.5,.3),(.9,.5,.1)\rangle$ is not Neu- α GSCS in E_1 .

Theorem 3.10.

Every Neu- α GS continuity maps is a Neu-GS continuity maps.

Proof.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS continuity maps. Let λ be a Neu-CS in E2. Then by hypothesis $f^{-1}(\lambda)$ Neu- α GSCS in E1. Since every Neu- α GSCS is a Neu-GSCS, $f^{-1}(\lambda)$ is a Neu-GSCS in E1. Hence f is a Neu-GS continuity maps.

Example 3.11.

Neu-GS continuity maps is not Neu- α GS continuity maps.

Let $E_1=\{a_1, a_2\}$, $E_2=\{b_1, b_2\}$, $U=< e_1,(.8,.5,.4)$, (.9,.5,.2)> and $V=< e_2,(.3,.5,.9)$, (0.1,.5,.9)>. Then $\tau_N=\{0_N,U,1_N\}$ and $\sigma_N=\{0_N,V,1_N\}$ are Neutrosophic Topologies on E_1 and E_2 respectively. Define a maps $f:(E_1,\tau_N)\rightarrow (E_2,\sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Since the Neutrosophic set $\lambda=< e_2,(.9,.5,.3)$, (.9,.5,.1)> is Neu-CS in E_2 , $f^1(\lambda)$ is Neu-GSCS in E_1 but not Neu- α GSCS in E_1 . Therefore f is a Neu-GS continuity maps but not a Neu- α GS continuity maps.

Remark 3.12.

Neu-P continuity maps and Neu- α GS continuity maps are independent of each other.

Example 3.13.

Neu-P continuity maps is not Neu- α GS continuity maps Let E₁={a₁, a₂}, E₂={b₁, b₂},U= < e₁, (.3,.5,.7),(.4,.5,.6)> and V=< e₂,(.8,.5,.3), (.9,.5,.2)>. Then τ_N ={0_N,U,1_N} and σ_N ={0_N, V, 1_N} are Neutrosophic Topologies on E₁ and E₂ respectively. Define a maps f:(E₁, τ_N) \rightarrow (E₂, σ_N)by f(a₁)=b₁ and f(a₂)=b₂.Since the Neutrosophic set λ =< e₂,(.3,.5, .8), (.2,.5, .9)> is Neu-CS in E₂, f⁻¹(λ) is Neu-PCS in E₁ but not Neu- α GSCS in E₁. Therefore f is a Neu-P continuity maps but not Neu- α GS continuity maps.

Example 3.14.

Neu-αGS continuity maps is not Neu-P continuity maps

Let $E_1=\{a_1, a_2\}$, $E_2=\{b_1, b_2\}$, $U=< e_1, (.4,.5,.8), (.5,.5,.7)>$ and $V=< e_1, (.5,.5,.7)$, (.6,.5,.6)> and $W=< e_2, (.8,.5,.4)$, (.5,.5,.7)>. Then $\tau_N=\{0_N,U,V,1_N\}$ and $\sigma_N=\{0_N,W,1_N\}$ are Neutrosophic Topologies on E_1 and E_2 respectively. Define a maps $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Since the Neutrosophic set $\lambda=< y$, (.4,.5,.8), (.7,.5,.5)> is Neu- α GSCS but not Neu-PCS in E_2 , $f^1(\lambda)$ is Neu- α GSCS in E_1 but not Neu-PCS in E_1 . Therefore f is a Neu- α GS continuity maps but not Neu-P continuity maps.

Theorem 3.15.

Every Neu- α GS continuity maps is a Neu- α G continuity maps.

Proof.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS continuity maps. Let λ be a Neu-CS in E_2 . Since f is Neu- α GS continuity maps, $f^1(\lambda)$ is a Neu- α GSCS in E_1 . Since every Neu- α GSCS is a Neu- α GCS, $f^1(\lambda)$ is a Neu- α GCS in E_1 . Hence f is a Neu- α G continuity maps.

Example 3.16.

Neu- αG continuity maps is not Neu- αGS continuity maps

Let $E_1=\{a_1, a_2\}$, $E_2=\{b_1, b_2\}$, $U=<e_1,(.1,.5,.7),(.3,.5,.6)>$ and $V=<e_2,(.7,.5,.4),$ (.6,.5,.5)>. Then $\tau_N=\{0_N,U,1_N\}$ and $\sigma_N=\{0_N,V,1_N\}$ are Neutrosophic Topologies on E_1 and E_2 respectively. Define a maps $f:(E_1, \tau_N)\rightarrow (E_2, \sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Since the Neutrosophic set $\lambda=<e_2,(.4,.5,.7),(.5,.5,.6)>$ is Neu-CS in E_2 , $f^1(\lambda)$ is Neu- α GCS in E_1 but not Neu- α GSCS in E_1 . Therefore f is a Neu- α G continuity maps but not a Neu- α GS continuity maps.

Theorem 3.17.

Every Neu- α GS continuity maps is a Neu-G α continuity maps.

Proof.

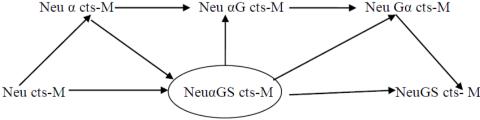
Let $f:(E_1, \tau_N) \to (E_2, \sigma_N)$ be a Neu- α GS continuity maps. Let λ be a Neu-CS in E_2 . Since f is Neu- α GS continuity maps, $f^1(\lambda)$ is a Neu- α GSCS in E_1 . Since every Neu- α GSCS is a Neu-G α CS, $f^1(\lambda)$ is a Neu-G α CS in E_1 . Hence f is a Neu-G α continuity maps.

Example 3.18.

Neu-G α continuity maps is not Neu- α GS continuity maps Let E₁={a₁, a₂}, E₂={b₁, b₂}, U=< e₁, (.5,.5,.7), (.3,.5, .9)> and V=< e₂, (.6,.5,.6), (.5,.5,.7)>. Then τ_N ={0 $_N$,U,1 $_N$ } and σ_N ={0 $_N$,V,1 $_N$ } are Neutrosophic Topologies on E₁ and E₂ respectively. Define a maps f:(E₁, τ_N) \rightarrow (E₂, σ_N)by f(a₁)=b₁ and f(a₂)=b₂. Since the Neutrosophic set λ =<y,(.6,.5,.6), (.7,.5, .5)> is Neu-CS in E₂, f¹(λ)is Neu-G α CS in E₁ but not Neu- α GSCS in E₁. Therefore f is a Neu-G α continuity maps but not a Neu- α GS continuity maps.

Remark 3.19.

We obtain the following diagram from the results we discussed above.



Theorem 3 20

A maps $f:(E_1,\tau_N)\to (E_2,\sigma_N)$ is Neu- α GS continuity if and only if the inverse image of each Neutrosophic set in E_2 is a Neu- α GSOS in E_1 .

Proof.

first part Let λ be a Neutrosophic set in E₂. This implies λ^{C} is Neu-CS in E₂. Since f is Neu- α GS continuity, $f^{-1}(\lambda^{C})$ is Neu- α GSCS in E₁. Since $f^{-1}(\lambda^{C})=(f^{-1}(\lambda))^{C}$, $f^{-1}(\lambda)$ is a Neu- α GSOS in E₁.

Converse part Let λ be a Neu-CS in E₂. Then λ^{C} is a Neutrosophic set in E₂. By hypothesis $f^{-1}(\lambda^{C})$ is Neu- α GSOS in E₁. Since $f^{-1}(\lambda^{C})=(f^{-1}(\lambda))^{C}$, $(f^{-1}(\lambda))^{C}$ is a Neu- α GSOS in E₁. Therefore $f^{-1}(\lambda)$ is a Neu- α GSCS in E₁. Hence f is Neu- α GS continuity.

Theorem 3.21.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a maps and $f^1(\lambda)$ be a Neu-RCS in E_1 for every Neu-CS λ in E_2 . Then f is a Neu- α GS continuity maps.

Proof.

Let λ be a Neu-CS in E₂ and f¹(λ) be a Neu-RCS in E₁. Since every Neu-RCS is a Neu- α GSCS, f¹(λ) is a Neu- α GSCS in E₁. Hence f is a Neu- α GS continuity maps.

Definition 3.22.

A Neutrosophic Topology (Ε, τ_N) is said to be an

- (i) Neu- α ga $U_{1/2}$ (in short Neu- α ga $U_{1/2}$) space , if every Neu- α GSCS in E is a Neu-CS in E,
- (ii)Neu- α gbU1/2(in short Neu- α gbU1/2) space ,if every Neu- α GSCS in E is a Neu-GCS in E,
- (iii)Neu- α gcU_{1/2}(in short Neu- α gcU_{1/2}) space, if every Neu- α GSCS in E is a Neu-GSCS in E.

Theorem 3.23.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS continuity maps, then f is a Neu-continuity maps if E_1 is a Neu- α ga $U_{1/2}$ space.

Proof.

Let λ be a Neu-CS in E₁. Then $f^{-1}(\lambda)$ is a Neu- α GSCS in E₁, by hypothesis. Since E₁ is a Neu- α gaU_{1/2}, $f^{-1}(\lambda)$ is a Neu-CS in E₁. Hence f is a Neu-continuity maps.

Theorem 3.24.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS continuity maps, then f is a Neu-G continuity maps if E_1 is a Neu- α gb $U_{1/2}$ space.

Proof.

Let λ be a Neu-CS in E₁. Then $f^{-1}(\lambda)$ is a Neu- α GSCS in E₁, by hypothesis. Since E₁ is a Neu- α gbU_{1/2}, $f^{-1}(\lambda)$ is a Neu-GCS in E₁. Hence f is a Neu-G continuity maps.

Theorem 3.25.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS continuity maps, then f is a Neu-GS continuity maps if E_1 is a Neu- α gcU_{1/2} space.

Proof.

Let λ be a Neu-CS in E₂. Then $f^{-1}(\lambda)$ is a Neu- α GSCS in E₁, by hypothesis. Since E₁ is a Neu- α gcU_{1/2}, $f^{-1}(\lambda)$ is a Neu-GSCS in E₁. Hence f is a Neu-GS continuity maps.

Theorem 3.26.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS continuity maps and $g:(E_2, \sigma_N) \rightarrow (E_3, \varrho_N)$ be an Neutrosophic continuity, then $g \circ f:(E_1, \tau_N) \rightarrow (E_3, \varrho_N)$ is a Neu- α GS continuity.

Proof.

Let λ be a Neu-CS in E₃. Then $g^{-1}(\lambda)$ is a Neu-CS in E₂, by hypothesis. Since f is a Neu- α GS continuity maps, $f^{-1}(g^{-1}(\lambda))$ is a Neu- α GSCS in E₁. Hence $g \circ f$ is a Neu- α GS continuity maps.

Theorem 3.27.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a maps from Neutrosophic Topology in E_1 in to a Neutrosophic Topology E_2 . Then the following conditions set are equivalent if E_1 is a Neu- $\alpha_{ga}U$ 1/2 space.

- (i) f is a Neu- α GS continuity maps.
- (ii) if μ is a Neutrosophic set in E₂ then f⁻¹(μ) is a Neu- α GSOS in E₁.
- (iii) $f^1(\text{Neu-int}(\mu)) \subseteq \text{Neu-int}(\text{Neu-Cl}(\text{Neu-int}(f^1(\mu))))$ for every Neutrosophic set μ in E_2 .

Proof.

(i) \rightarrow (ii): is obviously true.

- (ii) \rightarrow (iii): Let μ be any Neutrosophic set in E₂. Then Neu-int(μ) is a Neutrosophic set in E₂. Then f⁻¹(Neu-int(μ)) is a Neu- α GSOS in E₁. Since E₁ is a Neu- α gaU_{1/2} space, f⁻¹(Neu-int(μ))is a Neutrosophic set in E₁. Therefore f⁻¹(Neu-int(μ))=Neu-int(f⁻¹(Neu-int(μ))) \subseteq Neu-int(Neu-Cl(Neu-int(μ)))).
- (iii) \rightarrow (i) Let μ be a Neu-CS in E₂. Then its complement μ^{C} is a Neutrosophic set in E₂. By Hypothesis f⁻¹(Neu-int(μ^{C})) \subseteq Neu-int(Neu-Cl(Neu-int(μ^{C})))). This implies that f⁻¹(μ^{C}) \subseteq Neu-int(Neu-Cl(Neu-int(μ^{C}))))). Hence f⁻¹(μ^{C}) is a Neu- α OS in E₁. Since every Neu- α OS is a Neu- α GSOS, f⁻¹(μ^{C}) is a Neu- α GSOS in E₁. Hence f is a Neu- α GS continuity maps. **Theorem 3.28.**

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a maps. Then the following conditions set are equivalent if E_1 is a Neu- $agaU_{1/2}$ space.

- (i) f is a Neu- α GS continuity maps.
- (ii) $f^{-1}(\lambda)$ is a Neu- α GSCS in E₁ for every Neu-CS λ in E₂.
- (iii) Neu-Cl(Neu-int(Neu-Cl($f^1(\lambda)$))) $\subseteq f^1(\text{Neu-Cl}(\lambda))$ for every Neutrosophic set λ in E₂.

Proof.

- (i) \rightarrow (ii): is obviously true.
- (ii) \rightarrow (iii): Let λ be a Neutrosophic set in E2.Then Neu-Cl(λ) is a Neu-CS in E2. By hypothesis, f^{-1} (Neu-Cl(λ)) is a Neu- α GSCS in E1. Since E1 is a Neu- α gaU1/2 space, f^{-1} (Neu-Cl(λ)) is a Neu-CS in E1. Therefore Neu-Cl(f^{-1} (Neu-Cl(λ)))= f^{-1} (Neu-Cl(λ)). NowNeu-Cl(Neu-int(Neu-Cl(f^{-1} (λ)))) \subseteq Neu-Cl(λ)).
- (iii) \rightarrow (i): Let λ be a Neu-CS in E₂. By hypothesis Neu-Cl(Neu-int(Neu-Cl($f^1(\lambda)$))) $\subseteq f^1(\text{Neu-Cl}(\lambda))=f^1(\lambda)$. This implies $f^1(\lambda)$ is a Neu- α CS in E₁ and hence it is a Neu- α GSCS in E₁. Therefore f is a Neu- α GS continuity maps.

Definition 3.29.

Let (E, τ_N) be a Neutrospohic topology. The Neutrospohic alpha generalized semi closure (Neu- α GSCl(λ)in short) for any Neutrosophic set λ is Defined as follows. Neu- α GSCl(λ)= \cap { K|K is a Neu- α GSCS in E₁ and $\lambda \subseteq K$ }. If λ is Neu- α GSCS, then Neu- α GSCl(λ)= λ .

Theorem 3.30.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS continuity maps. Then the following conditions set are hold.

- (i) $f(\text{Neu-}\alpha GSCl(\lambda)) \subseteq \text{Neu-Cl}(f(\lambda))$, for every Neutrosophic set λ in E_1 .
- (ii) Neu- α GSCl(f⁻¹(μ)) \subseteq f⁻¹(Neu-Cl(μ)), for every Neutrosophic set μ in E₂.

Proof.

- (i) Since Neu-Cl($f(\lambda)$)is a Neu-CS in E2 and f is a Neu- α GS continuity maps, f^1 (Neu-Cl($f(\lambda)$))is Neu- α GSCS in
 - E₁. That is Neu- α GSCl(λ) \subseteq f¹(Neu-Cl(f(λ))). Therefore f(Neu- α GSCl(λ)) \subseteq Neu-Cl(f(λ)), for every Neutrosophic set λ in E₁.
- (ii) Replacing λ by $f^{\text{-1}}(\mu)$ in (i) we get $f(\text{Neu-}\alpha GSCl(f^{\text{-1}}(\mu))) \subseteq \text{Neu-Cl}(f(f^{\text{-1}}(\mu))) \subseteq \text{Neu-Cl}(\mu)$. Hence Neu- $\alpha GSCl(f^{\text{-1}}(\mu)) \subseteq \text{Neu-Cl}(\mu)$.
 - $f^{-1}(\mu))\subseteq f^{-1}(\text{Neu-Cl}(\mu))$, for every Neutrosophic set μ in E_2 .

4. Neutrosophic α-Generalized Semi Irresolute Maps

In this section we Introduce Neutrosophic α -generalized semi irresolute maps and study some of its characterizations.

Definition 4.1.

A maps $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ is called a Neutrosophic alpha-generalized semi irresolute (Neu- α GS irresolute) maps if $f^1(\lambda)$ is a Neu- α GSCSin (E_1, τ_N) for every Neu- α GSCS λ of (E_2, σ_N)

Theorem 4.2.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS irresolute, then f is a Neu- α GS continuity maps.

Proof.

Let f be a Neu- α GS irresolute maps. Let λ be any Neu-CS in E2. Since every Neu-CS is a Neu- α GSCS, λ is a Neu- α GSCS in E2. By hypothesis f⁻¹(λ) is a Neu- α GSCS in E2. Hence f is a Neu- α GS continuity maps.

Example 4.3.

Neu- α GS continuity maps is not Neu- α GS irresolute maps.

Let $E_1=\{a_1, a_2\}$, $E_2=\{b_1, b_2\}$, $U=\langle e_1, (.4,.5, .7), (.5,.5,.6)\rangle$ and $V=\langle e_2, (.8,.5,.3), (.4,.6, .7)\rangle$. Then $\tau_N=\{0_N,U,1_N\}$ and $\sigma_N=\{0_N,V,1_N\}$ are Neutrosophic Topologies on E_1 and E_2 respectively. Define a maps $f:(E_1, \tau_N)\rightarrow (E_2, \sigma_N)$ by $f(a_1)=b_1$ and $f(a_2)=b_2$. Then f is a Neu- α GS continuity. We have $\mu=\langle e_2, (.2,.5, .9), (.6,.5, .5)\rangle$ is a Neu- α GSCS in E_2 but $f^1(\mu)$ is not a Neu- α GSCS in E_1 . Therefore f is not a Neu- α GS irresolute maps.

Theorem 4.4.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS irresolute, then f is a Neutrosophic irresolute maps if E_1 is a Neu- α ga $U_{1/2}$ space.

Proof.

Let λ be a Neu-CS in E2. Then λ is a Neu- α GSCS in E2. Therefore $f^1(\lambda)$ is a Neu- α GSCS in E1, by hypothesis. Since E1 is a Neu- α gaU1/2 space, $f^1(\lambda)$ is a Neu-CS in E1. Hence f is a Neutrosophic irresolute maps.

Theorem 4.5.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ and $g:(E_2, \sigma_N) \rightarrow (E_3, \rho_N)$ be Neu- α GS irresolute maps, then $g \circ f:(E_1, \tau_N) \rightarrow (E_3, \rho_N)$ is a Neu- α GS irresolute maps.

Proof.

Let λ be a Neu- α GSCS in E₃. Then $g^{-1}(\lambda)$ is a Neu- α GSCS in E₂. Since f is a Neu- α GS irresolute maps. $f^{-1}((g^{-1}(\lambda)))$ is a Neu- α GSCS in E₁. Hence $g \circ f$ is a Neu- α GS irresolute maps.

Theorem 4.6.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS irresolute and $g:(E_2, \sigma_N) \rightarrow (E_3, \rho_N)$ be Neu- α GS continuity maps, then $g \circ f:(E_1, \tau_N) \rightarrow (E_3, \rho_N)$ is a Neu- α GS continuity maps.

Proof.

Let λ be a Neu-CS in E₃. Then $g^{-1}(\lambda)$ is a Neu- α GSCS in E₂. Since f is a Neu- α GS irresolute, $f^{-1}((g^{-1}(\lambda)))$ is a Neu- α GSCS in E₁. Hence $g \circ f$ is a Neu- α GS continuity maps.

Theorem 4.7.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a Neu- α GS irresolute, then f is a Neu-G irresolute maps if E_1 is a Neu-G space.

Proof.

Let λ be a Neu- α GSCS in E₁. By hypothesis, $f^{-1}(\lambda)$ is a Neu- α GSCS in E₁. Since E₁ is a Neu- α gbU_{1/2} space, $f^{-1}(\lambda)$ is a Neu-GCS in E₁. Hence f is a Neu-G irresolute maps.

Theorem 4.8.

Let $f:(E_1, \tau_N) \rightarrow (E_2, \sigma_N)$ be a maps from a Neutrosophic Topology E_1 Into a Neutrosophic Topology E_2 . Then the following conditions set are equivalent if E_1 and E_2 are Neu- $\alpha_{ga}U_{1/2}$ spaces.

- (i) f is a Neu- α GS irresolute maps.
- (ii) $f^{-1}(\mu)$ is a Neu- α GSOS in E₁ for each Neu- α GSOS μ in E₂.
- (iii) Neu-Cl($f^{-1}(\mu)$) $\subseteq f^{-1}$ (Neu-Cl(μ)) for each Neutrosophic set μ of E₂.

Proof.

- (i) \rightarrow (ii) : Let μ be any Neu- α GSOS in E₂. Then μ^{C} is a Neu- α GSCS in E₂. Since f is Neu- α GS irresolute, f⁻¹(μ^{C}) is a Neu- α GSCS in E₁. But f⁻¹(μ^{C})=(f⁻¹(μ))^C. Therefore f⁻¹(μ) is a Neu- α GSOS in E₁.
- (ii) \rightarrow (iii) : Let μ be any Neutrosophic set in E2and $\mu \subseteq \text{Neu-Cl}(\mu)$. Then $f^1(\mu) \subseteq f^1(\text{Neu-Cl}(\mu))$. Since NeuCl(μ) is a Neu-CS in E2, Neu-Cl(μ) is a Neu- α GSCS in E2. Therefore (Neu-Cl(μ))^C is a Neu- α GSOS in E2. By hypothesis, $f^1((\text{Neu-Cl}(\mu))^C)$ is a Neu- α GSOS in E1. Since $f^1((\text{Neu-Cl}(\mu))^C) = (f^1(\text{Neu-Cl}(\mu)))^C$, $f^1(\text{Neu-Cl}(\mu))$ is a Neu- α GSCS in E1. Since E1 is Neu- α gaU1/2 space, $f^1(\text{Neu-Cl}(\mu))$ is a Neu-CS in E1. Hence Neu-Cl($f^1(\mu)$) $\subseteq \text{Neu-Cl}(f^1(\text{Neu-Cl}(\mu))$.
- (iii) \rightarrow (i): Let μ be any Neu- α GSCS in E2. Since E2 is Neu- α gaU1/2 space, μ is a Neu-CS in E2 and Neu-Cl(μ)= μ .Hence f-1(μ)=f-1(Neu-Cl(μ)=Neu-Cl(f-1(μ)). But clearly f-1(μ)⊆Neu-Cl(f-1(μ)). Therefore Neu-Cl(f-1(μ))=f-1(μ). This implies f-1(μ) is a Neu- α GS and hence it is a Neu- α GSCS in E1. Thus f is a Neu- α GS irresolute maps.

Conclusion

In this research paper using Neu- α GSCS(Neutrosophic α gs-closed sets) we are defined Neu- α GS continuity maps and analyzed its properties.after that we were compared already existing Neutrosophic continuity maps to Neu- α GSCS continuity maps. Furthermore we were extended to this

maps to $\mbox{Neu-}\alpha\mbox{GS}$ irresolute maps , Finally This concepts can be extended to future Research for some mathematical applications.

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