Neutrosophic α^m-continuity

R. Dhavaseelan[1], R. Narmada Devi[2], S. Jafari[3] and Qays Hatem Imran[4]

[1] Department of Mathematics, Sona College of Technology Salem-636005,Tamil Nadu,India.

E-mail: dhavaseelan.r@gmail.com

[2] Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R & D Institute of Science and Technology, Chennai, India.

E-mail: narmadadevi23@gmail.com

[3] Department of Mathematics, College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark.

E-mail: jafaripersia@gmail.com

[4] Department of Mathematics, Al-Muthanna University, College of Education for Pure Science, Iraq.

E-mail:qays.imran@gmail.com

*Correspondence: Author (dhavaseelan.r@gmail.com)

Abstract: In this paper, we introduce and study a new class of neutrosophic closed set called neutrosophic α^m-closed set. In this respect, we introduce the concepts of neutrosophic α^m-continuous, strongly neutrosophic α^m continuous, neutrosophic α^m-irresolute and present their basic properties.

Keywords: Neutrosophic α^m-closed set, neutrosophic α^m-continuous, strongly neutrosophic α^m-continuous, neutrosophic α^m-irresolute.

1 Introduction

In 1965, Zadeh [21] studied the idea of fuzzy sets and its logic. Later, Chang [8] introduced the concept of fuzzy topological spaces. Atanassov [1] discussed the concepts of intuitionistic fuzzy set[[2],[3],[4]]. The concepts of strongly fuzzy continuous and fuzzy gc-irresolute are introduced by G. Balasubramanian and P. Sundaram [6]. The idea of α^m-closed in topological spaces was introduced by M. Mathew and R. Parimelazhagan[16]. He also introduced and investigated, α^m-continuous maps in topological spaces together with S. Jafari[17]. The concept of fuzzy α^m-continuous function was introduced by R. Dhavaseelan[13]. After the introduction of the concept of neutrosophy and neutrosophic set by F. Smarandache [[19], [20]], the concepts of neutrosophic crisp set and neutrosophic crisp topological space were introduced by A. A. Salama and S. A. Alblowi[18]. In this paper, a new class of neutrosophic closed set called neutrosophic α^m closed set is studied. Furthermore, the concepts of neutrosophic α^m-continuous, strongly neutrosophic α^m-continuous, neutrosophic α^m-irresolute are introduced and obtain some interesting properties. Throughout this paper neutrosophic topological spaces (briefly NTS) (S_1, ξ_1),(S_2, ξ_2) and (S_3, ξ_3) will be replaced by S_1,S_2 and S_3, respectively.

2 Preliminiaries

Definition 2.1. [19] Let T,I,F be real standard or non standard subsets of $]0^-, 1^+[$, with $sup_T = t_{sup}, inf_T = t_{inf}$
$sup_I = i_{sup}, inf_I = i_{inf}$
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T, I, F are neutrosophic components.

Definition 2.2. [19] Let S_1 be a non-empty fixed set. A neutrosophic set (briefly N-set) Λ is an object such that $\Lambda = \{\langle x, \mu_{_\Lambda}(x), \sigma_{_\Lambda}(x), \gamma_{_\Lambda}(x)\rangle : x \in S_1\}$ where $\mu_{_\Lambda}(x), \sigma_{_\Lambda}(x)$ and $\gamma_{_\Lambda}(x)$ which represents the degree of membership function (namely $\mu_{_\Lambda}(x)$), the degree of indeterminacy (namely $\sigma_{_\Lambda}(x)$) and the degree of non-membership (namely $\gamma_{_\Lambda}(x)$) respectively of each element $x \in S_1$ to the set Λ.

Remark 2.3. [19]

(1) An N-set $\Lambda = \{\langle x, \mu_{_\Lambda}(x), \sigma_{_\Lambda}(x), \Gamma_{_\Lambda}(x)\rangle : x \in S_1\}$ can be identified to an ordered triple $\langle \mu_{_\Lambda}, \sigma_{_\Lambda}, \Gamma_{_\Lambda}\rangle$ in $]0^-, 1^+[$ on S_1.

(2) In this paper, we use the symbol $\Lambda = \langle \mu_{_\Lambda}, \sigma_{_\Lambda}, \Gamma_{_\Lambda}\rangle$ for the N-set $\Lambda = \{\langle x, \mu_{_\Lambda}(x), \sigma_{_\Lambda}(x), \Gamma_{_\Lambda}(x)\rangle : x \in S_1\}$.

Definition 2.4. [18] Let $S_1 \neq \emptyset$ and the N-sets Λ and Γ be defined as
$\Lambda = \{\langle x, \mu_{_\Lambda}(x), \sigma_{_\Lambda}(x), \Gamma_{_\Lambda}(x)\rangle : x \in S_1\}$, $\Gamma = \{\langle x, \mu_{_\Gamma}(x), \sigma_{_\Gamma}(x), \Gamma_{_\Gamma}(x)\rangle : x \in S_1\}$. Then

(a) $\Lambda \subseteq \Gamma$ iff $\mu_{_\Lambda}(x) \leq \mu_{_\Gamma}(x)$, $\sigma_{_\Lambda}(x) \leq \sigma_{_\Gamma}(x)$ and $\Gamma_{_\Lambda}(x) \geq \Gamma_{_\Gamma}(x)$ for all $x \in S_1$;

(b) $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;

(c) $\bar{\Lambda} = \{\langle x, \Gamma_{_\Lambda}(x), \sigma_{_\Lambda}(x), \mu_{_\Lambda}(x)\rangle : x \in S_1\}$; [Complement of Λ]

(d) $\Lambda \cap \Gamma = \{\langle x, \mu_{_\Lambda}(x) \wedge \mu_{_\Gamma}(x), \sigma_{_\Lambda}(x) \wedge \sigma_{_\Gamma}(x), \Gamma_{_\Lambda}(x) \vee \Gamma_{_\Gamma}(x)\rangle : x \in S_1\}$;

(e) $\Lambda \cup \Gamma = \{\langle x, \mu_{_\Lambda}(x) \vee \mu_{_\Gamma}(x), \sigma_{_\Lambda}(x) \vee \sigma_{_\Gamma}(x), \Gamma_{_\Lambda}(x) \wedge \gamma_{_\Gamma}(x)\rangle : x \in S_1\}$;

(f) $[\ ]\Lambda = \{\langle x, \mu_{_\Lambda}(x), \sigma_{_\Lambda}(x), 1 - \mu_{_\Lambda}(x)\rangle : x \in S_1\}$;

(g) $\langle\rangle\Lambda = \{\langle x, 1 - \Gamma_{_\Lambda}(x), \sigma_{_\Lambda}(x), \Gamma_{_\Lambda}(x)\rangle : x \in S_1\}$.

Definition 2.5. [10] Let $\{\Lambda_i : i \in J\}$ be an arbitrary family of N-sets in S_1. Then

(a) $\bigcap \Lambda_i = \{\langle x, \wedge\mu_{_{\Lambda_i}}(x), \wedge\sigma_{_{\Lambda_i}}(x), \vee\Gamma_{_{\Lambda_i}}(x)\rangle : x \in S_1\}$;

(b) $\bigcup \Lambda_i = \{\langle x, \vee\mu_{_{\Lambda_i}}(x), \vee\sigma_{_{\Lambda_i}}(x), \wedge\Gamma_{_{\Lambda_i}}(x)\rangle : x \in S_1\}$.

In order to develop NTS we need to introduce the N-sets $0_{_N}$ and $1_{_N}$ in S_1 as follows:

Definition 2.6. [10] $0_{_N} = \{\langle x, 0, 0, 1\rangle : x \in S_1\}$ and $1_{_N} = \{\langle x, 1, 1, 0\rangle : x \in S_1\}$.

Definition 2.7. [10] A neutrosophic topology (briefly N-topology) on $S_1 \neq \emptyset$ is a family ξ_1 of N-sets in S_1 satisfying the following axioms:

(i) $0_{_N}, 1_{_N} \in \xi_1$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in \xi_1$,

(iii) $\cup G_i \in \xi_1$ for arbitrary family $\{G_i \mid i \in \Lambda\} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is called an NTS and each N-set in ξ_1 is called a neutrosophic open set (briefly N-open set) . The complement $\overline{\Lambda}$ of an N-open set Λ in S_1 is called a neutrosophic closed set (briefly N-closed set) in S_1.

Definition 2.8. [10] Let Λ be an N-set in an NTS S_1. Then

$Nint(\Lambda) = \bigcup\{G \mid G$ is an N-open set in S_1 and $G \subseteq \Lambda\}$ is called the neutrosophic interior (briefly N-interior) of Λ;

$Ncl(\Lambda) = \bigcap\{G \mid G$ is an N-closed set in S_1 and $G \supseteq \Lambda\}$ is called the neutrosophic closure (briefly N-cl) of Λ.

Definition 2.9. Let $S_1 \neq \emptyset$. If r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the N-set $x_{r,t,s}$ is called a neutrosophic point(briefly NP)in S_1 given by

$$x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & \text{if } x = x_p \\ (0,0,1), & \text{if } x \neq x_p \end{cases}$$

for $x_p \in S_1$ is called the support of $x_{r,t,s}$, where r denotes the degree of membership value, t denotes the degree of indeterminacy and s is the degree of non-membership value of $x_{r,t,s}$.

Now we shall define the image and preimage of N-sets. Let $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$ and $\Omega : S_1 \to S_2$ be a map.

Definition 2.10. [10]

(a) If $\Gamma = \{\langle y, \mu_{_\Gamma}(y), \sigma_{_\Gamma}(y), \Gamma_{_\Gamma}(y)\rangle : y \in S_2\}$ is an N-set in S_1, then the pre-image of Γ under Ω, denoted by $\Omega^{-1}(\Gamma)$, is the N-set in S_1 defined by
$\Omega^{-1}(\Gamma) = \{\langle x, \Omega^{-1}(\mu_{_\Gamma})(x), \Omega^{-1}(\sigma_{_\Gamma})(x), \Omega^{-1}(\Gamma_{_\Gamma})(x)\rangle : x \in S_1\}$.

(b) If $\Lambda = \{\langle x, \mu_{_\Lambda}(x), \sigma_{_\Lambda}(x), \Gamma_{_\Lambda}(x)\rangle : x \in S_1\}$ is an N-set in S_1, then the image of Λ under Ω, denoted by $\Omega(\Lambda)$, is the N-set in S_2 defined by
$\Omega(\Lambda) = \{\langle y, \Omega(\mu_{_\Lambda})(y), \Omega(\sigma_{_\Lambda})(y), (1 - \Omega(1 - \Gamma_{_\Lambda}))(y)\rangle : y \in S_2\}$. where

$$\Omega(\mu_{_\Lambda})(y) = \begin{cases} \sup_{x \in \Omega^{-1}(y)} \mu_{_\Lambda}(x), & \text{if } \Omega^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$\Omega(\sigma_{_\Lambda})(y) = \begin{cases} \sup_{x \in \Omega^{-1}(y)} \sigma_{_\Lambda}(x), & \text{if } \Omega^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$(1 - \Omega(1 - \Gamma_{_\Lambda}))(y) = \begin{cases} \inf_{x \in \Omega^{-1}(y)} \Gamma_{_\Lambda}(x), & \text{if } \Omega^{-1}(y) \neq \emptyset, \\ 1, & \text{otherwise,} \end{cases}$$

In what follows, we use the symbol $\Omega_{-}(\Gamma_{_\Lambda})$ for $1 - \Omega(1 - \Gamma_{_\Lambda})$.

Corollary 2.11. [10] Let Λ , $\Lambda_i(i \in J)$ be N-sets in S_1, Γ, $\Gamma_i(i \in K)$ be N-sets in S_1 and $\Omega : S_1 \rightarrow S_2$ a function. Then

(a) $\Lambda_1 \subseteq \Lambda_2 \Rightarrow \Omega(\Lambda_1) \subseteq \Omega(\Lambda_2)$,

(b) $\Gamma_1 \subseteq \Gamma_2 \Rightarrow \Omega^{-1}(\Gamma_1) \subseteq \Omega^{-1}(\Gamma_2)$,

(c) $\Lambda \subseteq \Omega^{-1}(\Omega(\Lambda))$ { If Ω is injective,then $\Lambda = \Omega^{-1}(\Omega(\Lambda))$ } ,

(d) $\Omega(\Omega^{-1}(\Gamma)) \subseteq \Gamma$ { If Ω is surjective, then $\Omega(\Omega^{-1}(\Gamma)) = \Gamma$ },

(e) $\Omega^{-1}(\bigcup \Gamma_j) = \bigcup \Omega^{-1}(\Gamma_j)$,

(f) $\Omega^{-1}(\bigcap \Gamma_j) = \bigcap \Omega^{-1}(\Gamma_j)$,

(g) $\Omega(\bigcup \Lambda_i) = \bigcup \Omega(\Lambda_i)$,

(h) $\Omega(\bigcap \Lambda_i) \subseteq \bigcap \Omega(\Lambda_i)$ { If Ω is injective,then $\Omega(\bigcap \Lambda_i) = \bigcap \Omega(\Lambda_i)$},

(i) $\Omega^{-1}(1_{_N}) = 1_{_N}$,

(j) $\Omega^{-1}(0_{_N}) = 0_{_N}$,

(k) $\Omega(1_{_N}) = 1_{_N}$, if Ω is surjective

(l) $\Omega(0_{_N}) = 0_{_N}$,

(m) $\overline{\Omega(\Lambda)} \subseteq \Omega(\overline{\Lambda})$, if Ω is surjective,

(n) $\Omega^{-1}(\overline{\Gamma}) = \overline{\Omega^{-1}(\Gamma)}$.

Definition 2.12. [11] An N-set Λ in an NTS (S_1, ξ_1) is called

1) a neutrosophic semiopen set (briefly N-semiopen) if $\Lambda \subseteq Ncl(Nint(\Lambda))$.

2) a neutrosophic α open set (briefly $N\alpha$-open set) if $\Lambda \subseteq Nint(Ncl(Nint(\Lambda)))$.

3) a neutrosophic preopen set (briefly N-preopen set) if $\Lambda \subseteq Nint(Ncl(\Lambda))$.

4) a neutrosophic regular open set (briefly N-regular open set) if $\Lambda = Nint(Ncl(\Lambda))$.

5) a neutrosophic semipre open or β open set (briefly $N\beta$-open set) if $\Lambda \subseteq Ncl(Nint(Ncl(\Lambda)))$.

An N-set Λ is called a neutrosophic semiclosed set, neutrosophic α closed set, neutrosophic preclosed set, neutrosophic regular closed set and neutrosophic β closed set, respectively, if the complement of Λ is an N-semiopen set, $N\alpha$-open set, N-preopen set, N-regular open set, and $N\beta$-open set, respectively.

Definition 2.13. [10] Let (S_1, ξ_1) be an NTS. An N-set Λ in (S_1, ξ_1) is said to be a generalized neutrosophic closed set (briefly g-N-closed set) if $Ncl\,(\Lambda) \subseteq G$ whenever $\Lambda \subseteq G$ and G is an N-open set. The complement of a generalized neutrosophic closed set is called a generalized neutrosophic open set (briefly g-N-open set).

Definition 2.14. [10] Let (S_1, ξ_1) be an NTS and Λ be an N-set in S_1. Then the neutrosophic generalized closure (briefly N-g-cl) and neutrosophic generalized interior (briefly N-g-Int) of Λ are defined by,

(i)$NGcl(\Lambda)$ = $\bigcap${G: G is a g-N-closed set in S_1 and $\Lambda \subseteq G$}.

(ii)$NGint(\Lambda)$ = $\bigcup${G: G is a g-N-open set in S_1 and $\Lambda \supseteq G$}.

3 Neutrosophic α^m continuous functions

Definition 3.1. An N-subset Λ of an NTS (S_1, ξ_1) is called neutrosophic α^m-closed set (briefly $N\alpha^m$-closed set) if $Nint(Ncl(\Lambda)) \subseteq U$ whenever $\Lambda \subset U$ and U is $N\alpha$-open.

Definition 3.2. An N-subset λ of an NTS (S_1, ξ_1) is called a neutrosophic αg-closed set (briefly $N\alpha g$-closed set) if $\alpha Ncl(\lambda) \subseteq U$ whenever $\lambda \subseteq U$ and U is an $N\alpha$-open set in S_1.

Definition 3.3. An N-subset λ of an NTS (S_1, ξ_1) is called a neutrosophic $g\alpha$-closed set (briefly $Ng\alpha$-closed set) if $\alpha Ncl(\Lambda) \subseteq U$ whenever $\Lambda \subseteq U$ and U is an N-open set in S_1.

Remark 3.4. In an NTS (S_1, ξ_1), the following statements are true:

(i) Every N-closed set is an Ng-closed set.

(ii) Every N-closed set is an $N\alpha$-closed set.

Remark 3.5. In an NTS (S_1, ξ_1), the following statements are true:

(i) Every Ng-closed set is an $Ng\alpha$-closed set.

(ii) Every $N\alpha$-closed set is an $N\alpha$g-closed set.

(iii) Every $N\alpha g$-closed set is an $Ng\alpha$-closed set.

Remark 3.6. In an NTS (S_1, ξ_1), the following statements are true:

(i) Every N-closed set is an $N\alpha^m$-closed set.

(ii) Every $N\alpha^m$-closed set is an $N\alpha$-closed set.

(iii) Every $N\alpha^m$-closed set is an $N\alpha$g-closed set.

(iv) Every $N\alpha^m$-closed set is an $Ng\alpha$-closed set.

Proof. (i) This follows directly from the definitions.
(ii) Let Λ be an $N\alpha^m$-closed set in S_1 and U a N-open set such that $\lambda \subseteq U$. Since every N-open set is an $N\alpha$-open set and Λ is a $N\alpha^m$-closed set, $Nint(Ncl(\Lambda)) \subseteq (Nint(Ncl(\Lambda))) \cup (Ncl(Nint(\Lambda))) \subseteq U$. Therefore, Λ is an $N\alpha$-closed set in S_1.
(iii) It is a consequence of (ii) and remark 3.5 (ii).
(iv) It is a consequence of (iii) and remark 3.5 (iii). □

Proposition 3.7. The intersection of an $N\alpha^m$-closed set and an N-closed set is an $N\alpha^m$-closed set.

Proof. Let Λ be an $N\alpha^m$-closed set and Ψ an N-closed set. Since Λ is an $N\alpha^m$-closed set, $Nint(Ncl(\Lambda)) \subseteq U$ whenever $\Lambda \subseteq U$, where U is an $N\alpha$-open set. To show that $\Lambda \cap \Psi$ is an $N\alpha^m$-closed set, it is enough to show that $Nint(Ncl(\Lambda \cap \Psi)) \subseteq U$ whenever $\Lambda \cap \Psi \subseteq U$, where U is an $N\alpha$-open set. Let $M = S_1 - \Psi$. Then $\Lambda \subseteq U \cup M$. Since M is an N-open set, $U \cup M$ is an $N\alpha$-open set and Λ is an $N\alpha^m$-closed set, $Nint(Ncl(\Lambda)) \subseteq U \cup M$. Now, $Nint(Ncl(\Lambda \cap \Psi)) \subseteq Nint(Ncl(\Lambda)) \cap Nint(Ncl(\Psi)) \subseteq Nint(Ncl(\Lambda)) \cap \Psi \subseteq (U \cup M) \cap \Psi \subseteq (U \cap \Psi) \cup (M \cap \Psi) \subseteq (U \cap \Psi) \cup 0_N \subseteq U$. This implies that $\Lambda \cap \Psi$ is an $N\alpha^m$-closed set. □

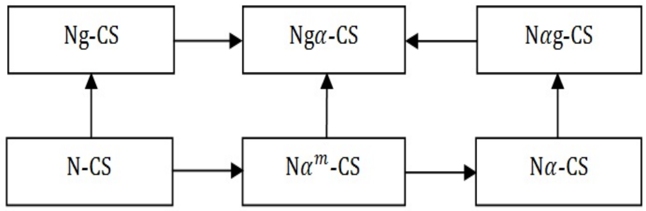

Figure 1: Implications of a neutrosophic α^m-closed set

Proposition 3.8. If Λ and Γ are two $N\alpha^m$-closed sets in an NTS (S_1, ξ_1), then $\Lambda \cap \Gamma$ is an $N\alpha^m$-closed set in S_1.

Proof. Let Λ and Γ be two $N\alpha^m$-closed sets in an NTS (S_1, ξ_1). Let U be a $N\alpha$-open set in S_1 such that $\Lambda \cap \Gamma \subseteq U$. Now, $Nint(Ncl(\Lambda \cap \Gamma)) \subseteq Nint(Ncl(\Lambda)) \cap Nint(Ncl(\Gamma)) \subseteq U$. Hence $\Lambda \cap \Gamma$ is an $N\alpha^m$-closed set. □

Proposition 3.9. Every $N\alpha^m$-closed set is $N\alpha$-closed set.

The converse of the above Proposition 3.9 need not be true.

Example 3.10. Let $S_1 = \{a, b, c\}$. Define the N-subsets Λ and Γ as follows
$\Lambda = \{x, (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.7}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5})\}$, $\Gamma = \{x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7})\}$. Then $\xi_1 = \{0_{S_1}, 1_{S_1}, \Lambda, \Gamma\}$ is an N-topology on S_1. Clearly (S_1, ξ_1) is an NTS. Observe that the N-subset $\Sigma = \{x, (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.7}, \frac{b}{0.4}, \frac{c}{0.5})\}$ is $N\alpha$-closed but it is not $N\alpha^m$-closed set.

Definition 3.11. Let (S_1, ξ_1) be an NTS and Λ an N-subset of S_1. Then the neutrosophic α^m-interior (briefly $N\alpha^m$-I) and the neutrosophic $N\alpha^m$-closure (briefly $N\alpha^m$-cl) of Λ are defined by,
$\alpha^m Nint(\Lambda) = \cup\{U | U \text{is } N\alpha^m\text{-open set in } S_1 \text{ and } \Lambda \supseteq U\}$
$\alpha^m Ncl(\Lambda) = \cap\{U | U \text{is } N\alpha^m\text{-closed set in } S_1 \text{ and } \Lambda \subseteq U\}$.

Proposition 3.12. If Λ is an $N\alpha^m$-c-set and $\Lambda \subseteq \Gamma \subseteq Nint(Ncl(\Lambda))$, then Γ is $N\alpha^m$-c-set.

Proof. Let Λ be an $N\alpha^m$-c-set such that $\Lambda \subseteq \Gamma \subseteq Nint(Ncl(\Lambda))$. Let U be an $N\alpha$-open set of S_1 such that $\Gamma \subseteq U$. Since Λ is $N\alpha^m$-c-set, we have $Nint(Ncl(\Lambda)) \subseteq U$, whenever $\Lambda \subseteq U$. Since $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq Nint(Ncl(\Lambda))$, then $Nint(Ncl(\Gamma)) \subseteq Nint(Ncl(Nint(Ncl(\Lambda)))) \subseteq Nint(Ncl(\lambda)) \subseteq U$. Therefore $Nint(Ncl(\Gamma)) \subseteq U$. Hence Γ is an $N\alpha^m$-c-set in S_1. □

Remark 3.13. The union of two $N\alpha^m$-c-sets need not be an $N\alpha^m$-c-set.

Remark 3.14. The following are the implications of an $N\alpha^m$-c-set and the reverses are not true.

Definition 3.15. Let (S_1, ξ_1) and (S_2, ξ_2) be any two NTS.

1) A map $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ is called neutrosophic α^m-continuous (briefly $N\alpha^m$-$cont$) if the inverse image of every N-closed set in (S_2, ξ_2) is $N\alpha^m$-c-set in (S_1, ξ_1).
Equivalently if the inverse image of every N-open set in (S_2, ξ_2) is $N\alpha^m$-open set in (S_1, ξ_1).

2) A map $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ is called neutrosophic α^m-irresolute (briefly $N\alpha^m$-I) if the inverse image of every $N\alpha^m$-c-set in (S_2, ξ_2) is $N\alpha^m$-c-set in (S_1, ξ_1).
Equivalently if the inverse image of every $N\alpha^m$-open set in (S_2, ξ_2) is $N\alpha^m$-open set in (S_1, ξ_1).

3) A map $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ is called strongly neutrosophic α^m-continuous (briefly $SN\alpha^m$-$cont$) if the inverse image of every $N\alpha^m$-c-set in (S_2, ξ_2) is N-closed set in (S_1, ξ_1).
Equivalently if the inverse image of every $N\alpha^m$-open set in (S_2, ξ_2) is N-open set in (S_1, ξ_1).

Proposition 3.16. Let (S_1, ξ_1) and (S_2, ξ_2) be any two NTS. If $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ is NC, then it is $N\alpha^m$-$cont$.

Proof. Let Λ be any N-closed set in (S_2, ξ_2). Since f is NC, $\Omega^{-1}(\Lambda)$ is N-closed in (S_1, ξ_1). Since every N-closed set is $N\alpha^m$-c-set, $\Omega^{-1}(\Lambda)$ is $N\alpha^m$-c-set in (S_1, ξ_1). Therefore Ω is $N\alpha^m$-$cont$. □

The converse of Proposition 3.16 need not be true as it is shown in the following example.

Example 3.17. Let $S_1 = \{a, b, c\}$ and $S_2 = \{a, b, c\}$. Define N-subsets E, F, G and D as follows $E = \{x, (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.3}), (\frac{a}{0.7}, \frac{b}{0.5}, \frac{c}{0.7})\}$, $F = \{x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.6})\}$, $G = \{x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5})\}$, and $D = \{x, (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.5}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.5}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.5})\}$. Then the family $\xi_1 = \{0_{S_1}, 1_{S_1}, E, F\}$ is an NT on S_1 and $\xi_2 = \{0_{S_2}, 1_{S_2}, G, D\}$ is an NT on S_2. Thus (S_1, ξ_1) and (S_2, ξ_2) are NTS. Define $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ as $\Omega(a) = a, \Omega(b) = c, \Omega(c) = b$. Clearly Ω is $N\alpha^m$-$cont$ but Ω is not NC since $\Omega^{-1}(D) \notin \xi_1$ for $D \in \xi_2$.

Proposition 3.18. Let (S_1, ξ_1) and (S_2, ξ_2) be any two neutrosophic NTS. If $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ is $N\alpha^m$-I, then it is $N\alpha^m$-$cont$.

Proof. Let Λ be an N-closed set in (S_2, ξ_2). Since every N-closed set is $N\alpha^m$-c-set, Λ is $N\alpha^m$-c-set in S_2. Since Ω is $N\alpha^m$-I, $\Omega^{-1}(\Lambda)$ is $N\alpha^m$-c-set in (S_1, ξ_1). Therefore Ω is $N\alpha^m$-$cont$. □

The converse of Proposition 3.18 need not be true.

Example 3.19. Let $S_1 = \{a, b, c\}$. Define the N-subsets E,F and G as follows $E = \{x, (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.4}), (\frac{a}{0.6}, \frac{b}{0.5}, \frac{c}{0.6})\}$, $F = \{x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5})\}$ and $G = \{x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.7}), (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.7}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.3})\}$. Then $\xi_1 = \{0_{S_1}, 1_{S_1}, E, F\}$ and $\xi_2 = \{0_{S_1}, 1_{S_1}, C\}$ are N-topologies on S_1. Define $\Omega : (S_1, \xi_1) \to (S_1, \xi_2)$ as follows $\Omega(a) = b, \Omega(b) = a, \Omega(c) = c$. Observe that Ω is $N\alpha^m$-continuous. But Ω is not $N\alpha^m$-I. Since $D = \{x, (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.2}), (\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.2}), (\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.8})\}$ is $N\alpha^m$-c-set in (S_1, ξ_2), $\Omega^{-1}(D)$ is not $N\alpha$-c-set in (S_1, ξ_1).

Proposition 3.20. Let (S_1, ξ_1) and (S_2, ξ_2) be any two NTS. If $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ is $SN\alpha^m$-I, then it is NC.

Proof. Let Λ be an N-closed set in (S_2, ξ_2). Since every N-closed set is $N\alpha^m$-c-set. Since Ω is $SN\alpha^m$-$cont$, $\Omega^{-1}(\Lambda)$ is N-closed set in (S_1, ξ_1). Therefore Ω is NC. □

The converse of Proposition 3.20 need not be true.

Example 3.21. Let $S_1 = \{a, b, c\}$. Define the N-subsets E,F and G as follows $E = \{x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{0.9})\}$, $F = \{x, (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0}), (\frac{a}{0.1}, \frac{b}{0.1}, \frac{c}{0}), (\frac{a}{0.9}, \frac{b}{0.9}, \frac{c}{1})\}$ and $G = \{x, (\frac{a}{0.1}, \frac{b}{0}, \frac{c}{0.1}), (\frac{a}{0.1}, \frac{b}{0}, \frac{c}{0.1}), (\frac{a}{0.9}, \frac{b}{1}, \frac{c}{0.9})\}$. Then $\xi_1 = \{0_{S_1}, 1_{S_1}, E, F\}$ and $\xi_2 = \{0_{S_1}, 1_{S_1}, G\}$ are N-topologies on S_1. Define $\Omega(S_1, \xi_1) \to (S_1, \xi_2)$ as follows $\Omega(a) = \Omega(b) = a, \Omega(c) = c$. Ω is NC but Ω is not $SN\alpha^m$-$cont$. Since $D = \{x, (\frac{a}{0.05}, \frac{b}{0}, \frac{c}{0.1}), (\frac{a}{0.05}, \frac{b}{0}, \frac{c}{0.1}), (\frac{a}{0.95}, \frac{b}{1}, \frac{c}{0.9})\}$ is $N\alpha^m$-c-set in (S_1, ξ_2), $\Omega^{-1}(D)$ is not N-closed set in (S_1, ξ_1).

Proposition 3.22. Let $(S_1, \xi_1), (S_2, \xi_2)$ and (S_3, ξ_3) be any three NTS. Suppose $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$, $\Xi : (S_2, \xi_2) \to (S_3, \xi_3)$ are maps. Assume Ω is $N\alpha^m$-I and Ξ is $N\alpha^m$-$cont$, then $\Xi \circ \Omega$ is $N\alpha^m$-$cont$.

Proof. Let Λ be an N-closed set in (S_3, ξ_3). Since Ξ is $N\alpha^m$-$cont$, $\Xi^{-1}(\Lambda)$ is $N\alpha^m$-c-set in (S_2, ξ_2). Since Ω is $N\alpha^m$-I,$\Omega^{-1}(\Xi^{-1}(\Lambda))$ is $N\alpha^m$-closed in (S_1, ξ_1). Thus $\Xi \circ \Omega$ is $N\alpha^m$-$cont$. $\square$

Proposition 3.23. Let $(S_1, \xi_1), (S_2, \xi_2)$ and (S_3, ξ_3) be any three NTS. Let $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ and $\Xi : (S_2, \xi_2) \to (S_3, \xi_3)$ be maps such that Ω is $SN\alpha^m$-$cont$ and Ξ is $N\alpha^m$-$cont$, then $\Xi \circ \Omega$ is NC.

Proof. Let Λ be an N-c-set in (S_3, ξ_3). Since Ξ is $N\alpha^m$-$cont$, $\Xi^{-1}(\Lambda)$ is $N\alpha^m$-c-set in (S_2, ξ_2). Moreover, since Ω is $SN\alpha^m$-$cont$, $\Omega^{-1}(\Omega^{-1}(\Lambda))$ is N-closed in (S_1, ξ_1). Thus $\Xi \circ \Omega$ is NC. $\square$

Proposition 3.24. Let $(S_1, \xi_1), (S_2, \xi_2)$ and (S_3, ξ_3) be any three NTS. Let $\Omega : (S_1, \xi_1) \to (S_2, \xi_2)$ and $\Xi : (S_2, \xi_2) \to (S_3, \xi_3)$ be two maps. Assume Ω and Ξ are $N\alpha^m$-I, then $\Xi \circ \Omega$ is $N\alpha^m$-I.

Proof. Let Λ be an $N\alpha^m$-c-set in (S_3, ξ_3). Since Ξ is $N\alpha^m$-I, $\Xi^{-1}(\Lambda)$ is $N\alpha^m$-c-set in (S_2, ξ_2). Since Ω is $N\alpha^m$-I, $\omega^{-1}(\Xi^{-1}(\Lambda))$ is an $N\alpha^m$-c-set in (S_1, ξ_1). Thus $\Xi \circ \Omega$ is $N\alpha^m$-I. $\square$

4 Conclusions

In this paper, a new class of neutrosophic closed set called neutrosophic α^m closed set is introduced and studied. Furthermore, the basic properties of neutrosophic α^m-continuity are presented with some examples.

References

[1] K. Atanassov. Intuitionistic fuzzy sets, in: Sgurev, Ed., VII ITKR's Session, Sofia (June 1983 Central Sci. and Techn. Library, Bulg. Academy of Sciences.,(1984).

[2] K. Atanassov. Intuitionistic fuzzy sets, Fuzzy Sets and Systems., 20 (1986),87–96.

[3] K. Atanassov. Review and new results on intuitionistic fuzzy sets, Preprint IM-MFAIS-1-88, Sofia., 1988.

[4] K. Atanassov and S. Stoeva. Intuitionistic fuzzy sets, in: Polish Syrup. on Interval & Fuzzy Mathematics, Poznan., (August 1983),23–26.

[5] K. K. Azad. On fuzzy semicontinuity, Fuzzy almost continuity and fuzzy weakly continuity, Journal of Mathematical Analysis and Applications, 82(1981), 14–32.

[6] G. Balasubramanian, P.Sundaram. On Some generalizations of fuzzy continuous functions, Fuzzy Sets and Systems, 86(1997), 93–100.

[7] A.S. Bin Shahna. On fuzzy strong semicontinuity and fuzzy precontinuity, Fuzzy Sets and Systems,44(1991),303–308.

[8] C. L. Chang. Fuzzy topological spaces,Journal of Mathematical Analysis and Applications, 24(1968),182–190.

[9] R. Dhavaseelan, E. Roja and M. K. Uma. Generalized intuitionistic fuzzy closed sets, Advances in Fuzzy Mathematics, 5(2010),152–172.

[10] R. Dhavaseelan and S. Jafari. Generalized neutrosophic closed sets, In New Trends in Neutrosophic Theory and Application; F. Smarandache and S. Pramanik (Editors), Pons Editions, Brussels, Belgium., Vol. 2, 2018, pp. 261–274.

[11] R. Dhavaseelan, M. Parimala, S. Jafari, F. Smarandache. On Neutrosophic semi-supra open set and neutrosophic semi-supra continuous functions,Neutrosophic Sets and Systems, 16(2017),39–43.

[12] R. Dhavaseelan, E. Roja and M. K. Uma. Intuitionistic fuzzy exponential map via generalized open set, International Journal of Mathematical Archive,3(2)(2012), 636–643.

[13] R. Dhavaseelan. Fuzzy α^m continuity,Asian Journal of Mathematics and Computer Research, 15(1)(2017),18–22.

[14] N. Levine. Decomposition of continuity in topological spaces, The American Mathematical Monthly, 60(1961), 44–46.

[15] N. Levine. Semi-open sets and semi-continuity in topological spaces, The American Mathematical Monthly, 70(1963), 36–41.

[16] M. Mathew and R. Parimelazhagan. α^m closed in topological spaces, International Journal of Mathematical Analysis, 8(47)(2014),2325–2329.

[17] M. Mathew, R. Parimelazhagan, and S.Jafari. α^m continuous maps in topological spaces, arXiv:1601.03518.

[18] A. A. Salama and S. A. Alblowi. Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Mathematics, 3(4)(2012),31–35.

[19] F. Smarandache , Neutrosophy and Neutrosophic Logic , First International Conference on Neutrosophy , Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002) , smarand@unm.edu

[20] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability., American Research Press, Rehoboth, NM,(1999).

[21] L. A. Zadeh. Fuzzy sets, Information and Control, 8(1965), 338–353.