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Neutrosophic Sets and Systems

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The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel’s dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $[0, 1]^3$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Multi-valued Neutrosophic Sets and its Application in Multi-criteria Decision-making Problems

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Abstract. In recent years, hesitant fuzzy sets and neutrosophic sets have aroused the interest of researchers and have been widely applied to multi-criteria decision-making problems. The operations of multi-valued neutrosophic sets are introduced and a comparison method is developed based on related research of hesitant fuzzy sets and intuitionistic fuzzy sets in this paper. Furthermore, some multi-valued neutrosophic number aggregation operators are proposed and the desirable properties are discussed as well. Finally, an approach for multi-criteria decision-making problems was explored applying the aggregation operators. In addition, an example was provided to illustrate the concrete application of the proposed method.

Keywords: Multi-valued neutrosophic sets; multi-criteria decision-making; aggregation operators

1. Introduction

Atanassov introduced intuitionistic fuzzy sets (AIFSs) [1-4], which an extension of Zadeh’s fuzzy sets (FSs) [5]. As for the present, AIFS has been widely applied in solving multi-criteria decision-making (MCDM) problems [6-10], neural networks [11, 12], medical diagnosis [13], color region extraction [14, 15], market prediction [16]. Then, AIFS was extended to the interval-valued intuitionistic fuzzy sets (AIVIFSs) [17]. AIFS took into account membership degree, non-membership degree and degree of hesitation simultaneously. So it is more flexible and practical in addressing the fuzziness and uncertainty than the traditional FSs. Moreover, in some actual cases, the membership degree, non-membership degree and hesitation degree of an element in AIFS may not be only one specific number. To handle the situations that people are hesitant in expressing their preference over objects in a decision-making process, hesitant fuzzy sets (HFSs) were introduced by Torra [18] and Narukawa [19]. Then generalized HFSs and dual hesitant fuzzy sets (DHFSs) were developed by Qian and Wang [20] and Zhu et al. [21] respectively.

Although the FS theory has been developed and generalized, it can not deal with all sorts of uncertainties in different real physical problems. Some types of uncertainties such as the indeterminate information and inconsistent information can not be handled. For example, when we ask the opinion of an expert about certain statement, he or she may say that the possibility that the statement is true is 0.6, the statement is false is 0.3 and the
degree that he or she is not sure is 0.2 [22]. This issue is beyond the field of the FSs and AIFSs. Therefore, some new theories are required.

Florentin Smarandache coined neutrosophic logic and neutrosophic sets (NSs) in 1995 [23, 24]. A NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies in \( [0, 1] \), the non-standard unit interval [25]. Obviously, it is the extension to the standard interval \([0, 1]\) as in the AIFS. And the uncertainty present here, i.e. indeterminacy factor, is independent of truth and falsity values while the incorporated uncertainty is dependent of the degree of belongingness and degree of non belongingness in AIFSs [26]. So for the aforementioned example, it can be expressed as \( x(0.6, 0.3, 0.2) \) in the form of NS.

However, without being specified, it is difficult to apply in the real applications. Hence, a single-valued neutrosophic sets (SVNSs) was proposed, which is an instance of the NSs [22, 26]. Furthermore, the information energy of SVNSs, correlation and correlation coefficient of SVNSs as well as a decision-making method based on SVNSs were presented [27]. In addition, Ye also introduced the concept of simplified neutrosophic sets (SNSs), which can be described by three real numbers in the real unit interval \([0,1]\), and proposed a MCDM using aggregation operators for SNSs [28]. Majumdar et al. introduced a measure of entropy of a SNS [26]. Wang et al. and Lupiáñez proposed the concept of interval-valued neutrosophic sets (IVNS) and gave the set-theoretic operators of IVNS [29, 30]. Furthermore, Ye proposed the similarity measures between SVNS and INSs based on the relationship between similarity measures and distances [31, 32].

However, in some cases, the operations of SNSs in Ref. [28] might be irrational. For instance, the sum of any element and the maximum value should be equal to the maximum one, while it does not hold with the operations in Ref. [28]. Furthermore, decision-makers also hesitant to express their evaluation values for each membership in SNS. For instance, in the example given above, if decision-maker think that the possibility that statement is true is 0.6 or 0.7, the statement is false is 0.2 or 0.3 and the degree that he or she is not sure is 0.1 or 0.2. Then how to handle these circumstances with SVNS is also a problem. At the same time, if the operations and comparison method of SVNSs are extended to multi-valued in SVNS, then there exist shortcomings else as we discussed earlier. Therefore, the definition of multi-valued neutrosophic sets (MVNSs) and its operations along with comparison approach between multi-valued neutrosophic numbers (MVNNs), and aggregation operators for MVNS are defined in this paper. Thus, a MCDM method is established based on the proposed operators, an illustrative example is given to demonstrate the application of the proposed method.
The rest of paper is organized as follows. Section 2 briefly introduces the concepts and operations of NSs and SNSs. The definition of MVNS along with its operations and comparison approach for MVNSs is defined on the basis of AIFS and HFSs in Section 3. Aggregation operators MVNNs are given and a MCDM method is developed in Section 4. In Section 5, an illustrative example is presented to illustrate the proposed method and the comparative analysis and discussion were given. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, definitions and operations of NSs and SNSs are introduced, which will be utilized in the rest of the paper.

Definition 1 [25]. Let $X$ be a space of points (objects), with a generic element in $X$ denoted by $x$. A NS $A$ in $X$ is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $\mathbb{R}$, that is,

\[ T_A(x) : X \rightarrow [0, 1], I_A(x) : X \rightarrow [0, 1], F_A(x) : X \rightarrow [0, 1] \]

are singleton subintervals/subsets in the real standard $[0, 1]$, which is called a SNS. It is a subclass of NSs. The operational relations of SNSs are also defined in Ref. [28].

Definition 2 [25]. A NS $A$ is contained in the other NS $B$, denoted as $A \subseteq B$, if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup I_A(x) \leq \sup I_B(x)$, $\sup F_A(x) \leq \sup F_B(x)$ for $x \in X$.

Since it is difficult to apply NSs to practical problems, Ye reduced NSs of nonstandard intervals into a kind of SNSs of standard intervals that will preserve the operations of the NSs [26].

Definition 3 [28]. Let $X$ be a space of points (objects), with a generic element in $X$ denoted by $x$. A NS $A$ in $X$ is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals/subsets in the real standard $[0, 1]$, that is $T_A(x) : X \rightarrow [0, 1]$, $I_A(x) : X \rightarrow [0, 1]$, and $F_A(x) : X \rightarrow [0, 1]$.

Then, a simplification of $A$ is denoted by

\[ A = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X \} \]

which is called a SNS. It is a subclass of NSs. The operational relations of SNSs are also defined in Ref. [28].

Definition 4 [31]. Let $A$ and $B$ are two SNSs. For any $x \in X$,

\[ A + B = \{T_A(x) + T_B(x) - T_A(x) \cdot T_B(x), T_A(x) + I_A(x) - I_A(x) \cdot I_B(x), T_A(x) + F_A(x) - F_A(x) \cdot F_B(x)\} \]

\[ A : B = \{T_A(x) \cdot T_B(x), T_A(x) \cdot I_B(x), T_A(x) \cdot F_B(x)\} \]

\[ A \cdot A = \{1 - (1 - T_A(x))^2, 1 - (1 - I_A(x))^2, 1 - (1 - F_A(x))^2\} \]

\[ A^\lambda = \{T_A(x)^\lambda, I_A(x)^\lambda, F_A(x)^\lambda\} \]

\[ A^{\lambda\beta} = \{T_A(x)^{\lambda\beta}, I_A(x)^{\lambda\beta}, F_A(x)^{\lambda\beta}\} \]

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It has some limitations in Definition 9.

1. In some situations, the operations, such as \( A + B \) and \( A \cdot B \), as given in Definition 9, might be irrational. This is shown in the example below.

Let \( a = < 0.5, 0.5, 0.5 >, \ a^* = < 1, 0, 0 > \) be two simplified neutrosophic numbers (SNNs). Obviously, \( a^* = < 1, 0, 0 > \) is the maximum of the SNS. It is notorious that the sum of any number and the maximum number should be equal to the maximum one. However, according to the equation (1) in Definition 9, \( a + b = < 1, 0.5, 0.5 > \neq b \).

Hence, the equation (1) does not hold. So does the other equations in Definition 9. It shows that the operations above are incorrect.

2. The correlation coefficient for SNSs in Ref. [27] on basis of the operations does not satisfy in some special cases.

Let \( a_1 = < 0.8, 0, 0 > \) and \( a_2 = < 0.7, 0, 0 > \) be two SNSs, and \( a^* = < 1, 0, 0 > \) be the maximum of the SNS. According to the MCDM based on the correlation coefficient for SNSs under the simplified neutrosophic environment in Ref. [29], we can obtain the result \( S_1(a_1, a^*) = S_2(a_2, a^*) = 1 \), that is, the alternative \( a_1 \) is equal to alternative \( a_2 \). We cannot distinguish the best one else. However, \( T_{a_1}(x) > T_{a_2}(x) \), \( I_{a_1}(x) > I_{a_2}(x) \) and \( F_{a_1}(x) < F_{a_2}(x) \), it is clear that the alternative \( a_2 \) is superior to alternative \( a_1 \).

3. If \( I_a = I_B \), then \( A \) and \( B \) are reduced to two AIFNs. However, above operations are not in accordance with the laws for two AIFSs in [4,6-10,30].

3. Multi-valued neutrosophic sets and their operations

In this section, MVNSs is defined, and its operations based on AIFSs [4,6-10,30] are developed as well.

Definition 5. Let \( X \) be a space of points (objects), with a generic element in \( X \) denoted by \( x \). A MVNS \( A \) in \( X \) is characterized by three functions \( \tilde{T}_a(x) \), \( \tilde{I}_a(x) \) and \( \tilde{F}_a(x) \) in the form of subset of \( [0, 1] \), which can be denoted as follows:
\[ A = \{<x, \tilde{T}_a(x), \tilde{I}_a(x), \tilde{F}_a(x) > | x \in X \} \]

where \( \tilde{T}_a(x), \tilde{I}_a(x), \) and \( \tilde{F}_a(x) \) are three sets of some values in \([0,1]\), denoting the truth-membership degree, indeterminacy-membership function and falsity-membership degree respectively, with the conditions:

\[ 0 \leq \gamma, \eta, \xi \leq 1, 0 \leq \gamma^+ + \eta^+ + \xi^+ \leq 3, \]

where \( \gamma \in \tilde{T}_a(x), \eta \in \tilde{I}_a(x), \xi \in \tilde{F}_a(x), \) and \( \gamma^+ = \text{sup} \tilde{T}_a(x), \eta^+ = \text{sup} \tilde{I}_a(x) \) and \( \xi^+ = \text{sup} \tilde{F}_a(x), \tilde{T}_a(x), \tilde{I}_a(x), \text{and} \) \( \tilde{F}_a(x) \) are set of crisp values between zero and one. For convenience, we call \( A = \{<\tilde{T}_a, \tilde{I}_a, \tilde{F}_a > \} \) the multi-valued neutrosophic number (MVNN). Apparently, MVNSs are an extension of NSs. Especially, if \( \tilde{T}_a, \tilde{I}_a, \) and \( \tilde{F}_a \) have only one value \( \gamma, \eta \) and \( \xi \), respectively, and \( 0 \leq \gamma + \eta + \xi \leq 3 \), then the MVNSs are reduced to SNS; If \( \tilde{I}_a = \emptyset \), then the MVNSs are reduced to DHFSs; If \( \tilde{I}_a = \tilde{F}_a = \emptyset \), then the MVNSs are reduced to HFSs. Thus the MVNSs are an extension of these sets above.

The operational relations of MVNSs are also defined as follows.

**Definition 6.** The complement of a MVNS

\[ A = \{<\tilde{T}_a, \tilde{I}_a, \tilde{F}_a > \} \text{ is denoted by } A^C \text{ and is defined by } 
\]

\[ A^C = \{<\gamma', \eta', \xi'| \text{ where } \gamma' = \inf \tilde{T}_a, \eta' = \inf \tilde{I}_a, \text{ and } \xi' = \inf \tilde{F}_a \} \).

**Definition 7.** The MVNS \( A = \{<\tilde{T}_a, \tilde{I}_a, \tilde{F}_a > \} \) is contained in the other MVNS \( B = \{<\tilde{T}_b, \tilde{I}_b, \tilde{F}_b > \} \), \( A \subseteq B \) if and only if \( \gamma_a \leq \gamma_b, \eta_a \geq \eta_b \text{ and } \xi_a \geq \xi_b \).

Where \( \gamma_a = \inf \tilde{T}_a, \gamma_a = \sup \tilde{T}_a, \eta_a = \inf \tilde{I}_a, \eta_a = \sup \tilde{I}_a \) and \( \xi_a = \inf \tilde{F}_a, \xi_a = \sup \tilde{F}_a \).

**Definition 8.** Let \( A = \{<\tilde{T}_a, \tilde{I}_a, \tilde{F}_a > \} \), \( B = \{<\tilde{T}_b, \tilde{I}_b, \tilde{F}_b > \} \) be two MVNNs, and \( \lambda > 0 \). The operations for MVNNs are defined as follows.

\[ (1) \quad A + B = \left\{ \begin{array}{ll} \bigcup_{\gamma \in \tilde{T}_a, \eta \in \tilde{I}_a, \xi \in \tilde{F}_a} (\gamma + \gamma_B - \gamma_A \cdot \gamma_B), & \eta = \eta + \eta_B - \eta_A \cdot \eta_B, \\ \eta = \eta + \eta_B - \eta_A \cdot \eta_B, & A \end{array} \right\}; \]

\[ (2) \quad A \cdot B = \left\{ \begin{array}{ll} \bigcup_{\gamma \in \tilde{T}_a, \eta \in \tilde{I}_a, \xi \in \tilde{F}_a} (\gamma \cdot \gamma_B), & \eta = \eta + \eta_B - \eta_A \cdot \eta_B, \\ \eta = \eta + \eta_B - \eta_A \cdot \eta_B, & A \end{array} \right\}; \]

\[ (3) \quad A = \{<\tilde{T}_a, \tilde{I}_a, \tilde{F}_a > \} \text{ are reduced to HFSs if } \tilde{I}_a = \emptyset, \text{ and the MVNSs are reduced to DHFSs if } \tilde{I}_a = \tilde{F}_a = \emptyset \text{, then the operations for MVNNs as follows:} \]

\[ (4) \quad A = \{<\tilde{T}_a, \tilde{I}_a, \tilde{F}_a > \} \text{ are reduced to the operations for MVNNs as follows:} \]

\[ (5) \quad A_B = \{<1 - (1 - T_A)^{1}, (I_A)^{1}, (F_A)^{1} > \}; \]

\[ (6) \quad A = \{<T_A^{1}, 1 - (1 - I_A)^{1}, 1 - (1 - F_A)^{1} > \}; \]

\[ (7) \quad A \cdot B = \{<T_A \cdot T_B - T_A \cdot I_B, I_A \cdot I_B, F_A \cdot F_B > \}; \]

\[ (8) \quad A \cdot B = \{<T_A \cdot I_B - I_A \cdot I_B, F_A + F_B - F_A \cdot F_B > \}. \]

Note that the operations for MVNNs are coincides with operations of AIFSs in Ref. [7,34].
Theorem 1. Let $A = \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle$, $B = \langle \tilde{T}_B, \tilde{I}_B, \tilde{F}_B \rangle$, $C = \langle \tilde{T}_C, \tilde{I}_C, \tilde{F}_C \rangle$ be three MVNNs, then the following equations are true.

1. $A + B = B + A$,
2. $A \cdot B = B \cdot A$,
3. $\lambda(A + B) = \lambda A + \lambda B, \lambda > 0$,
4. $(A \cdot B)^k = A^k + B^k, \lambda > 0$,
5. $\lambda A + \lambda B = (\lambda_1 + \lambda_2)A, \lambda_1 > 0, \lambda_2 > 0$,
6. $A^k \cdot A^k = A^{(k + k)} , \lambda_1 > 0, \lambda_2 > 0$,
7. $(A + B) + C = A + (B + C)$,
8. $(A \cdot B) \cdot C = A \cdot (B \cdot C)$.

3.2 Comparison rules

Based on the score function and accuracy function of AIFS [35-38], the score function, accuracy function and certainty function of a MVNN are defined in the following.

Definition 9. Let $A = \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle$ be a MVNN, and then the score function $s(A)$, accuracy function $a(A)$ and certainty function $c(A)$ of an MVNN are defined as follows:

\[
s(A) = \frac{1}{I_{\tilde{T}_A} \cdot I_{\tilde{I}_A} \cdot I_{\tilde{F}_A}} \times \sum_{\gamma_i, \eta_j, \xi_k} (\gamma_i + 1 - \eta_j + 1 - \xi_k)/3;
\]

\[
a(A) = \frac{1}{I_{\tilde{T}_A} \cdot I_{\tilde{F}_A}} \sum_{\gamma_i, \xi_k} (\gamma_i - \xi_k);
\]

\[
c(A) = \frac{1}{I_{\tilde{T}_A}} \sum_{\gamma_i} \gamma_i.
\]

Where $\gamma_i \in \tilde{T}_A, \eta_j \in \tilde{I}_A, \xi_k \in \tilde{F}_A$, $l_{\tilde{T}_A}, l_{\tilde{I}_A}$ and $l_{\tilde{F}_A}$ denotes the element numbers in $\tilde{T}_A, \tilde{I}_A$ and $\tilde{F}_A$, respectively.

The score function is an important index in ranking the MVNNs. For a MVNN $A$, the truth-membership $\tilde{T}_A$ is bigger, the MVNN is greater. And the indeterminacy-membership $\tilde{I}_A$ is more, the MVNN is greater. Similarly, the false-membership $\tilde{F}_A$ is smaller, the MVNN is greater.

For the accuracy function, if the difference between truth and falsity is bigger, then the statement is more affirmative.

That is, the larger the values of $\tilde{T}_A, \tilde{I}_A$ and $\tilde{F}_A$, the more the accuracy of the MVNN. As to the certainty function, the value of truth-membership $\tilde{T}_A$ is bigger, it means more certainty of the MVNN.

On the basis of Definition 9, the method to compare MVNNs can be defined as follows.

Definition 10. Let $A$ and $B$ be two MVNNs. The comparison methods can be defined as follows:

1. If $s(A) > s(B)$, then $A$ is greater than $B$, that is, $A$ is superior to $B$, denoted by $A > B$.
2. If $s(A) = s(B)$ and $a(A) > a(B)$, then $A$ is greater than $B$, that is, $A$ is superior to $B$, denoted by $A > B$.
3. If $s(A) = s(B)$, $a(A) = a(B)$ and $c(A) > c(B)$, then $A$ is greater than $B$, that is, $A$ is superior to $B$, denoted by $A > B$.
(4) If \( s(A) = s(B), a(A) = a(B) \) and \( c(A) = c(B) \), then \( A \) is equal to \( B \), that is, \( A \) is indifferent to \( B \), denoted by \( A \sim B \).

4. Aggregation operators of MVNNs and their application to multi-criteria decision-making problems

In this section, applying the MVNSs operations, we present aggregation operators for MVNNs and propose a method for MCDM by utilizing the aggregation operators.

4.1 MVNN aggregation operators

**Definition 11.** Let \( A_j = <\tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j}> \ (j = 1, 2, \ldots, n) \) be a collection of MVNNs, and let

\[
\text{MVNNWA} : \text{MVNN}^n \rightarrow \text{MVNN},
\]

\[
\text{MVNNWA} (A_1, A_2, \ldots, A_n) = \sum_{j=1}^{n} w_j A_j,
\]

then \( \text{MVNNWA} \) is called the multi-valued neutrosophic number weighted averaging operator of dimension \( n \), where \( W = (w_1, w_2, \ldots, w_n) \) is the weight vector of \( A_j \) \((j = 1, 2, \ldots, n)\), with \( w_j \geq 0 \) \((j = 1, 2, \ldots, n)\) and \( \sum_{j=1}^{n} w_j = 1 \).

**Theorem 2.** Let \( A_j = <\tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j}> \ (j = 1, 2, \ldots, n) \) be a collection of MVNNs, \( W = (w_1, w_2, \ldots, w_n) \) be the weight vector of \( A_j \) \((j = 1, 2, \ldots, n)\), with \( w_j \geq 0 \) \((j = 1, 2, \ldots, n)\) and \( \sum_{j=1}^{n} w_j = 1 \), then their aggregated result using the MVNNWA operator is also an MVNN, and

\[
\text{MVNNWA} (A_1, A_2, \ldots, A_n) = \left( \bigcup_{j=1}^{n} \tilde{T}_{A_j} \right) \left[ \left( \prod_{j=1}^{n} \tilde{I}_{A_j} \right) \left( \bigcup_{j=1}^{n} \tilde{F}_{A_j} \right) \right]
\]

(2)

Where \( W = (w_1, w_2, \ldots, w_n) \) is the vector of \( A_j(j = 1, 2, \ldots, n) \), \( W_j \in [0, 1] \) and \( \sum_{j=1}^{n} w_j = 1 \).

It is obvious that the MVNNWA operator has the following properties.

(1) (Idempotency): Let \( A_j(j = 1, 2, \ldots, n) \) be a collection of MVNNs. If all \( A_j(j = 1, 2, \ldots, n) \) are equal, i.e., \( A_j = A \), for all \( j \in \{1, 2, \ldots, n\} \), then

\[
\text{MVNNWA}(A_1, A_2, \ldots, A_n) = A.
\]

(2) (Boundedness): If \( A_j = <\tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j}> \ (j = 1, 2, \ldots, n) \) is a collection of MVNNs and \( \min_j, \max_j, \max_j \text{A}_j \) \((j = 1, 2, \ldots, n)\), then \( \text{MVNNWA}(A_1, A_2, \ldots, A_n) \) is a collection of MVNNs and

\[
A' = \left\{ \min_j T_{A_j}, \max_j I_{A_j}, \max_j F_{A_j} \right\},
\]

\[
A'' = \left\{ \max_j T_{A_j}, \min_j I_{A_j}, \min_j F_{A_j} \right\},
\]

for all \( j \in \{1, 2, \ldots, n\} \), then

\[
A' \subseteq \text{MVNNWA}(A_1, A_2, \ldots, A_n) \subseteq A''.
\]

(3) (Monotony): Let \( A_j(j = 1, 2, \ldots, n) \) a collection of MVNNs. If \( A_j \subseteq A'_j \), for \( j \in \{1, 2, \ldots, n\} \), then

\[
\text{SNWA}(A_1, A_2, \ldots, A_n) \subseteq \text{SNWA}(A'_1, A'_2, \ldots, A'_n).
\]

Juan-juan Peng and Jian-qiang Wang, Multi-valued Neutrosophic Sets and its Application in Multi-criteria Decision-making Problems
Definition 12. Let \( A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle \) (\( j = 1, 2, \cdots, n \)) be a collection of MVNNs, and let

\[
\text{MVNNWG : MVNN}^n \rightarrow \text{MVNN} : \\
\text{MVNNWG}_a (A_1, A_2, \ldots, A_n) = \prod_{j=1}^n A_j^{w_j} , \tag{3}
\]

then MVNNWG is called an multi-valued neutrosophic number weighted geometric operator of dimension \( n \), where \( A_{\sigma(j)} \) is the \( j \)-th largest value.

\[
W = (w_1, w_2, \ldots, w_n) \quad \text{is the weight vector of} \quad A_j \\
(j = 1, 2, \cdots, n) , \quad \text{with} \quad w_j \geq 0 \quad (j = 1, 2, \cdots, n) \quad \text{and} \quad \sum_{j=1}^n w_j = 1.
\]

Theorem 3. Let \( A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle \) (\( j = 1, 2, \cdots, n \)) be a collection of MVNNs, we have the following result:

\[
\text{MVNNWG}_a (A_1, A_2, \ldots, A_n) = \begin{cases} 
\bigcup_{a \leq t} \left\{ \prod_{j=1}^n (T_j)^{w_j} \right\}, \\
\bigcup_{n \leq t} \left\{ - \prod_{j=1}^n (1 - \eta_j)^{w_j} \right\}, \\
\bigcup_{a \leq f} \left\{ 1 - \prod_{j=1}^n (1 - \xi_j)^{w_j} \right\}, 
\end{cases} \tag{4}
\]

where \( W = (w_1, w_2, \ldots, w_n) \) is the vector of \( A_j (j = 1, 2, \cdots, n) \), \( w_j \in [0, 1] \) and \( \sum_{j=1}^n w_j = 1. \)

Definition 13. Let \( A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle \) (\( j = 1, 2, \cdots, n \)) be a collection of MVNNs, and let

\[
\text{MVNNOWA : MVNN}^n \rightarrow \text{MVNN} : \\
\text{MVNNOWA} (A_1, A_2, \ldots, A_n) = w_1 A_{\sigma(1)} + w_2 A_{\sigma(2)} + \cdots + w_n A_{\sigma(n)} = \sum_{j=1}^n w_j A_{\sigma(j)} \tag{5}
\]

then \( \text{MVNNOWA} \) is called the multi-valued neutrosophic number ordered weighted averaging operator of dimension \( n \), where \( A_{\sigma(j)} \) is the \( j \)-th largest value.

\[
W = (w_1, w_2, \ldots, w_n) \quad \text{is the weight vector of} \quad A_j \\
(j = 1, 2, \cdots, n) , \quad \text{with} \quad w_j \geq 0 \quad (j = 1, 2, \cdots, n) \quad \text{and} \quad \sum_{j=1}^n w_j = 1.
\]

Theorem 4. Let \( A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle \) (\( j = 1, 2, \cdots, n \)) be a collection of MVNNs, \( W = (w_1, w_2, \ldots, w_n) \) be the weight vector of \( A_j \) (\( j = 1, 2, \cdots, n \)), \( w_j \geq 0 \)

\[
(j = 1, 2, \cdots, n) \quad \text{and} \quad \sum_{j=1}^n w_j = 1, \quad \text{then their aggregated result using the MVNNOWA operator is also an MVNN, and}
\]

\[
\text{MVNNOWA}_a (A_1, A_2, \ldots, A_n) = \left\{ \bigcup_{j \leq f} \left\{ \prod_{j=1}^n (1 - \eta_j)^{w_j} \right\}, \\
\bigcup_{j \leq e} \left\{ \prod_{j=1}^n (1 - \xi_j)^{w_j} \right\}, \\
\bigcup_{j \leq d} \left\{ \prod_{j=1}^n (1 - \zeta_j)^{w_j} \right\} \right\}, \tag{6}
\]

where \( A_{\sigma(j)} \) is the \( j \)-th largest value according to the total order: \( A_{\sigma(1)} \geq A_{\sigma(2)} \geq \cdots \geq A_{\sigma(n)}. \)

Definition 14. Let \( A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle \) (\( j = 1, 2, \cdots, n \)) be a collection of MVNNs, and let

\[
\text{MVNNOWG : MVNN}^n \rightarrow \text{MVNN} : \\
\text{MVNNOWG} (A_1, A_2, \ldots, A_n) = \sum_{j=1}^n w_j A_{\sigma(j)}. \tag{7}
\]
Let $A_j = \langle T_j, I_j, F_j \rangle$ ($j=1,2,\cdots,n$) be a collection of MVNNs, and let

$$\text{MVNNHOWA} : \text{MVNN}^n \rightarrow \text{MVNN} :$$

$$\text{MVNNHOWA} (A_1, A_2, \cdots, A_n) = w_1 A_{1(j)} + w_2 A_{2(j)} + \cdots + w_n A_{n(j)} = \sum_{j=1}^{n} w_j A_{j(j)}$$

then \text{MVNNHOWA} is called the multi-valued neutrosophic number hybrid ordered weighted averaging operator of dimension $n$, where $A_{j(j)}$ is the $j$-th largest value of

$$w = (w_1, w_2, \cdots, w_n)$$

is the weight vector of $A_j$ ($j=1,2,\cdots,n$) and

$$\sum_{j=1}^{n} w_j = 1.$$
a collection of MVNNs, and let

$$\text{MVNNHOWG} : \text{MVNN}^n \rightarrow \text{MVNN}$$

$$\text{MVNNHOWG} (A_1, A_2, \cdots, A_n) = \prod_{j=1}^{n} \hat{A}_{n(j)}$$  \hspace{1cm} (11)$$

then MVNNHOWG is called the multi-valued neutrosophic number hybrid ordered weighted geometric operator of dimension $n$, where $\hat{A}_{n(j)}$ is the $j$-th largest of the weighted value $\hat{A}_j = (\hat{A}_j, \hat{A}_j^m, j = 1, 2, \cdots, n)$.

Similarly, it can be proved that the mentioned operators have the same properties as the MVNNWA operator.

4.2 Multi-criteria decision-making method based on the MVNN aggregation operators

Assume there are $n$ alternatives $A = \{a_1, a_2, \cdots, a_n\}$ and $m$ criteria $C = \{c_1, c_2, \cdots, c_m\}$, whose criterion weight vector is $w = (w_1, w_2, \cdots, w_n)$, where $w_j \geq 0$ ($j = 1, 2, \cdots, m$), $\sum_{j=1}^{m} w_j = 1$. Let $R = (a_{ij})_{n \times m}$ be the simplified neutrosophic decision matrix, where $a_{ij} = (T_{ij}, I_{ij}, F_{ij})$ is a criterion value, denoted by MVNN, where $T_{ij}$ indicates the truth-membership function that the alternative $a_i$ satisfies the criterion $c_j$, $I_{ij}$ indicates the indeterminacy-membership function that the alternative $a_i$ satisfies the criterion $c_j$ and $F_{ij}$ indicates the falsity-membership function that the alternative $a_i$ satisfies the criterion $c_j$.

In the following, a procedure to rank and select the most desirable alternative(s) is given.

**Step 1:** Aggregate the MVNNs.
Utilize the MVNNWA operator or the MVNNWG operator or MVNNHOWA operator or the MVNNHOWG operator to aggregate MVNNs and we can get the individual value of the alternative \( a_i \) \((i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)\).

\[
x_i = MVNNWA_w(a_{ij}, a_{ij}, \ldots, a_{im}), \text{ or}
\]

\[
x_i = MVNNWG_w(a_{ij}, a_{ij}, \ldots, a_{im}), \text{ or}
\]

\[
x_i = MVNNOWA_w(a_{ij}, a_{ij}, \ldots, a_{im}), \text{ or}
\]

\[
x_i = MVNNHOWA_w(a_{ij}, a_{ij}, \ldots, a_{im}), \text{ or}
\]

\[
x_i = MVNNHOWG_w(a_{ij}, a_{ij}, \ldots, a_{im}).
\]

**Step 2**: Calculate the score function value \( s(y_i) \), accuracy function value \( a(y_i) \) and certainty function value \( c(y_i) \) of \( y_i \) \((i = 1, 2, \ldots, m)\) by Definition 9.

**Step 3**: Rank the alternatives. According to Definition 10, we could get the priority of the alternatives \( a_i \) \((i = 1, 2, \ldots, m)\) and choose the best one.

**5. Illustrative example**

In this section, an example for the multi-criteria decision making problem of alternatives is used as the demonstration of the application of the proposed decision making method, as well as the effectiveness of the proposed method.

Let us consider the decision making problem adapted from Ref. [28]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money:

1. \( A_1 \) is a car company;
2. \( A_2 \) is a food company;
3. \( A_3 \) is a computer company;
4. \( A_4 \) is an arms company.

The investment company must take a decision according to the following three criteria:

1. \( C_1 \) is the risk analysis;
2. \( C_2 \) is the growth analysis;
3. \( C_3 \) is the environmental impact analysis, where \( C_1 \) and \( C_2 \) are benefit criteria, and \( C_3 \) is a cost criterion. The weight vector of the criteria is given by \( W = (0.35, 0.25, 0.4) \). The four possible alternatives are to be evaluated under the above three criteria by the form of MVNNs, as shown in the following simplified neutrosophic decision matrix \( D \):

\[
\begin{array}{c}
\langle 0.4, 0.5, 0.2 \rangle \\
\langle 0.6, 0.1, 0.2 \rangle \\
\langle 0.3, 0.4, 0.2 \rangle \\
\langle 0.7, 0.1, 0.2 \rangle
\end{array}
\begin{array}{c}
\langle 0.2, 0.3, 0.3 \rangle \\
\langle 0.5, 0.2, 0.3 \rangle \\
\langle 0.5, 0.2, 0.3 \rangle \\
\langle 0.6, 0.2, 0.3 \rangle
\end{array}
\begin{array}{c}
\langle 0.2, 0.2, 0.2 \rangle \\
\langle 0.1, 0.1, 0.2 \rangle \\
\langle 0.1, 0.1, 0.2 \rangle \\
\langle 0.1, 0.1, 0.2 \rangle
\end{array}
\]

The procedures of decision making based on MVNS are shown as following.

**Step 1**: Aggregate the MVNNs.
Utilize the MVNNWA operator or the MVNNWG operator to aggregate MVNNs of each decision maker, and we can get the individual value of the alternative $a_i$ ($i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$).

By using MVNNWA operator, the alternatives matrix $A_{WA}$ can be obtained:

$$A_{WA} = \begin{bmatrix}
\{0.327, 0.368\}, \{0.200, 0.221\}, \{0.368\} \\
\{0.563\}, \{0.132, 0.168\}, \{0.152, 0.200\} \\
\{0.438, 0.467\}, \{0.200, 0.235\}, \{0.255\} \\
\{0.574\}, \{0.155, 0.198\}, \{0.157\}
\end{bmatrix}.$$

With MVNNWG operator, the alternatives matrix $A_{WG}$ is as follows:

$$A_{WG} = \begin{bmatrix}
\{0.303, 0.328\}, \{0.200, 0.226\}, \{0.388\} \\
\{0.558\}, \{0.141, 0.176\}, \{0.161, 0.200\} \\
\{0.418, 0.462\}, \{0.200, 0.242\}, \{0.262\} \\
\{0.538\}, \{0.186, 0.219\}, \{0.166\}
\end{bmatrix}.$$

**Step 2:** Calculate the score function value, accuracy function value and certainty function value.

To the alternatives matrix $A_{WA}$, by using Definition 9, then we have:

$$s_{A_{WA}} = (0.590, 0.746, 0.660, 0.747).$$

Apparently, there is no need to compute accuracy function value and certainty function value.

To the alternatives matrix $A_{WG}$, by using Definition 10, the function matrix of $A_{WG}$ is as follows:

$$s_{A_{WG}} = (0.571, 0.739, 0.653, 0.723).$$

Apparently, there is no need to compute accuracy function value and certainty function value else.

**Step 3:** Get the priority of the alternatives and choose the best one.

According to Definition 10 and results in step 2, for $A_{WA}$, we have $a_4 \succ a_2 \succ a_3 \succ a_1$. Obviously, the best alternative is $a_4$. For $A_{WG}$, we have $a_2 \succ a_4 \succ a_3 \succ a_1$. Obviously, the best alternative is $a_2$.

Similarly, if the other two aggregation operators are utilized, then the results can be founded in Table 1.

From the results in Table 1, we can see that if the $MVNNOWA$ and $MVNNHOWA$ are utilized in Step 1, then we can obtain the results: $a_4 \succ a_2 \succ a_3 \succ a_1$. The best one is $a_4$ while the worst is $a_1$. If the $MVNNOWNG$ and $MVNNHOWG$ operators are used, then the final ranking is $a_2 \succ a_4 \succ a_3 \succ a_1$, the best one is $a_2$ while the worst one is $a_1$.

In most cases, the different aggregation operator may lead to different rankings. However, all weighted average operators and all geometry operators also lead to the same rankings respectively. So we have two ranks of four alternatives and the best one is always the $A_4$ or $A_2$, the worst one is always the $A_1$. At the same time, decision-makers can choose different aggregation operator according to their preference.
Table 1: The rankings as aggregation operator changes

<table>
<thead>
<tr>
<th>Operators</th>
<th>The final ranking</th>
<th>The best alternative(s)</th>
<th>The worst alternative(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVNNWA</td>
<td>( a_4 \succ a_2 \succ a_3 \succ a_1 )</td>
<td>( a_4 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>MVNNWG</td>
<td>( a_2 \succ a_4 \succ a_3 \succ a_1 )</td>
<td>( a_2 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>MVNHOWA</td>
<td>( a_4 \succ a_3 \succ a_2 \succ a_1 )</td>
<td>( a_4 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>MVNNOWG</td>
<td>( a_2 \succ a_4 \succ a_3 \succ a_1 )</td>
<td>( a_2 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>MVNNHOWA</td>
<td>( a_2 \succ a_4 \succ a_3 \succ a_1 )</td>
<td>( a_2 )</td>
<td>( a_1 )</td>
</tr>
<tr>
<td>MVNNHOWA</td>
<td>( a_2 \succ a_4 \succ a_3 \succ a_1 )</td>
<td>( a_2 )</td>
<td>( a_1 )</td>
</tr>
</tbody>
</table>

6. Conclusion

MVNSs can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications. However, as a new branch of NSs, there is no enough research about MVNSs. Especially, the existing literature does not put forward the aggregation operators and MCDM method for MVNSs. Based on the related research achievements in AIFSs, the operations of MVNSs were defined. And the approach to solve MCDM problem with MVNNs was proposed. In addition, the aggregation operators of MVNNWA, MVNNWG, MVNNOW, MVNNOWG, MVNNHOWA and MVNNHOWG were given. Thus, a MCDM method is established based on the proposed operators. Utilizing the comparison approach, the ranking order of all alternatives can be determined and the best one can be easily identified as well. An illustrative example demonstrates the application of the proposed decision making method, and the calculation is simple. In the further study, we will continue to investigate the related comparison method for MVNSs.

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References


http://www.hindawi.com/journals/jam/2012/879629/...


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More on neutrosophic soft rough sets and its modification

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Abstract. This paper aims to introduce and discuss anew mathematical tool for dealing with uncertainties, which is a combination of neutrosophic sets, soft sets and rough sets, namely neutrosophic soft rough set model. Also, its modification is introduced. Some of their properties are studied and supported with proved propositions and many counter examples. Some of rough relations are re-defined as a neutrosophic soft rough relations. Comparisons among traditional rough model, suggested neutrosophic soft rough model and its modification, by using their properties and accuracy measures are introduced. Finally, we illustrate that, classical rough set model can be viewed as a special case of suggested models in this paper.

Keywords: Neutrosophic set, soft set, rough set approximations, neutrosophic soft set, neutrosophic soft rough set approximations.

1 Introduction

In recent years, many theories based on uncertainty have been proposed, such as fuzzy set theory [36], intuitionistic fuzzy set theory [5], vague set theory [10] and interval-valued fuzzy set theory [11]. In 1982, Pawlak [22] initiated his rough set model, based on equivalence relations, as a new approach towards soft computing finding a wide application. Rough set model has been developed, in many papers, as a generalization models. These models based on reflexive relation, symmetric relation, preference relation, tolerance relation, any relation, coverings, different neighborhood operators, using uncertain function, etc. [12, 15, 16, 24, 25, 29, 32-34, 37]. Also, many papers, recently, have been appeared to apply it in many real life applications such as [2, 3, 7, 17, 27, 28, 30, 35]. In 1995, Smarandache, started his study of the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise data. Neutrosophic logic is a generalization of intuitionistic fuzzy logic. In neutrosophic logic a proposition is $\tau\%$ true, $i\%$ indeterminate, and $f\%$ false. For example, let’s analyze the following proposition: Let $(0.6,0.4,0.3)$ belongs to $A$ means, with probability of $60\%$ ($x$ in $A$), with probability of $30\%$ ($x$ not in $A$) and with probability of $40\%$ (undecidable).

Soft set theory [21], proposed by Molodtsov in 1999, is also a mathematical tool for dealing with uncertainties. Recently, traditional soft model has been developed and applied in some decision making problems in many papers such as [1, 4, 6, 8, 13, 14, 18, 19, 31].


In this paper, we introduce a combination of neutrosophic sets, soft sets and rough sets, called neutrosophic soft rough set model. Also, a modification of it is introduced. Basic properties and concepts of suggested models are deduced. We compare between traditional rough model and proposed models to illustrate that traditional rough model is a special case of these proposed models.

2 Preliminaries

In this section we recall some definitions and properties regarding rough set, neutrosophic set, soft set and neutrosophic soft set theories required in this paper. The following definitions and proposition are given in [22], as follows

Definition 2.1 An equivalence class of an element $x \in U$, determined by the equivalence relation $E$ is

$$[x]_E = \{x' \in U : E(x) = E(x')\}. $$

Definition 2.2 Lower, upper and boundary approximations of a subset $X \subseteq U$ are defined as

$$\underline{E}(X) = \cup\{[x]_E : [x]_E \subseteq X\}, $$

$$\bar{E}(X) = \cup\{[x]_E : [x]_E \cap X \neq \emptyset\}. $$
Definition 2.3 [23] An information system is a quadruple 
\( IS = (U, A, V, f) \), where \( U \) is a non-empty finite set of objects, \( A \) is a non-empty finite set of attributes, \( V = \bigcup \{ V_e, e \in A \} \), \( V_e \) is the value set of attribute \( e \), and \( f : U \times A \rightarrow V \), is called an information (knowledge) function.

Definition 2.4 [23] An information system is a quadruple 
\[ IS = (U, A, V, f) \]
where \( U \) is a non-empty finite set of objects, \( A \) is a non-empty finite set of attributes, 
\[ V = \bigcup \{ V_e, e \in A \} \]
\( V_e \) is the value set of attribute \( e \), and 
\[ f : U \times A \rightarrow V \]
is called an information (knowledge) function.

Definition 2.2 Let \( (U, E) \) be a Pawlak approximation space and let \( X, Y \subseteq U \). Then,
(a) \( E(X) \subseteq X \subseteq \overline{E}(X) \).
(b) \( E(\phi) = \phi = \overline{E}(\phi) \) and \( E(U) = U = \overline{E}(U) \).
(c) \( \overline{E}(X \cup Y) = \overline{E}(X) \cup \overline{E}(Y) \).
(d) \( \overline{E}(X \cap Y) = \overline{E}(X) \cap \overline{E}(Y) \).
(e) \( X \subseteq Y \), then \( \overline{E}(X) \subseteq \overline{E}(Y) \) and \( \overline{E}(X) \subseteq \overline{E}(Y) \).
(f) \( E(X \cup Y) \supseteq E(X) \cup E(Y) \).
(g) \( \overline{E}(X \cap Y) \subseteq \overline{E}(X) \cap \overline{E}(Y) \).
(h) \( E(\overline{X}^c) = [\overline{E}(X)]^c \), \( \overline{X}^c \) is the complement of \( X \).
(i) \( \overline{E}(\overline{X}^c) = [\overline{E}(X)]^c \).
(j) \( \overline{E}(E(X)) = \overline{E}(E(X)) = \overline{E}(X) \).
(k) \( \overline{E}(\overline{E}(X)) = E(\overline{E}(X)) = \overline{E}(X) \).

Definition 2.5 [21] Let \( U \) be an initial universe set, \( E \) be a set of parameters, \( A \subseteq E \) and let \( P(U) \) denotes the power set of \( U \). Then, a pair \( S = (F, A) \) is called a soft set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow P(U) \). In other words, a soft set over \( U \) is a parameterized family of subsets of \( U \). For \( e \in A \), \( F(e) \) may be considered as the set of \( e \)-approximate elements of \( S \).

Definition 2.6 [26] A neutrosophic set \( A \) on the universe of discourse \( U \) is defined as
\[ A = \{ (x, T_A(x), I_A(x), F_A(x)) : x \in U \}, \]
where \( 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \), where
\[ T, I, F : U \rightarrow ]0,1[. \]

Definition 2.7 [20] Let \( U \) be an initial universe set and \( E \) be a set of parameters. Consider \( A \subseteq E \), and let \( P(U) \) denotes the set of all neutrosophic sets of \( U \). The collection \( (F, A) \) is termed to be the neutrosophic soft set over \( U \), where \( F \) is a mapping given by
\[ F : A \rightarrow P(U). \]

3 Neutrosophic soft lower and upper concepts and their properties

In this section, neutrosophic soft rough lower and upper approximations are introduced and their properties are deduced and proved. Moreover, many counter examples are obtained.

For more illustration the meaning of neutrosophic soft set, we consider the following example

Example 3.1 Let \( U \) be a set of cars under consideration and \( E \) is the set of parameters (or qualities). Each parameter is a generalized neutrosophic word or sentence involving generalized neutrosophic words. Consider \( E = \{ \text{beautiful, cheap, expensive, wide, modern, in good repair, costly, comfortable} \} \). In this case, to define a neutrosophic soft set means to point out beautiful cars, cheap cars and so on. Suppose that, there are five cars in the universe \( U \), given by, \( U = \{ e_1, e_2, e_3, e_4, e_5 \} \) and the set of parameters \( A = \{ e_1, e_2, e_3, e_4, e_5 \} \), where each \( e_i \) is a specific criterion for cars: \( e_1 \) stands for (beautiful), \( e_2 \) stands for (cheap), \( e_3 \) stands for (modern), \( e_4 \) stands for (comfortable). Suppose that,
\[ F(\text{beautiful}) = \{ \langle h_1, 0.6, 0.6, 0.2 \rangle, \langle h_2, 0.4, 0.6, 0.6 \rangle, \langle h_3, 0.6, 0.4, 0.2 \rangle, \langle h_4, 0.6, 0.3, 0.3 \rangle, \langle h_5, 0.8, 0.2, 0.3 \rangle \} \]
\[ F(\text{cheap}) = \{ \langle h_1, 0.8, 0.4, 0.3 \rangle, \langle h_2, 0.6, 0.2, 0.4 \rangle, \langle h_3, 0.8, 0.1, 0.3 \rangle, \langle h_4, 0.8, 0.2, 0.2 \rangle, \langle h_5, 0.8, 0.3, 0.2 \rangle \} \]
and the parameter, a neutrosophic right, may not be symmetric relation. Hence, may be not symmetric relation.

Table 1, represents the reflexive relation. The family of all neutrosophic right, and falsity membership value of . For any element ,

\[
\begin{array}{cccc}
U & e_1 & e_2 & e_3 & e_4 \\
\h_1 & (0.6, 0.6, 0.2) & (0.8, 0.4, 0.3) & (0.7, 0.4, 0.3) & (0.8, 0.6, 0.4) \\
\h_2 & (0.4, 0.6, 0.6) & (0.6, 0.2, 0.4) & (0.6, 0.4, 0.3) & (0.7, 0.6, 0.6) \\
\h_3 & (0.6, 0.4, 0.2) & (0.8, 0.1, 0.3) & (0.7, 0.2, 0.5) & (0.7, 0.6, 0.4) \\
\h_4 & (0.6, 0.3, 0.3) & (0.8, 0.2, 0.2) & (0.5, 0.2, 0.6) & (0.7, 0.5, 0.6) \\
\h_5 & (0.8, 0.2, 0.3) & (0.8, 0.3, 0.2) & (0.7, 0.3, 0.4) & (0.9, 0.5, 0.7)
\end{array}
\]

Table 1: Tabular representation of \((F, A)\) of Example 3.1.

**Definition 3.1** Let \((G, A)\) be a neutrosophic soft set on a universe \(U\). For any element \(h \in U\), a neutrosophic right neighborhood, with respect to \(e \in A\) is defined as follows

\[
h_e = \{ h_i \in U : T_e(h_i) \geq T_e(h), I_e(h_i) \geq I_e(h), F_e(h_i) \leq F_e(h) \}.
\]

**Definition 3.2** Let \((G, A)\) be a neutrosophic soft set on a universe \(U\). For any element \(h \in U\), a neutrosophic right neighborhood, with respect to all parameters \(A\) is defined as follows

\[
h_A = \cap \{ h_i : e_i \in A \}.
\]

For more illustration of Definitions 3.1 and 3.2, the following example is introduced.

**Example 3.2** According Example 3.1, we can deduce the following results:

\[
\langle h_1, 0.7, 0.4, 0.3 \rangle, \langle h_2, 0.6, 0.4, 0.3 \rangle, \langle h_3, 0.7, 0.2, 0.5 \rangle, \langle h_4, 0.5, 0.2, 0.6 \rangle, \langle h_5, 0.7, 0.3, 0.4 \rangle.
\]

\[
F(\text{comfortable}) = \langle h_1, 0.8, 0.6, 0.4 \rangle, \langle h_2, 0.7, 0.6, 0.6 \rangle, \langle h_3, 0.7, 0.6, 0.4 \rangle, \langle h_4, 0.7, 0.5, 0.6 \rangle, \langle h_5, 0.9, 0.5, 0.7 \rangle.
\]

In order to store a neutrosophic soft set in a computer, we could represent it in the form of a table as shown in Table 1 (corresponding to the neutrosophic soft set in Example 3.1). In this table, the entries are \(c_{ij} \) corresponding to the car \(h_i\) and the parameter \(e_j\), where \(c_{ij} = (\text{true membership value of } h_i, \text{indeterminacy-membership value of } h_i, \text{falsity membership value of } h_i)\) in \(F(\text{comfortable})\). Table 1, represents the neutrosophic soft set \((F, A)\) as follows

\[
h_{1e} = h_{1e_1} = h_{1e_2} = h_{1e_3} = \{ h_1 \}, h_{2e} = h_{2e_1} = h_{2e_2} = h_{2e_3} = \{ h_1, h_2 \}, h_{3e} = h_{3e_1} = h_{3e_2} = h_{3e_3} = \{ h_1, h_3 \}, h_{4e_1} = h_{4e_2} = \{ h_1, h_4 \}, h_{4e_3} = \{ h_1, h_4 \}, h_{5e_1} = h_{5e_2} = h_{5e_3} = h_{5e_4} = \{ h_1, h_5 \}, h_{5e_5} = \{ h_1, h_5 \}.
\]

It follows that, \(h_1 \in A\), \(h_2 \in A\) = \{ \{ h_1, h_2 \}, h_3 \} = \{ h_1, h_3 \}, h_4 \in A\) and \(h_5 \in A\) = \{ \{ h_1, h_5 \} \}.

**Proposition 3.1** Let \((G, A)\) be a neutrosophic soft set on a universe \(U\), \(\xi\) is the family of all neutrosophic right neighborhoods on it, and let \(R_e : U \rightarrow \xi\), \(R_e(h) = h_e\). Then,

(a) \(R_e\) is reflexive relation.

(b) \(R_e\) is transitive relation.

(c) \(R_e\) may be not symmetric relation.

**Proof** Let

\[
\langle h_1, T(\text{h}_1), I(\text{h}_1), F(\text{h}_1) \rangle, \langle h_2, T(\text{h}_2), I(\text{h}_2), F(\text{h}_2) \rangle\]

and \(\langle h_3, T(\text{h}_3), I(\text{h}_3), F(\text{h}_3) \rangle \in G(A)\). Then,

(a) Obviously,

\[
T(\text{h}_1) = T(\text{h}_1), I(\text{h}_1) = I(\text{h}_1)\] and \(F(\text{h}_1) = F(\text{h}_1)\). Hence, for every \(e \in A\), \(h_1 \in h_1e_1\). Then \(h_1 R_e h_1\) and then \(R_e\) is reflexive relation.

(b) Let \(h_1 R_e h_2\) and \(h_2 R_e h_3\). Then, \(h_2 \in h_1\) and \(h_3 \in h_2\). Hence, \(T(\text{h}_2) \geq T(\text{h}_1), I(\text{h}_2) \geq I(\text{h}_1), F(\text{h}_2) \leq F(\text{h}_1)\) and \(T(\text{h}_3) \geq T(\text{h}_2), I(\text{h}_3) \geq I(\text{h}_2), F(\text{h}_3) \leq F(\text{h}_2)\). Consequently, we have \(T(\text{h}_3) \geq T(\text{h}_1), I(\text{h}_3) \geq I(\text{h}_1)\) and \(F(\text{h}_3) \leq F(\text{h}_1)\). It follows that, \(h_3 \in h_3e_1\).
Then \( h_i \). Then \( h_i \sim R \sim h_i \) and then \( R \) is transitive relation. Consequently, \( h_i \sim h_i \). Hence, \( (h_i, h_i) \in R \), but \( (h_i, h_i) \notin R \). Then, \( R \) isn’t symmetric relation.

Neutrosophic soft lower and upper approximations are defined as follows

**Definition 3.3.** Let \((G, A)\) be a neutrosophic soft set on \( U \). Then, neutrosophic soft lower, upper and boundary approximations of \( X \subseteq U \), respectively, are

\[
\overline{NRX} = \bigcup_{A} \{ h : h \in U, h \subsetneq X \},
\]

\[
\overline{NRX} = \bigcup_{A} \{ h : h \in U, h \cap X \neq \emptyset \},
\]

\[
b_{NR} X = \overline{NRX} - \overline{NRX}.
\]

Properties of neutrosophic soft rough set approximations are introduced in the following proposition.

**Proposition 3.2** Let \((G, A)\) be a neutrosophic soft set on \( U \), and let \( X, Z \subseteq U \). Then the following properties hold

(a) \( \overline{NRX} \subseteq X \subseteq \overline{NRX} \).

(b) \( \overline{NR\emptyset} = \overline{NR\emptyset} = \emptyset \).

(c) \( \overline{NRU} = \overline{NRU} = U \).

(d) \( X \subseteq Z \Rightarrow \overline{NRX} \subseteq \overline{NRZ} \).

(e) \( X \subseteq Z \Rightarrow \overline{NRX} \subseteq \overline{NRZ} \).

(f) \( \overline{NR}(X \cap Z) = \overline{NRX} \cap \overline{NRZ} \).

(g) \( \overline{NR}(X \cup Z) \supseteq \overline{NRX} \cup \overline{NRZ} \).

(h) \( \overline{NR}(X \cap Z) \subseteq \overline{NRX} \cap \overline{NRZ} \).

(i) \( \overline{NR}(X \cup Z) = \overline{NRX} \cup \overline{NRZ} \).

**Proof**

(a) From Definition 3.3, obviously, we can deduce that, \( \overline{NRX} \subseteq X \). Also, let \( h \in X \), but \( R \), defined in Proposition 3.1, is reflexive relation. Then, for all \( e \in A \), there exists \( h \) such that, \( h \in h \), and then \( h \in h \).

So \( h \subseteq X \neq \emptyset \). Hence, \( h \in \overline{NRX} \). Therefor \( \overline{NRX} \subseteq \overline{NRX} \).

(b) Proof of (b), follows directly, from Definition 3.3 and Property (a).

(c) From Property (a), we have \( U \subseteq \overline{NRU} \), but \( U \) is the universe set, then \( \overline{NRU} = U \). Also, from Definition 3.3, we have \( \overline{NRU} = \bigcup_{A} \{ h : h \subseteq U \} \), but for all \( h \in U \), we have \( h \subseteq U \). Hence, \( \overline{NRU} = U \). Therefor \( \overline{NRU} = \overline{NRU} = U \).

(d) Let \( X \subseteq Z \) and \( p \in \overline{NRX} \). Then, there exists \( h \) such that, \( p \in h \subseteq X \). But \( X \subseteq Z \), then \( p \in h \subseteq Z \). Hence, \( p \in \overline{NRZ} \).

Therefor \( \overline{NRX} \subseteq \overline{NRZ} \).

(e) Let \( X \subseteq Z \) and \( p \in \overline{NRX} \). Then, there exists \( h \) such that, \( p \in h \subseteq X \). But \( X \subseteq Z \), then \( h \subseteq Z \). Hence, \( p \in \overline{NRZ} \).

Therefor \( \overline{NRX} \subseteq \overline{NRZ} \).

(f) Let \( p \in \overline{NR}(X \cap Z) = \bigcup_{A} \{ h : h \subseteq (X \cap Z) \} \). So, there exists \( h \) such that, \( p \in h \subseteq (X \cap Z) \), then \( p \in h \subseteq X \) and \( p \in h \subseteq Z \). Consequently, \( p \in \overline{NRX} \) and \( p \in \overline{NRZ} \). Thus, \( \overline{NR}(X \cap Z) \subseteq \overline{NRX} \cap \overline{NRZ} \). Conversely, let \( p \in \overline{NRX} \cap \overline{NRZ} \). Hence \( p \in \overline{NRX} \) and \( p \in \overline{NRZ} \). Then there exists \( h \) such that, \( p \in h \subseteq X \) and \( p \in h \subseteq Z \), then \( p \in h \subseteq (X \cap Z) \).

Consequently, \( p \in \overline{NR}(X \cap Z) \), it follows that \( \overline{NRX} \)
\( \cap NRZ \subseteq NR(X \cap Z) \). Therefore, \( NR(X \cap Z) = NR \cap NRZ \).

(g) Let \( p \notin NR(X \cup Z) = \cup \{ h \_A \cap h \_A \cup X \cup Z \} \). So, for all \( h \_A \), such that \( p \in h \_A \), we have \( h \_A \subseteq X \cup Z \), then for all \( h \_A \) containing \( p \), we have \( h \_A \subseteq X \) and \( h \_A \subseteq Z \).

Consequently, \( p \notin NRX \) and \( p \notin NRZ \), then \( p \notin NRX \cap NRZ \). Therefore, \( NR(X \cup Z) \supseteq NRX \cap NRZ \).

(h) Let \( p \in \overline{NR}(X \cap Z) = \cup \{ h \_A \cap (X \cap Z) = \emptyset \} \). So, there exists \( h \_A \) such that, \( p \in h \_A \) and \( h \_A \cap (X \cap Z) \neq \emptyset \). Consequently, \( p \in \overline{NRX} \) and \( p \in \overline{NRZ} \), then \( p \in \overline{NRX} \cap \overline{NRZ} \). Therefore, \( \overline{NR}(X \cap Z) \subseteq \overline{NRX} \cap \overline{NRZ} \).

(i) Let \( p \notin \overline{NR}(X \cup Z) = \cup \{ h \_A \cap (X \cup Z) = \emptyset \} \). So, for all \( h \_A \) containing \( p \), we have

\( h \_A \cap (X \cup Z) = \emptyset \), then for all \( h \_A \) containing \( p \), we have \( h \_A \cap X = \emptyset \) and \( h \_A \cap Z = \emptyset \).

Consequently, \( p \notin \overline{NRX} \) and \( p \notin \overline{NRZ} \), then \( p \notin \overline{NRX} \cup \overline{NRZ} \). Conversely, let \( p \in \overline{NR}(X \cup Z) \). Then, there exists \( h \_A \) such that, \( p \in h \_A \) and \( h \_A \cap (X \cup Z) \neq \emptyset \), it follows that, \( h \_A \cap X \neq \emptyset \) or \( h \_A \cap Z \neq \emptyset \). Consequently, \( p \in \overline{NRX} \) or \( p \in \overline{NRZ} \), hence, \( p \in \overline{NRX} \cup \overline{NRZ} \), then \( \overline{NRX} \cup \overline{NRZ} \supseteq \overline{NR}(X \cup Z) \). Therefore, \( \overline{NRX} \cup \overline{NRZ} = \overline{NR}(X \cup Z) \).

The following example illustrates that, containments of Property (a), may be proper.

**Example 3.4** From Example 3.1, If \( X = \{ h \_1 \} \) and \( Z = \{ h \_2 \} \), then \( \overline{NRX} = \{ h \_1 \} \) and \( \overline{NRZ} = \{ h \_2 \} \). Consequently, \( \overline{NRX} \neq X \) and \( X \neq \overline{NRX} \).

The following example illustrates that, containments of Properties (d) and (e), may be proper.

**Example 3.5** From Example 3.1, If \( X = \{ h \_1 \} \) and \( Z = \{ h \_2 , h \_4 \} \), then \( \overline{NRX} = \emptyset \), \( \overline{NRZ} = \{ h \_1 \} \), \( \overline{NRZ} = \{ h \_1 , h \_2 \} \) and \( \overline{NRZ} = \{ h \_1 , h \_2 , h \_4 \} \). Hence, \( \overline{NRX} \neq \overline{NRZ} \) and \( \overline{NRX} \neq \overline{NRZ} \).

The following example illustrates that, a containment of Property (g), may be proper.

**Example 3.6** From Example 3.1, If \( X = \{ h \_1 \} \) and \( Z = \{ h \_2 \} \), then \( \overline{NRX} = \{ h \_1 \} \), \( \overline{NRZ} = \emptyset \) and \( \overline{NR}(X \cup Z) = \{ h \_1 , h \_2 \} \). Therefore, \( \overline{NR}(X \cup Z) \neq \overline{NRX} \cup \overline{NRZ} \).

The following example illustrates that, a containment of Property (h), may be proper.

**Example 3.7** From Example 3.1, If \( X = \{ h \_1 , h \_4 \} \) and \( Z = \{ h \_2 , h \_4 \} \), then \( \overline{NRX} = \{ h \_1 , h \_2 , h \_3 , h \_4 \} \), \( \overline{NRZ} = \{ h \_1 , h \_2 , h \_3 , h \_4 \} \) and \( \overline{NRZ} = \{ h \_4 \} \). Therefore, \( \overline{NR}(X \cap Z) \neq \overline{NRX} \cap \overline{NRZ} \).

**Proposition 3.3** Let \( (G,A) \) be a neutrosophic soft set on a universe \( U \), and let \( X , Z \subseteq U \). Then the following properties hold.

(a) \( NR \overline{NRX} = \overline{NRX} \).

(b) \( NR \overline{NRX} = \overline{NRX} \).

**Proof**

(a) Let \( W = \overline{NRX} \) and \( p \in W = \cup \{ h \_A \cap h \_A \subseteq X \} \).

Then, there exists some \( h \_A \) containing \( p \), such that \( h \_A \subseteq W \). So, \( p \in \overline{NRW} \). Hence, \( W \subseteq \overline{NRW} \).
Thus, \(NR_X \subseteq NR \setminus NR_X\). Also, from Property (a), of Proposition 3.2, we have \(NR_X \subseteq X\) and by using Property (d), of Proposition 3.2, we get \(NR \setminus NR_X \subseteq NR_X\). Therefor \(NR \setminus NR_X = NR_X\).

(b) Let \(W = \overline{NR_X}\), by using Property (a), of Proposition 3.2, we have \(NRW \subseteq W\). Conversely, let \(p \in W = \cup \{h\}_{\partial} : h \cap X \neq \emptyset\}, hence there exists \(h\) containing \(p\) such that, \(p \in h\}_{\partial} \subseteq W\), it follows that, \(p \in NRW\). Consequently, \(W \subseteq NRW\), then \(W = NRW\). Thus, \(NRX = NRX\).

**Proposition 3.4** Let \((G, A)\) be a neutrosophic soft set on \(U\), and let \(X, Z \subseteq U\). Then, the following properties don’t hold

(a) \(NR \setminus NRX = \overline{NRX}\).

(b) \(NR \setminus NRX = NRX\).

(c) \(NR_X^c = \overline{[NRX]^c}\).

(d) \(NRX^c = [NRX]^c\).

(e) \(NR(X - Z) = NRX - NRZ\).

(f) \(NR(X - Z) = \overline{NRX} - \overline{NRZ}\).

The following example proves (a) of Proposition 3.4.

**Example 3.8** From Example 3.1, if \(X = \{h_1\}\), then \(NR_X = \{h_1, h_2\}\) and \(NR \setminus NR_X = \{h_1, h_2, h_3\}\).

Hence, \(NR \setminus NRX \neq \overline{NRX}\).

The following example proves (b) of Proposition 3.4.

**Example 3.9** From Example 3.1, if \(X = \{h_1\}\), then \(NR_X = \{h_1\}\) and \(NR \setminus NRX = \{h_1, h_2, h_3\}\).

Hence, \(NR \setminus NRX \neq \overline{NRX}\).

The following example proves (c) of Proposition 3.4.

**Example 3.10** From Example 3.1, if \(X = \{h_1\}\), then \(NRX^c = \{h_1, h_3, h_4, h_5\}\) and \([NRX]^c = \{h_3, h_4, h_5\}\). Therefor \(NRX^c \neq [NRX]^c\).

The following example proves (d) of Proposition 3.4.

**Example 3.11** From Example 3.1, if \(X = \{h_1, h_2\}\) and \(Z = \{h_1, h_3\}\), then \(NRX = \{h_1, h_2\}\), \(NRZ = \{h_1, h_3\}\), \(NR(X - Z) = \emptyset\), \(NRX = \{h_1, h_2, h_3\}\), \(NRZ = \{h_1, h_2, h_3\}\), \(NR(X - Z) = \{h_1, h_2\}\).

Hence, \(NRX - NRZ \neq \overline{NRX} - \overline{NRZ}\).

**4 Modification of suggested neutrosophic soft rough set approximations**

In this section, we introduce a modification of suggested neutrosophic soft rough set approximations, introduced in Section 3. Some basic properties are introduced and proved. Finally, a comparison among traditional rough set model, suggested neutrosophic soft rough set model and its modification, by using their properties.

Modified neutrosophic soft lower and upper approximations are defined as follows

**Definition 4.1** Let \((G, A)\) be a neutrosophic soft set on \(U\). Then, modified neutrosophic soft lower, upper and boundary approximations of \(X \subseteq U\), respectively, are

\[
N_{\partial X} = \cup \{h\}_{\partial} : h \in U, h \subseteq X\},
\]

\[
N^R_X = [N_{\partial X}]^c,
\]

\[
b_{\partial X} = N^R_X - N_{\partial X}.
\]

Modified neutrosophic soft lower and upper approximations properties are introduced in the following proposition.

**Proposition 4.1** Let \((G, A)\) be a neutrosophic soft set on
\( U \), and let \( X, Z \subseteq U \). Then the following properties hold:

(a) \( N^R_X X \subseteq X \subseteq N^R_X X \).

(b) \( N^R_{\emptyset} = N^R_{\emptyset} = \emptyset \).

(c) \( N^R_U = N^R_U = U \).

(d) \( X \subseteq Z \implies N^R_X X \subseteq N^R_{N^R_Z} \).

(e) \( X \subseteq Z \implies N^R_X X \subseteq N^R_{Z} \).

(f) \( N^R_{(X \cap Z)} = N^R_X \cap N^R_{Z} \).

(g) \( N^R_{(X \cup Z)} \supseteq N^R_X \cup N^R_{Z} \).

(h) \( N^R_{(X \cap Z)} \subseteq N^R_X \cap N^R_{Z} \).

(i) \( N^R_{(X \cup Z)} = N^R_X \cup N^R_{Z} \).

(j) \( N^R_{N^R_X} X = N^R_X \).

(k) \( N^R_{N^R_X} X = N^R_X \).

(l) \( N^R_{X^c} = [N^R_X]^c \).

(m) \( N^R_{X^c} = [N^R_X]^c \).

**Proof**

Properties (a)-(i) are proved at the same way as Proposition 3.2.

(j) Let

\[ W = N^R_X X \quad \text{and} \quad p \in W = \cup \left\{ h_{\bar{A}} : h_{\bar{A}} \subseteq X \right\} \]

Then, there exists some \( h_{\bar{A}} \) containing \( p \), such that \( h_{\bar{A}} \subseteq W \). So, \( p \in N^R_{N^R_W} W \). Hence, \( W \subseteq N^R_{N^R_W} W \).

Thus, \( N^R_X X \subseteq N^R_{N^R_X} X \). Also, from Property (a), of Proposition 3.2, we have \( N^R_X X \subseteq X \) and by using Property (d), of Proposition 3.2, we can deduce that \( N^R_X X \subseteq N^R_X X \). Therefore, \( N^R_{N^R_X} N^R_X X = N^R_X X \).
\[ N^R (X \cap Z) \neq N^R X \cap N^R Z. \]

**Proposition 4.2** Let \((G, A)\) be a neutrosophic soft set on a universe \(U\), and let \(X, Z \subseteq U\). Then, the following properties don’t hold

(a) \(N^R N^R X = N^R X\).

(b) \(N^R N^R X = N^R X\).

(c) \(N^R (X - Z) = N^R X - N^R Z\).

(d) \(N^R (X - Z) = N^R X - N^R Z\).

The following example proves (a) of Proposition 4.2.

**Example 4.5** From Example 3.1, If \(X = \{h_2\}\), then

\[ N^R X = \{h_2\} \text{ and } N^R X = \emptyset. \]

Hence, \(N^R X \neq N^R X\).

The following example proves (b) of Proposition 4.2.

**Example 4.6** From Example 3.1, If \(X = \{h_1\}\), then

\[ N^R X = \{h_1\} \text{ and } N^R X = \{h_1, h_2, h_3\}. \]

Hence, \(N^R X \neq N^R X\).

The following example proves (c), (d) of Proposition 4.2.

**Example 4.7** From Example 3.1, If \(X = \{h_1, h_2\}\) and \(Z = \{h_1, h_3\}\), then \(N^R X = \{h_1, h_2\}\), \(N^R Z = \{h_1, h_3\}\), \(N^R (X - Z) = \emptyset\), \(N^R X = \{h_1, h_2, h_3\}\), \(N^R Z = \{h_1, h_2, h_3\}\), \(N^R (X - Z) = \{h_2\}\).

Therefore \(N^R (X - Z) \neq N^R X - N^R Z\) and \(N^R (X - Z) \neq N^R X - N^R Z\).

**Remark 4.1** A comparison among traditional rough model, suggested neutrosophic soft rough model and its modification, by using their properties, is concluded in Table 2, where traditional rough are symboled by (T), neutrosophic soft rough by (N), its modification by (M) and (*) means that, this property is satisfied, as follows

**Table 2:** Comparison among traditional rough and suggested models, by using their properties.

<table>
<thead>
<tr>
<th>Rough properties</th>
<th>T</th>
<th>N</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E(\emptyset) = \mathcal{E}(\emptyset) = \emptyset)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>(E(U) = \mathcal{E}(U) = U)</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>(E(X) \subseteq X \subseteq \mathcal{E}(X))</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>(E(X \cup Y) = \mathcal{E}(X) \cup \mathcal{E}(Y))</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>(E(X \cap Y) = \mathcal{E}(X) \cap \mathcal{E}(Y))</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>(E(X \cup Y) \supseteq \mathcal{E}(X) \cup \mathcal{E}(Y))</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>(E(X \cap Y) \subseteq \mathcal{E}(X) \cap \mathcal{E}(Y))</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

To compare between suggested neutrosophic soft upper approximation and its modification, the following proposition is introduced.

**Proposition 4.3** Let \((G, A)\) be a neutrosophic soft set on a universe \(U\). For any considered set \(X \subseteq U\), the following property holds

\[ N^R X \subseteq \mathcal{N} X. \]
Proof Obvious.

The following example illustrates that a containment relationship between suggested neutrosophic soft upper and its modification, may be proper.

Example 4.7 According to Example 3.1, Table 3 can be created as follows

<table>
<thead>
<tr>
<th>$X$</th>
<th>$N^R X$</th>
<th>$\overline{NRX}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${h_2}$</td>
<td>${h_2}$</td>
<td>${h_1, h_2}$</td>
</tr>
<tr>
<td>${h_3}$</td>
<td>${h_3}$</td>
<td>${h_1, h_3}$</td>
</tr>
<tr>
<td>${h_2, h_3}$</td>
<td>${h_2, h_3}$</td>
<td>${h_1, h_2, h_3}$</td>
</tr>
<tr>
<td>${h_2, h_4}$</td>
<td>${h_2, h_4}$</td>
<td>${h_1, h_2, h_4}$</td>
</tr>
<tr>
<td>${h_2, h_5}$</td>
<td>${h_2, h_5}$</td>
<td>${h_1, h_2, h_5}$</td>
</tr>
<tr>
<td>${h_3, h_4}$</td>
<td>${h_3, h_4}$</td>
<td>${h_1, h_3, h_4}$</td>
</tr>
<tr>
<td>${h_3, h_5}$</td>
<td>${h_3, h_5}$</td>
<td>${h_1, h_3, h_5}$</td>
</tr>
<tr>
<td>${h_2, h_3, h_4}$</td>
<td>${h_2, h_3, h_4}$</td>
<td>${h_1, h_2, h_3, h_4}$</td>
</tr>
<tr>
<td>${h_2, h_3, h_5}$</td>
<td>${h_2, h_3, h_5}$</td>
<td>${h_1, h_2, h_3, h_5}$</td>
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<tr>
<td>${h_2, h_4, h_5}$</td>
<td>${h_2, h_4, h_5}$</td>
<td>${h_1, h_2, h_4, h_5}$</td>
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<td>${h_2, h_3, h_4, h_5}$</td>
<td>${h_2, h_3, h_4, h_5}$</td>
<td>${h_1, h_2, h_3, h_4, h_5}$</td>
</tr>
<tr>
<td>${h_2, h_3, h_4, h_5}$</td>
<td>${h_2, h_3, h_4, h_5}$</td>
<td>${h_1, h_2, h_3, h_4, h_5}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$U$</th>
<th>$\overline{NRX}$</th>
</tr>
</thead>
</table>

Table 3: Comparison between suggested upper approximation and its modification.

From Table 3, we can deduce that, for any considered set $X$, the modified upper approximation is decreased. It follows that its boundary region is decreased.

5 Neutrosophic soft rough concepts and their modification

In this section, some of neutrosophic soft rough concepts are defined as a generalization of rough concepts. Their modification are introduced and compare with them.

Neutrosophic soft rough $NR$-definability and $N^R$-definability of any subset $X \subseteq U$, is defined as follows

**Definition 5.1.** Let $(G, A)$ be a neutrosophic soft set on $U$, and let $X \subseteq U$. A subset $X \subseteq U$, is called

(a) $NR$-definable, if $\overline{NRX} = NRX = X$.

(b) $N^R$-definable, if $N^R X = X$.

(c) Internally $NR$-definable, if $NRX = X$ and $\overline{NRX} \neq X$.

(d) Internally $N^R$-definable, if $N^R X = X$ and $N^R X \neq X$.

(e) Externally $NR$-definable, if $\overline{NRX} \neq X$ and $\overline{NRX} = X$.

(f) Externally $N^R$-definable, if $N^R X \neq X$ and $N^R X = X$.

(g) $NR$-rough, if $\overline{NRX} \neq X$ and $\overline{NRX} \neq X$.

(h) $N^R$-rough, if $N^R X \neq X$ and $N^R X \neq X$.

**Proposition 5.1** Let $(G, A)$ be a neutrosophic soft set on $U$. For any considered set $X \subseteq U$, the following properties hold

(a) $X$ is $NR$-definable set $\rightarrow X$ is $N^R$-definable set.

(b) $X$ is externally $NR$-definable set $\rightarrow X$ is externally $N^R$-definable set.

(c) $X$ is $N^R$-rough set $\rightarrow X$ is $NR$-rough set.

**Proof** Obvious.

The following example proves that the inverse of Proposition 5.1, does not hold.

Example 5.1 According to Example 3.1, Table 4 can be created, where (Ex) means externally definable and (R) means rough as follows
Table 4: Comparison between $NR$-definability and its modification.

<table>
<thead>
<tr>
<th>Ex- $NR$</th>
<th>Ex- $N_R$</th>
<th>$N_R$ -R</th>
<th>$NR$ -R</th>
</tr>
</thead>
<tbody>
<tr>
<td>${h_2}$</td>
<td>${h_2}$</td>
<td>${h_2}$</td>
<td>${h_2}$</td>
</tr>
<tr>
<td>${h_3}$</td>
<td>${h_3}$</td>
<td>${h_3}$</td>
<td>${h_3}$</td>
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<tr>
<td>${h_2, h_3}$</td>
<td>${h_2, h_3}$</td>
<td>${h_2, h_3}$</td>
<td>${h_2, h_3}$</td>
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<tr>
<td>${h_2, h_4}$</td>
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<td>${h_2, h_4}$</td>
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<tr>
<td>${h_3, h_4}$</td>
<td>${h_3, h_4}$</td>
<td>${h_3, h_4}$</td>
<td>${h_3, h_4}$</td>
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<tr>
<td>${h_3, h_5}$</td>
<td>${h_3, h_5}$</td>
<td>${h_3, h_5}$</td>
<td>${h_3, h_5}$</td>
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<tr>
<td>${h_2, h_3, h_4}$</td>
<td>${h_2, h_3, h_4}$</td>
<td>${h_2, h_3, h_4}$</td>
<td>${h_2, h_3, h_4}$</td>
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<tr>
<td>${h_2, h_3, h_5}$</td>
<td>${h_2, h_3, h_5}$</td>
<td>${h_2, h_3, h_5}$</td>
<td>${h_2, h_3, h_5}$</td>
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<tr>
<td>${h_2, h_4, h_5}$</td>
<td>${h_2, h_4, h_5}$</td>
<td>${h_2, h_4, h_5}$</td>
<td>${h_2, h_4, h_5}$</td>
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<td>${h_3, h_4, h_5}$</td>
<td>${h_3, h_4, h_5}$</td>
<td>${h_3, h_4, h_5}$</td>
</tr>
<tr>
<td>${h_2, h_3, h_4, h_5}$</td>
<td>${h_2, h_3, h_4, h_5}$</td>
<td>${h_2, h_3, h_4, h_5}$</td>
<td>${h_2, h_3, h_4, h_5}$</td>
</tr>
</tbody>
</table>

In the following definition neutrosophic soft rough inclusion relations and their modifications are defined.

**Definition 5.3** Let $(G, A)$ be a neutrosophic soft set on $U$, and let $X \subseteq U$, $X \subseteq \subseteq U$. Then

$$X \subseteq_{NR} Z, \text{if } NRX \subseteq N_{R}Z,$$

$$X \subseteq_{NR} Z, \text{if } N_{R}X \subseteq NRZ,$$

$$X \subseteq_{NR} Z, \text{if } N^{R}X \subseteq N^{R}Z.$$
**Definition 5.4** Let \((G, A)\) be a neutrosophic soft set on a universe \(U\), and let \(X, Z \subseteq U\). Then

\[
X = \begin{cases} \if \NR X = \NR Z, \\ \end{cases}
\]

\[
X = \begin{cases} \if \NR Z, \\ \end{cases}
\]

\[
X = \begin{cases} \if N^R X = N^R Z, \\ \end{cases}
\]

\[
X = \begin{cases} \if \NR Z, \\ \end{cases}
\]

\[
X \approx \begin{cases} \if \NR Z, \\ \end{cases}
\]

\[
X \approx \begin{cases} \if \NR Z, \\ \end{cases}
\]

The following examples illustrate Definition 5.4.

**Example 5.3** In Example 3.1, if

\[
X_1 = \{h_2\}, \quad X_2 = \{h_3\}, \quad X_3 = \{h_1, h_2\} \text{ and } X_4 = \{h_1, h_3\}. \text{ Then, } \NR X_1 = \NR X_2 = \emptyset \text{ and } \NR X_3 = \NR X_4 = \{h_1, h_2, h_3\}. \text{ Consequently, } X_1 \approx X_2,
\]

\[
X_3 \approx X_4 \text{ and } X_3 \approx X_4.
\]

**Example 5.4** According to Example 3.1, if \(A' = \{e_1, e_3\}\). Tabular representation of Neutrosophic soft set \((G, A)\) can be seen in Table 5, as follows

<table>
<thead>
<tr>
<th>(A)</th>
<th>(h_1)</th>
<th>(h_2)</th>
<th>(h_3)</th>
<th>(h_4)</th>
<th>(h_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e_1)</td>
<td>(6, 6, 2)</td>
<td>(4, 6, 6)</td>
<td>(6, 4, 2)</td>
<td>(6, 3, 3)</td>
<td>(8, 2, 3)</td>
</tr>
<tr>
<td>(e_3)</td>
<td>(7, 4, 3)</td>
<td>(6, 4, 3)</td>
<td>(7, 2, 5)</td>
<td>(5, 2, 6)</td>
<td>(7, 3, 4)</td>
</tr>
</tbody>
</table>

**Table 5**: Tabular representation of neutrosophic soft set in Example 5.4.

It follows that,

\[
\{h_1\}_A = \{h_1\}, \quad \{h_2\}_A = \{h_1, h_2\}, \quad \{h_3\}_A = \{h_1, h_2, h_3\}, \quad \{h_4\}_A = \{h_1, h_2, h_3, h_4\}, \quad \{h_5\}_A = \{h_1, h_2, h_3, h_4, h_5\}.
\]

If we take \(X_1 = \{h_3\}\) and \(X_2 = \{h_3, h_4\}\), then

\[
\NR X_1 = \NR X_2 = \emptyset \text{ and } \NR X_1 = \NR X_2 = \{h_1, h_3, h_4\} \text{ and } N^R X_1 = N^R X_2 = \{h_1, h_3\}\].

Therefore \(X_1 \approx X_2 \text{ and } X_2 \approx X_2\).

**Proposition 5.3** Let \((G, A)\) be a neutrosophic soft set on a universe \(U, X, Z \subseteq U\) and let \(I \in \{NR, N^R\}\). Then,

(a) \(X = \NR X\).

(b) \(X = N^R X\).

(c) \(X = N^R X\).

(d) \(X = Y \rightarrow X \approx Z\).

(e) \(X \subseteq Z \rightarrow X \subseteq Z \text{ and } X \subseteq Z\).

(f) \(X \subseteq Z \rightarrow Z = \emptyset \rightarrow X = \emptyset\).

(g) \(X \subseteq Z \rightarrow X = U \rightarrow Z = U\).

(h) \(X \subseteq Z \rightarrow \emptyset \rightarrow X = \emptyset\).

(i) \(X \subseteq Z \rightarrow U \rightarrow Z = U\).

**Proof** From Propositions 3.2, 3.3 and 4.1, we get the proof, directly.

We can determine the degree of neutrosophic soft

\(NR\)-definability and \(N^R\)-definability of \(X \subseteq U\), by using their accuracy measures denoted by \(C_{NR} X\) and \(C_{N^R} X\), respectively, which are defined as follows

**Definition 5.5** Let \((G, A)\) be a neutrosophic soft set on \(U\) and let \(X \subseteq U\). Then,
Proposition 5.4 Let \((G, A)\) be a neutrosophic soft set on \(U\) and let \(X \subseteq U\), the following statements are satisfied:

(a) \(0 \leq C_{NR} (X) \leq C_{NR} (X) \leq 1\).

(b) \(X\) is \(NR\)-definable, if and only if, \(C_{NR} (X) = 1\).

(c) \(X\) is \(N\)-definable, if and only if, \(C_{NR} (X) = 1\).

Proof From Definitions 3.3, 4.1, 5.1 and 5.5, we get the proof, directly.

A comparison between suggested neutrosophic soft rough model and its modification, by using their accuracy measures, is concluded in Table 6.

Example 5.5 From Example 3.1, we can create Table 6, as follows:

<table>
<thead>
<tr>
<th>Accuracy measures</th>
<th>(C_{NR})</th>
<th>(C_{NR})</th>
</tr>
</thead>
<tbody>
<tr>
<td>({h_3, h_4})</td>
<td>0.33</td>
<td>0.50</td>
</tr>
<tr>
<td>({h_2, h_4})</td>
<td>0.33</td>
<td>0.50</td>
</tr>
<tr>
<td>({h_2, h_5})</td>
<td>0.33</td>
<td>0.50</td>
</tr>
<tr>
<td>({h_2, h_4, h_5})</td>
<td>0.50</td>
<td>0.67</td>
</tr>
<tr>
<td>({h_3, h_4, h_5})</td>
<td>0.50</td>
<td>0.67</td>
</tr>
<tr>
<td>({h_2, h_3, h_4, h_5})</td>
<td>0.40</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Table 6: Comparison between suggested neutrosophic soft rough model and its modification, by using their accuracy measures.

From Table 6, by using suggested modified approximations, the degree of definability of all these subsets is increased. It means that, when we use suggested modified approximations, we notice that, for any considered neutrosophic soft rough set, its boundary region is decreased. It leads to more accurate results of any real life application.

Remark 5.1 Let \((G, A)\) be a neutrosophic soft set on a universe \(U\), and let \(h \in U\), \(X \subseteq U\). If we consider the following case: If

\[ T_i(h) > 0.5, \text{ then } e(h) = 1, \text{ otherwise, } e(h) = 0. \]

Hence, neutrosophic right neighborhood of an element \(h\) is replaced by the following equivalence class

\[ [h] = \{ h_i \in U: e(h_i) = e(h), e \in A \}. \]

It follows that, neutrosophic soft rough set approximations will be returned to Pawlak’s rough set approximations. Consequently, all properties of traditional rough set approximations will be satisfied. Hence, Pawlak’s approach to rough sets is a special case of the proposed approaches in this paper.

Conclusion

The difference in neutrosophic logic is that there is a component of indeterminate \(I\), which means, for example in decision making and control theory, that we have \((I\%)\) hesitating to take a decision. It follows that proposed models, in this paper, are more realistic than Pawlak’s model. Pawlak’s approach to rough sets can be viewed as a special case of neutrosophic soft approach to rough sets. Our future work, aims to apply them in solving many practical problems in medical science.

References

[7] X. Ge, X. Bai, Z. Yun, Topological characterizations of covering for special covering-based upper approximation...


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Solution of Multi-Criteria Assignment Problem using Neutrosophic Set Theory

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Abstract: Assignment Problem (AP) is a very well-known and also useful decision making problem in real life situations. It becomes more effective when different criteria are added. To solve Multi-Criteria Assignment Problem (MCAP), the different criteria have been considered as neutrosophic elements because Neutrosophic Set Theory (NST) is a generalization of the classical sets, conventional fuzzy sets, Intuitionistic Fuzzy Sets (IFS) and Interval Valued Fuzzy Sets (IVFS). In this paper two different methods have been proposed for solving MCAP. In the first method, we have calculated evaluation matrix, score function matrix, accuracy matrix and ranking matrix of the MCAP. The rows represent the alternatives and columns represent the projects of the MCAP. From the ranking matrix, the ranking order of the alternatives and the projects are determined separately. From the above two matrices, composite matrix is formed and it is solved by Hungarian Method to get the optimal assignment.

In the second one, Cosine formula for Vector Similarity Measure [1] on neutrosophic set is used to calculate the degree of similarity between each alternative and the ideal alternative. From the similarity matrix, the ranking order of the alternatives and the projects are determined in the same way as above. Finally the problem is solved by Hungarian Method to obtain the optimal solution.

Keywords: Assignment, Neutrosophic Set, Similarity Measures.

1. Introduction:-

NST is a powerful formal framework which generalizes the concepts of classical set, fuzzy set, IFS, IVFS etc. In the year 1965 Zadeh [2] first introduced the concept of fuzzy set which is a very effective tool to measure uncertainty in real life situation. After two decades, Turksen [3] proposed the concept of IVFS. Atanassov [4] introduced IFS which not only describes the degree of membership, but also the degree of non-membership function. Wang et. al [5] proposed a different concept of imprecise data which gives indeterminate information. F. Smarandache introduced the degree of indeterminacy/neutrality [6] as independent concept in 1995 (published in 1998) and defined the neutrosophic set. He coined the words ‘neutrosophy’ and ‘neutrosophic’. In 2013, he refined the neutrosophic set to ‘n’ components: t1, t2,...... ; i1, i1,......; f1, f2,......

Different authors have solved Multi-Criteria Decision Making (MCDM) problems in different ways. But in neutrosophy, MCAP has not been solved earlier. In real life situation, truth value and falsity (membership and non-membership function) are not sufficient; indeterminacy is also a very important part for decision making problem. NST is a different and more practical concept of fuzzy set where degree of truth value, falsity and indeterminacy are all considered and so it is more relevant to solve MCDM problems.

Several mathematicians have worked on the concept of similarity measures of fuzzy sets. Xu. Z. S [7] used similarity measures of IFS and their applications to multiple attribute DM problems. Li et. al [8] also...

In this paper we have developed two methods to solve MCAP. One is based on score function and another one is on vector similarity measure for neutrosophic set. The methods have been demonstrated by a numerical example. The paper is organized as follows - In section 2 preliminaries have been given; section 3 describes the MCAP method and its solution procedures along with the two algorithms. Section 4 illustrates the numerical example and finally section 5 concludes the paper.

2. Preliminaries:

2.1 Neutrosophic Set:

Let U be the space of points (or objects) with generic element ‘x’. A neutrosophic set A in U is characterized by a truth membership function $T_A$, and indeterminacy function $I_A$ and a falsity membership function $F_A$, where $T_A$, $I_A$ and $F_A$ are real standard or non-standard sub-intervals of $[0, 1]^3$, i.e $T_A: x \rightarrow [0, 1]^3$, $I_A: x \rightarrow [0, 1]^3$, $F_A: x \rightarrow [0, 1]^3$.

A neutrosophic set A upon U as an object is defined as -

$$\frac{x}{T_A(x), I_A(x), F_A(x)} = \{ \frac{x}{T_A, I_A, F_A} : x \in U \}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are sub-intervals or union of sub-intervals of $[0, 1]$.

2.2 Algebraic Operations with Neutrosophic Set:

For two neutrosophic sets A and B where

\[ a) \text{ Complement of A} \]
\[ A' = \{ \frac{x}{T_{A'}, I_{A'}, F_{A'}} : T = 1 - T_A, I = 1 - I_A, F = 1 - F_A \} \]

\[ b) \text{ Intersection of A and B} \]
\[ A \cap B = \{ \frac{x}{T_{A'}, I_{A'}, F_{A'}} : T = T_A T_B, I = I_A I_B, F = F_A F_B \} \]

\[ c) \text{ Union of A and B} \]
\[ A \cup B = \{ \frac{x}{T_{A'}, I_{A'}, F_{A'}} : T = T_A + T_B - T_A T_B, I = I_A + I_B - I_A I_B, F = F_A + F_B - F_A F_B \} \]

\[ d) \text{ Cartesian Product of A and B} \]
\[ A \times B = \{ (\frac{x}{T_A, I_A, F_A}, \frac{y}{T_B, I_B, F_B}) \mid \frac{x}{T_A, I_A, F_A} \in A, \frac{y}{T_B, I_B, F_B} \in B \} \]

\[ e) \text{ A is a subset of B} \]
\[ A \subseteq B \forall \frac{x}{T_A, I_A, F_A} \in A \text{ and } \frac{y}{T_B, I_B, F_B} \in B \]

\[ f) \text{ Difference of A and B} \]
\[ A \setminus B = \{ \frac{x}{T_A, I_A, F_A} : T = T_A - T_B, I = I_A - I_B, F = F_A - F_B \} \]

NST can be used in assignment problem (AP) and Generalized Assignment Problem (GAP).

2.3 Cosine formula for vector similarity measure:

Cosine formula for vector similarity measure is
WS\v(a_i, A^*) = 
\frac{\sum_{j=1}^{n} w_j [a_j a_j^* + b_j b_j^* + c_j c_j^*]}{\sqrt{\sum (a_j^2 + b_j^2 + c_j^2) \sum (a_j^2 + b_j^2 + c_j^2)}}

\text{………………..}[1]

Where A_i is the alternative, A^* is the ideal alternative, w_j represents the weight of the alternatives s.t. \sum_{j=1}^{n} w_j = 1.

The criteria are divided into two types – one is cost criterion and the other is benefit criterion (profit, efficiency, quality etc). For these two types ideal alternatives have been defined as –

\text{a}> Ideal alternative for cost criterion, A^* is
\text{a}^*_i = < (a_j^*, b_j^*, c_j^*) >= 
\left\{ \begin{array}{l}
\min_i (a_j), \max_i (b_j), \max_i (c_j) \\
\end{array} \right.

\text{………………..}[2]

\text{b}> Ideal alternative for benefit criterion, A^* is
\text{a}^*_i = < (a_j^*, b_j^*, c_j^*) >= 
\left\{ \begin{array}{l}
\max_i (a_j), \min_i (b_j), \min_i (c_j) \\
\end{array} \right.

\text{………………..}[3]

3. MCAP using NST:-

In this section we have formulated the MCAP using NST. The AP has been solved by different mathematicians in various ways [14], [15]. Here we have proposed two methods – \text{i> In the first method, to compute the best final result, the evaluation of the alternatives with regard to each criteria are must. So from the decision matrix, evaluation matrix has been calculated. Then score function of each alternative has been computed. To find the degree of accuracy (H(A_i)) of neutrosophic elements, accuracy matrix has been evaluated. The larger value of H(A_i), the more is the degree of accuracy of an alternative A_i. To evaluate all the above matrices weights must be considered because the larger the value of W(E(A_i)), the more is the suitability to which the alternative A_i satisfies the decision maker’s requirement. Using the above, ranking matrix has been computed. Then the alternatives (Teams) are ranked with respect to the criteria (Projects) row-wise and the opposite is done column-wise. From the above two matrices, composite matrix has been formed. Finally assignment is done using Hungarian method. ii> By using the cosine formula for vector similarity measure on neutrosophic elements. Similarity matrix is computed and the Hungarian method, as mentioned earlier, is again used to get the optimal assignment.

3.1 Solution procedure for MCAP:-

Method 1:

To solve MCAP we have considered the elements of the criteria as neutrosophic elements (T, I, F), where T is the truth membership degree, I is indeterminacy and F represents falsity degree. From the input data, evaluation matrix E(A), score function matrix S(A) and accuracy matrix H(A) of the alternatives are determined. Algorithm 1 is applied to find the ranking matrix R(A) using the above three matrices and weights of the criteria.

\textbf{Algorithm 1:}

\textbf{Step 1:} Construct the matrix of neutrosophic MCAP.

\textbf{Step 2:} Determine the evaluation matrix E(A) = (E(A_j))_{max} of the alternatives as E(A) = \left[ T_A^I, T_A^u \right]

where

\left[ T_A^I, T_A^u \right] =
\left[ \begin{array}{l}
\min((\frac{\text{T}_{A_i} + \text{I}_{A_i}}{2}, (\frac{1 - \text{F}_{A_i} + \text{I}_{A_i}}{2})),
\max((\frac{\text{T}_{A_i} + \text{I}_{A_i}}{2}, (\frac{1 - \text{F}_{A_i} + \text{I}_{A_i}}{2})))
\end{array} \right]
\text{………}[4]

\textbf{Step 3:} Compute the score function matrix S(A) = (S(A_j))_{max} of an alternative using the formula

S(A) = 2 \left[ T_A^U - T_A^L \right] \text{where } 0 \leq S(A_i) \leq 1 \text{………}[5]
Step 4: Compute Accuracy matrix \( H(A) = (H(A_{ij}))_{mn} \) to evaluate degree of accuracy of the neutrosophic elements as –
\[
H(A) = 0.5 \left[ T_{A}^U + T_{A}^L \right] \]

Step 5: Using \( E(A) \), \( H(A) \), \( S(A) \) and \( w_j \), ranking matrix \( R(A) = (R(A_{ij}))_{mn} \) of the alternatives is determined by the formula
\[
R(A) = \sum_{j=1}^{n} \left( S(A_{ij})^2 - \frac{1 - H(A_{ij})}{2} \right) w_j \]

Step 6: Form \( R_1 \) matrix by considering the rank of the teams and \( R_2 \) matrix for the projects.

Step 7: Form the composite matrix by taking the product of \( R_1 \) and \( R_2 \).

Step 8: Solve the composite matrix by Hungarian method to get the optimal assignment.

Step 9: End.

Method 2:

This method is based on the concept of similarity measures. Here the cosine formula, previously mentioned, has been used. The ideal alternatives for the two types of criteria (cost and benefit) have been defined in the equations [2] and [3]. The cosine formula for similarity measures which is defined in equation [1] has been used to find the degree of similarity and the ranking matrix has been evaluated. The alternative which has the maximum value of the degree of similarity is more similar to the ideal alternative \( A^* \) and can be considered as the best choice.

Algorithm 2:

Step 1: Categorize the criteria in two ways – cost criterion and benefit criterion.

Step 2: Determine the ideal alternative for both of the types of criteria defined as in equation [2] and [3].

Step 3: Consider the weights of the criteria \( w_j \) and use cosine formula (equation [1]) for vector similarity measures on NS to find the similarity matrix.

Step 4: Follow steps 6 to 8 of Method 1 Algorithm 1.

Step 5: End.

Numerical Example:

Let us consider an AP consisting of three projects and four teams with three criteria. The three criteria are – cost, profit and efficiency of the team which are considered as neutrosophic elements and the data are as follows.

<table>
<thead>
<tr>
<th>Projects</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teams</td>
<td>c1</td>
<td>c2</td>
<td>c3</td>
</tr>
<tr>
<td>A</td>
<td>(0.75, 0.39, 0.1)</td>
<td>(0.6, 0.5, 0.25)</td>
<td>(0.8, 0.4, 0.2)</td>
</tr>
<tr>
<td>B</td>
<td>(0.8, 0.68, 0.15)</td>
<td>(0.6, 0.4, 0.1)</td>
<td>(0.5, 0.2, 0.3)</td>
</tr>
<tr>
<td>C</td>
<td>(0.4, 0.75, 0.5)</td>
<td>(0.8, 0.9, 0.1)</td>
<td>(0.5, 0.25, 0.4)</td>
</tr>
<tr>
<td>D</td>
<td>(0.4, 0.5, 0.15)</td>
<td>(0.6, 0.3, 0.5)</td>
<td>(0.5, 0.25, 0.4)</td>
</tr>
</tbody>
</table>
The weights of the criteria are \( w_1 = 0.35 \), \( w_2 = 0.40 \) and \( w_3 = 0.25 \) such that \( \sum_{j=1}^{3} w_j = 1. \)

The problem is to find the optimal assignment.

**Solution:**

**Method 1:**

First we calculate the evaluation matrix \( E(A_i) \) of the alternatives by applying formula [4] –

**Table 2:**

<table>
<thead>
<tr>
<th>Project Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(0.57,0.645), (0.55,0.625)</td>
<td>(0.25,0.35), (0.7,0.75)</td>
<td>(0.15,0.4), (0.375,0.475)</td>
</tr>
<tr>
<td>B</td>
<td>(0.7,0.725), (0.57,0.63)</td>
<td>(0.2,0.45), (0.45, 0.45)</td>
<td>(0.25,0.4), (0.4,0.45)</td>
</tr>
<tr>
<td>C</td>
<td>(0.6,0.675), (0.825,0.925)</td>
<td>(0.225,0.4), (0.4,0.45)</td>
<td>(0.55,0.7), (0.45, 0.65)</td>
</tr>
<tr>
<td>D</td>
<td>(0.5,0.65), (0.35, 0.45)</td>
<td>(0.225,0.4), (0.45, 0.45)</td>
<td>(0.35,0.45), (0.25,0.45)</td>
</tr>
</tbody>
</table>

The score function matrix is calculated by the formula [5].

**Table 3:**

<table>
<thead>
<tr>
<th>Project Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(0.15,0.15,0)</td>
<td>(0.2,0.1,0.3)</td>
<td>(0.5,0.2,0)</td>
</tr>
<tr>
<td>B</td>
<td>(0.05,0.12,0.5)</td>
<td>(0.5,0.3,0)</td>
<td>(0.3,0.1,0.3)</td>
</tr>
<tr>
<td>C</td>
<td>(0.15,0.2,1)</td>
<td>(0.35,1,0.2)</td>
<td>(0.3,0.1,0.4)</td>
</tr>
<tr>
<td>D</td>
<td>(0.3,0.3,0.4)</td>
<td>(0.35,0.7)</td>
<td>(0.2,0.4,0.6)</td>
</tr>
</tbody>
</table>

Accuracy matrix \( H(A_i) \) has been calculated by formula [6].

**Table 4:**

<table>
<thead>
<tr>
<th>Project Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(0.6075,0.5875,0.6)</td>
<td>(0.3,0.725,0.325)</td>
<td>(0.275,0.425,0.4)</td>
</tr>
<tr>
<td>B</td>
<td>(0.7125,0.6,0.4)</td>
<td>(0.325,0.475,0.45)</td>
<td>(0.325,0.425,0.175)</td>
</tr>
<tr>
<td>C</td>
<td>(0.6375,0.875,5.05)</td>
<td>(0.3125,0.47,5.05)</td>
<td>(0.625,0.425,0.55)</td>
</tr>
<tr>
<td>D</td>
<td>(0.575,0.375,0.45)</td>
<td>(0.3125,0.45,0.575)</td>
<td>(0.4,0.35,0.5)</td>
</tr>
</tbody>
</table>

Now we calculate the ranking matrix \( R(A_i) \) using formula [7].

**Table 5:**

<table>
<thead>
<tr>
<th>Project Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>- 0.184</td>
<td>- 0.222</td>
<td>- 0.075</td>
</tr>
<tr>
<td>B</td>
<td>- 0.136</td>
<td>- 0.169</td>
<td>- 0.279</td>
</tr>
<tr>
<td>C</td>
<td>0.124</td>
<td>0.165</td>
<td>- 0.161</td>
</tr>
<tr>
<td>D</td>
<td>- 0.161</td>
<td>- 0.118</td>
<td>- 0.129</td>
</tr>
</tbody>
</table>
Table 6:
Ranking Indices ($R_1$) of the project w.r.t the team

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 7:
Ranking Indices ($R_2$) of the team w.r.t the project

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 8:
Composite matrix $R_1R_2$

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>8</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>2</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>2</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>9</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

To solve the above matrix, a dummy column has been added and the AP is solved by Hungarian method to get the optimal assignment.

Table 9:
Solution Matrix (a)

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>6</td>
<td>11</td>
<td>[0]</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>[0]</td>
<td>5</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>0</td>
<td>[0]</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>[0]</td>
</tr>
</tbody>
</table>

Therefore optimal assignment is A→III, B→I, C→II, and D→IV.

Method 2:
Here the three criteria are –
c_1 → Cost, c_2 → Profit, c_3 → Efficiency

For cost criterion, ideal alternative $A^*$ is –

$$a_j^* = < (a_j^*, b_j^*, c_j^*)> = \left[ \min_i (a_{ij}), \max_i (b_{ij}), \max_i (c_{ij}) \right]$$

For benefit criteria (profit and efficiency), ideal alternative $A^*$ is –

$$a_j^* = < (a_j^*, b_j^*, c_j^*)> = \left[ \max_i (a_{ij}), \min_i (b_{ij}), \min_i (c_{ij}) \right]$$

Therefore cosine formula for similarity measure is –

$$WS_c (A_i, A^*) = \sum_{j=1}^{n} w_j [a_{ij}a_j^* + b_{ij}b_j^* + c_{ij}c_j^*]$$

$$\sqrt{\sum (a_{ij}^2 + b_{ij}^2 + c_{ij}^2) \sum (a_{ij}^*2 + b_{ij}^*2 + c_{ij}^*2)}$$

where weights of the criteria $c_1$, $c_2$ and $c_3$ are $w_1 = 0.35$, $w_2 = 0.40$ and $w_3 = 0.25$ such that $\sum_{j=1}^{3} w_j = 1$.

Table 10:
Degree of similarity matrix ($WS_c (A_i, A^*)$)

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>0.289</td>
<td>0.323</td>
<td>0.441</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>0.303</td>
<td>0.275</td>
<td>0.293</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>0.280</td>
<td>0.251</td>
<td>0.264</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>0.244</td>
<td>0.263</td>
<td>0.274</td>
<td></td>
</tr>
</tbody>
</table>
Table 11:
Ranking Indices ($R_3$) of the project w.r.t the team

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Table 12:
Ranking Indices ($R_4$) of the team w.r.t the project

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Table 13:
Composite matrix $R_3R_4$

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>12</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>12</td>
<td>6</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Solving composite matrix by Hungarian method-

Table 14:

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>10</td>
<td>7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>11</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 15:
Solution Matrix (b)

<table>
<thead>
<tr>
<th>Projects</th>
<th>Teams</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7</td>
<td>[0]</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>[0]</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>8</td>
<td>5</td>
<td>[0]</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>11</td>
<td>2</td>
<td>[0]</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Therefore optimal assignment is A→II, B→I, C→IV, and D→III.

4. Conclusion:-
This paper proposes two different approaches to solve MCAP. Both of them are simple but very efficient and have not been used earlier. The numerical example demonstrates the application and effectiveness of the methods with the incomplete and indeterminate information which exist commonly in real life situations.

References:-


[7] Xu Z. S, Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute

Supriya Kar, Kajla Basu, Sathi Mukherjee, Solution of Multi-Criteria Assignment Problem using Neutrosophic Set Theory


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Taylor Series Approximation to Solve Neutrosophic Multiobjective Programming Problem

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Abstract. In this paper, Taylor series is used to solve neutrosophic multi-objective programming problem (NMOPP). In the proposed approach, the truth membership, indeterminacy membership, falsity membership functions associated with each objective of multi-objective programming problems are transformed into a single objective linear programming problem by using a first order Taylor polynomial series. Finally, to illustrate the efficiency of the proposed method, a numerical experiment for supplier selection is given as an application of Taylor series method for solving neutrosophic multi-objective programming problem at end of this paper.

Keywords: Taylor series; Neutrosophic optimization; Multiobjective programming problem.

1 Introduction

In 1995, Smarandache [1] starting from philosophy (when he fretted to distinguish between absolute truth and relative truth or between absolute falsehood and relative falsehood in logics, and respectively between absolute membership and relative membership or absolute non-membership and relative non-membership in set theory) [1] began to use the non-standard analysis. Also, inspired from the sport games (winning, defeating, or tie scores), from votes (pro, contra, null/black votes), from positive/negative/zero numbers, from yes/no/NA, from decision making and control theory (making a decision, not making, or hesitating), from accepted/rejected/pending, etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics. [1] combined the non-standard analysis with a tri-component logic/set/probability theory and with philosophy. How to deal with all of them at once, is it possible to unity them? [1]. The words “neutrosophy” and “neutrosophic” were invented by F. Smarandache in his 1998 book [1]. Etymologically, “neu-tro-sophy” (noun) [French neutre < Latin neuter, neutral, and Greek sophia, skill / wisdom] means knowledge of neutral thought. While “neutrosophic” (adjective), means having the nature of, or having the characteristic of Neutrosophy. Neutrosophic theory means Neutrosophy applied in many fields in order to solve problems related to indeterminacy. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every entity <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. entities supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every entity <A> tends to be neutralized and balanced by <antiA> and <nonA> entities - as a state of equilibrium. In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Hence, in one hand, the Neutrosophic Theory is based on the triad <A>, <neutA>, and <antiA>. In the other hand, Neutrosophic Theory studies the indeterminacy, labeled as I, with In = 1 for n ≥ 1, and mI + nI = (m+n)I, in neutrosophic structures developed in algebra, geometry, topology etc. The most developed fields of Neutrosophic theory are Neutrosophic Sets, Neutrosophic Logic, Neutrosophic Probability, and Neutrosophic Statistics - that started in 1995, and recently Neutrosophic Precalculus and...
Neutrosophic Calculus, together with their applications in practice. Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of ]0, 1[. Multi-objective linear programming problem (MOLPP) a prominent tool for solving many real decision making problems like game theory, inventory problems, agriculture based management systems, financial and corporate planning, production planning, marketing and media selection, university planning and student admission, health care and hospital planning, air force maintenance units, bank branches etc.

Our objective in this paper is to propose an algorithm to the solution of neutrosophic multi-objective programming problem (NMOPP) with the help of the first order Taylor’s theorem. Thus, neutrosophic multi-objective linear programming problem is reduced to an equivalent multi-objective linear programming problem. An algorithm is proposed to determine a global optimum to the problem in a finite number of steps. The feasible region is a bounded set. In the proposed approach, we have attempted to reduce computational complexity in the solution of (NMOPP). The proposed algorithm is applied to supplier selection problem.

The rest of this article is organized as follows. Section 2 gives brief some preliminaries. Section 3 describes the formation of the problem. Section 4 presents the implementation and validation of the algorithm with practical application. Finally, Section 5 presents the conclusion and proposals for future work.

2 Some preliminaries

Definition 1. [1] A triangular fuzzy number \( \tilde{J} \) is a continuous fuzzy subset from the real line \( R \) whose triangular membership function \( \mu_{\tilde{J}}(\tilde{J}) \) is defined by a continuous mapping from \( R \) to the closed interval [0,1], where

\[
\mu_{\tilde{J}}(\tilde{J}) = \begin{cases} 
0 & \text{for all } \tilde{J} \in (-\infty, a_1], \\
J - a_1 & \text{for } a_1 \leq \tilde{J} \leq m, \\
(a_2 - J) & \text{for } m \leq \tilde{J} \leq a_2, \\
0 & \text{otherwise}.
\end{cases}
\]

(1)

Figure 1: Membership Function of Fuzzy Number \( \tilde{J} \).

where \( m \) is a given value and \( a_1 \), \( a_2 \) denote the lower and upper bounds. Sometimes, it is more convenient to use the notation explicitly highlighting the membership function parameters. In this case, we obtain

\[
\mu(\tilde{J}; a_1, m, a_2) = \max \left\{ \min \left\{ \frac{J - a_1}{m - a_1}, \frac{a_2 - J}{a_2 - m} \right\}, 0 \right\}.
\]

(2)

In what follows, the definition of the \( \alpha \)-level set or \( \alpha \)-cut of the fuzzy number \( \tilde{J} \) is introduced.

Definition 2. [1] Let \( X = \{x_1, x_2, ..., x_{|a|}\} \) be a fixed non-empty universe. An intuitionistic fuzzy set IFS \( A \) in \( X \) is defined as

\[
A = \left\{ (x, \mu_A(x), \nu_A(x)) | x \in X \right\}
\]

(3)

which is characterized by a membership function \( \mu_A : X \rightarrow [0,1] \) and a non-membership function \( \nu_A : X \rightarrow [0,1] \) with the condition

\[
0 \leq \mu_A(x) + \nu_A(x) \leq 1
\]

for all \( x \in X \) where \( \mu_A \) and \( \nu_A \) represent, respectively, the degree of membership and non-membership of the element \( x \) to the set \( A \). In addition, for each IFS \( A \) in \( X \), \( \tau_A(x) = 1 - \mu_A(x) - \nu_A(x) \) for all \( x \in X \) is called the degree of hesitation of the...
element $x$ to the set $A$. Especially, if $\pi_A(x) = 0$, then the IFS $A$ is degraded to a fuzzy set.

**Definition 3.** [4] The $\alpha$-level set of the fuzzy parameters $\bar{J}$ in problem (1) is defined as the ordinary set $L_\alpha(\bar{J})$ for which the degree of membership function exceeds the level, $\alpha$, $\alpha \in [0,1]$, where:

$$L_\alpha(\bar{J}) = \left\{ J \in R \mid \mu_J(J) \geq \alpha \right\}$$

(4)

For certain values $\alpha^*$ to be in the unit interval,

**Definition 4.** [1] Let $X$ be a space of points (objects) and $x \in X$. A neutrosophic set $A$ in $X$ is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. It has been shown in figure 2. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $\mathbb{J}$. That is $T_A(x)X \rightarrow [0,1]$ and $I_A(x)X \rightarrow [0,1]$ and $F_A(x)X \rightarrow [0,1]$. There is not restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0 \leq \sup T_A(x) \leq \sup I_A(x) \leq \sup F_A(x) \leq 3$.

In the following, we adopt the notations $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ instead of $T_A(x), I_A(x)$ and $F_A(x)$, respectively. Also we write SVN numbers instead of single valued neutrosophic numbers.

**Definition 5.** [10] Let $X$ be a universe of discourse. A single valued neutrosophic set $A$ over $X$ is an object having the form

$A = \{x, \mu_A(x), \sigma_A(x), \nu_A(x) \mid x \in X\}$

where $\mu_A(x) : X \rightarrow [0,1]$, $\sigma_A(x) : X \rightarrow [0,1]$ and $\nu_A(x) : x \rightarrow [0,1]$ with $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$ for all $x \in X$. The intervals $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of $x$ to $A$, respectively.

For convenience, a SVN number is denoted by $A = (a,b,c)$, where $a,b,c \in [0,1]$ and $a+b+c \leq 3$.

**Definition 6**

Let $\bar{J}$ be a neutrosophic triangular number in the set of real numbers $R$, then its truth-membership function is defined as

$$T_j(J) = \frac{J-a_1}{a_2-a} \quad a_1 \leq J \leq a_2,$$

(5)

$$I_j(J) = \frac{a_2-b}{b_3-b} \quad a_2 \leq J \leq b_3,$$

(6)

and its indeterminacy-membership function is defined as

$$F_j(J) = \frac{J-b_1}{c_2-c} \quad b_2 \leq J \leq c_2,$$

(7)

otherwise.

Figure 2: Neutrosophication process [11]
3 Formation of The Problem

The multi-objective linear programming problem and the multi-objective neutrosophic linear programming problem are described in this section.

A. Multi-objective Programming Problem (MOPP)

In this paper, the general mathematical model of the MOPP is as follows [6]:

\[
\min / \max \left[ z_1(x_1, \ldots, x_n), z_2(x_1, \ldots, x_n), \ldots, z_p(x_1, \ldots, x_n) \right]
\]

subject to \( x \in S, x \in R^n \)

\[
S = x \in R^n \left( \begin{array}{c} \leq b \end{array} \right), X \geq 0.
\]

B. Neutrosophic Multi-objective Programming Problem (NMOPP)

If an imprecise aspiration level is introduced to each of the objectives of MOPP, then these neutrosophic objectives are termed as neutrosophic goals.

Let \( z_i \in [z_i^L, z_i^U] \) denote the imprecise lower and upper bounds respectively for the \( i^{th} \) neutrosophic objective function.

For maximizing objective function, the truth membership, indeterminacy membership, falsity membership functions can be expressed as follows:

\[
\mu^I_i(z_i) = \begin{cases} 1, & \text{if } z_i \leq z_i^L, \\ \frac{z_i^L - z_i}{z_i^L - z_i}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 0, & \text{if } z_i \geq z_i^U, \end{cases}
\]

\[
\sigma^I_i(z_i) = \begin{cases} 0, & \text{if } z_i \leq z_i^L, \\ \frac{z_i^L - z_i}{z_i^L - z_i}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 1, & \text{if } z_i \geq z_i^U, \end{cases}
\]

\[
u^I_i(z_i) = \begin{cases} 0, & \text{if } z_i \leq z_i^L, \\ \frac{z_i^L - z_i}{z_i^L - z_i}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 1, & \text{if } z_i \geq z_i^U, \end{cases}
\]

for minimizing objective function, the truth membership, indeterminacy membership, falsity membership functions can be expressed as follows:

\[
\mu^I_i(z_i) = \begin{cases} 1, & \text{if } z_i \geq z_i^U, \\ \frac{z_i - z_i^L}{z_i^U - z_i}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 0, & \text{if } z_i \leq z_i^L, \end{cases}
\]

\[
\sigma^I_i(z_i) = \begin{cases} 0, & \text{if } z_i \geq z_i^U, \\ \frac{z_i - z_i^L}{z_i^U - z_i}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 1, & \text{if } z_i \leq z_i^L, \end{cases}
\]

\[
u^I_i(z_i) = \begin{cases} 0, & \text{if } z_i \geq z_i^U, \\ \frac{z_i - z_i^L}{z_i^U - z_i}, & \text{if } z_i^L \leq z_i \leq z_i^U, \\ 1, & \text{if } z_i \leq z_i^L, \end{cases}
\]

4 Algorithm for Neutrosophic Multi-Objective Programming Problem

The computational procedure and proposed algorithm of presented model is given as follows:

**Step 1.** Determine \( x^*_i = (x^*_1, x^*_2, \ldots, x^*_m) \) that is used to maximize or minimize the \( i^{th} \) truth membership function \( \mu^I_i(X) \), the indeterminacy membership \( \sigma^I_i(X) \), and
the falsity membership functions  \( u_i^f(X) \), \( i=1,2,...,p \) and \( n \) is the number of variables.

**Step 2.** Transform the truth membership, indeterminacy membership, falsity membership functions by using first-order Taylor polynomial series

\[
\mu_i^f(x) = \mu_i^f(x_i^*) + \sum_{j=1}^{n} \mu_i^f(x_j^*) \frac{\partial \mu_i^f(x_i^*)}{\partial x_j}(x_j-x_j^*)
\]

(16)

\[
\sigma_i^f(x) = \sigma_i^f(x_i^*) + \sum_{j=1}^{n} \sigma_i^f(x_j^*) \frac{\partial \sigma_i^f(x_i^*)}{\partial x_j}(x_j-x_j^*)
\]

(17)

\[
u_i^f(x) = \nu_i^f(x_i^*) + \sum_{j=1}^{n} \nu_i^f(x_j^*) \frac{\partial \nu_i^f(x_i^*)}{\partial x_j}(x_j-x_j^*)
\]

(18)

**Step 3.** Find satisfactory \( x_i^* = (x_{i1}^*, x_{i2}^*, ..., x_{in}^*) \) by solving the reduced problem to a single objective for the truth membership, indeterminacy membership, falsity membership functions respectively.

\[
p(x) = \sum_{i=1}^{p} \mu_i^f(x_i^*) + \sum_{j=1}^{n} \mu_i^f(x_j^*) \frac{\partial \mu_i^f(x_i^*)}{\partial x_j}(x_j-x_j^*)
\]

(19)

\[
q(x) = \sum_{i=1}^{p} \sigma_i^f(x_i^*) + \sum_{j=1}^{n} \sigma_i^f(x_j^*) \frac{\partial \sigma_i^f(x_i^*)}{\partial x_j}(x_j-x_j^*)
\]

\[
h(x) = \sum_{i=1}^{p} \nu_i^f(x_i^*) + \sum_{j=1}^{n} \nu_i^f(x_j^*) \frac{\partial \nu_i^f(x_i^*)}{\partial x_j}(x_j-x_j^*)
\]

Thus neutrosophic multiobjective linear programming problem is converted into a new mathematical model and is given below:

Maximize or Minimize \( p(x) \)

Maximize or Minimize \( q(x) \)

Maximize or Minimize \( h(x) \)

Where \( \mu_i^f(X) \), \( \sigma_i^f(X) \) and \( \nu_i^f(X) \) calculate using equations (10), (11), and (12) or equations (13), (14), and (15) according to type functions maximum or minimum respectively.

**4.1 Illustrative Example**

A multi-criteria supplier selection is selected from [2]. For supplying a new product to a market assume that three suppliers should be managed. The purchasing criteria are net price, quality and service. The capacity constraints of suppliers are also considered.

It is assumed that the input data from suppliers’ performance on these criteria are not known precisely. The neutrosophic values of their cost, quality and service level are presented in Table 1.

The multi-objective linear formulation of numerical example is presented as:

\[
\begin{align*}
\text{min } z_1 &= 5x_1 + 7x_2 + 4x_3, \\
\text{max } z_2 &= 0.80x_1 + 0.90x_2 + 0.85x_3, \\
\text{max } z_3 &= 0.90x_1 + 0.80x_2 + 0.85x_3, \\
\text{s.t.:} &
\end{align*}
\]

\[
x_1 + x_2 + x_3 = 800, \\
x_1 \leq 400, \\
x_2 \leq 450, \\
x_3 \leq 450, \\
x_j \geq 0, & \quad j=1,2,3.
\]

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Z1 Cost</th>
<th>Z2 Quality (%)</th>
<th>Z3 Service (%)</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>5</td>
<td>0.80</td>
<td>0.90</td>
<td>400</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>7</td>
<td>0.90</td>
<td>0.80</td>
<td>450</td>
</tr>
<tr>
<td>Supplier 3</td>
<td>4</td>
<td>0.85</td>
<td>0.85</td>
<td>450</td>
</tr>
</tbody>
</table>

The truth membership, indeterminacy membership, falsity membership functions were considered to be neutrosophic triangular. When they depend on three scalar parameters \((a_1, m, a_2)\), \(z\) depends on neutrosophic aspiration levels \((3550, 4225, 4900)\), when \(z\) depends on neutrosophic aspiration levels \((660, 681.5, 702.5)\), and \(z\) depends on neutrosophic aspiration levels \((657.5, 678.75, 700)\).

The truth membership functions of the goals are obtained as follows:

\[
\mu_i^f(z_1) = \begin{cases} 
0, & \text{if } z_1 \leq 3550, \\
\frac{4225-z_1}{4225-3550}, & \text{if } 3550 \leq z_1 \leq 4225, \\
\frac{4900-z_1}{4900-4225}, & \text{if } 4225 \leq z_1 \leq 4900, \\
0, & \text{if } z_1 \geq 4900 
\end{cases}
\]

\[
\mu_i^f(z_2) = \begin{cases} 
0, & \text{if } z_2 \leq 702.5, \\
\frac{z_2-681.5}{702.5-681.5}, & \text{if } 681.5 \leq z_2 \leq 702.5, \\
0, & \text{if } z_2 \leq 681.5, \\
\frac{z_2-660}{681.5-660}, & \text{if } 660 \leq z_2 \leq 681.5 
\end{cases}
\]
In the similar way, we get the optimal solution for the Indeterminacy membership model is obtained as follows: 
\((x_1, x_2, x_3) = (350, 0.450, z_1 = 3550, z_2 = 662.5, z_3 = 697.5)
and the Indeterminacy membership values are 
\(\mu_1 = 1, \mu_2 = 0.1163, \mu_3 = 0.894\). The Indeterminacy membership function values show that both goals \(z_1\) and \(z_2\) are satisfied with 100%, 11.63% and 89.4% respectively for the obtained solution which is \(x_1 = 350; x_2 = 0, x_3 = 450\).

In the similar way, we get the optimal solution for the falsity membership model is obtained as follows: 
\((x_1, x_2, x_3) = (350, 0.450, z_1 = 3550, z_2 = 662.5, z_3 = 697.5)
and the falsity membership values are 
\(\mu_1 = 0, \mu_2 = 0.8837, \mu_3 = 0.106\). The falsity membership function values show that both goals \(z_1\) and \(z_2\) are satisfied with 0%, 88.37% and 10.6% respectively for the obtained solution which is \(x_1 = 350; x_2 = 0, x_3 = 450\).

5 Conclusions and Future Work
In this paper, we have proposed a solution to Neutrosophic Multiobjective programming problem (NMOPP). The truth membership, Indeterminacy membership, falsity membership functions associated with each objective of the problem are transformed by using the first order Taylor polynomial series. The neutrosophic multi-objective programming problem is reduced to an equivalent multiobjective programming problem by the proposed method. The solution obtained from this method is very near to the solution of MOPP. Hence this method gives a more accurate solution as compared with other methods. Therefore the complexity in solving NMOPP, has reduced to easy computation. In the future studies, the proposed algorithm can be solved by metaheuristic algorithms.

Reference


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Decision Making Based on Some similarity Measures under Interval Rough Neutrosophic Environment

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Abstract: This paper is devoted to propose cosine, Dice and Jaccard similarity measures of interval rough neutrosophic set and interval neutrosophic mean operator. Some of the properties of the proposed similarity measures have been established. We have proposed multi attribute decision making approaches based on proposed similarity measures. To demonstrate the applicability and efficiency of the proposed approaches, a numerical example is solved and comparison has been done among the proposed the approaches.

Keywords: Tangent similarity measure, Single valued neutrosophic set, Cosine similarity measure, Medical diagnosis

1 Introduction

The concept of neutrosophic set was grounded by one of the greatest mathematician and philosopher Smarandache [1, 2, 3, 4, 5]. The root of neutrosophic set is the neutrosophy, a new branch of philosophy initiated by Smarandache [1]. Neutrosophy studies the ideas and notions that are neutral, indeterminate, unclear, vague, ambiguous, incomplete, contradictory, etc. Inherently, neutrosophic set is capable of dealing with uncertainty, indeterminate and inconsistent information. Smarandache endeavored to propagate the concept of neutrosophic set in all branches of sciences, social sciences and humanities. To use neutrosophic sets in practical fields such as real scientific and engineering applications, Wang et al.[6] extended the concept of neutrosophic set to single valued neutrosophic sets (SVNSs) and studied the set theoretic operators and various properties of SVNSs. Recently, single valued neutrosophic set has caught much attention to the researcher on various topics such as artificial intelligence [7], conflict resolution [8], education [9, 10], decision making [11-27] medical diagnosis [28], social problems [29, 30], etc. Smarandache’s original ideas blossomed into a comprehensive corpus of methods and tools for dealing with membership degrees of truth, falsity, indeterminacy and non-probabilistic uncertainty. In essence, the basic concept of neutrosophic set is a generalization of classical set or crisp set [31, 32], fuzzy set [33], intuitionistic fuzzy set [34]. The field has experienced an enormous development, and Smarandache’s seminal concept of neutrosophic set [1] has naturally evolved in different directions. Different sets were quickly proposed in the literature such as neutrosophic soft set [35], weighted neutrosophic soft sets [36], generalized neutrosophic soft set [37], Neutrosophic parametrized soft set [38], Neutrosophic soft expert sets [39, 40], neutrosophic refined sets [41, 42], Neutrosophic soft multi-set [43], neutrosophic bipolar set (44), neutrosophic cubic set (45, 46), neutrosophic complex set (47), rough neutrosophic set (48, 49), interval rough neutrosophic set [50], Interval-valued neutrosophic soft rough sets [51, 52], etc.

Broumi et al. [48, 49] recently proposed new hybrid intelligent structure namely, rough neutrosophic set combining the concept of rough set theory [53] and the concept of neutrosophic set theory to deal with uncertainty and incomplete information. Rough neutrosophic set [48, 49] is the generalization of rough fuzzy sets [54], [55] and rough intuitionistic fuzzy sets [56]. Several studies of rough neutrosophic sets have been reported in the literature. Mondal and Pramanik [57] applied the concept of rough neutrosophic set in multi-attribute decision making based on grey relational analysis. Pramanik and Mondal [58] presented cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [59] also proposed some rough neutrosophic similarity measures namely Dice and Jaccard similarity measures of rough neutrosophic environment. Mondal and Pramanik [60] proposed rough neutrosophic multi attribute decision making based on rough score accuracy function. Pramanik and Mondal [61] presented cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis.
In 2015, Broumi and Smarandache [50] combined the concept of rough set theory [53] and interval neutrosophic set theory [62] and defined interval rough neutrosophic set.

In this paper, we develop some similarity measures namely, cCosine, Dice, Jaccard similarity measures based on interval rough neutrosophic sets [50]. Rest of the paper is organized in the following way. Section 2 describes preliminaries of neutrosophic sets and rough neutrosophic sets and interval rough neutrosophic sets. Section 3 presents definitions and propositions of the proposed functions. Section 4 is devoted to present multi attribute decision-making method based on similarity functions. In Section 5, we provide a numerical example of the proposed approaches. Section 6 presents the comparison of results of the three proposed approaches. Finally section 7 presents concluding remarks and future scopes of research.

2 Mathematical preliminaries

2.1 Neutrosophic set

Definition 2.1[1] Let U be an universe of discourse. Then the neutrosophic set A can be presented of the form:

\[ A = \{< x, T_A(x), I_A(x), F_A(x)>, x \in U \} \]

where the functions \(T, I, F: U \rightarrow [0, 1]\) define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element \(x \in U\) to the set A satisfying the following conditions.

\[ \forall x \in U \Rightarrow T_A(x) + I_A(x) + F_A(x) = 1 \]

For two neutrosophic sets (NSs), \(A_{NS} = \{< x, T_A(x), I_A(x), F_A(x)>, x \in U \}\) and \(B_{NS} = \{< x, T_B(x), I_B(x), F_B(x)>, x \in U \}\) the two relations are defined as follows:

1. \(A_{NS} \subseteq B_{NS}\) if and only if \(T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)\)
2. \(A_{NS} = B_{NS}\) if and only if \(T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)\)

2.2 Single valued neutrosophic set (SVNS)

Definition 2.2 [6]

From philosophical point of view, the neutrosophic set assumes the value from real standard or non-standard subsets of \([0, 1]\). So instead of \([0, 1]\) one needs to take the interval \([0, 1]\) for technical applications, because \([0, 1]\) will be difficult to apply in the real applications such as scientific and engineering problems. Wang et. al [6] introduced single valued neutrosophic set (SVNS).

Let X be a space of points with generic element \(x \in X\). A SVNS A in X is characterized by a truth-membership function \(T_A(x)\), an indeterminacy-membership function \(I_A(x)\), and a falsity membership function \(F_A(x)\), for each point \(x \in X\), \(T_A(x)\), \(I_A(x)\), \(F_A(x)\) \(\in [0, 1]\). When X is continuous, a SVNS A can be written as follows:

\[ A = \oplus_{x \in X} \{< T_A(x), I_A(x), F_A(x) > : x \in X \} \]

When X is discrete, a SVNS A can be written as follows:

\[ A = \sum_{x \in X} \{< T_A(x), I_A(x), F_A(x) > : x \in X \} \]

For two SVNSs, \(A_{SVNS} = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X \}\) and \(B_{SVNS} = \{< x, T_B(x), I_B(x), F_B(x) > | x \in X \}\) the two relations are defined as follows:

1. \(A_{SVNS} \subseteq B_{SVNS}\) if and only if \(T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)\).
2. \(A_{SVNS} = B_{SVNS}\) if and only if \(T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)\).

2.3 Interval neutrosophic sets

Definition 2.3.1 [62]

Let X be a space of points (objects) with generic element \(x \in X\). An interval neutrosophic set (INS) A in X is characterized by truth-membership function \(T_A(x)\), indeterminacy-membership function \(I_A(x)\), and falsity-membership function \(F_A(x)\). For each point \(x \in X\), we have, \(T_A(x), I_A(x), F_A(x) \in [0, 1]\).

For two IVNS, \(A_{INS}, B_{INS}\) are respectively defined as:

\[ A_{INS} = \{< x, [T^L_A(x), T^U_A(x)], [I^L_A(x), I^U_A(x)], [F^L_A(x), F^U_A(x)] > | x \in X \} \]

\[ B_{INS} = \{< x, [T^L_B(x), T^U_B(x)], [I^L_B(x), I^U_B(x)], [F^L_B(x), F^U_B(x)] > | x \in X \} \]

the two relations are defined as follows:

1. \(A_{INS} \subseteq B_{INS}\) if and only if \(T^L_A(x) \leq T^L_B(x), T^U_A(x) \leq T^U_B(x), I^L_A(x) \geq I^L_B(x), I^U_A(x) \geq I^U_B(x), F^L_A(x) \geq F^L_B(x), F^U_A(x) \geq F^U_B(x)\)
2. \(A_{INS} = B_{INS}\) if and only if \(T^L_A(x) = T^L_B(x), T^U_A(x) = T^U_B(x), I^L_A(x) = I^L_B(x), I^U_A(x) = I^U_B(x), F^L_A(x) = F^L_B(x), F^U_A(x) = F^U_B(x)\).

2.4 Rough neutrosophic set

Definition 2.4.1 [48, 49]: Let Z be a non-zero set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function \(T_P(x)\), indeterminacy-function \(I_P(x)\) and non-membership function \(F_P(x)\). The lower and the upper approximations of P in the approximation (Z, R) denoted by \(\tilde{P}(P)\) and \(\bar{P}(P)\) are respectively defined as follows:

\[ \tilde{P}(P) = \{x \in X \mid T_{\tilde{P}(x)}(x), I_{\tilde{P}(x)}(x), F_{\tilde{P}(x)}(x) > \} \]

\[ \bar{P}(P) = \{x \in X \mid T_{\bar{P}(x)}(x), I_{\bar{P}(x)}(x), F_{\bar{P}(x)}(x) > \} \]
The symbols \( \vee \) and \( \wedge \) denote “max” and “min” operators respectively. \( T_p(z) \), \( I_p(z) \) and \( F_p(z) \) are the membership, indeterminacy and non-membership of \( z \) with respect to \( P \). It is easy to see that \( \overline{N}(P) \) and \( \overline{N}(P) \) are two neutrosophic sets in \( Z \).

Thus NS mapping \( \overline{N}, \overline{N} : N(Z) \rightarrow N(Z) \) are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair \( (N(P), \overline{N}(P)) \) is called the rough neutrosophic set in \( (Z, R) \).

From the above definition, it is seen that \( N(P) \) and \( \overline{N}(P) \) have constant membership on the equivalence classes of \( R \) if \( N(P) = N(P) \) and \( \overline{N}(P) = \overline{N}(P) \).

\[
N(P) = \{ \{x \in Z \mid T_p(x) = T_p(x) \} \mid x \in Z \},
\]

\[
\overline{N}(P) = \{ \{x \in Z \mid T_p(x) = T_p(x) \} \mid x \in Z \}.
\]

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function \( \mu_A : U \rightarrow [0, 1] \) and non-membership function \( \nu_A : U \rightarrow [0, 1] \) such that \( \mu_A(RX) = 1 \) and \( \nu_A(RX) = 0 \) for every \( x \in RX \) and \( \mu_A(U - RX) = 0 \) and \( \nu_A(U - RX) = 1 \). If \( x \in RX \), then \( \mu_A(x) = 1 \) and \( \nu_A(x) = 0 \).

2.5 Basic concept of rough approximations of an interval valued neutrosophic set and their properties

**Definition 2.5.3** [50]

Assume that, \( (U, R) \) be a Pawlak approximation space, for an interval neutrosophic set

\[
A = \{ x, [T^L_A(x), T^U_A(x)], [I^L_A(x), I^U_A(x)], [F^L_A(x), F^U_A(x)] \mid x \in U \}
\]

The lower approximation \( A_R \) and the upper approximation \( \overline{A}_R \) of A in the Pawlak approximation space \( (U, R) \) are expressed as follows:

\[
A_R = \{ x \mid [\land_{y \in [x]} T^L_A(y), [\land_{y \in [x]} T^U_A(y)], [\lor_{y \in [x]} I^L_A(y), [\lor_{y \in [x]} I^U_A(y)], [\lor_{y \in [x]} F^L_A(y), [\lor_{y \in [x]} F^U_A(y)] \mid x \in U \}
\]

\[
\overline{A}_R = \{ x \mid [\lor_{y \in [x]} T^L_A(y), [\lor_{y \in [x]} T^U_A(y)], [\land_{y \in [x]} I^L_A(y), [\land_{y \in [x]} I^U_A(y)], [\land_{y \in [x]} F^L_A(y), [\land_{y \in [x]} F^U_A(y)] \mid x \in U \}
\]

The symbols \( \land \) and \( \lor \) indicate “min” and “max” operators respectively. \( R \) denotes an equivalence relation for interval neutrosophic set A. Here \( [x]_R \) is the equivalence class of the element \( x \). It is obvious that

\[
[\land_{y \in [x]} T^L_A(y), [\land_{y \in [x]} T^U_A(y)] = [0, 1],
\]

\[
[\lor_{y \in [x]} I^L_A(y), [\lor_{y \in [x]} I^U_A(y)] = [0, 1],
\]

and

\[
0 \leq [\lor_{y \in [x]} T^L_A(y)] + [\lor_{y \in [x]} I^L_A(y)] + [\lor_{y \in [x]} F^L_A(y)] \leq 3
\]

Then \( A_R \) is an interval neutrosophic set (INS)

Similarly, we have

\[
[\lor_{y \in [x]} T^U_A(y)] + [\lor_{y \in [x]} I^U_A(y)] + [\lor_{y \in [x]} F^U_A(y)] \leq 3
\]

2.5 Interval neutrosophic rough sets [50]

Interval neutrosophic rough set [50] is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache [50] interval neutrosophic rough set is the generalizations of interval valued intuitionistic fuzzy rough set [63].

**Definition 2.5.1** [53]

Let \( R \) be an equivalence relation on the universal set \( U \). Then the pair \( (U, R) \) is called a Pawlak approximation space [5, 6]. An equivalence class of \( R \) containing \( x \) will be denoted by \( [x]_R \) for \( x \in U \), the lower and upper approximation of \( X \) with respect to \( (U, R) \) are denoted by respectively \( R^*_X \) and \( R^X \) and are defined by

\[
R^*_X = \{ x \in U \mid [x]_R \subseteq X \},
\]

\[
R^X = \{ x \in U \mid [x]_R \cap X \neq \emptyset \}.
\]

Now if \( R^*_X = R^X \), then \( X \) is called a rough set.

**Definition 2.5.2** [50]
and

\[ 0 \leq \vee_{x \in [a, b]} [\mathbf{I}_A^U(y)] + \wedge_{y \in [a, b]} [\mathbf{I}_B^U(y)] + \wedge_{y \in [a, b]} [\mathbf{I}_C^U(y)] \leq 3 \]

Then \( \overline{A}_R = \overline{A}_R \) is an interval neutrosophic set.

If \( \overline{A}_R = \overline{A}_R \) then \( A \) is a definable set, otherwise \( A \) is an interval valued neutrosophic rough set. Here, \( \overline{A}_R \) and \( \overline{A}_R \) are called the lower and upper approximations of interval neutrosophic set with respect to approximation space \((U, R)\) respectively. \( \overline{A}_R \) and \( \overline{A}_R \) are simply denoted by \( A \) and \( \overline{A} \) respectively.

**Proposition 1** [50]: Let \( A \) and \( B \) be two interval neutrosophic sets and \( A \) and \( \overline{A} \) the lower and upper approximation of interval neutrosophic set \( A \) with respect to approximation space \((U, R)\) respectively. \( \overline{B} \) and \( \overline{B} \) are the lower and upper approximation of interval neutrosophic set \( B \) with respect to approximation space \((U, R)\) respectively. Then the following relations hold good.

1. \( A \subseteq A \subseteq \overline{A} \)
2. \( \overline{A} \cup \overline{B} = \overline{A} \cup \overline{B} \) and \( \overline{A} \cap \overline{B} = \overline{A} \cap \overline{B} \)
3. \( \overline{A} \cap \overline{B} = \overline{A} \cap \overline{B} \) and \( \overline{A} \cup \overline{B} = \overline{A} \cup \overline{B} \)
4. \( \overline{A} = \overline{A} = \overline{A} \) and \( \overline{A} = \overline{A} = \overline{A} \)
5. \( \overline{A} = \overline{A} \) and \( \overline{A} = \overline{A} \)
6. If \( A \subseteq B \) then, \( A \subseteq B \) and \( \overline{A} \subseteq \overline{B} \)
7. \( A^c = \overline{A}^c \) and \( \overline{A}^c = \overline{A}^c \)

**Definition 2.5.4** [50]

Assume that, \((U, R)\) be a Pawlak approximation space and \( A \) and \( B \) are two interval neutrosophic sets over \( U \).

If \( A = B \) then \( A \) and \( B \) are called interval neutrosophic lower rough equal. If \( \overline{A} = \overline{B} \) , then \( A \) and \( B \) are called interval neutrosophic upper rough equal.

If \( \overline{A} = \overline{B} \), \( \overline{A} = \overline{B} \), then \( A \) and \( B \) are called interval neutrosophic rough equal.

**Proposition 2** [50]

Assume that \((U, R)\) be a Pawlak approximation space and \( A \) and \( B \) two interval neutrosophic sets over \( U \). then

1. \( A = B \Rightarrow A \cap B = A \) and \( A \cup B = B \)
2. \( \overline{A} = \overline{B} \Rightarrow \overline{A} \cup \overline{B} = \overline{A} \) and \( \overline{A} \cap \overline{B} = \overline{B} \)
3. \( A = A \) and \( B = B \) \( \Rightarrow \overline{A} \cup \overline{B} = A \cup B \)
4. \( \overline{A} = \overline{A} \) and \( \overline{B} = \overline{B} \) \( \Rightarrow \overline{A} \cap \overline{B} = A \cap B \)
5. \( A \subseteq B \) and \( B = \phi \) then \( A = \phi \)
6. \( A \subseteq B \) and \( B = U \) then \( A = U \)
7. \( B = \phi \) and \( A = \phi \) then \( A \cap B = \phi \)
8. \( \overline{A} = \overline{U} \) and \( \overline{B} = \overline{U} \Rightarrow A \cup B = U \)
9. \( \overline{A} = \overline{U} \Rightarrow A = B \)
10. \( \overline{A} = \overline{\phi} \Rightarrow A = \phi \)

**3. Cosine, Dice, Jaccard similarity measures of interval rough neutrosophic environment**

Cosine, Dice and Jaccard similarity measure are proposed in interval rough neutrosophic environment in the following subsections.

**3.1 Cosine similarity measure of interval rough neutrosophic environment**

**Definition 3.1.1**: Assume that there are two interval rough neutrosophic sets

\[ A = \left( \left\{ \mathbf{I}_A(x_i)^U \right\}, \left\{ \mathbf{I}_A(x_i)^U \right\}, \left\{ \mathbf{I}_A(x_i)^U \right\} \right) \]

\[ B = \left( \left\{ \mathbf{I}_B(x_i)^U \right\}, \left\{ \mathbf{I}_B(x_i)^U \right\}, \left\{ \mathbf{I}_B(x_i)^U \right\} \right) \]

in \( X = \{x_1, x_2, \ldots, x_n\} \).

A cosine similarity measure between interval rough neutrosophic sets \( A \) and \( B \) is defined as follows:

\[ C_{\text{IRNS}}(A, B) = \frac{1}{n} \sum \left( \frac{\Delta T_A(x_i) \Delta T_B(x_i) + \Delta I_A(x_i) \Delta I_B(x_i)}{\sqrt{(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2} \sqrt{(\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2}} \right) \]
\[
\Delta T_B(x_i) = \left( \frac{[T_B(x_i)]^4 + [T_B(x_i)]^U + [\bar{T}_B(x_i)]^4 + [\bar{T}_B(x_i)]^U}{4} \right),
\]

\[
\Delta I_A(x_i) = \left( \frac{[I_A(x_i)]^4 + [I_A(x_i)]^U + [\bar{I}_A(x_i)]^4 + [\bar{I}_A(x_i)]^U}{4} \right),
\]

\[
\Delta I_B(x_i) = \left( \frac{[I_B(x_i)]^4 + [I_B(x_i)]^U + [\bar{I}_B(x_i)]^4 + [\bar{I}_B(x_i)]^U}{4} \right),
\]

\[
\Delta F_A(x_i) = \left( \frac{[F_A(x_i)]^4 + [F_A(x_i)]^U + [\bar{F}_A(x_i)]^4 + [\bar{F}_A(x_i)]^U}{4} \right),
\]

\[
\Delta F_B(x_i) = \left( \frac{[F_B(x_i)]^4 + [F_B(x_i)]^U + [\bar{F}_B(x_i)]^4 + [\bar{F}_B(x_i)]^U}{4} \right).
\]

**Proposition 3**

Let \( A \) and \( B \) be interval rough neutrosophic sets then

1. \( 0 \leq C_{IRNS}(A, B) \leq 1 \)
2. \( C_{IRNS}(A, B) = C_{IRNS}(B, A) \)
3. \( C_{IRNS}(A, B) = 1 \) if \( A = B \)

**Proofs:**

1. It is obvious because all positive values of cosine function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When \( A = B \), then obviously \( C_{IRNS}(A, B) = 1 \). On the other hand if \( C_{IRNS}(A, B) = 1 \) then,

\[
\Delta T_A(x_i) = \Delta T_B(x_i),
\]

\[
\Delta I_A(x_i) = \Delta I_B(x_i),
\]

\[
\Delta F_A(x_i) = \Delta F_B(x_i) \text{ ie,}
\]

\[
[I_A(x_i)]^4 + [I_A(x_i)]^U + [\bar{I}_A(x_i)]^4 + [\bar{I}_A(x_i)]^U,
\]

\[
[T_A(x_i)]^4 + [T_A(x_i)]^U,
\]

\[
[\bar{T}_A(x_i)]^4 + [\bar{T}_A(x_i)]^U,
\]

\[
[F_A(x_i)]^4 + [F_A(x_i)]^U + [\bar{F}_A(x_i)]^4 + [\bar{F}_A(x_i)]^U,
\]

\[
[I_A(x_i)]^4 + [I_A(x_i)]^U + [\bar{I}_A(x_i)]^4 + [\bar{I}_A(x_i)]^U.
\]

**Proposition 4**

1. \( 0 \leq C_{WIRNS}(A, B) \leq 1 \)
2. \( C_{WIRNS}(A, B) = C_{WIRNS}(B, A) \)
3. \( C_{WIRNS}(A, B) = 1 \) if \( A = B \)

**Proof:**

The proofs of above properties are similar to the proofs of the properties of the proposition (3).

### 3.2 Dice similarity measure of interval rough neutrosophic environment

**Definition 3.2.2**

A Dice similarity measure between interval rough neutrosophic sets \( A \) and \( B \) (defined in 3.1.1) is defined as follows:

\[
\text{DIC}_{IRNS}(A, B) = \frac{2[\Delta T_A(x_i) \Delta T_B(x_i) + \Delta I_A(x_i) \Delta I_B(x_i)]}{\sqrt{[\Delta T_A(x_i)]^2 + [\Delta I_A(x_i)]^2 + [\Delta F_A(x_i)]^2} + \sqrt{[\Delta T_B(x_i)]^2 + [\Delta I_B(x_i)]^2 + [\Delta F_B(x_i)]^2}}.
\]

(6)

Where, \( \Delta T_A(x_i) = \frac{[T_A(x_i)]^4 + [\bar{T}_A(x_i)]^4 + [T_A(x_i)]^U + [\bar{T}_A(x_i)]^U}{4} \),

\[
\Delta T_B(x_i) = \frac{[T_B(x_i)]^4 + [\bar{T}_B(x_i)]^4 + [T_B(x_i)]^U + [\bar{T}_B(x_i)]^U}{4},
\]

\[
\Delta I_A(x_i) = \frac{[I_A(x_i)]^4 + [\bar{I}_A(x_i)]^4 + [I_A(x_i)]^U + [\bar{I}_A(x_i)]^U}{4},
\]

\[
\Delta I_B(x_i) = \frac{[I_B(x_i)]^4 + [\bar{I}_B(x_i)]^4 + [I_B(x_i)]^U + [\bar{I}_B(x_i)]^U}{4},
\]

\[
\Delta F_A(x_i) = \frac{[F_A(x_i)]^4 + [\bar{F}_A(x_i)]^4 + [F_A(x_i)]^U + [\bar{F}_A(x_i)]^U}{4},
\]

\[
\Delta F_B(x_i) = \frac{[F_B(x_i)]^4 + [\bar{F}_B(x_i)]^4 + [F_B(x_i)]^U + [\bar{F}_B(x_i)]^U}{4},
\]

\[
\Delta F_A(x_i) = \frac{[F_A(x_i)]^4 + [\bar{F}_A(x_i)]^4 + [F_A(x_i)]^U + [\bar{F}_A(x_i)]^U}{4},
\]

\[
\Delta F_B(x_i) = \frac{[F_B(x_i)]^4 + [\bar{F}_B(x_i)]^4 + [F_B(x_i)]^U + [\bar{F}_B(x_i)]^U}{4}.
\]
Proposition 5
Let A and B be interval rough neutrosophic sets then
1. 0 ≤ DIC IRNS(A, B) ≤ 1
2. DIC IRNS(A, B) = DIC IRNS(B, A)
3. DIC IRNS(A, B) = 1, iif A = B

Proofs:
1. It is obvious because all positive values of Dice function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously DIC IRNS(A, B) = 1. On the other hand if DIC IRNS(A, B) = 1 then,
   \[ \Delta I_A(x_i) = \left( \frac{[I_A(x_i)]^L + [I_A(x_i)]^U + [\bar{I}_A(x_i)]^L + [\bar{I}_A(x_i)]^U}{4} \right), \]
   \[ \Delta I_B(x_i) = \left( \frac{[I_B(x_i)]^L + [I_B(x_i)]^U + [\bar{I}_B(x_i)]^L + [\bar{I}_B(x_i)]^U}{4} \right), \]
   \[ \Delta F_A(x_i) = \left( \frac{[F_A(x_i)]^L + [\bar{F}_A(x_i)]^L + [F_A(x_i)]^U + [\bar{F}_A(x_i)]^U}{4} \right), \]
   \[ \Delta F_B(x_i) = \left( \frac{[F_B(x_i)]^L + [\bar{F}_B(x_i)]^L + [F_B(x_i)]^U + [\bar{F}_B(x_i)]^U}{4} \right). \]

This implies that A = B.

If we consider the weight \( w_i \) of each element \( x_i \), a weighted interval rough Dice similarity measure between interval rough neutrosophic sets A and B is defined as follows:

\[
\text{DIC WIRNS}(A, B) = 2 \sum_{i=1}^{n} \left[ \Delta T_A(x_i) \Delta T_B(x_i) + \Delta A_A(x_i) \Delta A_B(x_i) \right] \left( \frac{\left( \Delta T_A(x_i) \right)^2 + \left( \Delta A_A(x_i) \right)^2 + \left( \Delta A_B(x_i) \right)^2}{\left( \Delta T_B(x_i) \right)^2 + \left( \Delta A_B(x_i) \right)^2 + \left( \Delta A_B(x_i) \right)^2} + \right. \\
\left. \Delta A_A(x_i) \Delta A_B(x_i) + \Delta A_B(x_i) \Delta A_B(x_i) \right] \\
\sum_{i=1}^{n} w_i = 1.
\]

where
\[ \Delta T_A(x_i) = \left( \frac{[T_A(x_i)]^L + [\bar{T}_A(x_i)]^L + [T_A(x_i)]^U + [\bar{T}_A(x_i)]^U}{4} \right), \]

3.3 Jaccard similarity measure of interval rough neutrosophic environment

Definition 3.3.1 A Jaccard similarity measure between interval rough neutrosophic sets A and B (defined in 3.1.1) is defined as follows:

\[
\text{JAC IRNS}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{[\Delta T_A(x_i)]^2 + [\Delta A_A(x_i)]^2 + [\Delta A_B(x_i)]^2}{[\Delta T_B(x_i)]^2 + [\Delta A_B(x_i)]^2 + [\Delta A_B(x_i)]^2} + \right. \\
\left. \frac{[\Delta A_A(x_i)]^2 + [\Delta A_B(x_i)]^2 + [\Delta A_B(x_i)]^2}{[\Delta T_B(x_i)]^2 + [\Delta A_B(x_i)]^2 + [\Delta A_B(x_i)]^2} \right],
\]

where
\[ \Delta T_A(x_i) = \left( \frac{[T_A(x_i)]^L + [\bar{T}_A(x_i)]^L + [T_A(x_i)]^U + [\bar{T}_A(x_i)]^U}{4} \right), \]
3. JACIRNS(A, B) = 1, iff A = B

Proofs

Proposition 7
Let A and B be interval rough neutrosophic sets then

1. \( 0 \leq \text{JACIRNS}(A, B) \leq 1 \)
2. \( \text{JACIRNS}(A, B) = \text{JACIRNS}(B, A) \)
3. \( \text{JACIRNS}(A, B) = 1 \), iff A = B

Proofs:
1. It is obvious because all positive values of Jaccard function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously \( \text{JACIRNS}(A, B) = 1 \). On the other hand if \( \text{JACIRNS}(A, B) = 1 \) then,

\[
\begin{align*}
\Delta T_A(x_i) &= \left( \frac{[T_B(x_i)]^L + [T_B(x_i)]^U + [\bar{T}_B(x_i)]^L + [\bar{T}_B(x_i)]^U}{4} \right), \\
\Delta I_A(x_i) &= \left( \frac{[I_A(x_i)]^L + [I_A(x_i)]^U + [\bar{I}_A(x_i)]^L + [\bar{I}_A(x_i)]^U}{4} \right), \\
\Delta I_B(x_i) &= \left( \frac{[I_B(x_i)]^L + [I_B(x_i)]^U + [\bar{I}_B(x_i)]^L + [\bar{I}_B(x_i)]^U}{4} \right), \\
\Delta F_A(x_i) &= \left( \frac{[F_A(x_i)]^L + [F_A(x_i)]^U + [\bar{F}_A(x_i)]^L + [\bar{F}_A(x_i)]^U}{4} \right), \\
\Delta F_B(x_i) &= \left( \frac{[F_B(x_i)]^L + [F_B(x_i)]^U + [\bar{F}_B(x_i)]^L + [\bar{F}_B(x_i)]^U}{4} \right).
\end{align*}
\]

This implies that A = B.

If we consider the weight \( w_i \) of each element \( x_i \), a weighted interval rough Jaccard similarity measure between interval rough neutrosophic sets A and B can be defined as follows:

\[
\text{JACWIRNS}(A, B) = \frac{\sum_{i=1}^{n} w_i \Delta A(x_i) \Delta B(x_i) + \Delta I_A(x_i) \Delta I_B(x_i) + \Delta F_A(x_i) \Delta F_B(x_i)}{\sum_{i=1}^{n} w_i \Delta A(x_i)^2 + \Delta I_A(x_i)^2 + \Delta F_A(x_i)^2 + \Delta B(x_i)^2 + \Delta I_B(x_i)^2 + \Delta F_B(x_i)^2 + \Delta A(x_i) \Delta B(x_i) + \Delta I_A(x_i) \Delta I_B(x_i) + \Delta F_A(x_i) \Delta F_B(x_i)}
\]

\[
\sum_{i=1}^{n} w_i = 1 \text{. If we take } w_i = \frac{1}{n},
\]

i = 1, 2, ..., n, then \( \text{JACWIRNS}(A, B) = \text{JACIRNS}(A, B) \)

The weighted interval rough Jaccard similarity measure between two interval rough neutrosophic sets A and B also satisfies the following properties:

Proposition 8

1. \( 0 \leq \text{JACWIRNS}(A, B) \leq 1 \)
2. \( \text{JACWIRNS}(A, B) = \text{JACWIRNS}(B, A) \)
3. \( \text{JACWIRNS}(A, B) = 1 \), iff A = B

Proof:
The proofs of above properties are similar to the proofs of the properties of proposition (7).

4. Decision making based on cosine, Dice and Jaccard hamming similarity operator under interval rough neutrosophic environment

In this section, we apply interval rough similarity measures between IRNSs to the multi-criteria decision making problem. Assume that, \( A = \{ A_1, A_2, ..., A_m \} \) be a set of alternatives and \( C = \{ C_1, C_2, ..., C_n \} \) be the set of attributes.

The proposed decision making approach is described using the following steps.

Step 1: Construct the decision matrix with interval rough neutrosophic number

The decision maker forms a decision matrix with respect to \( m \) alternatives and \( n \) attributes in terms of interval rough neutrosophic numbers (see the Table 1).
Table 1: Interval rough neutrosophic decision matrix

\[
D = \left( \begin{array}{cccc}
\bar{d}_{11} & d_{12} & \cdots & d_{1n} \\
\. & \ddots & \ddots & \ddots \\
\bar{d}_{m1} & \bar{d}_{m2} & \cdots & \bar{d}_{mn}
\end{array} \right)
\]

Table 1: Interval rough neutrosophic decision matrix

<table>
<thead>
<tr>
<th>A_1</th>
<th>C_1</th>
<th>C_2</th>
<th>\ldots</th>
<th>C_n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\bar{d}<em>{11} &amp; d</em>{12} &amp; \cdots</td>
<td>d_{1n}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\ldots</td>
<td>\ddots &amp; \ddots &amp; \ddots &amp; \ddots</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A_m</td>
<td>\bar{d}<em>{m1} &amp; \bar{d}</em>{m2} &amp; \cdots</td>
<td>\bar{d}_{mn}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Here \( \bar{d}_{ij} \) is the interval rough neutrosophic number according to the \( i \)-th alternative and the \( j \)-th attribute.

Step 2: Determine interval rough neutrosophic mean operator (IRNMO)

\[
\begin{align*}
\langle \Delta T(x_i), \Delta I(x_i), \Delta F(x_i) \rangle &= \\
&= \left( \begin{array}{c}
\frac{[T(x_i)]^U + [I(x_i)]^U + [F(x_i)]^U}{4}, \\
\frac{[I(x_i)]^U + [T(x_i)]^U + [F(x_i)]^U}{4}, \\
\frac{[F(x_i)]^U + [I(x_i)]^U + [T(x_i)]^U}{4}
\end{array} \right)
\end{align*}
\]  

(11)

Step 3: Determine the weights of the attributes

Assume that the weight of the attributes \( C_j \) (\( j = 1, 2, \ldots, n \)) considered by the decision-maker is \( w_j \) (\( j = 1, 2, \ldots, n \)). Where, all \( w_j \) \( \in \) belongs to \([0, 1]\)

And \( \sum_{j=1}^{n} w_j = 1 \).

Step 4: Determine the benefit type attributes and cost type attributes

The evaluation attribute can be categorized into two types: benefit attribute and cost attribute. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all the alternatives. Therefore, we define an ideal alternative as follows.

\[
A^* = \{ C_1^*, C_2^*, \ldots, C_m^* \}
\]

Where benefit attribute

\[
C_j^* = \left[ \max_i T_{C_j}(A_i), \min_i I_{C_j}(A_i), \min_i F_{C_j}(A_i) \right]
\]

The cost attribute

\[
C_j^* = \left[ \min_i T_{C_j}(A_i), \max_i I_{C_j}(A_i), \max_i F_{C_j}(A_i) \right]
\]

Step 5: Determine the weighted interval rough neutrosophic similarity measure of the alternatives

Using the equations (5), (7), and (9), the weighted interval rough neutrosophic similarity functions can be written as follows.

\[
C_{\text{WRNS}}(A, B) = \sum_{j=1}^{n} w_j C_{\text{IRNS}}(A, B)
\]

(14)

\[
D{\text{IC}_{\text{WRNS}}}(A, B) = \sum_{j=1}^{n} w_j D{\text{IC}_{\text{IRNS}}}(A, B)
\]

(15)

\[
J{\text{AC}_{\text{WRNS}}}(A, B) = \sum_{j=1}^{n} w_j J{\text{AC}_{\text{IRNS}}}(A, B)
\]

(16)

Step 6: Rank the alternatives

Through the weighted interval rough neutrosophic similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined based on the descending order of similarity measures.

Step 7: End

5. Numerical Example

Assume that a decision maker intends to select the most suitable laptop for random use from the four initially chosen laptops (S_1, S_2, S_3) by considering four attributes namely: features C_1, reasonable Price C_2, Customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 4, the considered problem is solved by the following steps:

Step 1: Construct the decision matrix with interval rough neutrosophic number

The decision maker forms a decision matrix with respect to three alternatives and four attributes in terms of interval rough neutrosophic numbers as follows.
Table 2. Decision matrix with interval rough neutrosophic number

\[ d_s = \left[ (N(P^L), N(P^U)) \right]_{3 \times 4} = \]

<table>
<thead>
<tr>
<th></th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>[0.6,0.7],[0.3,0.5], [0.3,0.4]</td>
<td>[0.5,0.7],[0.3,0.4], [0.1,0.2]</td>
<td>[0.5,0.6],[0.4,0.5], [0.4,0.6]</td>
<td>[0.8,0.9],[0.3,0.4], [0.5,0.6]</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>[0.8,0.9],[0.1,0.3], [0.1,0.2]</td>
<td>[0.7,0.9],[0.3,0.5], [0.3,0.4]</td>
<td>[0.7,0.8],[0.2,0.4], [0.3,0.5]</td>
<td>[0.7,0.8],[0.3,0.5], [0.3,0.5]</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>[0.7,0.8],[0.2,0.3], [0.0,0.2]</td>
<td>[0.6,0.7],[0.1,0.2], [0.1,0.2]</td>
<td>[0.5,0.7],[0.2,0.3], [0.7,0.8], [0.3,0.5], [0.1,0.3]</td>
<td>[0.0,0.2]</td>
</tr>
</tbody>
</table>

(17)

**Step 2: Determine the interval rough neutrosophic mean operator (IRNMO)**

**Step 3: Determine the weights of attributes**
The weight vectors considered by the decision maker are 0.35, 0.25, 0.25 and 0.15 respectively.

**Step 4: Determine the benefit type attribute and cost type attribute**
Here three benefit type attributes \( C_1, C_2, C_3 \) and one cost type attribute \( C_4 \). Using equation (12), (13) and (18) we calculate the ideal alternative as follows.

\( A^* = \left[ (0.775, 0.700, 0.650, 0.600), (0.200, 0.175, 0.150, 0.225), (0.325, 0.350, 0.375, 0.425), (0.125, 0.150, 0.175, 0.200), (0.775, 0.700, 0.650, 0.600), (0.200, 0.175, 0.150, 0.225), (0.325, 0.350, 0.375, 0.425) \right] \)

**Step 5: Calculate the weighted interval rough neutrosophic similarity scores of the alternatives**
Calculated values of weighted interval rough neutrosophic similarity values presented as follows.

\[ C_{\text{WRNS}}(A^*, A_1) = 0.9754 \]
\[ C_{\text{WRNS}}(A^*, A_2) = 0.9979 \]
\[ C_{\text{WRNS}}(A^*, A_3) = 0.9878 \]
\[ \text{DIC}_{\text{WRNS}}(A^*, A_1) = 0.9716 \]
\[ \text{DIC}_{\text{WRNS}}(A^*, A_2) = 0.9971 \]
\[ \text{DIC}_{\text{WRNS}}(A^*, A_3) = 0.9835 \]

Using IRNMO, the transferred decision matrix is as follows.

**Table 3: Transformed decision matrix**

<table>
<thead>
<tr>
<th></th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>0.750, 0.300, 0.250</td>
<td>0.700, 0.375, 0.250</td>
<td>0.650, 0.375, 0.425</td>
<td>0.800, 0.375, 0.475</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0.775, 0.200, 0.125</td>
<td>0.650, 0.175, 0.150</td>
<td>0.675, 0.350, 0.225</td>
<td>0.675, 0.475, 0.225</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0.700, 0.250, 0.150</td>
<td>0.650, 0.250, 0.225</td>
<td>0.700, 0.325, 0.375</td>
<td>0.600, 0.325, 0.275</td>
</tr>
</tbody>
</table>

\[ J_{\text{WRNS}}(A^*, A_1) = 0.9448 \]
\[ J_{\text{WRNS}}(A^*, A_2) = 0.9943 \]
\[ J_{\text{WRNS}}(A^*, A_3) = 0.9678 \]

**Step 6: Rank the alternatives**
Ranking the alternatives is prepared based on the descending order of similarity measures (see the table 6). Highest value reflects the best alternative. Hence, the laptop \( A_2 \) is the best alternative for random use.

<table>
<thead>
<tr>
<th></th>
<th>Weighted interval rough similarity measures</th>
<th>Measured value</th>
<th>Ranking order</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>( C_{\text{WRNS}}(A_1, A^*) ) = 0.9754</td>
<td>( C_{\text{WRNS}}(A_2, A^*) ) = 0.9979</td>
<td>( A_2 \succ A_1 \succ A_3 )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( C_{\text{WRNS}}(A_2, A^*) ) = 0.9979</td>
<td>( C_{\text{WRNS}}(A_3, A^*) ) = 0.9878</td>
<td></td>
</tr>
<tr>
<td>( A_3 )</td>
<td>( C_{\text{WRNS}}(A_3, A^*) ) = 0.9878</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>measure</th>
<th>DWIRNS(A1, A*) = 0.9716</th>
<th>A2 &gt; A3 &gt; A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted interval rough Dice similarity measure</td>
<td>DWIRNS(A2, A*) = 0.9971</td>
<td>A2 &gt; A3 &gt; A1</td>
</tr>
<tr>
<td></td>
<td>DWIRNS(A3, A*) = 0.9835</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>measure</th>
<th>JWIRNS(A1, A*) = 0.9448</th>
<th>A2 &gt; A3 &gt; A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted interval rough Jaccard similarity measure</td>
<td>JWIRNS(A2, A*) = 0.9943</td>
<td>A2 &gt; A3 &gt; A1</td>
</tr>
<tr>
<td></td>
<td>JWIRNS(A3, A*) = 0.9678</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion

In this paper, we have proposed cosine, Dice and Jaccard similarity measures of interval rough neutrosophic set and proved some of their basic properties. We have presented an application, namely selection of best laptop for random use. The thrust of the concept presented in the paper will be in pattern recognition, medical diagnosis, personnel selection, etc. in interval neutrosophic environment.

References


25. P. Liu, Y. Chu, Y. Li, Y. Chen. Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making, 16 (2014), 242–255.


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Abstract. This study is inspired by Neutrosophy theory (Smarandache 1995, 1998), a new concept of states treatment with a generous applicability to logic, communication theory and applied linguistics, among other sciences. Neutrosophy considers a proposition, theory, concept, event A in relation to its opposite Anti-A which is not A, Non-A on that which is neither A nor Anti-A, denoted by “Neut A”. Together, A, Anti-A and Neut-A combined two by two and also all three of them form the Neutro-Synthesis. The classical reasoning development about evidences –the triad thesis-anti-thesis-synthesis- known as dialectics is extended in the Neutrosophy by the tetrad thesis-anti-thesis-neutro-thesis-neutro-synthesis, which carries on the unification on synthesis regarding the opposites and their neutrals. Neutrosophic logic also makes a distinction between a ‘relative truth’ and an ‘absolute truth’, while fuzzy logic (Zadeh 1965) does not.

Our aim is to analyze a series of Romanian printed press chronicles reflecting the same event of the political stage but each in a different view and positioning (from neutrality to polemic attitude). Methods for text examination are speech acts and modality analysis, exploring how the author is discursively positioned in the sample text material. The study tries to argue that the paradox of journalistic communication lies in the double constraint the authors of news articles have to face: to be convincing (i.e. argumentative) while keeping their credibility. They have to be neutral about the facts presented and the political agents implied, unless they are accused of taking sides. There is no credibility without neutrality, but, on the other hand, without a definite position on the part of journalists, they will not succeed in passing their messages along to the public.

Keywords: argumentation, subjectivity, neutrality, neutrosophy, press articles.
est transmise continuellement, massivement, sur tous les canaux médiatiques, à l’échelle planétaire, de sorte qu’on parle de plus en plus souvent de la société informationnelle, à savoir une société dont l’existence et le fonctionnement ne seraient possibles sans l’accès immédiat et massif à l’information. Avec l’extension à l’échelle planétaire des systèmes de communication par voie électronique, des réseaux de communications tels que l’Internet ou l’Xnet, on parle de société informatisée, qui permet la circulation instantanée de l’information de tous les domaines, d’un fournisseur situé à n’importe quel point du globe terrestre jusqu’au bénéficiaire situé à n’importe quel autre point. Ce qui plus est, les réseaux électroniques de communication englobent maintenant tous les autres formes et moyens de communication – la presse écrite, la radio, la télévision- étant capables de retransmettre des éditions de journaux, des émissions de radio et de télévision, etc. En outre, Internet peut contenir des sites d’informations spécialisés dans divers domaines, ainsi que des bibliothèques virtuelles. Ainsi, de la diffusion de l’information au niveau régional et national on en est arrivé, en quelques décennies seulement, à une diffusion globale. Ce phénomène de globalisation requiert une forme plus simple et plus directe de transmission de l’information.

2 La transmission de l’information: de la neutralité à la polémique

2.1 Le style neutre– l’idéal de la presse écrite

Pour amorcer, dans ce contexte, une discussion sur le style, nous avouerons notre attente que celui-ci soit également le plus simple, direct et efficace possible, autrement dit qu’on communique le maximum d’information en un minimum de mots. Mais en réalité, sur la majorité des sites Internet, la préoccupation pour le style est tout à fait marginale et ceux qui transmettent l’information le font d’une manière plutôt négligente et même familière. Le style de la presse écrite conserve, en général, les propriétés linguistiques et communicatives nécessaires à une transmission efficace de l’information, étant dans la plupart des cas simple, clair et direct. Sa propriété primordiale est la concision, dont le premier effet est l’efficacité. Le style neutre, concis et direct est spécifique des agences de presse professionnelles. En dehors de la concision, l’efficacité provient aussi de la clarté de l’expression et de la formulation. La clarté et la concision, l’objectivité entendue comme absence de l’immixtion émotionnelle sont doublées par la propriété et la précision des termes utilisés. La propriété des termes est définie comme leur qualité d’exprimer avec exactitude le concept ou l’idée visés. La précision, quant à elle, concerne l’adéquation des termes à l’information qui doit être transmise. Enfin, le style neutre est en relation directe avec l’objectivité, la transmission de l’information étant ainsi mise à l’abri des risques et des complications. Le style neutre caractérise non seulement la langue de la presse, mais aussi celle de l’administration, vouée à un public de niveau culturel moyen, qui valorise l’utilité de l’information. Ayant le plus haut degré d’adressabilité, le style neutre manifeste une tendance à l’universalité. Il s’oppose au style imagé, qui présente, comme on verra plus tard, des tendances de personnalisation.

À vrai dire, le journaliste est un « parent pauvre » de l’écrivain, n’étant plus guère qu’un capteur de l’information et un facteur de (re)structuration et de transmission de celle-ci, à travers le processus de rédaction des nouvelles, des articles, surtout dans le cas des agences de presse, des matériels écrits « à chaud » et sous la pression de la diffusion rapide. Dans ce cas-là, la rédaction prévaut sur l’élaboration, sur le commentaire et sur l’expression des opinions. Le style neutre suppose une économie de moyens et de matériel linguistique et conséquemment une rigueur de la rédaction et un aspect soutenu, clair et concis. Ce type de message comporte une dimension dénouative accentuée et une capacité accrue de pénétration de tous les milieux sociaux, car il est le résultat d’un effort de médiation. On peut conclure que le style neutre est subordonné à la communication référentielle.

2.2 Attitude, prise en charge et argumentativité

Le caractère informatif d’un texte de presse n’exclut pas pourtant son caractère argumentatif. En fait, le caractère argumentatif est intriqué au réseau informatif du texte, d’une manière naturelle. Tout énoncé est pourvu d’une orientation argumentative. Comme Anscombe et Ducrot l’ont montré dès les années ’80, l’argumentativité est un trait inhérent de tout discours. Mais lorsque l’attitude du journaliste devient visiblement subjective et il perd sa neutralité, le texte passe de l’information à l’opinion, manifestation d’une prise en charge argumentative du contenu du message. Par ce type de texte, son auteur essaye d’influencer, délibérément, la conscience de ses lecteurs et de les convaincre d’adopter son opinion ou d’admettre sa thèse. On peut constater une gradualité de la prise en charge du message par le journaliste, une échelle qui va de l’attitude objective, caractérisée sur le plan de l’expression par le style neutre, à l’attitude argumentative et critique, voire combative, produite entretenue par l’existence d’un conflit d’opinions. Ainsi, en dehors de la force argumentative découlant de l’organisation discursive et dépendant du statut sémantique et pragmatique des arguments proprement dits (éléments appartenant au plan idéatique-logique qui s’adressent à l’intellect), il s’y glisse une intention de persuader, d’emporter l’adhésion des lecteurs par des moyens qui appartiennent moins à la logique et à la raison qu’à l’émotion et à l’irrationnel. L’action de persuader tient au désir de convaincre quelqu’un, de le faire croire et agir de la manière dont nous souhaitons qu’il le fasse. Dans le cas du journaliste, cela signifie qu’il voudrait dé-
terminer le lecteur à adopter ses convictions, ses attitudes par rapport aux faits qu’il présente, sans lui donner l’occasion, la chance de se former une opinion personnelle sur lesdits faits, après la lecture de l’article. La persuasion relève donc de la capacité ou, si l’on peut dire, le talent d’influencer l’auditeur/ le lecteur pour qu’il adopte notre point de vue/ notre thèse. L’auteur d’un article d’opinion doit, bien sûr, apporter des arguments en faveur de sa thèse, mais comme en politique les arguments factuels ou les preuves sont parfois difficiles à procurer, il recourt aux hypothèses, aux suppositions et aux insinuations. On voit donc comment, de l’argumentation rationnelle, qui reste en grande partie objective (se rapprochant parfois des strategies de réfutation. C’est une stratégie argumentative, réactive, par laquelle le sujet exprime son désaccord et apporte des objections contre un acte ou un contenu exprimé au préalable par l’interlocuteur. L’étiquette de ‘polémique’ s’applique à une interaction verbale (discursive ou textuelle), de nature argumentative, qui se définit par un conflit ou un désaccord par rapport à un contenu, une situation, etc. La polémique vise à la disqualification de l’adversaire et, dans ce but, tend à manipuler les contenus par la déviation des sens. Dans la presse, les articles polémiques peuvent être plus tempérés ou plus agressifs, leur violence étant en étroite relation avec la force contestataire, la virulence du style, l’abondance des actes de langage offensifs (la négation polémique, contestation, la réfutation, le démenti) etc. Parfois le discours polémique revêt la forme du pamphlet, aspect outrageux, violent, offensant, illustré dans la presse écrite roumaine après ’90 par les journaux România Mare de Corneliu Vadim Tudor.

3 Bref aperçu de la vie politique et des partis roumains après 1989

La scène politique roumaine après ’90 est partagée entre plusieurs partis se réclamant de la gauche sociale-démocrate, du centre et de la droite modérée (du libéralisme) qui se succèdent l’un à l’autre au gouvernement, ou qui forment des alliances plus ou moins opportunistes afin de s’assurer la majorité dans le parlement. Le Parti social-démocrate (PSD) est un parti politique fondé en 1992, héritier du Parti de la Démocratie Sociale de Roumanie (PDSR), parti issu du Front du Salut National (Frontul Salvaării Naţionale) premier formation politique au pouvoir en Roumanie après 1989. Ses opposants l’accusent d’abriter des anciens du Parti Communiste Roumain, le parti unique entre 1948 et 1989, même s’il n’y a pas de lien organique entre les deux partis, et de perpétrer certaines mentalités et coutumes spécifiques à l’époque communiste.

Depuis février 2011, il est allié au Parti National libéral et au Parti Conservateur au sein de l’Union sociale libérale. Les sociaux-démocrates, en coalition avec le Parti National Libéral et avec l’Union Démocratique des Ma-

4 Hypothèse et corpus de la recherche

L’hypothèse de notre recherche est que l’usage de l’argumentation dans les articles de presse détermine le lecteur à faire certaines inférences et associations pour arriver aux conclusions poursuivies par l’auteur, d’une manière implicite. La présentation tendancieuse des faits de la réalité sociopolitique conduit à une interprétation (prédéterminée) apte à susciter chez le lecteur les attitudes et les sentiments que l’auteur de l’article désire éveiller. Par contre, dans le cas de l’attitude et du style neutres, la façon de présenter l’information (les événements et les déclarations des acteurs politiques) fait appel à la raison et au discernement des lecteurs, répondant aux besoins d’un public qui se considère comme étant constitué de citoyens réflexifs. Comme Habermas le montre, la fonction de la communication dans la sphère publique est la construction des identités sociales et des relations viables au sein d’une société démocratique. Ainsi, la communication est constituée d’éléments linguistiques capables de servir les positions des participants, fonction essentielle dans la construction des rôles de citoyens actifs (Fairclough, 1992). Nous avons choisi pour l’illustration de notre hypothèse une série d’articles de presse qui reflètent tous les mêmes événements (évolutions) sur l’échiquier de la vie politique roumaine, mais d’une manière très différente. Tandis que les agences de presse Mediafax et Agerpres se contentent de raconter les événements d’une manière neutre et de reprendre les déclarations des acteurs politiques impliqués sans les commenter, les journaux d’opinion tels que Gândul transmettent des échos variés des événements en question, illustrant des positions qui vont du scepticisme à l’optimisme, soutenues par des argumentations plus ou moins subjectives.

5 Analyse du corpus de presse

L’analyse des actes de langage accomplis dans le dis-
La stratégie argumentative indirecte qui mettra en évidence le degré de prise en charge des contenus informationnels véhiculés par le discours de l’article de presse et la colorature affective imprimée aux énoncés. La modalité a affaire au rapport que le locuteur entretient avec le contenu propositionnel et contribue à la construction des identités discursives et sociales. Par la manière dont il communiquent les faits et par l’expression de son attitude envers ceux-ci, les évaluations que le journaliste donne de la réalité sont projetées dans l’univers du récepteur. Ainsi, la liberté d’interprétation du lecteur peut être sérieusement affectée par la présence d’une attitude explicite du journaliste dans le message. Les éléments linguistiques qui peuvent véhiculer l’expression de l’attitude du journaliste sont : les auxi-verbes modaux (M. Tuţescu 2005) savoir, croire, pouvoir, devoir, falloir, sembler, etc.; les modes et les temps verbaux (à comparer par exemple l’emploi de l’indicatif présent par rapport au conditionnel journalistique); les adverbes modalisateurs d’énoncé et d’énonciation ; les adjectifs évaluatifs ou appréciatifs (C. Kerbrat-Orecchioni 1980, 1999) et la modalisation autonymique (italique, guillemets, incises, etc.).

5.1. Le premier extrait que nous avons soumis à l’analyse a été publié sur le site de l’agence de presse Mediafax à la veille des élections au sein du PSD. Son titre a la forme d’une citation d’un candidat à la présidence du parti, Liviu Dragnea. La neutralité ressort de l’absence de commentaires sur les déclarations reprises et de l’emploi de verbes de citation et d’autres marqueurs évidentiels spécifiques du discours rapporté neutre : Dragnea, despre candidatura sa la șefia PSD : [...] afirmă, a spus, a explicitat, a precizat. Il y a quand même des marqueurs (verbes de déclaration) qui expriment un commentaire critique des déclarations respectives : a evitat, a recunoscut. À la fin de l’article, une précision faite sur un ton sec rappelle que le recours du candidat en question est en train d’être jugé et qu’il avait été condamné en première instance à une année de prison avec suspension dans un dossier de fraude au referendum national. Ce commentaire a le rôle d’informer le lecteur sur le statut judiciaire du candidat, mais aussi de mettre une distance entre le journaliste-énonceur et les propos qu’il vient de rapporter.

5.2. L’extrait suivant, intitulé Dragnea, stafia lui Ceaușescu (Dragnea, le fantôme de Ceausescu) est un article d’opinion publié après les élections dans le journal en ligne www.gândul.info. L’auteur donne une évaluation positive de la situation, argumentant que l’élection de Dragnea constitue un vrai changement du paradigme des chefs du parti social-démocrate. La sympathie de l’auteur pour le personnage transparaît, bien qu’elle ne soit pas avouée explicitement, à travers les dénominations qu’il emploie, les actions et les qualités attribuées à Dragnea qu’il choisit de mettre en relief, les adjectifs évaluatifs et axiologiques, etc. La stratégie argumentative indirecte qu’il adopte est, à notre avis, d’une grande efficacité et possède un pouvoir persuasif nettement supérieur aux stratégies directes ou à la démonstration. L’auteur commence par citer les détracteurs de Dragnea, pour qui celui-ci est un « étranger » (roum. « venetic ») et un « trans-fuge » d’un autre parti. Mais, dans la bouche des conservateurs du PSD, représentés par la personnalité controversée de l’ancien président Ion Iliescu, connu pour son attachement à la gauche communiste, cet appellatif devient un argument favorable, un atout de Dragnea. Le journaliste a du mal à cacher son enthousiasme pour l’élection du premier président du PSD qui « ne porte pas dans son ADN politique le gène modifié de l’activiste du PCR ». Afin d’emporter l’adhésion des lecteurs à sa thèse, il reçoit à plusieurs stratégies de persuasion : l’emploi de la première personne du pluriel, qui inclut l’interlocuteur, la métaphore (ADN-ul său politic, gena modificată a activistului pecerist), l’ironie amicale (copilul „[din trandafiri” al lui Ion Iliescu), „Titulescu lui Năstase”, „copilul răzvrătit al vechilor emancipări pediste”) et la suggestion d’une connivence entre le public lecteur et le personnage du nouveau chef du PSD :

Ne vine să credem sau nu, asta e situația. După 25 de ani de la materializarea primei emanații revoluționare, partidul lui Ion Iliescu va fi condus de un cetățean care nu poartă în ADN-ul său politic gena modificată a activistului pecerist, campat în Kiseleff, ci, mai degrabă, pe cea cu parfumul vag al cozieriei casei de oaspeți a Lupeascăi, din Modrogan. Nu se poate să nu vi-l amintiți, de exemplu, în cadrul acela, remarcabil, din „Noaptea președintelui Geoană”, când le ținea spatele liderului-blitz și Mihaeliei, dragostea lui! Nu se poate să-i fi uitat privirea-lamă, „Gillette Stainless Steel”!

Il compare l’aspect de l’homme politique à celui d’un agent secret ou des acteurs ayant incarné des gardes du corps et des super héros dans les films d’aventure produits à Hollywood, pour insinuer en ce qui suit qu’il a la taille d’un vrai homme d’État, qui a joué jusqu’alors le rôle de l’homme de main (en roum. „omul din umbră”), du lieutenant, en attendant que sa chance arrive. L’argumentation, très bien conduite, opère tout à la fois par dichotomisation, polarisation et procédés rhétoriques.
variiş: déricațion (ex: „Încet dar sigur, PSD a trecut în ne-fiinţă. Firește că nu se află intins pe năsălie, ca să mergem să-l apleaăm cu cozi de tranfădrir în palme), ironie, apeluri la pathos și chiar la război. Le „coup de grâce” de acesta argumentație (en faveur de Dragnea și în contra viei garde communiste al PSD) este reprezentat de paragraful final care citea o rețetă de Dragnea la Iliescu, rețetă care îmbogățește o adevărată scena te-ătrală (la metaphore du rideau y est d’ailleurs convoquée) : Atac căruia Dragnea i-a răspuns, sec, ca o cădere de cortină peste trecutul comunist al PSD. Sau ca o „dez impedieare”, pur și simplu: ”Nu știi ce și-dorea Ceaușescu, pentru că nu l-am cunoscut foarte bine și nu pot să mă pronunț.”

5.3. Le troisième extrait soumis à l’analyse est un article du même journal qui se situe sur une position antag-onique à celle de l’article précédent. L’auteur soutient la thèse que l’élection de Dragnea à la tête du PSD n’a rien changé aux mœurs des membres du parti et que même son nouveau leader affiche un masque dont on n’est pas dupe. La polémique est entamée depuis le titre de l’article : que les déclarations du nouveau chef du PSD ne vehicule d’une manière expressive la thèse soutenue par ce parti ne sera jamais reformė. Le discours polémique est amorcé par une négation polémique explicite :

“Ce am înțeles noi astăzi, din Congresul PSD? Că niciodată acest partid al nemuririi comuniste nu se va schimba; nu se va reforma, nu va cunoaște beneficiul exorcizării, nu se va rupe în fighur „joyer du théâtre, poser”. Ce titre, d’un grand effet rhétorique, véhicule d’une manière expressive la thèse soutenue par l’article : que les déclarations du nouveau chef du PSD ne sont que de la poudre aux yeux du public et qu’en réalité ce parti ne sera jamais reformé. Le discours polémique est amorcé par une négation polémique explicite :

„Ce am înțeles noi astăzi, din Congresul PSD? Că niciodată acest partid al nemuririi comuniste nu se va schimba; nu se va reforma, nu va cunoaște beneficiul exorcizării, nu se va rupe în fighur „joyer du théâtre, poser”.

L’argumentation fait usage de stratégies directes : la refutation, l’interrogation, l’interpellation de l’adversaire et l’exemple. Pour éviter la situation ingrate du discours monogéré où le polémiste est seul maître à bord, l’auteur simule un échange avec Dragnea, en reprenant quelques déclarations de celui-ci auxquelles il répond par des contre-arguments. Il ne s’agit pas, en ce cas, de persuader l’adversaire, mais de s’adresser au lecteur, qui assiste à l’échange polémique et dont les vues, susceptibles de vaciller, sont en attente d’être confirmées et nourries. Le journaliste termine son argumentation par une interpellation directe de Liviu Dragnea contenant un dernier argument, destiné à renforcer la réfutation de la thèse adverse :

Nu, domnu’ Dragnea, asta nu e despărtirea de comunism, ci doar o încercare rizibilă de a deprimar partidul de imaginea lui Ion Iliescu. Desprinderea de comunism ar fi fost aia remarcată de Țuțea: “A te opune comunismului înseamnă a apăra puritatea Codului Penal”.

5.4. Le quatrième article sur les élections au sein du PSD, publié toujours dans le journal en ligne Gândul, porte le titre ironique O exercizare ratată (Une exercisation ratée), se référant aux efforts de Dragnea pour cosmetiser l’image du parti sans rompre véritablement avec le passé communiste de celui-ci. La position soutenue est la même que celle de l’article de sous 5.3 mais, si le ton de l’extrait précédent est sérieux et indigné, le ton du texte signé par Clarice Dinu est sarcasmatique et sa rhétorique est basée sur de nombreuses allusions au passé : emploi des termes traditionnellement associés aux leaders communistes comme tătuc, stalinism, baron ; évocation des anciens présidents communistes Nicolae Ceaușescu et Ion Iliescu et des anciens leaders du parti, Adrian Năstase, Mircea Geoană et Victor Ponta, qui ont perdu aux élections présidentielles à cause de l’image du parti, compromise par la corruption de ses membres notoires.

L’allégorie de l’exorcisation, d’une grande force argumentative, est soutenue par une isotopie dont nous signalons les éléments les plus saillants : preot, drac, a păcătui, a scoate dracui, a dezgropa, nefacută. Nous considérons que la stratégie argumentative choisie par l’auteur relève plutôt de la persuasion que de la rationalité. Le “coup de grâce” de cette argumentation (en faveur de Dragnea et contre la vieille garde communiste du PSD) est représenté par le paragraphe final qui cite la réplique

„Nici oa trebue să te lasă fără cuvinte.”


Alice Ionescu, Neutralité neutrosophique et expressivité dans le style journalistique
Le titre fait allusion au geste de Mihai Sturzu, leader du TSD (Organisation de la jeunesse social-démocrate) qui a dénoncé l’existence d’une seule candidature pour la fonction de président aux élections au sein du PSD comme coutume communiste, fait qui risquerait d’invalider le résultat de celles-ci. Ce geste, longuement commenté par la presse, a été sanctionné par la direction du parti avec l’exclusion de Sturzu, elle aussi très commentée. L’auteur fait le point des opinions véhiculées dans la presse :

Iar aici, părerele jurnaliştilor au fost împărțite: „PSD poate avea viitor, prin curajul tinerilor săi”, au spus unii; „Sturzu a fost năvăl crezând în democrația de partid, clamată de noul lider”, au considerat alții; „Liderul PSD a fost pus ca botul pe lab după atacul declanșat împotriva lui Liviu Dragnea”, a conchis ai treilea val; „Sturzu a spus ceea ce i-s-a dictat să spună, încercând să schimbe instaurarea dictaturii în partid”, au mai adăugat vreo câțiva. Ei bine, dacă nu l-au făcut pe Sturzu, introdus în lume de o anume aroganță, ai fi putut crede că omul a prins o mână grandioasă și pluszează cinstit, convins că va sala potul. [...] Ensuite il met en scène la voix d’un autre jeune social-démocrate, qui exprime un point de vue opposé sur Sturzu.

Astfel, Gabriel Petrea scrie într-un comunicat bine sințit că Mihai Sturzu, impus de Victor Ponta, în urmă cu doi ani, la șefia TSD, nu i-a reprezentat pe tinerii din partid și că, la Congresul PSD, acesta a avut cel mai ipocrit discurs din istoria partidului, vorbind fără jenă în numele propriului interes. “Duminică, 18 octombrie, Mihai Sturzu, uitând de toate aceste valori, a ținut, de la tribuna Congresului PSD, cel mai ipocrit discurs din istoria partidului. Impus de Victor Ponta la șefia PSD, acum exact doi ani de zile, în urma unui congres în care a fost singurul candidat, Mihai Sturzu acuză, fără jenă, congresele fără competiție. După ce doi ani la rând nu a fost vocea tinerilor în PSD, nici duminică el nu a reprezentat vocea organizației. Să nu se spună că Sturzu a vorbit în numele Tineretului Social Democrat! A vorbit în numele propriului interese, pentru obiective pe care nu le cunoaștem și de care nici nu ne pasă”, susține tânărul secretar general.

avant de donner son verdict :

Ulterior! Știu, de mici, se mănâncă între ei, ca o haiată de lupi flămânzi. Ce-mi era Sturzu, pentru care „dictatura” e bună numai și numai dacă dictează el, și ce-mi este Petrea, care a acceptat dictatura, ca statuia lui Lenin, până când Dragnea, „dictatorul-dictatorilor”, l-a cerut să revină, și el, la demnitatea și la aroganța lui, și să-l înfierce, cu mânie proletară, pe cântăreț. „Neles! Trăiți!!”

Bun tineret, distins partid.

Le sacrifice de la dernière phrase -ou plutôt antiphrase- est évident. La conclusion que l’auteur de l’article veut partager aux lecteurs est pessimiste : l’avenir du parti de gauche le plus important de la Roumanie post-communiste est compromis par l’opportunisme et le manque de scrupules de ses plus jeunes membres.

6 Conclusion

Le choix des articles de presse reflétant les dernières élections au sein du PSD met en évidence la multitude des points de vue sur le même contenu et la variété des interprétations que les journalistes donnent des faits et des évolutions respectives. Nous avons sélectionné un article informatif, qui présente les événements d’une manière objective, rédigé dans un style neutre et quatre articles d’opinion dont le degré de prise en charge du contenu transmis est variable.

On pourrait représenter le degré d’adhésion du locuteur au contenu propositionnel des énoncés par un axe allant de la neutralité totale à la prise de prise en charge :

Figure 1:

Neutralité absolue Prise en charge maximale

Sur cet axe, les extraits choisis pourraient se succéder de gauche à droite, le premier tendant à la neutralité et les autres se situant plus du côté de la prise en charge maximale. L’axe ci-dessus pourrait se rapporter aussi à une échelle des traitements des contenus informatifs allant de l’objectif (attitude zéro) au subjectif (prise en charge totale). L’interprétation donnée aux faits commentés varie elle aussi : ainsi, l’évaluation que les journalistes en font va du négatif au positif et leur attitude oscille entre l’optimisme modéré et le pessimisme extrême. Les mêmes faits, événements, gestes et déclarations donnent naissance à une multitude d’interprétations, et chacune se réclame « vraie », « juste » et « bien fondée ». À chacun sa vérité, car toute interprétation est singulière. Cette variété des points de vue met en exergue un autre phénomène scalaire spécifique aux langues naturelles : la vérité en langue est une vérité subjective, car prise en charge par un locuteur. À notre avis, dans la communication journalistique il y a toujours une attitude, plus ou moins manifeste, à l’égard de l’information présente. L’information dans la presse ne joue plus sur les oppositions classiques du vrai et du faux, mais sur le vraisemblable. L’objectif de la presse n’est pas seulement d’informer, mais aussi d’influencer l’opinion publique et de divertir le lecteur. La dramatisation, procédé consacré par le fait divers et la presse tabloïde, est devenue la façon habituelle de pimenter l’information dans tous les journaux d’information et d’opinion. Le paradoxe de la communication journalistique consiste, selon nous, dans
la double contrainte à laquelle l’auteur d’un article de presse est soumis : être persuasif (lire argumentatif) tout en gardant sa crédibilité. Il doit rester neutre par rapport aux faits présentes et aux acteurs politiques impliqués, sinon il risque de se faire accuser de parti pris. Sans neutralité, il n’a pas de crédibilité, mais dans l’absence d’une prise de position, il ne réussit pas à faire passer son message auprès du public.

Références


Articles de presse:


3. Liviu Dragnea s-a rupt (în figuri) de comunism Mis en ligne le 18.10.2015
http://www.gandul.info/puterea-gandului/liviu-dragnea-s-a-rupt-in-figuri-de-comunism-14853337

4. O exorcizare rataată, mis en ligne le 18.10.2015
http://www.gandul.info/puterea-gandului/o-exorcizare-ratata-14853194

5. PSD, next gen, mis en ligne le 21.10.2015
http://www.gandul.info/puterea-gandului/psd-next-gen-14856277

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Alice Ionescu, Neutralité neutrosophique et expressivité dans le style journalistique
Neutrosophic Semilattices and Their Properties

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Abstract: In this paper authors study neutrosophic semilattices and their properties. These neutrosophic semilattices are built using either $\cup$ or $\cap$ operation only. This application of these concepts is also discussed in this paper.

Keywords: Neutrosophic semi-lattices, pure neutrosophic semi-lattice.

1 Introduction

In this paper the new notion of neutrosophic semilattices is introduced for the first time. However the study of neutrosophic lattices started in 2004 [5,6]. But those neutrosophic lattices are of a special type as they were mainly defined to cater to the needs of applications in fuzzy models. This paper has three sections. Section one is introductory in nature. In section two different types of neutrosophic semilattices are defined and their properties developed.

Section three gives the probable applications of these concepts in data mining, sorting etc. Finally we give the conclusions based on this work.

2 Neutrosophic semilattices of different types and their properties

In this section neutrosophic semilattices of various types are defined and described. This study is new and innovative and certainly can provide lots of applications to various fields where semilattices are applied.

Example 2.1: Let $S(P) = \{1, a, aI, I, 0\}$ be the neutrosophic set, $\{S(P), \cap\}$ is a neutrosophic semilattice whose diagram is as follows.

Example 2.2. Let $\{S(P), \cap\}$ be the neutrosophic semilattice given by the following example.

Example 2.3. Let $\{S(P), \cap\}$ be the neutrosophic semilattice given by the following example.

In view of this the following neutrosophic semilattice is defined.

Definition 2.1: Let $\{S(P), \cap\}$ be the partially ordered neutrosophic set with 0, $\{S(P), \cap\}$ is defined as the neutrosophic semilattice if

$\min\{x, y\} = x \cap y \in S(P)$

for all $x, y \in S(P)$.
The examples given above are neutrosophic semilattices of finite order.
Similarly one can define \{S(P), \lor\} the neutrosophic semilattice.
Now examples of neutrosophic semilattice \{S(P), \lor\} are as follows.

**Example 2.4.** Let \{S(P), \lor\} be the neutrosophic semilattice which has the following figure.

![Figure 2.4](image)

**Example 2.5.** Let \{S(P), \lor\} be the neutrosophic semilattice which has the following figure.

![Figure 2.5](image)

**Example 2.6.** Let \{S(P), \lor\} be the neutrosophic semilattice given by the following figure.

![Figure 2.6](image)

The above neutrosophic semilattice is not pure for it contains 1, b_1, and b_2 as elements of S(P).
Thus the notion of neutrosophic pure semilattice is one in which all elements of S(P) are only neutrosophic elements.

**Example 2.7.** Let \{S(P), \lor\} be the pure vertex neutrosophic semilattice whose figure is given in the following.

![Figure 2.7](image)

Next the notion of edge neutrosophic semilattice under \{S, \lor\} and \{S, \land\} are defined and described in the following.

**Definition 2.2:** \{S, \lor\} or \{S, \land\} is defined to be the edge neutrosophic semilattice if all elements in S are real and is a partial ordered set. There are some edges which are neutrosophic are indeterminate.

Examples of them are given in the following.

**Example 2.8.** Let \{S, \lor\} be the edge neutrosophic semilattice which is given by the following figure.

![Figure 2.8](image)

**Example 2.9.** The following figure gives the edge neutrosophic semilattice \{S, \land\}.

![Figure 2.9](image)

**Example 2.10.** Let \{S, \land\} be the edge neutrosophic semilattice given by the following figure.
Next the notion of pure neutrosophic semilattice is defined in the following.

**Definition 2.3:** Let \( (S, \cup) \) (or \( (S, \cap) \) be the partial ordered set all of its vertex elements are neutrosophic and if every edge is also neutrosophic or an indeterminacy then \( (S, \cup) \) (or \( (S, \cap) \)) is defined as the pure neutrosophic semilattice.

Examples of pure neutrosophic semilattices are given below.

**Example 2.11:** Let \( \{S(P), \cup\} \) be the pure neutrosophic semilattice the figure of which is as follows;

![Figure 2.11](image)

The above semilattice is a pure neutrosophic semilattice whose cardinality is 10.

**Example 2.12.** \( \{S(P), \cap\} \) be the pure neutrosophic semilattice whose Hasse diagram is as follows.

![Figure 2.12](image)

This is again a pure neutrosophic semilattice of order 13.

Now having seen examples of pure vertex neutrosophic semilattices, edge neutrosophic semilattices and pure neutrosophic semilattices, examples of neutrosophic subsemilattices are provided.

**Example 2.13.** Let \( \{S(P), \cup\} \) be the vertex neutrosophic semilattice whose Hasse diagram is as follows.

![Figure 2.13](image)

Clearly the following figures gives the vertex neutrosophic subsemilattice.
Next examples of edge neutrosophic semilattice and their subsemilattices are obtained.

**Example 2.14.** Let \( \{S(P), \cap\} \) be the edge neutrosophic semilattice whose figure is given below.

![Figure 2.15](image1)

The subsemilattices of \( \{S(P), \cap\} \) need not in general be edge neutrosophic subsemilattices. They can be usual subsemilattices as well as edge neutrosophic subsemilattices.

The figures associated with subsemilattices of \( \{S(P), \cap\} \) is as follows.

![Figure 2.16](image2)

Fig 2.16(a) and (b) are edge neutrosophic subsemilattices whereas 2.16(c) is a usual subsemilattice.

Next the subsemilattices of a pure neutrosophic semilattices is described by the following example.

**Example 2.15:** Let \( \{S(P), \cup\} \) be the pure neutrosophic semilattice whose figure is given below.

![Figure 2.17](image3)

All subsemilattices of \( S(P) \) are pure neutrosophic subsemilattices only.

In view of this the following theorem is proved.

**Theorem 2.1:** Let \( \{S(P), \cup\} \) or \( \{S(P), \cap\} \) be the pure neutrosophic semilattice. Then every subsemilattice of \( \{S(P), \cup\} \) or \( \{S(P), \cap\} \) are also pure neutrosophic.
Proof: Follows from the fact in a pure neutrosophic semilattice all vertices and edges are neutrosophic. Hence the claim.

**Proposition 2.1:** Let \( \{S(P), \lor \} \) (or \( \{S(P), \land \} \) be a edge neutrosophic semilattice. Every subsemilattice need not be a edge neutrosophic subsemilattice.

**Proof:** Follows from the fact a edge neutrosophic semilattice can have subsemilattices which are not in general neutrosophic edges subsemilattice.

**Proposition 2.2:** Let \( \{S(P), \lor \} \) (or \( \{S(P), \land \} \) be a neutrosophic semilattice. Every subsemilattice of a neutrosophic semilattice need not be a neutrosophic subsemilattice.

**Proof:** Evident from the examples given.

### 3 Application of Neutrosophic Semilattices

In this section applications of neutrosophic semilattices is briefly given.

Infact all neutrosophic semilattices are neutrosophic trees. So these will find application in all places where neutrosophic trees find their applications.

So one can apply these neutrosophic semilattices when the research or the investigator feels that indeterminacy is present in that analysis.

### Conclusion

For the first time the new notion of neutrosophic semilattices is introduced and their properties are discussed. These neutrosophic lattices are also neutrosophic trees and they find their application in data mining and sorting.

### References


[4]. Iqbal Unnisa, W.B. Vasantha Kandasamy and Florentin Smarandache, Super modular Lattices, Educational Publisher, Ohio, 2012.


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Abstract. This study is an application of Neutrosophy in the sphere of liminality. First, the aim of this study is to underline the importance of the concept of Neutrosophy that was introduced by the professor Florentin Smarandache correlated with the concept of the liminality. According to Arnold Van Gennep and Victor Turner, in the liminality, the rituals are conducted to put the people in an ambiguous state where everything there is not true or neither false and meaning that the threshold state is neutral. Rituals, myths or rites are representing indeed a form of communication, but on an unclear level, determined by the uncertainty. Liminality has a part which is working under the uncertainty’s rules of Neutrosophy: when a person is participating in the rituals, he is searching a truth and risk a false. This means that the threshold state is improving the perception of the people from the moment when starts a ritual. But the threshold state can be generated also by the media. Rituals of the mass media are created in order to change the society’s perception, persuading the idea of what is true and false.

Keywords: Neutrosophy, liminality, rituals, uncertainty, media.

1 Introduction

Professor Florentin Smarandache introduced the concept of Neutrosophy as part of thinking discourse which studies what is the nature of neutralities (Smarandache, 2002; Smarandache, 2010; Smarandache, 2015). The uncertainty between the two rituals is correlated with the people’s participation. Our society is conducted by a series of opinions and belief, however, they are not only true and false, but also they have a numerous series of neutral variables. The most important element in the society is represented by the man. He is the entropy inductor (Smarandache & Vladutescu, 2014; Smarandache, Vladutescu, Dima & Voinea, 2015). In the relation with other persons, the man becomes more aware about their opinion and he respects them in order to receive the same. In our society, the probabilities of the neutral variables are determined by the conditions of what is true and false. Neutrosophy is a part of dialectics which reveals paradoxes and logics. From the moment when a person is born, he enters into reality and generates communication from every action. A communication act is created by a ritual. This religious act can be divided through a threshold state where everything is ambiguous or neutral. At the moment when a person is entering a liminal space, he will not know the actual situation. He will be put under a series of neutral variables dedicated to destabilizing the person’s entire world. Describing the necessity to attend a daily communication is represented in fact the manifestation of a threshold state in order to be a part of the rituals. Based on these things, a person is condemned to become aware of the uncertainty that will come. In the threshold state, all the neutralities are transformed in order to create an exit from the liminality; the ritual is not finished until the knowledge conquers the ambiguity. The consequence of not knowing what is happening in the ambiguous state –the liminality-determines the transparency between false and truth.

There are some philosophers who wrote about the thesis and antithesis as: Georg Wilhelm Friedrich Hegel (1770-1831), Karl Marx (1818-1883), Friedrich Engels (1820-1895), Immanuel Kant (1724-1804), Johann Gottlieb Fichte (1762-1814), and Thomas Schelling (born 1921). If we think about the question of Aristotle “what is the nature of things?”, we may find different responses based on science or religion. Our society creates every day new form of communication and improves the technology step by step. Regarding on the communication, through the actions, a man is generating emotions that can help supporting the basis of human kind. Therefore, in the daily rituals, there is a neutral connection that sustains the ambiguity. Liminality represents more than a simple concept. It’s the important factor that can transform the perception of people. In the liminal stage or threshold state, the people’s absence of knowledge embraces the need for information and communication, even everything there can be true or false. Neither of the activities that are taking place there are particularly true. On the threshold stage, the information is neutral. The communication in the rituals is consisting in a set of neutrosophic meanings. In a ritual, the communication between the person and the others creates a bond based on neutral manifestation. Everything here doesn’t have any particularly elements of
sensations or feelings, except the uncertainty. A person caught in the threshold stage will receive a feedback from the initiate of the rituals after the steps will be finished. Society is creating rituals in order to involve people in solid action with the purpose to persuade the population’s mind. People are participating in a ritual without comprehending what has happened to them (Beech, 2010). So, the rituals are representing more than a choice, some of them are instead a cultural obligation. Rituals involve the participation of every person. But, in the ritual, when a person doesn’t understand the meaning of threshold state regarding his transformation into a new person, the situation becomes more incomprehensible. His ambition to achieve the final result of the liminal space, determines the person to act properly, even he is in an ambiguous state. However, in this case, the people who participate into rituals are allocating a very large surplus of energy in order to understand the meaning of it. Based on this, liminality defines the actual situation as sacral event where the knowledge is persisting as a secondary act. The first act is all about the power to dispose the ambiguous state continue with the second act that insists on developing the knowledge after the ritual is over. Arnold Van Gennep introduced the concept of liminality to mark the importance of people’s metamorphosis.

Liminality is a threshold state or a bond between two worlds where everything we see is just a vapid perception of ours. Nothing that we see in the threshold state is true or false. Victor Turner (1969, 1977) claimed that the liminality doesn’t have a limited period of time, it depends on major factors, for example: when we are taking an exam, we participate in a common ritual for the students, however, the time here is something we all know, 1 or 2 hours. This means that the liminal space lasts 1 or 2 hours. At this time, we are caught under some rules that can have the power to subordinate us. If we don’t act like we are supposed to, we may lose to possibility to take the exam and go further with our lives. And we may be caught again in the liminality, but this time without the possibility to know exactly how long it will last the threshold state. We act properly; we get out of the threshold state faster. It’s simple. But in this period, we don’t have the chance to know exactly if we chose the correct answers (Ślusarczyk & Broniszewska, 2014).

The threshold state has numerous neutral values of exam’s answer, determined by the uncertainty. Here, we are condemned to a series of manifestation in order to make us to be seen as pawns in a strange round of chess. If we are just pawns, it means that the rest of the characters are representing the leaders (Voinea, 2013; Stanescu, 2015; Voinea, 2015). However, in this case, we have a series of moves limited. They determine our idea that correlate with the strict rules that game has.

2 Neutrosophy versus Liminality

The concept of liminality can be determined by neutrosophy, because the uncertainty that is maintained on an unknown level. When a ritual start the person who participate in it, must relinquish his past life and pass through the threshold state in order to start a new life. An important factor about the threshold state is that here, the person can be seen as equal by the members of the community, but with one differentiated conditions, it doesn’t have any rights. In the liminality, a person is facing three stages; the first one is separated from his life and common things where he is induced in a new world, apart from what he knew. Here, in the same thing he is introduced in an ambiguous state, but he remained watchful with what it has happened. Nothing about what was the meaning of his life is now true. In fact, the uncertainty remains a long period.

In the liminal space, the individual starts to ask himself question about what is the difference between true and false or how his life maybe was a lie until this moment. Depending of the ritual that determines the individual to conquer a new step in his life, the threshold state becomes his new home. For example: the enter in a political party represents a ritual. The determination of the person to become a member of a political party has to be much clarified in order to obtain this statute. Or another example, we can find Van Gennep’s traditional society in the tribes from Africa. There, people literally renounce their values and were put under some rigorous rules with the purpose to metamorphosis their life (Cerban & Panea, 2011). Therefore, in that limited or unlimited period of time, the liminal space inducted the future members to an unknown world where they didn’t do know what is true or false. The series of neutral values are the one responsible for the people’s hunger in finding their self or finding the truth.

The second stage of the liminality is determined by the possibility of the future members to adapt to their new conduct of life. He becomes aware of the new truth and can see the numerous possibilities that he has in the threshold state. In between true and false are a series of values that are not overlapped with each other. In fact, the true and false can’t be a presence in the liminality. A person caught in the threshold state will approach to what he finds unclear in order to achieve the knowledge. It has resulted, that the uncertainty prevails for the liminal space. He accumulates the necessary information from the group and improve our values, norms and rules in the form of their. The person in the threshold state is there to understand the reality better. He recreates his own life in function of the new set of other’s values (Budica, Busu, Dumitru & Purcaru, 2015).

The final stage of the ritual is the pre-integration where the person can be seen as more than prepared to go out from the liminality. But how we can say that he is prepared? The change must come from him. This time he is leaving the uncertainty and knows exactly what he wants.
without the possibility to be put again questions his choices (Grabara, Kolcun & Kot, 2014).

According to Victor Turner’s idea of liminality, even our common things like going to school or taking an exam are in facts rituals (Turner, 1977). So, everything we do is an on and on ritual. The incomprehensible becomes understood at every final destination of the liminality. However, at the final stage of the rituals, it appears another. We can say that the life is a circle composed of rituals: when one is finished, the other starts.

Liminality is a part of neutrosophy; it is constructed with different forms, but at the end all the rituals have the same path. After every ritual, a person is gaining knowledge, he understands the way of life and for the most of the time, he is the one who enters another ritual. Every ritual which a person is passing, it means a gain for the human kind (Negrea, 2013; Dima, Grabara & Vladu tescu, 2014; Negrea, 2015).

In our modern society, the time goes faster and faster and the people are changing unwillingly. Even if the concept of liminality introduced by Van Gennep was for the traditional society and Victor Turner named liminoid for our modern times, the idea remained the same. The Turner’s term “liminoid” (Turner, 1977) didn’t have much success, many scholars named the modern rituals as liminality or liminal space. In fact, both represent the path that every ritual has, starting with the peoples’ wishes to change and entering in the liminality and finished with the perception of the participations changed. Everything is changing, even our life.

The determination of our perception is based on the mass media. Media is creating the society and has the power to influence it how it wants. Mass media are developing rituals through television, radio and internet (social media). The last is seen as a giant source of information, but the real truth about what is behind the scene is unknown by the media’s audience. The daily media rituals are not only put us to liminality, but also to the neutrosophic theory. When people are watching the daily news, they are entering into a liminal space where everything they see may seem true, but if we analyze the situation carefully, we can discover that everything that the media generates is composed of neutral sets. Nothing we see on television is true or false (Coman, 1994; Coman, 2008; Thomassen, 2009). The story that news tell are more particularly between true and false, for example: if the news is about a terrible accident where 2 or 3 people were wounded, but they are out of danger, the audience will receive an information that these people are seriously hurt and they are in danger. Media system has the power to improve its information depending on the audience’s impact (Ionescu, 2013).

3 Conclusion

Liminality in the Neutrosophy generates the idea that the uncertainty can be exceeded by knowledge only when a ritual is finished. The threshold state is metamorphosis the perception of the people through rituals, determined unclear moments at that time. Every ritual is ambiguous and it means that in the first moment when a person is entering in the liminality, their knowledge becomes uncertainty. After the ritual is finished and the exit of threshold state comes, the uncertainty becomes knowledge. Our society is conducted by rituals every day: starting with going to work or having an exam to entering in a political party and so on. We can say that our society is conducted by a cycle of rituals. Through mass media’s rituals, society is changing every day.

The true and false state cannot be sustained by liminality, because the threshold state generated only neutral values and underlines the power of uncertainty in the people’s mind through rituals.

References


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Application of Extended Fuzzy Programming Technique to a real life Transportation Problem in Neutrosophic environment

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Abstract. Here this paper focuses on solving the transportation problems with neutrosophic data for the first time. The indeterminacy factor has been considered in Transportation Problems (TP). The two methods of linear programming – Fuzzy Linear Programming (FLP) and Crisp Linear Programming (CLP) are discussed with reference to neutrosophic transportation problems. The first method uses the membership, non-membership and indeterminacy degrees separately to find the crisp solution using the Fuzzy Programming Technique and then the optimal solution is calculated in terms of neutrosophic data with the help of defined cost membership functions. The satisfaction degree is then calculated to check the better solution. The second method directly solves the TP to find crisp solution considering a single objective function. The cost objective function is taken as neutrosophic data and the methods have been used as such for the first time. Both the methods have been illustrated with the help of a numerical example and these are then applied to solve a real life multi-objective and multi-index transportation problem. Finally the results are compared.

Keywords: Neutrosophic Transportation Problem; Fuzzy Linear Programming; Crisp Linear Programming; Fuzzy Programming Technique; indeterminacy degree

1 Introduction

The basic transportation problem was originally developed by Hitchcock [1]. There are several classical methods to solve such transportation problems where data is given in a precise way. But in real life transportation problems, data may not be known with certainty. In such cases, the imprecise data can be considered as interval valued or fuzzy data. Fuzzy set theory was introduced by Zadeh [2]. Zimmermann [3] introduced fuzzy linear programming (LP) problems. Zimmermann [4] considered LP with fuzzy goal and fuzzy constraints and used linear membership function and min operator as an aggregator of these functions. Thus Fuzzy Linear Programming (FLP) problem was formulated. Further, Fuzzy set theory was applied to solve LPP with several objectives functions. The fuzzified constraint and objective functions were used to solve the multi-objective linear programming problems. Chanas [5] focused on Fuzzy Linear Programming model for solving transportation problems with crisp cost coefficients and fuzzy supply and demand values. Chanas and Kuchta [6] developed an algorithm for the optimal solution of TP with fuzzy coefficients which are expressed as L-R fuzzy numbers. Chanas and Kuchta [7] developed an algorithm for solving integer fuzzy transportation problem with fuzzy supply and demand. Bit and Biswal [8] applied the fuzzy programming technique with linear membership function to solve Multi-objective transportation problem (MOTP). Bit and Biswal [9] proposed an additive fuzzy programming model that considered weights and priorities for all non equivalent objectives for the transportation planning problems. Li and Lai [10] developed a fuzzy compromise programming method to obtain a non-dominated compromise solution to the MOTP in which various objectives were synthetically considered with marginal evaluation for individual objectives and the global evaluation for all objective functions. A real life multi-index multi-objective transportation problem was solved by Kour, Mukherjee and Basu in [11],[12],[13],[14] and [15] using different approaches. Intuitionistic fuzzy sets (IFS) were introduced as generalization of fuzzy set (FS). Here membership and non-membership degree were used instead of exact numbers. Intuitionistic fuzzy sets (IFSs) were introduced by Atanassov [16]. Atanassov & Gargov [17] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) as a further generalization of that of IFSs. Atanassov [18] also defined some operational laws of IVIFSs. Angelov [19] reformulated the optimization problems in an intuitionistic fuzzy environment. Several works have been done taking the triangular and trapezoidal intuitionistic fuzzy number. Gani and Abbas [20] proposed a new method for intuitionistic...
fuzzy transportation problem using triangular intuitionistic fuzzy number. Hussain and Kumar [21] applied the fuzzy zero point method to find optimal solution of intuitionistic fuzzy transportation problems Antony [22] also developed the VAMs method for TP for triangular intuitionistic fuzzy number. Aggarwal and Gupta [23] solved the TP for generalized trapezoidal intuitionistic fuzzy number by ranking method. P. P. Angelov first introduced the Intuitionistic fuzzy optimization (IFO) in his paper [19] and solved the transportation problem with crisp data by this method. The concept of Neutrosophic set was introduced as a generalization of crisp, fuzzy, intuitionistic, interval valued Intuitionistic Fuzzy number by Smarandache[24]. The Indeterminacy function (I) was added to the two available parameters: Truth (T) and Falsity (F) membership functions. In Neutrosophic Set, the indeterminacy is quantified explicitly and truth - membership, indeterminacy membership and false - membership are completely independent. In intuitionistic fuzzy sets, the indeterminacy is 1- T (x) - F (x) (i.e. hesitancy or unknown degree) by default. In Neutrosophy, the indeterminacy membership (I(x)) is introduced as a new subcomponent so as to include the degree to which the decision maker is not sure. This type of treatment of the problem was out of scope of intuitionistic fuzzy sets. Wang et al. [25] introduced the concept of single valued neutrosophic set (SVNS).

The present paper presents the solution of transportation problems with neutrosophic data using linear programming methods. It deals with cost objective function as neutrosophic data and the Neutrosophic TP has been solved using two methods. In the first method, fuzzy linear programming (FLP) has been extended for the neutrosophic data and the second method uses the crisp linear programming method (CLP).

The formulations and solutions are illustrated with the help of solved example and then the results are compared. The uncertainties of the real life problems are considered in the form of neutrosophic data. In transportation problems, the cost of transportation, the demand and the supply may not be known exactly as crisp numbers. Thus the uncertainties can be considered in terms of their degrees of acceptance, degrees of indeterminacy and degrees of rejection. That is, neutrosophic fuzzy numbers can be used for representing the imprecise data of cost of transportation or demand or supply or all in a transportation problem. This can be explained with the help of an example. If the transportation cost is taken in terms of the neutrosophic fuzzy number (0.8,0.1,0.2), that means the degree of acceptance of the available cost is 0.8, degrees of indeterminacy is 0.1 while the degree of rejection of the available cost is 0.2.

Finally the methods are applied to solve a real life multi-objective and multi-index neutrosophic transportation problem for the first time. The problem is solved to optimize the three objectives simultaneously namely, transportation cost, deterioration rate and underused capacity with neutrosophic data. The paper presents a better application of the method for multi-objective transportation problems.

2 Preliminaries

2.1 Single Valued Neutrosophic Set (SVNS)

An SVNS A in X is characterized by a truth-membership function \( T_A(x) \), an indeterminacy-membership function \( I_A(x) \), and a falsity-membership function \( F_A(x) \) for each point \( x \) in \( X \). When \( X \) is continuous, an SVNS A can be written as

\[
A = \left\{ x \in X \mid T_A(x), I_A(x), F_A(x) \in [0, 1] \right\}
\]

When \( X \) is discrete, an SVNS A can be written as

\[
A = \sum_{x \in X} \left\{ T_A(x_i), I_A(x_i), F_A(x_i) \right\}
\]

3 Problem description and methodology

3.1 Problem Description

The transportation problem is meant for minimization of transportation cost from different sources to different destinations.

- Classical transportation problems:

  In the classical transportation problem cost objective function and the constraints are considered as crisp values. Therefore it is required to calculate the optimal solution which minimizes the cost objective functions and satisfies all the constraints.

  Minimize \( f(x) \)

  Subject to \( g_j(x) \leq 0, j = 1,2,\ldots,q \)
The degree of satisfaction of the objective function and the constraints is maximized to find the optimal solution.

Intuitionistic fuzzy transportation problem: Then the intuitionistic fuzzy transportation problem (IFTP) was considered [19, 20, 21, 23]. In such case the degree of rejection \( v_i(x) \) is also considered along with the degree of acceptance \( \mu_i(x) \) of the cost objective function and the constraints. The degree of acceptance is maximized and the degree of rejection is minimized to find the optimal solution in such problems.

Neutrosophic transportation problem: In a transportation problems with neutrosophic data , the indeterminacy factor has been considered for the first time. The degree of indeterminacy \( r_i(x) \) was also considered along with the two available parameters, degree of acceptance \( \mu_i(x) \) and degree of rejection \( v_i(x) \) of the cost objective function and the constraints. The problem is to maximize degree of acceptance and minimize the degree of rejection and indeterminacy.

\[ \sum_{i} S_i \leq \sum_{j} x_{ij} \]  
where \( S_i \) denotes the supply of source \( i \),

\[ \sum_{j} D_j \leq \sum_{i} x_{ij} \]  
where \( D_j \) denotes the demand of destination \( j \),

\[ x_{ij} \geq 0 \]  

\[ \text{For Multi-objective TP, we obtain a set of similar three equations for each of the objective functions} \]

\[ \text{Model 2. Crisp Linear Programming Model (for Neutrosophic data)} \]

For Single objective transportation problems, the model is

Maximize \( Z = \sum \sum (\mu_{ij} - v_{ij} - r_{ij})x_{ij} \)
subject to \( 0 \leq \mu(x), r(x), v(x) \leq 1 \), and other constraints mentioned in Equation(2) (3).

For Multi-objective transportation problems, we obtain a set of similar equations for each objective function.

3.2 Methodology

3.2.1 Fuzzy Linear Programming

The Transportation problem with neutrosophic data has been formulated as a multi-objective transportation problem as in Model 1 and has been solved by Fuzzy Linear Programming Technique (Das[26], Zimmermann [3]).

Extended Fuzzy Programming Technique
Step 1: Solve the multi-objective transportation problem as a single objective transportation problem using each time only one objective and ignoring others.

Step 2: From the results of Step 1, determine the corresponding values for every objective at each solution derived.

Then find the lower and upper bounds , \( Z_k^L \) and \( Z_k^U \) (k=1,2,3,...,K).

Step 3 : Linear Membership Function

A Linear membership function \( \mu_k(x) \) corresponding to \( k^{th} \) objective for the minimization problem is defined as

\[ \mu_k(x) = \begin{cases} \frac{x_k - L_k}{U_k - L_k} & \text{if } Z_k^L \leq Z_k < Z_k^U \\ \frac{x_k - U_k}{Z_k - L_k} & \text{if } Z_k^U < Z_k \leq Z_k^L \\ 0 & \text{if } Z_k \geq Z_k^U \end{cases} \]  

(4)

Similarly, a linear membership function can be defined for maximization problem as
The linear programming problem can further be simplified as in Model 3:

**Model 3:**
Maximize $\lambda$
subject to $Z_k + \lambda (Z_k^U - Z_k^L) \leq Z_k^U$
for minimization problem and
$Z_k + \lambda (Z_k^U - Z_k^L) \geq Z_k^L$
for maximization problem

with the given constraints and non-negativity restriction as in Model 1 and $\lambda \geq 0$.

Thus the Step 3 gives the values of the three objective functions, $Z_1$, $Z_2$ and $Z_3$ as in Model 1.

Step 4:

$$Z = Z_1 - Z_2 - Z_3$$

This provides the crisp optimal value for the objective functions. Then using the definition of cost membership function, the satisfaction, indeterminacy and rejection degrees of membership function of the solution are obtained as Single Valued Neutrosophic Set (SVNS).

### 3.2.2 Crisp Linear Programming

The Model 1 can further be formulated as a single-objective linear programming problem as in Model 2 and is solved as usual by standard software. The solution gives the optimal value of cost objective function $Z$ as a crisp value. For multi-objective transportation problems, it forms a set of objective functions in equations which can be solved by fuzzy programming technique and the optimal solution can be obtained as crisp value for each objective function.

### 4 Numerical Illustration

#### 4.2.1 Example 1

The problem is taken as a neutrosophic transportation problem (NTP) in which each transportation cost is taken as neutrosophic data representing the degree of acceptance, degree of indeterminacy and degree of rejection of the cost as in Table 1. The demand and the capacity are considered as crisp values.

<table>
<thead>
<tr>
<th>Market 1</th>
<th>Market 2</th>
<th>Market 3</th>
<th>Market 4</th>
<th>Capacity $(S_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Port 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.6,0.1,0.2)(0.7,0.2,0.1)(0.3,0.3,0.1)(0.8,0.1,0.1)$ 400</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Port 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.5,0.2,0.3)(0.4,0.1,0.1)(0.5,0.3,0.1)(0.3,0.3,0.2)$ 150</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Port 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.4,0.3,0.2)(0.3,0.2,0.2)(0.6,0.3,0.1)(0.7,0.3,0.2)$ 300</td>
</tr>
<tr>
<td>Demand 200</td>
<td>200</td>
<td>100</td>
<td>350</td>
<td>$(D_j)$</td>
</tr>
</tbody>
</table>

Table 1: Data for NFTP.

The objective for this problem can be determined by degree of acceptance $\mu_o(x)$, degree of indeterminacy $r_o(x)$ and degree of rejection $v_o(x)$ of the cost function defined as follows:

$$
\mu_o(x) = \begin{cases} 
1, & \sum_{i=1}^{3} \sum_{j=1}^{4} C_i x_{ij} < 200 \\
\frac{(350 - \sum_{i=1}^{3} \sum_{j=1}^{4} C_i x_{ij})}{150}, & 200 \leq \sum_{i=1}^{3} \sum_{j=1}^{4} C_i x_{ij} \leq 350 \\
0, & \sum_{i=1}^{3} \sum_{j=1}^{4} C_i x_{ij} > 350 
\end{cases}
$$

(10)
where costs are considered in terms of thousand dollars.

4.2 Solution

The given problem is a neutrosophic transportation problem (NTP) and is solved by the above mentioned methods and the results are obtained.

Solution with data from Table 1 by the method based on FLP

Substituting the neutrosophic data from Table 1 in the Model 1, we get three different objective functions as

Maximize

\[ Z_1 = 0.6x_{11} + 0.7x_{12} + 0.3x_{13} + 0.8x_{14} + 0.5x_{21} + 0.4x_{22} + 0.5x_{23} + 0.3x_{24} + 0.4x_{31} + 0.3x_{32} + 0.6x_{33} + 0.7x_{34} \]

Minimize

\[ Z_2 = 0.1x_{11} + 0.2x_{12} + 0.3x_{13} + 0.1x_{14} + 0.2x_{21} + 0.1x_{22} + 0.3x_{23} + 0.3x_{24} + 0.3x_{31} + 0.2x_{32} + 0.3x_{33} + 0.3x_{34} \]

Minimize

\[ Z_3 = 0.2x_{11} + 0.1x_{12} + 0.1x_{13} + 0.1x_{14} + 0.3x_{21} + 0.1x_{22} + 0.1x_{23} + 0.2x_{24} + 0.2x_{31} + 0.2x_{32} + 0.1x_{33} + 0.2x_{34} \]

subject to

\[ \sum_{j} x_{ij} = S_i \]

where \( S_i \) denotes the supply of source \( i \) given in Table 1

\[ \sum_{i} x_{ij} = D_j \]

where \( D_j \) denotes the demand of destination \( j \) given in Table 1

\( x_{ij} \geq 0 \)

Step 1: The problem is solved considering as single objective taking only one objective function and neglecting others. The solution sets are obtained as:

\( Z_1 = 565 \), \( Z_2 = 180 \), \( Z_3 = 140 \).

Step 2: For each solution set, the values for the other two objective functions are obtained as:

\( Z_1 = 565 \) (for \( Z_2 \) solution set), \( Z_1 = 505 \) (for \( Z_3 \) solution set)

\( Z_2 = 140 \), \( Z_2 = 180 \) (for \( Z_1 \) solution set), \( Z_2 = 150 \) (for \( Z_3 \) solution set)

\( Z_3 = 105 \), \( Z_3 = 140 \) (for \( Z_1 \) solution set), \( Z_3 = 110 \) (for \( Z_2 \) solution set)

For each objective, the best and worst values are given as

\( Z_1^U = 565 \), \( Z_1^L = 505 \), \( Z_2^U = 180 \), \( Z_2^L = 140 \), \( Z_3^U = 140 \), \( Z_3^L = 105 \).

Step 3: Using the values obtained in Step 2 in the Equations (6) and (7) obtained from Model 3 of Section 4.2, the final solution is obtained as

\( \lambda = 0.9090909 \), \( Z_1 = 508.64 \), \( Z_2 = 143.64 \), \( Z_3 = 108.18 \)

Step 4: Using the values obtained in Equation (9), \( Z = 256.82 \)
Also the degree of acceptance, indeterminacy and rejection of cost objective functions are obtained using Equations (10),(11) and (12) as

\[ \mu_o = 0.62, r_o = 0.0047, \nu_o = 0.14 \]

i.e. \((\mu_o, r_o, \nu_o) = (0.62, 0.0047, 0.14)\)

Solution with data from Table 1 by the method based on CLP

Substituting the neutrosophic data from Table 1 in the Model 2, we get

Minimize

\[ Z = 0.3x_{11} + 0.4x_{12} - 0.9x_{13} + 0.6x_{14} 
+ 0x_{21} + 0.2x_{22} + 0.1x_{23} - 0.8x_{24} 
- 0.1x_{31} - 0.1x_{32} + 0.2x_{33} + 0.2x_{34} \]

subject to all the constraints in Equation (13)

The transportation problem is solved as single objective TP by crisp linear programming as in Model 2 and the crisp optimal solution is obtained as \( Z = 260 \).

The degree of acceptance, indeterminacy and rejection of cost objective functions are obtained as

\[ \mu_o = 0.6, r_o = 0.01, \nu_o = 0.16 \]

i.e. \((\mu_o, r_o, \nu_o) = (0.6, 0.01, 0.16)\)

5 Real Life Illustration

5.1 Real life multi-objective multi-index transportation problem

To illustrate the application of the proposed approach for a real life multi-objective multi-index transportation problem, following numerical example from Kour, Mukherjee and Basu [11] is considered, previously taken as approximate past records from DSP Plant, Durgapur, West Bengal, INDIA.

The problem deals with the solution of the multi-objective multi-index real life transportation problem focusing on the minimization of the transportation cost, deterioration rate and underused capacity of the transported raw materials like coal, iron ore, etc from different sources to different destination sites at Durgapur Steel Plant (DSP) by different transportation modes like train, trucks, etc. The problem is formulated by taking different parameters in the objective function as neutrosophic data and supply and demand as crisp numbers.

Consider a problem in which we have three raw materials (m=3) i.e. q=1(Coal), 2(Iron-ore), 3(Limestone). These raw materials are transported from different \(i^{th}\) sources to \(j^{th}\) destination sites by different transportation modes \(h\) as per Table 2.
The data in the form of crisp numbers for supply and demand are as follows:

<table>
<thead>
<tr>
<th>q</th>
<th>h</th>
<th>l</th>
<th>$S_{qp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>182.5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>107.5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>59</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>30.5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>77</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>89.5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>51.25</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>78.05</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>47.75</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>122.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
<td>147.5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 4 Supply data

<table>
<thead>
<tr>
<th>Q</th>
<th>h</th>
<th>j</th>
<th>$D_{qj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>195</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>81</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>88</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>129</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>49</td>
</tr>
</tbody>
</table>
The data for supply $S_{ij}$, $\forall i,j$ are given in the Table 3. The data for demand $D_{ij}$, $\forall j,q$ are given in the Table 4. The neutrosophic objective for this problem can be determined by degree of acceptance $\mu_o(x)$, degree of indeterminacy $r_o(x)$ and degree of rejection $\nu_o(x)$ of the three objective functions defined as follows:

For Transportation Cost:

$$
\mu_o(x) = \begin{cases}
1, & \frac{3}{4} \sum_{i=1}^{3} \sum_{j=1}^{4} C_{ij} y_{ij} < 200 \\
\frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} \leq 350, & 200 \leq \frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} \\
0, & \frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} > 350 \end{cases}
$$

For Underused capacity:

$$
\mu_o(x) = \begin{cases}
1, & \frac{3}{4} \sum_{i=1}^{3} \sum_{j=1}^{4} C_{ij} y_{ij} < 210 \\
\frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} \leq 350, & 210 \leq \frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} \\
0, & \frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} > 350 \end{cases}
$$

For Deterioration Rate:

$$
\mu_o(x) = \begin{cases}
1, & \frac{3}{4} \sum_{i=1}^{3} \sum_{j=1}^{4} C_{ij} y_{ij} < 150 \\
200 \leq \frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} \leq 350, & \frac{3}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij} y_{ij} > 350 \end{cases}
$$

D.Kour, K.Basu, Application of Extended Fuzzy Programming Technique to a real life Transportation Problem in Neutrosophic environment
The given problem is first written in the form of the formulated model, Model 4 as:

Minimize \( Z_1 = \sum_{q=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{o} \sum_{h=1}^{p} C_{ijh}^q X_{ijh}^q \) \hspace{1cm} (23)

Minimize \( Z_2 = \sum_{q=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{o} \sum_{h=1}^{p} R_{ijh}^q X_{ijh}^q \) \hspace{1cm} (24)

Minimize \( Z_3 = \sum_{q=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{o} \sum_{h=1}^{p} U_{ijh}^q X_{ijh}^q \) \hspace{1cm} (25)

Subject to \( \sum_{j} \sum_{h} X_{ijh}^q \geq S_{iq}, \quad \forall \ i, q \) \hspace{1cm} (26)

\( \sum_{i} \sum_{h} X_{ijh}^q \leq D_{jq}, \quad \forall \ j, q \) \hspace{1cm} (27)

\( X_{ijh}^q \geq 0. \) \hspace{1cm} (28)

where \( q = \text{type of raw material}; \ m = \text{number of raw materials}; \)
\( n = \text{number of sources}; \)
\( o = \text{number of destination sites}; \)
\( h = \text{transportation modes}; \ p = \text{number of transportation modes}. \)

\( X_{ijh}^q = \text{Quantity to be transported of } q^{th} \text{ raw material from } i^{th} \text{ source to } j^{th} \text{ destination by transportation mode ‘h’;} \)
\( C_{ijh}^q = \text{Transportation cost (in billion rupees per metric tonne) of transportation of } q^{th} \text{ raw material from } i^{th} \text{ source to } j^{th} \text{ destination by transportation mode ‘h’ as neutrosophic set} \)
\( R_{ijh}^q = \text{Deterioration rate (in tonnes per million metric tonne) while transporting } q^{th} \text{ raw material from } i^{th} \text{ source to } j^{th} \text{ destination by transportation mode ‘h’; as neutrosophic set} \)
\( U_{ijh}^q = \text{Underused capacity (in tonnes per thousand metric tonne) while transporting } q^{th} \text{ raw material from } i^{th} \text{ source to } j^{th} \text{ destination by transportation mode ‘h’; as neutrosophic set} \)
\( S_{iq} = \text{Supplied quantity of } q^{th} \text{ raw material from } i^{th} \text{ source (Availability) (in million metric tonnes)} \)
\( D_{jq} = \text{Demand of } q^{th} \text{ raw material at } j^{th} \text{ destination (Requirement) (in million metric tonnes)} \)
\( Z_1, Z_2, Z_3 \) are the minimal values of the neutrosophic Transportation Cost, Deterioration rate and Underused capacity.

5.2 Solution
The given problem is a neutrosophic transportation problem (NTP) and is solved by the above mentioned methods.

**Solution by the method based on FLP:**

Substituting the above neutrosophic data in the Model 1, three different objective functions for each of the Equations (23), (24) and (25) are obtained. Then the problem can be solved using Extended neutrosophic fuzzy programming technique.

**Step 1:** The problem is solved considering as single objective taking only one objective function and neglecting others. The solution sets are obtained as:

1. \( Z_{11} = 419.66 \), \( Z_{12} = 21.43 \), \( Z_{13} = 166.02 \)
2. \( Z_{21} = 411.94 \), \( Z_{22} = 76.64 \), \( Z_{23} = 158.32 \)
3. \( Z_{31} = 326.51 \), \( Z_{32} = 34.66 \), \( Z_{33} = 229.94 \)

**Step 2:** For each solution set, the values for the other objective functions can be obtained. The best and worst values for each objective are obtained as:

\[ Z_1^L = 364.58, Z_1^U = 828.49, Z_1^U - Z_1^L = 463.91 \]
\[ Z_2^L = 21.43, Z_2^U = 42.13, Z_2^U - Z_2^L = 20.7 \]
\[ Z_3^L = 103.13, Z_3^U = 216.79, Z_3^U - Z_3^L = 113.66 \]
\[ Z_{21}^L = 376.66, Z_{21}^U = 843.75, Z_{21}^U - Z_{21}^L = 467.09 \]
\[ Z_{22}^L = 35.43, Z_{22}^U = 85.42, Z_{22}^U - Z_{22}^L = 49.99 \]
\[ Z_{23}^L = 64.12, Z_{23}^U = 191.88, Z_{23}^U - Z_{23}^L = 127.76 \]
\[ Z_{31}^L = 326.51, Z_{31}^U = 714.85, Z_{31}^U - Z_{31}^L = 388.28 \]
\[ Z_{32}^L = 31.22, Z_{32}^U = 56.14, Z_{32}^U - Z_{32}^L = 24.92 \]
\[ Z_{33}^L = 91.65, Z_{33}^U = 255.72, Z_{33}^U - Z_{33}^L = 164.07 \]

**Step 3:** Corresponding to the three objective functions, a linear membership function can be defined. Then the problem can be solved using Equations (6) and (7) from Model 4 of Section 4.2 and the final solution is obtained as \( \lambda = 0.8365686 \).

\[ Z_{11} = 328.75, Z_{12} = 0 \]
\[ Z_{13} = 75.63, Z_{21} = 284.38, Z_{22} = 35.45 \]
\[ Z_{23} = 85, Z_{31} = 263.25, Z_{32} = 0, Z_{33} = 105.25 \]

**Step 4:** Using the values obtained in Equation (12), \( Z_1 = 253.12, Z_2 = 163.93, Z_3 = 158 \)

The degree of acceptance, indeterminacy and rejection of different objective functions are obtained as:

- **Transportation cost:** \( \mu_o = 0.65, r_o = 0.095, \nu_o = 0.13 \)
  i.e. \( (\mu_o, r_o, \nu_o) = (0.65, 0.095, 0.13) \)

- **Deterioration rate:** \( \mu_o = 0.93, r_o = 0.0021, \nu_o = 0.005 \)
  i.e. \( (\mu_o, r_o, \nu_o) = (0.93, 0.0021, 0.005) \)

- **Underused capacity:** \( \mu_o = 0.8, r_o = 0.001, \nu_o = 0.037 \)
  i.e. \( (\mu_o, r_o, \nu_o) = (0.8, 0.001, 0.037) \)

**Solution by the method based on CLP:**

Substituting the neutrosophic data in the Equation (3) in the Model 2, a set of three similar equations is obtained which form a multi-objective transportation problem and thus can be solved by fuzzy programming technique. This gives the optimal solution for
each objective function. The final crisp optimal solution is obtained as

\[ \lambda = 0.4075449 \quad , \quad Z_1 = 217.665 \quad , \quad Z_2 = 329.12 \quad , \quad Z_3 = 169.355 \]

The degree of acceptance, indeterminacy and rejection of different objective functions are obtained as

- **Transportation cost**: \[ \mu_o = 0.88, r_o = 0.003, \nu_o = 0.014 \]
  
  i.e. \((\mu_o, r_o, \nu_o) = (0.88, 0.003, 0.014)\)

- **Deterioration rate**: \[ \mu_o = 0.104, r_o = 0.7, \nu_o = 0.8 \]
  
  i.e. \((\mu_o, r_o, \nu_o) = (0.104, 0.7, 0.8)\)

- **Underused capacity**: \[ \mu_o = 0.77, r_o = 0.0059, \nu_o = 0.053 \]
  
  i.e. \((\mu_o, r_o, \nu_o) = (0.77, 0.0059, 0.053)\)

### 6. Results and Discussions

- The two methods are introduced for neutrosophic transportation problems and illustrated by an example in Section 4. The method is then applied for a real life multi-objective and multi-index neutrosophic transportation problem in Section 5 for the first time.

- The optimal solution for the neutrosophic transportation problems in Section 4 is obtained by the above two methods, i.e. by FLP and the other by CLP. The crisp optimal solution for the cost objective function of the given neutrosophic fuzzy transportation problem in Section 4 is obtained by FLP method using linear membership function as 256.82 (thousand dollars) as in Table 5. The degree of acceptance, indeterminacy and rejection of the obtained solution is calculated as (0.62, 0.0047, 0.14). Thus the satisfaction degree of the solution is 62% acceptable 0.4% indeterminant (not known) and 76% rejectable.

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(Z_1)</th>
<th>(Z_2)</th>
<th>(Z_3)</th>
<th>((\mu_o, r_o, \nu_o))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.909</td>
<td>508.64</td>
<td>143.6108.2</td>
<td>256.8(0.6,0.0047,0.14)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5**: Solution of Example in Section 4 using Linear membership function.

The degree of satisfaction of the optimal solution depends upon the respective defined membership, indeterminacy and non-membership function in the given problems. The degree of satisfaction and the degree of rejection need not be complement to each other. The crisp optimal solution for the cost objective transportation problem of the given neutrosophic transportation problem is obtained by crisp linear programming method as 260 (thousand dollars). The satisfaction degree of this solution is 0.6 which means the solution is 60% acceptable 0.01% indeterminant (not known) and 0.16% rejectable.

<table>
<thead>
<tr>
<th>(\lambda)</th>
<th>(Z_1)</th>
<th>(Z_2)</th>
<th>(Z_3)</th>
<th>((\mu_o, r_o, \nu_o))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.62</td>
<td>0.0047</td>
<td>0.14</td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6**: Comparison of the obtained neutrosophic solutions using FLP and CLP methods.

The crisp optimal solution for the different objective functions – transportation cost, deterioration rate and underused capacity of the given real life neutrosophic transportation problem in Section 5 is obtained by FLP method using linear membership function as 253.12, 163.93 and 158 as in Table 8. The degree of acceptance, indeterminacy and rejection of the obtained solution for
The transportation cost, deterioration rate and underused capacity is calculated as (0.65, 0.095, 0.13), (0.93, 0.0021, 0.005) and (0.8, 0.001, 0.037). Thus the satisfaction degree of the three solutions are 0.65, 0.93 and 0.859 which means the first transportation cost solution is 65% acceptable, 9% indeterminant and 13% rejectable. The second deterioration rate solution is 93% acceptable, 0.2% indeterminant and 0.5% rejectable and the third underused capacity solution is 80% acceptable, 0.1% indeterminant and 3.7% rejectable.

Thus the FLP method appears to be better method as it gives more optimal solution as compared to the crisp linear programming method.

### 7. Conclusions

- In this paper, the Neutrosophic Transportation Problem (NTP) is solved by two methods- FLP method and CLP method.
- The first method, FLP method gives the solution as crisp and then as SVNS which represent the degree of acceptance, indeterminacy and rejection of the solution obtained from the defined membership function for a particular problem.
- The second method, i.e., CLP method gives the solution as crisp number only. Then the degree of the acceptance, indeterminacy and rejection is calculated
- The FLP method can be seen as a better method and it gives more optimal solution.
- The SVNS data can represent real life uncertainties and so depicts more practical solutions of the problem as it helps to determine the degree of acceptance, indeterminacy and rejection of the obtained solution.
- A real life multi-objective and multi-index Neutrosophic transportation problem has also been solved in Section 5 other than the numerical example in Section 4 to illustrate the two proposed methods. The results and comparisons of the large scale problem are shown in the Table 5, Table 6, Table 7 and Table 8. The results obtained are compared and the FLP method proves to give better solution compared to the CLP method for most of the circumstances.

### Table 7: Solution of real life example in Section 5 using Linear membership function.

<table>
<thead>
<tr>
<th>λ</th>
<th>Z₁</th>
<th>Z₂</th>
<th>Z₃</th>
<th>Z</th>
<th>( \mu_o, r_o, \nu_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.909</td>
<td>328.8</td>
<td>0</td>
<td>75.63</td>
<td>253.1</td>
<td>(0.65, 0.09, 0.13)</td>
</tr>
<tr>
<td>284.38</td>
<td>35.45</td>
<td>85</td>
<td>163.9</td>
<td>(0.93, 0.00, 2.005)</td>
<td></td>
</tr>
<tr>
<td>263.25</td>
<td>0</td>
<td>105.2</td>
<td>158</td>
<td>(0.8, 0.001, 0.04)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 8: Comparison of the obtained neutrosophic solutions using FLP and CLP methods.

<table>
<thead>
<tr>
<th></th>
<th>Fuzzy linear programming</th>
<th>Crisp linear programming</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transportation cost</td>
<td>(0.65, 0.09, 0.13)</td>
<td>(0.88, 0.003, 0.014)</td>
</tr>
<tr>
<td>Deterioration rate</td>
<td>(0.93, 0.00, 21, 0.005)</td>
<td>(0.104, 0.7, 0.8)</td>
</tr>
<tr>
<td>Underused capacity</td>
<td>(0.8, 0.001, 0.037)</td>
<td>(0.77, 0.0059, 0.053)</td>
</tr>
</tbody>
</table>
The solution obtained by the proposed approaches has not been compared with any of the existing approaches for NTPs, as no work has been done for neutrosophic transportation problem. It is a new type of problem.

The application of the methods to a real life multi-objective and multi-index neutrosophic transportation problem is also a new field itself.

References:


TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-attribute Decision Making Problems

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Abstract. In the paper, we propose Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) technique for solving single valued neutrosophic soft set expert based multi-attribute decision making problems. Single valued neutrosophic soft expert sets are combination of single valued neutrosophic sets and soft expert sets. In the decision making process, the ratings of alternatives with respect to the parameters are expressed in terms of single valued neutrosophic soft expert sets to deal with imprecise or vague information. The unknown weights of the parameters are derived from maximizing deviation method. Then, we determine the rank of the alternatives and choose the best one by using TOPSIS method. Finally, a numerical example for teacher selection is presented to demonstrate the applicability and effectiveness of the proposed approach.

Keywords: Single valued neutrosophic sets, single valued neutrosophic soft expert sets, TOPSIS, multi-attribute decision making.

1 Introduction

Hwang and Yoon [1] grounded the technique for order preference by similarity to ideal solution (TOPSIS) method for solving conventional multi-attribute decision making (MADM) problems. The basic concept of TOPSIS is straightforward. It is developed from the concept of a displaced ideal point from which the compromise solution has the shortest distance. Hwang and Yoon [1] proposed that the ranking of alternatives would be based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). TOPSIS approach simultaneously considers the distances to both PIS and NIS, and a preference order is ranked based on their relative closeness, and a combination of these two distance measures.

MADM is the process of identifying the most suitable alternative from a finite set of feasible alternatives with respect to numerous usually conflicting attributes. MADM has been applied to various practical problems such as learning management system evaluation [2], project portfolio selection [3], electric utility resource planning [4], economics, military affairs, etc. However, in practical decision making situation, the information about the rating of the alternative with respect to the attributes cannot be assessed due to imprecise source of information. So, traditional MADM methods are not capable to solve these types of problems.

In 1965, Zadeh [5] proposed fuzzy set which is characterized by membership function to deal with problems with imprecise information. Atanassov [6] defined intuitionistic fuzzy set by incorporating non-membership function. However, for proper description of an object in uncertain and complex environment, we require to deal with indeterminate and inconsistent information. So, Smarandache [7, 8, 9, 10] extended the concept of Atanassov [6] by introducing indeterminacy membership function as an independent component and defined neutrosophic set for dealing with the problems with incomplete, imprecise, inconsistent information. Thereafter, Wang et al. [11] defined single valued neutrosophic set (SVNS) as an instance of neutrosophic set for dealing with real scientific and engineering problems.
Molodtsov [12] initiated the concept of soft set theory for dealing with uncertainty and vagueness in 1999. Soft set is free from the limitation of variety of theories such as probability theory, fuzzy theory, rough set theory, vague set theory and it is easy to implement in practical problems. After the pioneering work of Molodtsov [12], many researchers developed diverse mathematical hybrid models such as fuzzy soft sets [13, 14, 15], intuitionistic fuzzy soft set theory [16, 17, 18], possibility fuzzy soft set [19], generalized fuzzy soft sets [20, 21], generalized intuitionistic fuzzy soft set [22], possibility intuitionistic fuzzy soft set [23], vague soft set [24], possibility vague soft set [25], neutrosophic soft set [26], weighted neutrosophic soft sets [27], generalized neutrosophic soft set [28], intuitionistic neutrosophic soft set [29, 30], etc in order to solve different practical problems. However, most of the models consider only one expert and this creates difficulties for the researchers who employ questionnaires for his/her works and studies [31]. In order to overcome the difficulties, Alkhazaleh and Salleh [31] developed soft expert sets in 2011 where the researcher can observe the opinions of all experts in one model without any operations. They defined basic operations of soft expert sets and studied some of their properties and then applied the concept in decision making problem. Alkhazaleh and Salleh [32] also defined fuzzy soft expert set which is an extension of soft expert set and fuzzy set. Hazaymey et al. [33] introduced generalized fuzzy soft expert set by combining soft expert set due to Alkhazaleh and Salleh [31] and general soft set due to Majumdar and Samanta [21]. Hazaymey et al. [34] also incorporated fuzzy parameterized fuzzy soft expert set by extending the concept of fuzzy soft expert set by providing a membership value of each parameter in a set of parameters. Later, many authors have developed soft expert sets in different environment to form different structures such as vague soft expert set [35], generalized vague soft expert set [36], fuzzy parameterized soft expert set [37], possibility fuzzy soft expert set [38], intuitionistic fuzzy soft expert set [39], etc and the concepts of soft expert sets are applied to different practical problems [40, 41, 42]. Recently, Şahin et al. [43] incorporated neutrosophic soft expert set as a combination of neutrosophic neutrosophic set and soft expert set to deal with indeterminate and inconsistent information. Later, Broumi and Smarandache [44] explored the concept of single valued neutrosophic soft expert set (SVNSSE) which is an extension of fuzzy soft expert sets and intuitionistic fuzzy soft expert sets and they investigated some related properties with supporting proofs.

In the paper, we have developed a new method for solving SVNSSE based MADM problem through TOPSIS technique.

The content of the paper is constructed as follows. Section 2 presents some basic definitions concerning neutrosophic sets, SVNSs, soft sets, soft expert. Section 3 is devoted to present TOPSIS method for SVNSES based MADM problems. Section 4 presents an algorithm of the proposed method. A hypothetical problem regarding teacher selection is solved in Section 5 to illustrate the applicability of the proposed method. Finally, Section 6 presents conclusions and future scope research.

2 Preliminaries

We present basic definitions regarding neutrosophic sets, soft sets, soft expert sets and SVNSES in this Section as follows:

2.1 Neutrosophic Sets [7, 8, 9, 10]

Consider \( X \) be a space of objects with a generic element of \( X \) denoted by \( x \). Then, a neutrosophic set \( N \) on \( X \) is defined as follows:

\[
N = \{ x, \langle T_N(x), I_N(x), F_N(x) \rangle \mid x \in X \}
\]

where, \( T_N(x) \), \( I_N(x) \), \( F_N(x) : X \to [0, 1] \) represent respectively the degrees of truth-membership, indeterminacy-membership, and falsity-membership of a point \( x \in X \) to the set \( N \) with the condition \( 0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3 \).

2.2 Single valued neutrosophic Sets [11]

Let \( X \) be a universal space of points with a generic element of \( X \) denoted by \( x \), then a SVN S \( S \) is presented as follows:

\[
S = \{ x, \langle T_S(x), I_S(x), F_S(x) \rangle \mid x \in X \}
\]

where, \( T_S(x) \), \( I_S(x) \), \( F_S(x) : X \to [0, 1] \) and \( 0 \leq T_S(x) + I_S(x) + F_S(x) \leq 3 \) for each point \( x \in X \).

Definition 1 [45] The Hamming distance between two SVNSESs \( S_i = \{ x, \langle T_{S_i}(x), I_{S_i}(x), F_{S_i}(x) \rangle \mid x \in X \} \) and \( S_j = \{ x, \langle T_{S_j}(x), I_{S_j}(x), F_{S_j}(x) \rangle \mid x \in X \} \) is defined as follows:

\[
L_{\text{Ham}}(S_i, S_j) = \sum_{j \neq i} \left[ \frac{|T_{S_i}(x_j) - T_{S_j}(x_j)|}{F_{S_j}(x_j)} + \frac{|I_{S_i}(x_j) - I_{S_j}(x_j)|}{F_{S_j}(x_j)} + \frac{|F_{S_i}(x_j) - F_{S_j}(x_j)|}{F_{S_j}(x_j)} \right] \tag{1}
\]

Definition 2 [45] The normalized Hamming distance between two SVNSESs \( S_i = \{ x, \langle T_{S_i}(x), I_{S_i}(x), F_{S_i}(x) \rangle \mid x \in X \} \) and \( S_j = \{ x, \langle T_{S_j}(x), I_{S_j}(x), F_{S_j}(x) \rangle \mid x \in X \} \) is defined as follows:

\[
L_{\text{Ham}}(S_i, S_j) = \sum_{j \neq i} \left[ \frac{|T_{S_i}(x_j) - T_{S_j}(x_j)|}{F_{S_j}(x_j)} + \frac{|I_{S_i}(x_j) - I_{S_j}(x_j)|}{F_{S_j}(x_j)} + \frac{|F_{S_i}(x_j) - F_{S_j}(x_j)|}{F_{S_j}(x_j)} \right] \tag{2}
\]
\[ x_j \in \mathcal{X} \] and \( S_2 = \{ x_j \mid T_1(x_j), I_1(x_j), F_1(x_j) \mid x_j \in \mathcal{X} \} \) is defined as follows:

\[
\gamma_{L_{Ham}}(S_1, S_2) = \frac{1}{3n} \sum_{j=1}^{n} \left( |T_1(x_j) - T_2(x_j)| + |I_1(x_j) - I_2(x_j)| + |F_1(x_j) - F_2(x_j)| \right)
\]

**Definition 3** [45] The Euclidean distance between two SVNSs \( S_1 = \{ x_j \mid T_1(x_j), I_1(x_j), F_1(x_j) \mid x_j \in \mathcal{X} \} \) and \( S_2 = \{ x_j \mid T_2(x_j), I_2(x_j), F_2(x_j) \mid x_j \in \mathcal{X} \} \) is defined as follows:

\[
L_{\text{Eucl}}(S_1, S_2) = \frac{1}{2n} \sum_{j=1}^{n} \left( (T_1(x_j) - T_2(x_j))^2 + (I_1(x_j) - I_2(x_j))^2 + (F_1(x_j) - F_2(x_j))^2 \right)
\]

**Definition 4** [45] The normalized Euclidean distance between two SVNSs \( S_1 = \{ x_j \mid T_1(x_j), I_1(x_j), F_1(x_j) \mid x_j \in \mathcal{X} \} \) and \( S_2 = \{ x_j \mid T_2(x_j), I_2(x_j), F_2(x_j) \mid x_j \in \mathcal{X} \} \) is defined as follows:

\[
\gamma_{L_{\text{Eucl}}} (S_1, S_2) = \frac{1}{2n} \sum_{j=1}^{n} \left( (T_1(x_j) - T_2(x_j))^2 + (I_1(x_j) - I_2(x_j))^2 + (F_1(x_j) - F_2(x_j))^2 \right)
\]

### 2.3 Soft set [12]

Let \( X \) be a universal set and \( E \) be a set of parameters. Assume that \( P(X) \) represents power set of \( X \). Also, let \( A \) be a non-empty set, where \( A \subseteq E \). Then, a pair \((M, A)\) is called a soft set over \( X \), where \( M \) is a mapping given by \( M : A \rightarrow P(X) \), where \( P(X) \) represents power set of \( X \).

### 2.4 Neutrosophic soft set [26]

Let, \( X \) be an initial universal set. Also, let \( E \) be a set of parameters and \( A \) be a non-empty set such that \( A \subseteq E \). NS \((X)\) represents the set of all neutrosophic subsets of \( X \). Then, a pair \((M, A)\) is termed to be the neutrosophic soft set over \( X \), where \( M \) is a mapping given by \( M : A \rightarrow \text{NS}(X) \).

### 2.5 Soft expert set [31]

Consider \( X \) an initial universal set, \( E \) be the set of parameters, \( Z \) be a set of experts (agents) and \( O = \{ \text{agree} = 1, \text{disagree} = 0 \} \) be a set of opinions. Let, \( W = E \times Z \times O \), \( A \subseteq W \). Then, a pair \((M, A)\) is called soft expert set over \( X \), where \( M \) is a mapping given by \( M : A \rightarrow \text{NS}(X) \), where \( \text{NS}(X) \) represents power set of \( X \).

**Definition 5** [31] An agree-soft expert set \((M, A)_1\) over \( X \) is a soft expert subset of \((M, A)_1\) is defined as follows:

\[
(M, A)_1 = \{ M(\beta) : \beta \in E \times Z \times \{1\} \}
\]

**Definition 6** [31] An disagree-soft expert set \((M, A)_0\) over \( X \) is a soft expert subset of \((M, A)_0\) is defined as follows:

\[
(M, A)_0 = \{ M(\beta) : \beta \in E \times Z \times \{0\} \}
\]

### 2.6 Single valued neutrosophic soft expert set [44]

Consider \( X = \{ x_1, x_2, \ldots, x_n \} \) be a universal set of objects, \( E = \{ e_1, e_2, \ldots, e_n \} \) be the set of parameters, \( Z = \{ z_1, z_2, \ldots, z_n \} \) be a set of experts (agents) and \( O = \{ \text{agree} = 1, \text{disagree} = 0 \} \) be a set of opinions. Let, \( W = E \times Z \times O \), and \( A \) be a non-empty set such that \( A \subseteq W \). A pair \((M, A)\) is said to be SVNSES over \( X \), where \( M \) is a mapping given by \( M : A \rightarrow \text{SVNSES}(X) \), where SVNSES \((X)\) represents all single valued neutrosophic subsets of \( X \).

**Example:** Let \( X \) be the set of objects under consideration and \( E \) be the set of parameters, where every parameter is a neutrosophic word or sentence concerning neutrosophic words. Suppose there are three objects in the universe \( X \) given by \( X = \{ x_1, x_2, x_3 \} \), \( E = \{ \text{costly, beautiful} \} = \{ e_1, e_2 \} \) be the set of decision parameters and \( Z = \{ z_1, z_2 \} \) be a set of experts. Suppose \( M : A \rightarrow \text{SVNSES}(X) \) is defined as follows:

\[
M(e_1, z_1, 1) = \{ \{ x_1, 0.2, 0.5, 0.7 \}, \{ x_2, 0.4, 0.2, 0.5 \}, \{ x_3, 0.6, 0.3, 0.4 \} \}
\]

\[
M(e_2, z_1, 1) = \{ \{ x_1, 0.5, 0.1, 0.2 \}, \{ x_2, 0.5, 0.2, 0.4 \}, \{ x_3, 0.6, 0.2, 0.2 \} \}
\]

\[
M(e_1, z_2, 1) = \{ \{ x_1, 0.7, 0.1, 0.3 \}, \{ x_2, 0.8, 0.3, 0.1 \}, \{ x_3, 0.8, 0.2, 0.4 \} \}
\]

\[
M(e_2, z_2, 1) = \{ \{ x_1, 0.9, 0.1, 0.2 \}, \{ x_2, 0.3, 0.3, 0.2 \}, \{ x_3, 0.4, 0.3, 0.1 \} \}
\]

\[
M(e_1, z_1, 0) = \{ \{ x_1, 0.3, 0.5, 0.1 \}, \{ x_2, 0.5, 0.2, 0.1 \}, \{ x_3, 0.4, 0.3, 0.2 \} \}
\]

\[
M(e_2, z_1, 0) = \{ \{ x_1, 0.7, 0.1, 0.5 \}, \{ x_2, 0.6, 0.3, 0.4 \}, \{ x_3, 0.6, 0.5, 0.4 \} \}
\]

\[
M(e_1, z_2, 0) = \{ \{ x_1, 0.2, 0.1, 0.4 \}, \{ x_2, 0.6, 0.5, 0.4 \}, \{ x_3, 0.5, 0.6, 0.3 \} \}
\]
\(M(e_2, z_2, 0) = \{\{x_1, 0.8, 0.4, 0.2\}, \{x_2, 0.7, 0.5, 0.4\}, \{x_3, 0.5, 0.3, 0.3\}\}.

Then, \((M_1, A)\) is a SVNSES over the soft universe.

**Definition 7** [44]: Let \((M_1, A)\) and \((M_2, B)\) be two SVNSESs over a common soft universe. Then, \((M_1, A)\) is said to be single valued neutrosophic soft expert subset of \((M_2, B)\) if

(i). \(B \subseteq A\)

(ii). \(M_1(\beta)\) is a single valued neutrosophic subset \(M_2(\delta)\), \(\forall \delta \in A\).

**Definition 8** [44]: A null SVNSES \((\varnothing, A)\) is defined as follows:

\[ (\varnothing, A) = M(\beta) \text{ where } \beta \in W. \]

Where \(M(\beta) = <0, 0, 1>, \text{ that is } T_{M(\beta)} = 0, I_{M(\beta)} = 0, F_{M(\beta)} = 1, \forall \beta \in W. \]

**Definition 9** [44]: The complement of a SVNSES \((M, A)\) is defined as follows:

\[ (M, A)^C = \tilde{C}(M(\beta)) \forall \beta \in X. \]

Where, \(\tilde{C}\) represents single valued neutrosophic complement.

**Definition 10** [44]: Consider \((M_1, A)\) and \((M_2, B)\) be two SVNSESs over a common soft universe. The union \((M_1, A) \cup (M_2, B) = (M_3, C)\), where \(C = A \cup B\) and is defined as follows:

\[ M_3(\beta) = M_1(\beta) \cup M_2(\beta), \forall \beta \in C. \]

Where, \(M_3(\beta) = \begin{cases} M_1(\beta), \beta \in A - B \\ M_2(\beta), \beta \in B - A \\ M_1(\beta) \cup M_2(\beta), \beta \in A \cap B \end{cases} \)

where \(M_1(\beta) \cup M_2(\beta) = \{<x, \text{ Max } \{T_{M_1(\beta)}, T_{M_2(\beta)}\}, \text{ Min } \{I_{M_1(\beta)}, I_{M_2(\beta)}\}, \text{ Min } \{F_{M_1(\beta)}, F_{M_2(\beta)}\}> : x \in X\} \).

**Definition 11** [44]: Suppose \((M_1, A)\) and \((M_2, B)\) are two SVNSESs over a common soft universe. The intersection \((M_1, A) \cap (M_2, B) = (M_4, D)\), where \(D = A \cap B\) and is defined as follows:

\[ M_4(\beta) = M_1(\beta) \cap M_2(\beta), \forall \beta \in D. \]

Here, \(M_4(\beta) = \begin{cases} M_1(\beta), \beta \in A - B \\ M_2(\beta), \beta \in B - A \\ M_1(\beta) \cap M_2(\beta), \beta \in A \cap B \end{cases} \)

where \(M_1(\beta) \cap M_2(\beta) = \{<x, \text{ Max } \{T_{M_1(\beta)}, T_{M_2(\beta)}\}, \text{ Min } \{I_{M_1(\beta)}, I_{M_2(\beta)}\}, \text{ Min } \{F_{M_1(\beta)}, F_{M_2(\beta)}\}> : x \in X\} \).

### 3 TOPSIS method for MADM with single valued neutrosophic soft expert information

Let \(C = \{C_1, C_2, \ldots, C_m\}\), \((m \geq 2)\) be a discrete set of \(m\) feasible alternatives, \(A = \{a_1, a_2, \ldots, a_n\}\), \((n \geq 2)\) be a set of parameters under consideration and \(w = (w_1, w_2, \ldots, w_n)^T\) be the unknown weight vector of the attributes with \(0 \leq w_j \leq 1\) and \(\sum_{j=1}^{n} w_j = 1\). Let, \(Z = [z_1, z_2, \ldots, z_t]\) be a set of \(t\) experts (agents), where we consider the weights of the experts are equal and \(O = \{\text{agree} = 1, \text{ disagree} = 0\}\) be a set of opinions. The rating of performance value of alternative \(C_i\), \(i = 1, 2, \ldots, m\) with respect to the parameters is presented by the experts and they can be expressed in terms of SVNSs. Therefore, the proposed methodology for solving single valued neutrosophic soft expert MADM problem based on TOPSIS method is presented as follows:

**Step 1. Formulation of decision matrix with SVNSs**

Let, the rating of alternative \(C_i\) \((i = 1, 2, \ldots, m)\) with respect to the parameter provided by the experts is represented by SVNSS \((M, A)\) and they can be presented in matrix form as follows:

\[ D_{SVNS} = \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1q} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \ldots & d_{mq} \end{bmatrix} \]

Here, \(d_{ij} = (T_{ij}, I_{ij}, F_{ij})\) where \(T_{ij}, I_{ij}, F_{ij} \in [0, 1]\) and \(0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3\), \(i = 1, 2, \ldots, m, j = 1, 2, \ldots, q\) where \(q = n \times t \times 2\).

**Step 2. Determination of unknown weights of the parameters**

In the selection process, we assume that the importance (weight) of the attributes is not same and the weights of the attributes are completely unknown. Therefore, we employ...
maximizing deviation method due to Yang [46] in order to
to obtain the unknown weights. The deviation values of
alternative C_i (i = 1, 2, ..., m) to all other alternatives under
the attribute A_j (j = 1, 2, ..., q) can be defined as Y_j (w_j) = \sum_{k=1}^{n} y_{i,k}(d_{ij}, d_{k})w_j \text{, then } Y_j (w_j) = \sum_{i=1}^{m} Y_i w_j = \sum_{j=1}^{q} \sum_{i=1}^{m} y_{i,k}(d_{ij}, d_{k})w_j\text{ denotes the total deviation values of all}
alternatives to the other alternatives for the attribute A_j (j = 1, 2, ..., q).

Step 4. Determination of single valued neutrosophic
relative positive ideal solution (SVNRPIS) and single
valued neutrosophic relative negative ideal solution
(SVNRNIS)
The parameters are generally classified into two categories
namely benefit type attributes (a_j) and cost type attributes
(a_j). Consider R_{SVNRPIS} \text{ and R}_{SVNRNIS} be the single valued
neutrosophic relative positive ideal solution (SVNRPIS)
and single valued neutrosophic relative negative ideal
solution (SVNRNIS). Then, R_{SVNRPIS} \text{ and R}_{SVNRNIS} can
be defined as follows:

R_{SVNRPIS} = \left( \langle T^{u_i}_1, I^{u_i}_1, F^{u_i}_1 \rangle, \langle T^{u_i}_2, I^{u_i}_2, F^{u_i}_2 \rangle, \ldots, \langle T^{u_i}_q, I^{u_i}_q, F^{u_i}_q \rangle \right)

R_{SVNRNIS} = \left( \langle T^{l_i}_1, I^{l_i}_1, F^{l_i}_1 \rangle, \langle T^{l_i}_2, I^{l_i}_2, F^{l_i}_2 \rangle, \ldots, \langle T^{l_i}_q, I^{l_i}_q, F^{l_i}_q \rangle \right)

Step 3. Construction of weighted decision matrix
We obtain aggregated weighted decision matrix by
multiplying weights (w_j) [48] of the parameters and
aggregated decision matrix \left( \langle d_{ij}^w \rangle \right)_{m \times q} is presented as follows:

D^w_{SVNSES} = D_{SVNSES} \otimes w = \left( d^w_{ij} \right)_{m \times q} \otimes w_j

\left( \langle d_{ij}^w \rangle \right)_{m \times q} = \begin{bmatrix}
d_{11}^w & d_{12}^w & \ldots & d_{1q}^w \\
\vdots & \ddots & \ddots & \vdots \\
d_{m1}^w & \ldots & \ldots & d_{mq}^w
\end{bmatrix}

Here, \left( d_{ij}^w \right) = \left( T^{u_i}_j, I^{u_i}_j, F^{u_i}_j \right) where T^{u_i}_j, I^{u_i}_j, F^{u_i}_j \in [0, 1]
and 0 \leq T^{u_i}_j + I^{u_i}_j + F^{u_i}_j \leq 3, i = 1, 2, ..., m, j = 1, 2, ..., q.

Step 5. Computation of distance measure of each
alternative from RPIIS and RNIS
The normalized Euclidean measure of each
alternative \left( \langle T^{u_i}_j, I^{u_i}_j, F^{u_i}_j \rangle \right) from the
SVNRPIS

\left( \langle T^{u_i}_j, I^{u_i}_j, F^{u_i}_j \rangle \right) \text{ for } i = 1, 2, ..., m; j = 1, 2, ..., q \text{ can be}
defined as follows:

L^i_{N} \left( \langle d_{ij}^w \rangle, \langle d_{ij}^w \rangle \right) = \frac{1}{3q} \sqrt{\sum_{i=1}^{m} \left( T^{u_i}_j(x_i) - T^{u_i}_j(x_i) \right)^2 + \left( I^{u_i}_j(x_i) - I^{u_i}_j(x_i) \right)^2 + \left( F^{u_i}_j(x_i) - F^{u_i}_j(x_i) \right)^2}

Similarly, normalized Euclidean measure of each
alternative \left( \langle T^{u_i}_j, I^{u_i}_j, F^{u_i}_j \rangle \right) from the
SVNRNIS

\left( \langle T^{u_i}_j, I^{u_i}_j, F^{u_i}_j \rangle \right) \text{ for } i = 1, 2, ..., m; j = 1, 2, ..., q \text{ can be}
written as follows:
Step 6. Calculation of the relative closeness co-efficient to the neutrosophic ideal solution
The relative closeness co-efficient of each alternative $C_i$, ($i = 1, 2, ..., m$) with respect to the SVNRPIS is defined as follows:

$$\tau^*_i = \frac{L_N^+(d^+_n, d^{-+}_n)}{L_N^+(d^+_n, d^{-+}_n) + L_N^-(d^+_n, d^{-+}_n)}$$

(9)

where, $0 \leq \tau^*_i \leq 1$, $i = 1, 2, ..., m$.

Step 7. Rank the alternatives
We rank the alternatives according to the values of $\tau^*_i$, $i = 1, 2, ..., m$ and bigger value of $\tau^*_i$ ($i = 1, 2, ..., m$) reflects the best alternative.

4 Proposed algorithm for MADM problem with single valued neutrosophic soft expert information

An algorithm for MADM problem with single valued neutrosophic soft expert information through TOPSIS method is given using the following steps.

Step 1. Construct the decision matrix $D_{SVNSES}$.

Step 2. Determine the unknown weight ($w_j$) of the attributes by using Eq. (6).

Step 3. Formulate the weighted aggregated decision matrix $D^*_w = \{d^*_w\}_{m \times q}$.

Step 4. Recognize the SVNRPIS ($R^+_{SVNRPIS}$) and SVNRNIS ($R^-_{SVNRNIS}$).

Step 5. Calculate the distance of each alternative from SVNRPIS ($R^+_{SVNRPIS}$) and SVNRNIS ($R^-_{SVNRNIS}$) using Eqs. (7) and (8) respectively.

Step 6. Determine the relative closeness co-efficient $\tau^*_i$ ($i = 1, 2, ..., m$) using Eq. (9) of each alternative $C_i$.

Step 7. Rank the preference order of alternatives in accordance with the order of their relative closeness.

5 A numerical example

In this section, we solve a hypothetical problem to show the effectiveness of the proposed approach. Suppose that a school authority is going to recruit an assistant teacher in Mathematics to fill the vacancy on contractual basis for six months. After preliminary screening, three candidates (alternatives) $C_1, C_2, C_3$ are short-listed for further assessment. A committee consisting of two members namely ‘Senior Mathematics teacher ($z_1$)’ and ‘an external expert on the relevant subject’ ($z_2$) is formed to conduct the interview in order to select the most suitable teacher and $O = \{1 = agree, 0 = disagree\}$ be the set of opinions of the selection committee members. The committee considers two parameters $a_i$, $i = 1, 2$, where $a_1$ denotes ‘pedagogical knowledge’ and $a_2$ denotes ‘personality’. After the interview of the candidates, the select committee provides the following SVNSESs.

$$\begin{align*}
M(a_1, z_1, 1) &= \{x_1, 0.7, 0.5, 0.2\}, \{x_2, 0.6, 0.2, 0.3\}, \{x_3, 0.8, 0.3, 0.3\}, \\
M(a_2, z_1, 1) &= \{x_1, 0.5, 0.1, 0.4\}, \{x_2, 0.9, 0.2, 0.2\}, \{x_3, 0.8, 0.1, 0.2\}, \\
M(a_1, z_2, 1) &= \{x_1, 0.7, 0.3, 0.5\}, \{x_2, 0.9, 0.2, 0.1\}, \{x_3, 0.7, 0.1, 0.4\}, \\
M(a_2, z_2, 1) &= \{x_1, 0.6, 0.2, 0.3\}, \{x_2, 0.9, 0.1, 0.1\}, \{x_3, 0.8, 0.3, 0.2\}, \\
M(a_1, z_1, 0) &= \{x_1, 0.3, 0.4, 0.3\}, \{x_2, 0.5, 0.3, 0.2\}, \{x_3, 0.2, 0.3, 0.5\}, \\
M(a_2, z_1, 0) &= \{x_1, 0.4, 0.1, 0.3\}, \{x_2, 0.3, 0.3, 0.1\}, \{x_3, 0.4, 0.3, 0.4\}, \\
M(a_1, z_2, 0) &= \{x_1, 0.5, 0.1, 0.2\}, \{x_2, 0.4, 0.2, 0.3\}, \{x_3, 0.5, 0.1, 0.4\}, \\
M(a_2, z_2, 0) &= \{x_1, 0.5, 0.2, 0.3\}, \{x_2, 0.3, 0.3, 0.2\}, \{x_3, 0.5, 0.2, 0.5\}. 
\end{align*}$$

Then, the proposed procedure for solving the problem is provided using the following steps.

Step 1: Formulation of decision matrix
We present the SVNSESs in the tabular form (see the table 1) as given below (see Table 1)

Step 2. Calculation of the weights of the attributes
We use Hamming distance and obtained the weights of the parameters using Eq. (6) as follows:

$$\begin{align*}
w_1 &= 0.12, w_2 = 0.14, w_3 = 0.16, w_4 = 0.14, w_5 = 0.12, w_6 = 0.12, w_7 = 0.08, w_8 = 0.12, \text{ where } \sum_{j=1}^{8} w_j = 1. 
\end{align*}$$

Step 3. Construction of weighted decision matrix
The tabular form of the weighted decision matrix is presented the Table 2.
Step 4. Determination of SVNRPIS and SVNRNIS
The SVNRPIS ($R_{SVNRPIS}^*$) and SVNRNIS ($R_{SVNRNIS}^*$) can be obtained from the weighted decision matrix (see Table 2) as follows:

$R_{SVNRPIS}^* = < (0.176, 0.824, 0.824); (0.276, 0.724, 0.798); (0.308, 0.692, 0.692); (0.276, 0.724, 0.724); (0.08, 0.865, 0.824); (0.059, 0.758, 0.758); (0.054, 0.832, 0.879); (0.08, 0.824, 0.824) >$,

$R_{SVNRNIS}^* = < (0.104, 0.92, 0.865); (0.092, 0.798, 0.88); (0.175, 0.825, 0.895); (0.12, 0.845, 0.845); (0.026, 0.896, 0.92); (0.042, 0.865, 0.896); (0.04, 0.879, 0.929); (0.042, 0.865, 0.92) >$. 

Step 5. Compute the distance measure of each alternative from the SVNRPIS and SVNRNIS
The Euclidean distance measures of each alternative from the SVNRPIS are calculated by using Eq. (8) as follows:

$L_N^{i*} = 0.1542, L_N^{1*} = 0.0393, L_N^{3*} = 0.0753.$

Similarly, the Euclidean distance measures of each alternative from the SVNRNIS are determined by using Eq. (8) as follows:

$L_N^1 = 0.1736, L_N^2 = 0.1565, L_N^3 = 0.1542.$

Step 6. Calculation of the relative closeness coefficient
We calculate the relative closeness co-efficient $\tau_i$ (i = 1, 2, 3) by using Eq. (9) as shown as follows:

$\tau_1^* = 0.5296, \tau_2^* = 0.7993, \tau_3^* = 0.6719.$

Step 7. Rank the alternatives
The ranking order of alternatives according to the relative closeness coefficient is presented as follows:

$C_2 \succ C_3 \succ C_1.$

Consequently, $C_2$ is the best candidate.

6 Conclusion
SVNSES is an effective and useful decision making tool to describe indeterminate and inconsistent information and it is also possible for a user to view the opinions of all experts in a single model. In this study, we have investigated a TOPSIS method for solving MADM problems with single valued neutrosophic soft expert information. The rating of performance values of the alternatives with respect to the parameters are presented in terms of SVNSESs. We determine the weights of the parameters by maximizing deviation method and formulate weighted decision matrix. We identify SVNRPIS and SVNRNIS from the weighted decision matrix and normalized Euclidean distance measure is used to calculate distances of each alternative from SVNRPISs as well as SVNRNISs. Relative closeness co-efficient of each alternative is then calculated to select the most desirable alternative. Finally, an application of the proposed method for teacher selection is given.

In future, the proposed method can be used for dealing with interval-valued neutrosophic soft expert based MADM problems and different practical problems such as pattern recognition, medical diagnosis, information fusion, supplier selection, etc.

References


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Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers, Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers

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Abstract. In this paper, we introduce for the first time the neutrosophic quadruple numbers (of the form $a + bT + cI + dF$) and the refined neutrosophic quadruple numbers.
Then we define an absorbance law, based on a prevalence order, both of them in order to multiply the neutrosophic components $T, I, F$ or their sub-components $T_j, I_k, F_l$ and thus to construct the multiplication of neutrosophic quadruple numbers.

Keywords: neutrosophic quadruple numbers, refined neutrosophic quadruple numbers, absorbance law, multiplication of neutrosophic quadruple numbers, multiplication of refined neutrosophic quadruple numbers.

1  Neutrosophic Quadruple Numbers

Let’s consider an entity (i.e. a number, an idea, an object, etc.) which is represented by a known part (a) and an unknown part $(bT + cI + dF)$.

Numbers of the form:
\[ NQ = a + bT + cI + dF, \]
where a, b, c, d are real (or complex) numbers (or intervals or in general subsets), and

- T = truth / membership / probability,
- I = indeterminacy,
- F = false / membership / improbability,
are called Neutrosophic Quadruple (Real respectively Complex) Numbers (or Intervals, or in general Subsets).

“a” is called the known part of $NQ$, while “$bT + cI + dF$” is called the unknown part of $NQ$.

2  Operations

Let
\[ NQ_1 = a_1 + b_1T + c_1I + d_1F, \]
\[ NQ_2 = a_2 + b_2T + c_2I + d_2F \]
and $\alpha \in \mathbb{R}$ (or $\alpha \in \mathbb{C}$) a real (or complex) scalar.

Then:

2.1 Addition
\[ NQ_1 + NQ_2 = (a_1 + a_2) + (b_1 + b_2)T + (c_1 + c_2)I + (d_1 + d_2)F. \]

2.2 Substraction
\[ NQ_1 - NQ_2 = (a_1 - a_2) + (b_1 - b_2)T + (c_1 - c_2)I + (d_1 - d_2)F. \]

2.3 Scalar Multiplication
\[ \alpha \cdot NQ = NQ \cdot \alpha = \alpha a + \alpha bT + \alpha cI + \alpha dF. \]

3  Refined Neutrosophic Quadruple Numbers

Let us consider that Refined Neutrosophic Quadruple Numbers are numbers of the form:
\[ R\!N\!Q = a + \sum_{l=1}^{p} b_l T_l + \sum_{j=1}^{r} c_j I_j + \sum_{k=1}^{s} d_k F_k, \]
where a, all $b_l$, all $c_j$, and all $d_k$ are real (or complex) numbers, intervals, or, in general, subsets.

- $T_1, T_2, \ldots, T_p$ are refinements of $T$;
- $I_1, I_2, \ldots, I_r$ are refinements of $I$;
- $F_1, F_2, \ldots, F_s$ are refinements of $F$.

There are cases when the known part (a) can be refined as well as $a_1, a_2, \ldots$.
The operations are defined similarly.
\[ RNQ^{(u)} = a^{(u)} + \sum_{i=1}^{p} b_{i}^{(u)} T_{i} + \sum_{j=1}^{r} c_{j}^{(u)} I_{j} + \sum_{k=1}^{s} d_{k}^{(u)} F_{k}, \]  
\text{(12)}

for \( u = 1 \) or \( 2 \).

Then:

3.1 Addition

\[ RNQ^{(1)} + RNQ^{(2)} = [a^{(1)} + a^{(2)}] + \sum_{i=1}^{p} [b_{i}^{(1)} + b_{i}^{(2)}] T_{i} + \sum_{j=1}^{r} [c_{j}^{(1)} + c_{j}^{(2)}] I_{j} + \sum_{k=1}^{s} [d_{k}^{(1)} + d_{k}^{(2)}] F_{k}. \]  
\text{(13)}

3.2 Subtraction

\[ RNQ^{(1)} - RNQ^{(2)} = [a^{(1)} - a^{(2)}] + \sum_{i=1}^{p} [b_{i}^{(1)} - b_{i}^{(2)}] T_{i} + \sum_{j=1}^{r} [c_{j}^{(1)} - c_{j}^{(2)}] I_{j} + \sum_{k=1}^{s} [d_{k}^{(1)} - d_{k}^{(2)}] F_{k}. \]  
\text{(14)}

3.3 Scalar Multiplication

For \( \alpha \in \mathbb{R} \) (or \( \alpha \in \mathbb{C} \)) one has:

\[ \alpha \cdot RNQ^{(1)} = \alpha \cdot a^{(1)} + \alpha \cdot \sum_{i=1}^{p} b_{i}^{(1)} T_{i} + \alpha \cdot \sum_{j=1}^{r} c_{j}^{(1)} I_{j} + \alpha \cdot \sum_{k=1}^{s} d_{k}^{(1)} F_{k}. \]  
\text{(15)}

4 Absorbance Law

Let \( S \) be a set, endowed with a total order \( x \preceq y \), named “\( x \) prevailed by \( y \)” or “\( x \) less preferred than \( y \)” or “\( x \) less preferred than \( y \)”.

We consider \( x \preceq y \) as “\( x \) prevailed by or equal to \( y \)” “\( x \) less than or equal to \( y \)” or “\( x \) less preferred than or equal to \( y \)”.

For any elements \( x, y \in S \), with \( x \preceq y \), one has the absorbance law:

\[ x \cdot y = y \cdot x = \text{absorb}(x, y) = \max\{x, y\} = y, \]

which means that the bigger element absorbs the smaller element (the big fish eats the small fish!).

Clearly,

\[ x \cdot x = \text{absorb}(x, x) = \max\{x, x\} = x, \]

and

\[ x_{1} \cdot x_{2} \cdot \ldots \cdot x_{n} = \text{absorb}(\ldots \text{absorb}(\text{absorb}(x_{1}, x_{2}), x_{3}), \ldots, x_{n}) = \max\{\ldots \max\{\max\{x_{1}, x_{2}, x_{3}\}, \ldots, x_{n}\}\} = \max\{x_{1}, x_{2}, \ldots, x_{n}\}. \]

Analogously, we say that \( x \succ y \) and we read: “\( x \) prevails to \( y \)” or “\( x \) is stronger than \( y \)”.

Also, \( x \succcurlyeq y \), and we read: “\( x \) prevails or is equal to \( y \)” “\( x \) is stronger than or equal to \( y \)” or “\( x \) is preferred or equal to \( y \)”.

5 Multiplication of Neutrosophic Quadruple Numbers

It depends on the prevalence order defined on \( \{T, I, F\} \).

Suppose in an optimistic way the neutrosophic expert considers the prevalence order \( T > I > F \). Then:

\[ NQ_{1} \cdot NQ_{2} = (a_{1} + b_{1} T + c_{1} I + d_{1} F) \cdot (a_{2} + b_{2} T + c_{2} I + d_{2} F) = a_{1} a_{2} + (a_{1} b_{2} + a_{2} b_{1} + b_{1} b_{2} + b_{1} c_{2} + c_{1} b_{2} + b_{2} d_{1} + d_{1} d_{2}) T + (a_{1} c_{2} + a_{2} c_{1} + c_{1} d_{2} + c_{2} d_{1}) I + (a_{1} d_{2} + a_{2} d_{1} + d_{1} d_{2}) F, \]

since \( T I = I T = T, T F = F T = I F = F I = F I = I, \)
while \( T^{2} = T, I^{2} = I, F^{2} = F. \)

Suppose in an pessimistic way the neutrosophic expert considers the prevalence order \( F > I > T \). Then:

\[ NQ_{1} \cdot NQ_{2} = (a_{1} + b_{1} T + c_{1} I + d_{1} F) \cdot (a_{2} + b_{2} T + c_{2} I + d_{2} F) = a_{1} a_{2} + (a_{1} b_{2} + a_{2} b_{1} + b_{1} b_{2} + b_{1} c_{2} + c_{1} b_{2} + b_{2} d_{1} + d_{1} d_{2}) T + (a_{1} c_{2} + a_{2} c_{1} + c_{1} d_{2} + c_{2} d_{1}) I + (a_{1} d_{2} + a_{2} d_{1} + d_{1} d_{2}) F, \]

since \( F I = I F = F, F \cdot T = T \cdot F = F, I \cdot T = T \cdot I = I \)
while similarly \( F^{2} = F, I^{2} = I, T^{2} = T. \)

5.1 Remark

Other prevalence orders on \( \{T, I, F\} \) can be proposed, depending on the application/problem to solve, and on other conditions.
6 Multiplication of Refined Neutrosophic Quadruple Numbers

Besides a neutrosophic prevalence order defined on \{T, I, F\}, we also need a sub-prevalence order on \{T_1, T_2, \ldots, T_p\}, a sub-prevalence order on \{I_1, I_2, \ldots, I_r\}, and another sub-prevalence order on \{F_1, F_2, \ldots, F_s\}.

We assume that, for example, if \(T > I > F\), then \(T_j > I_k > F_l\) for any \(j \in \{1, 2, \ldots, p\}\), \(k \in \{1, 2, \ldots, r\}\), and \(l \in \{1, 2, \ldots, s\}\). Therefore, any prevalence order on \{\(T, I, F\)\} imposes a prevalence suborder on their corresponding refined components.

Without loss of generality, we may assume that \(T_1 > T_2 > \cdots > T_p\) (if this was not the case, we re-number the subcomponents in a decreasing order).

Similarly, we assume without loss of generality that:

\(I_1 > I_2 > \cdots > I_r\), and
\(F_1 > F_2 > \cdots > F_s\).

6.1 Exercise for the Reader

Let’s have the neutrosophic refined space

\(NS = \{T_1, T_2, T_3, I, F_1, F_2\}\),

with the prevalence order \(T_1 > T_2 > T_3 > I > F_1 > F_2\).

Let’s consider the refined neutrosophic quadruples

\(NA = 2 - 3T_1 + 2T_2 + T_3 - I + 5F_1 - 3F_2\), and
\(NB = 0 + T_1 - T_2 + 0 \cdot T_3 + 5I - 8F_1 + 5F_2\).

By multiplication of sub-components, the bigger absorbs the smaller. For example:

\(T_2 \cdot T_3 = T_2\),
\(T_1 \cdot F_1 = T_1\),
\(I \cdot F_2 = I\),
\(T_2 \cdot F_1 = T_2\), etc.

Multiply \(NA\) with \(NB\).

References


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On Refined Neutrosophic Algebraic Structures

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Abstract. The objective of this paper is to develop refined neutrosophic algebraic structures. In particular, we study refined neutrosophic group and we present some of its elementary properties.

Keywords: neutrosophic logic, neutrosophic set, refined neutrosophic algebraic structures, refined neutrosophic group, refined neutrosophic numbers.

1 Introduction

In neutrosophic logic, each proposition is approximated to have the percentage of truth in a subset (T), the percentage of indeterminacy in a subset (I), and the percentage of falsity in a subset (F), where T, I, F are standard or non-standard subsets of the non-standard unit interval J. The concept of neutrosophic numbers of the form a + bI, where I is the indeterminacy with $I' = I$, and, a and b are real or complex numbers, was introduced by Kandasamy and Smarandache in 2003. In the same year, Kandasamy and Smarandache introduced the concept of neutrosophic algebraic structures by combining the indeterminate element I with the elements of a given algebraic structure (X, ·) to form a new algebraic structure (X(I, ·) = (X, (a, bI)), (1)

It can be shown from (1) and (2) that:

$I_1 = I$;
$I_2 = I$;
$I_3 = aI_b$;
$I_4 = aI_b + bI_a$;
$I_5 = I_1I_2 = I_1$;
$I_6 = I_2I_3 = I_2$.

Now, let X be a nonempty set and let I and I_2 be two indeterminacies. Then the set $X(I_1, I_2) = \{X, I_1, I_2\} = \{(x, yI_1, zI_2): x, y, z \in X\}$ is called a refined neutrosophic set generated by X, I_1 and I_2. In [5], Smarandache introduced the refined neutrosophic logic and neutrosophic set where it was shown that it is possible to split the components $<T, I, F>$ into the form $<T_1, T_2, \ldots, T_n; I_1, I_2, \ldots, F_n; I_1, F_2, \ldots, F_2>$. Also in [6], Smarandache extended the neutrosophic numbers a + bI into refined neutrosophic numbers of the form a + b_1I_1 + b_2I_2 + \ldots + b_nI_n, where a, b_1, b_2, \ldots, b_n are real or complex numbers and considered the refined neutrosophic set based on these refined neutrosophic numbers.

2 Refined Neutrosophic Algebraic Structures

Consider the split of the indeterminacy I into two indeterminacies I_1 and I_2 defined as follows:

$I_1 = \text{contradiction (true (T) and false (F))}$,
$I_2 = \text{ignorance (true (T) or false (F))}$

For any two elements $a, b \in X(I_1, I_2)$, we define

$(a, bI_1, cI_2) \ast (d, eI_1, fI_2) = (g, (a + d, bI_1, cI_2)) \ast (h, (eI_1, fI_2))$.

Definition 2.1.

Let $(X(I_1, I_2), +, \cdot)$ be any refined neutrosophic algebraic structure where + and \cdot are ordinary addition and multiplication respectively. For any two elements $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, we define

$(a, bI_1, cI_2) + (d, eI_1, fI_2) = (a + d, (b + e)I_1, c + fI_2)$.
Refined neutrosophic group

Example 2.

Groups.

2.2.

(\(G(I), \ast\)) is called a finite refined neutrosophic group

Example 4.

Then:

\[
\{x, y : x \in G(I), y \in H(I)\}
\]

Definition 2.6.

2.3.

Example 3.

Then \(\mathbb{Z}(I_1, I_2)\) is a commutative refined neutrosophic group.

\[
G(I) = (a, bI_1, cI_2, d, eI_1, fI_2) = (ad, (ae + bd + be + bf + ce)I_1, (af + cd + cf)I_2).
\]

\[
\begin{pmatrix}
M_{R}^\mathbb{Z}(I_1, I_2, \ast) = \left\{ \begin{bmatrix} x & y \\ z \end{bmatrix} : x, y, z \in \mathbb{R}(I_1, I_2) \right\}
\end{pmatrix}
\]

\[
M_{R}^\mathbb{Z}(I_1, I_2) = \left\{ \begin{bmatrix} x & y \\ z \end{bmatrix} : x, y, z \in \mathbb{R}(I_1, I_2) \right\}
\]

is a non-commutative refined neutrosophic group.

Theorem 2.7.

\[
A(I_1, I_2) = \{a_1, a_2, a_3, a_4\}
\]

\[
G(I) = (a, bI_1, cI_2, d, eI_1, fI_2) = (ad, (ae + bd + be + bf + ce)I_1, (af + cd + cf)I_2).
\]

\[
\begin{pmatrix}
AF(G(I_1, I_2), \ast) = \left\{ (a, bI_1, cI_2, d, eI_1, fI_2) : a, b, c, d, e, f \in \mathbb{R} \right\}
\end{pmatrix}
\]

Corollary 2.4.

Every refined neutrosophic group \((G(I), I_2, \ast)\) is a group.

Theorem 2.5.

Let \((G(I), I_2, \ast)\) and \((H(I), I_2, \ast)\) be two refined neutrosophic groups. Then \(G(I_1, I_2) \times H(I_1, I_2) = \{(x, y) : x \in G(I_1, I_2), y \in H(I_1, I_2)\}\) is a refined neutrosophic group.

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Let \((G(I_1, I_2), *)\) and \((H(I_1, I_2), *)\) be two refined neutrosophic groups. The mapping \(\varphi : (G(I_1, I_2), *) \rightarrow (H(I_1, I_2), *)\) is called a neutrosophic homomorphism if the following conditions hold:

\[
\begin{align*}
(1) & \quad \varphi(x * y) = \varphi(x) * \varphi(y), \\
(2) & \quad \varphi(I_k) = I_k \quad \forall x, y \in G(I_1, I_2) \text{ and } k = 1, 2.
\end{align*}
\]

The image of \(\varphi\) is defined by the set

\[
\text{Im} \varphi = \{y \in H(I_1, I_2) : y = \varphi(x), \text{ for some } x \in G(I_1, I_2)\}.
\]

If \(G(I_1, I_2)\) and \(H(I_1, I_2)\) are additive refined neutrosophic groups, then the kernel of the neutrosophic homomorphism \(\varphi : (G(I_1, I_2), +) \rightarrow (H(I_1, I_2), +)\) is defined by the set

\[
\text{Ker} \varphi = \{x \in G(I_1, I_2) : \varphi(x) = (0, 0, 0)\}.
\]

Epimorphism, monomorphism, isomorphism, endomorphism and automorphism of \(\varphi\) have the same definitions as those of the classical cases.

Example 6.

Let \((G(I_1, I_2), *)\) and \((H(I_1, I_2), *)\) be two refined neutrosophic groups. Let \(\varphi : (G(I_1, I_2), +) \times H(I_1, I_2) \rightarrow G(I_1, I_2)\) be a mapping defined by \(\varphi(x, y) = x\) and let \(\psi : G(I_1, I_2) \times H(I_1, I_2) \rightarrow H(I_1, I_2)\) be a mapping defined by \(\psi(x, y) = y\). Then \(\varphi\) and \(\psi\) are refined neutrosophic group homomorphisms.

Theorem 2.11.

Let \(\varphi : (G(I_1, I_2), +) \rightarrow (H(I_1, I_2), +)\) be a refined neutrosophic group homomorphism. Then \(\text{Im} \varphi\) is a neutrosophic subgroup of \(H(I_1, I_2)\).

Theorem 2.12.

Let \(\varphi : (G(I_1, I_2), +) \rightarrow (H(I_1, I_2), +)\) be a refined neutrosophic group homomorphism. Then \(\text{Ker} \varphi\) is a subgroup of \(G\) and not a neutrosophic subgroup of \(G(I_1, I_2)\).

Example 7.

Let \(\varphi : \mathbb{Z}_{2}(I_1, I_2) \times \mathbb{Z}_{2}(I_1, I_2) \rightarrow \mathbb{Z}_{2}(I_1, I_2)\) be a neutrosophic group homomorphism defined by \(\varphi(x, y) = x\) for all \(x, y \in \mathbb{Z}_{2}(I_1, I_2)\). Then

\[
\text{Im} \varphi = \{(0, 0, 0), (1, 0, 0), (0, I_1, 0), (0, 0, I_2),
(0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}
\]

\[
\text{Ker} \varphi = \{(0, 0, 0), (0, 0, 0), (0, 0, 0), (0, 0, 0), (0, 0, I_2), (0, 0, 0), (1, 0, I_2), (0, 0, 0), (1, 0, 0), (1, I_1, 0), (0, 0, 0), (1, I_1, I_2)\}.
\]

Conclusion

By splitting the usual indeterminacy \(I\) into two indeterminacies \(I_1\) and \(I_2\), we have developed a new neutrosophic set \(X(I_1, I_2)\) called a refined neutrosophic set and we have generated a new neutrosophic algebraic structure \((X(I_1, I_2), \ast)\) from \(X, I_1\) and \(I_2\) which we called a refined neutrosophic algebraic structure. In particular, we have studied refined neutrosophic group and we have presented some of its elementary properties.

Using the same approach as in this paper, other refined neutrosophic algebraic structures involving rings, fields, vector spaces, modules, group rings, loops, hypergroups, hyperrings, algebras, and so on could be developed. We hope to look into these in our future papers.

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References


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Neutrosophic Actions, Prevalence Order, Refinement of Neutrosophic Entities, and Neutrosophic Literal Logical Operators

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Abstract. In this paper, we define for the first time three neutrosophic actions and their properties. We then introduce the prevalence order on \{T, I, F\} with respect to a given neutrosophic operator "o", which may be subjective - as defined by the neutrosophic experts; and the refinement of neutrosophic entities \(<A>\), \(<\text{neut}A>\), and \(<\text{anti}A>\). Then we extend the classical logical operators to neutrosophic literal logical operators and to refined literal logical operators, and we define the refinement neutrosophic literal space.

Keywords: neutrosophic logic, neutrosophic actions, prevalence order, neutrosophic operator, neutrosophic literal logical operators, refined literal logical operators, refinement neutrosophic literal space, neutrosophic conjunction, neutrosophic Sheffer’s stroke, neutrosophic equivalence.

1 Introduction

In Boolean Logic, a proposition \(P\) is either true (T), or false (F). In Neutrosophic Logic, a proposition \(P\) is either true (T), false (F), or indeterminate (I).

For example, in Boolean Logic the proposition \(P_1: 1 + 1 = 2\) (in base 10) is true, while the proposition \(P_2: 1 + 1 = 3\) (in base 10) is false.

In neutrosophic logic, besides propositions \(P_1\) (which is true) and \(P_2\) (which is false), we may also have proposition \(P_3: 1 + 1 = ?\) (in base 10), which is an incomplete/indeterminate proposition (neither true, nor false).

1.1 Remark

All conjectures in science are indeterminate at the beginning (researchers not knowing if they are true or false), and later they are proved as being either true, or false, or indeterminate in the case they were unclearly formulated.

1.2 Notations

In order to avoid confusions regarding the operators, we note them as:

a. Boolean (classical) logic:

\(\neg, \land, \lor, \rightarrow, \leftrightarrow\)

b. Fuzzy logic:

\(\neg, \land, \lor, \rightarrow, \leftrightarrow\)

\(V, V', \overline{V}, F, F', \overline{F}\)

c. Neutrosophic logic:

\(\neg, \land, \lor, \rightarrow, \leftrightarrow\)

N', N', N', N', N'

2 Three Neutrosophic Actions

In the frame of neutrosophy, we have considered [1995] for each entity \(<A>\), its opposite \(<\text{anti}A>_1\), and their neutrality \(<\text{neut}A>_1\) \{i.e. neither (\(A\)), nor \(<\text{anti}A>_1\)\}.

Also, by \(<\text{non}A>_1\) we mean what is not \(<A>_1\), i.e. its opposite \(<\text{anti}A>_1\), together with its neutral(ity) \(<\text{neut}A>_1\); therefore:

\(<\text{non}A>_1 = <\text{neut}A>_1 \lor <\text{anti}A>_1>\).

Based on these, we may straightforwardly introduce for the first time the following neutrosophic actions with respect to an entity \(<A>_1>:

1. To neutralize (or to neuter, or simply to neutize) the entity \(<A>_1>. [As a noun: neutralization, or neuter-ization, or simply neut-i-zation.]

We denote it by \(<\text{neut}A>_1>\) or neut\(A)_1>.

2. To antithetic-ize (or to anti-ize) the entity \(<A>_1. [As a noun: antithetic-ization, or anti-ization.]

We denote it by \(<\text{anti}A>_1\) or anti\(A)_1>.

This action is 100% opposition to entity \(<A>_1>\) (strong opposition, or strong negation).

3. To non-ize the entity \(<A>_1. [As a noun: non-i-zation.]

We denote it by \(<\text{non}A>_1\) or non\(A)_1>.

It is an opposition in a percentage between (0, 100\%) to entity \(<A>_1>\) (weak opposition).

Of course, not all entities \(<A>_1>\) can be neutralized, or antithetic-ized, or non-ized.
2.2 Example
Let
\( \langle A \rangle = \text{"Phoenix Cardinals beats Texas Cowboys"} \).
Then,
\( \langle \text{neut}A \rangle = \text{"Phoenix Cardinals has a tie game with Texas Cowboys"}; \)
\( \langle \text{anti}A \rangle = \text{"Phoenix Cardinals is beaten by Texas Cowboys"}; \)
\( \langle \text{non}A \rangle = \text{"Phoenix Cardinals has a tie game with Texas Cowboys, or Phoenix Cardinals is beaten by Texas Cowboys"}. \)

3 Properties of the Three Neutrosophic Actions
\[
\begin{align*}
\text{neut}(\langle \text{anti}A \rangle) &= \text{neut}(\langle \text{neut}A \rangle) = \langle A \rangle; \\
\text{anti}(\langle \text{anti}A \rangle) &= \langle A \rangle \text{ or } \langle \text{anti}A \rangle; \\
\text{non}(\langle \text{anti}A \rangle) &= \langle A \rangle \text{ or } \langle \text{neut}A \rangle; \\
\langle \text{neut}A \rangle &= \langle A \rangle \text{ or } \langle \text{anti}A \rangle;
\end{align*}
\]

4 Neutrosophic Actions’ Truth-Value Tables
Let’s have a logical proposition P, which may be true (T), indeterminate (I), or false (F) as in previous example. One applies the neutrosophic actions below.

4.1 Neutralization (or Indetermination) of P
\[
\begin{array}{c|ccc}
\text{neut}(P) & T & I & F \\
\hline
& I & I & I
\end{array}
\]

4.2 Antitheticization (Neutrosophic Strong Opposition to P)
\[
\begin{array}{c|ccc}
\text{anti}(P) & T & I & F \\
\hline
& F & T \lor F & T
\end{array}
\]

4.3 Non-ization (Neutrosophic Weak Opposition to P)
\[
\begin{array}{c|ccc}
\text{non}(P) & T & I & F \\
\hline
& I \lor F & T \lor F & T \lor I
\end{array}
\]

5 Refinement of Entities in Neutrosophy
In neutrosophy, an entity \( \langle A \rangle \) has an opposite \( \langle \text{anti}A \rangle \) and a neutral \( \langle \text{neut}A \rangle \). But these three categories can be refined in sub-entities \( \langle A \rangle_1, \langle A \rangle_2, ..., \langle A \rangle_m \) and respectively \( \langle \text{neut}A \rangle_1, \langle \text{neut}A \rangle_2, ..., \langle \text{neut}A \rangle_n \), and also \( \langle \text{anti}A \rangle_1, \langle \text{anti}A \rangle_2, ..., \langle \text{anti}A \rangle_p \), where \( m, n, p \) are integers \( \geq 1 \), but \( m + n + p \geq 4 \) (meaning that at least one of \( \langle A \rangle \), \( \langle \text{anti}A \rangle \) or \( \langle \text{neut}A \rangle \) is refined in two or more sub-entities).

For example, if \( \langle A \rangle = \text{white color}, \)
then \( \langle \text{anti}A \rangle = \text{black color}, \)
while \( \langle \text{neut}A \rangle = \text{colors different from white and black} \).

If we refine them, we get various nuances of white color: \( \langle A \rangle_1, \langle A \rangle_2, ..., \) and various nuances of black color: \( \langle \text{anti}A \rangle_1, \langle \text{anti}A \rangle_2, ..., \) and the colors in between them (red, green, yellow, blue, etc.): \( \langle \text{neut}A \rangle_1, \langle \text{neut}A \rangle_2, ..., \).

Similarly as above, we want to point out that not all entities \( \langle A \rangle \) and/or their corresponding (if any) \( \langle \text{neut}A \rangle \) and \( \langle \text{anti}A \rangle \) can be refined.

6 The Prevalence Order
Let’s consider the classical literal (symbolic) truth (T) and falsehood (F).

In a similar way, for neutrosophic operators we may consider the literal (symbolic) truth (T), the literal (symbolic) indeterminacy (I), and the literal (symbolic) falsehood (F).

We also introduce the prevalence order on \( \{T,I,F\} \) with respect to a given binary and commutative neutrosophic operator “o”.

The neutrosophic operators are: neutrosophic negation, neutrosophic conjunction, neutrosophic disjunction, neutrosophic exclusive disjunction, neutrosophic Sheffer’s stroke, neutrosophic implication, neutrosophic equivalence, etc.

The prevalence order is partially objective (following the classical logic for the relationship between \( T \) and \( F \)), and partially subjective (when the indeterminacy \( I \) interferes with itself or with \( T \) or \( F \)).

For its subjective part, the prevalence order is determined by the neutrosophic logic expert in terms of the application/problem to solve, and also depending on the specific conditions of the application/problem.

For \( X \neq Y \), we write \( X \ominus Y \), or \( X \succ Y \), and we read “X” prevails to Y with respect to the neutrosophic binary commutative operator “o”, which means that \( X \ominus Y = X \).

Let’s see the below examples. We mean by “o”: conjunction, disjunction, exclusive disjunction, Sheffer’s stroke, and equivalence.

7 Neutrosophic Literal Operators & Neutrosophic Numerical Operators

7.1 If we mean by neutrosophic literal proposition, a proposition whose truth value is a letter: either T or I or F. The operators that deal with such logical propositions are called neutrosophic literal operators.

7.2. And by neutrosophic numerical proposition, a proposition whose truth value is a triple of numbers (or in general of numerical subsets of the interval \([0, 1]\)), for examples \( A(0.6, 0.1, 0.4) \) or \( B([0, 0.2], \{0.3, 0.4, 0.6\}, (0.7, 0.8)) \).
The operators that deal with such logical propositions are called **neutrosophic numerical operators**.

### 8 Truth-Value Tables of Neutrosophic Literal Operators

In Boolean Logic, one has the following truth-value table for negation:

#### 8.1 Classical Negation

<table>
<thead>
<tr>
<th>¬</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

In Neutrosophic Logic, one has the following neutrosophic truth-value table for the neutrosophic negation:

#### 8.2 Neutrosophic Negation

<table>
<thead>
<tr>
<th>¬&lt;sub&gt;N&lt;/sub&gt;</th>
<th>T</th>
<th>I</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
<td>I</td>
<td>T</td>
</tr>
</tbody>
</table>

So, we have to consider that the negation of I is I, while the negations of T and F are similar as in classical logic.

In classical logic, one has:

#### 8.3 Classical Conjunction

<table>
<thead>
<tr>
<th>∧</th>
<th>T</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

In neutrosophic logic, one has:

#### 8.4 Neutrosophic Conjunction (\(\text{AND}_N\)), version 1

<table>
<thead>
<tr>
<th>∧&lt;sub&gt;N&lt;/sub&gt;</th>
<th>T</th>
<th>I</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>I</td>
<td>F</td>
</tr>
<tr>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>I</td>
<td>F</td>
</tr>
</tbody>
</table>

The objective part (circled literal components in the above table) remains as in classical logic, but when indeterminacy I interferes, the neutrosophic expert may choose the most fit prevalence order.

There are also cases when the expert may choose, for various reasons, to entangle the classical logic in the objective part. In this case, the prevalence order will be totally subjective.

The prevalence order works for classical logic too. As an example, for classical conjunction, one has \(F \geq_T T\), which means that \(F \land_T T = F\), while the prevalence order for the neutrosophic conjunction in the above tables was:

\[I \geq_T F \geq_T T,\]

which means that \(I \land_N F = I\), and \(I \land_N T = I\).

Other prevalence orders can be used herein, such as:

\[F \geq_N I \geq_N T,\]

and its corresponding table would be:

#### 8.5 Neutrosophic Conjunction (\(\text{AND}_N\)), version 2

<table>
<thead>
<tr>
<th>∧&lt;sub&gt;N&lt;/sub&gt;</th>
<th>T</th>
<th>I</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>I</td>
<td>F</td>
</tr>
<tr>
<td>I</td>
<td>I</td>
<td>I</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

which means that \(F_{\land N} I = F\) and \(I_{\land N} I = I\); or another prevalence order:

\[F \geq_N T \geq_N I,\]

and its corresponding table would be:

#### 8.6 Neutrosophic Conjunction (\(\text{AND}_N\)), version 3

<table>
<thead>
<tr>
<th>∧&lt;sub&gt;N&lt;/sub&gt;</th>
<th>T</th>
<th>I</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>I</td>
<td>I</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>I</td>
<td>F</td>
</tr>
</tbody>
</table>

which means that \(F_{\land N} I = F\) and \(T_{\land N} I = T\).

If one compares the three versions of the neutrosophic literal conjunction, one observes that the objective part remains the same, but the subjective part changes.
The subjective of the prevalence order can be established in an optimistic way, or pessimistic way, or according to the weights assigned to the neutrosophic literal components T, I, F by the experts.

In a similar way, we do for disjunction.

In classical logic, one has:

8.7 Classical Disjunction

\[
\begin{array}{ccc}
\lor & T & F \\
T & T & T \\
F & T & F \\
\end{array}
\]

In neutrosophic logic, one has:

8.8 Neutrosophic Disjunction (\(OR_N\))

\[
\begin{array}{ccc}
\lor_N & T & I & F \\
T & \mathbb{T} & T & \mathbb{T} \\
I & T & I & F \\
F & \mathbb{T} & F & \mathbb{T} \\
\end{array}
\]

where we used the following prevalence order:

\(T \succ_F \succ_I\).

but the reader is invited (as an exercise) to use another prevalence order, such as:

\(T \succ_I \succ_F\),

or \(I \succ_T \succ_F\), etc.,

for all neutrosophic logical operators presented above and below in this paper.

In classical logic, one has:

8.9 Classical Exclusive Disjunction

\[
\begin{array}{ccc}
\rightarrow & T & F \\
T & F & T \\
F & T & F \\
\end{array}
\]

In neutrosophic logic, one has:

8.10 Neutrosophic Exclusive Disjunction

\[
\begin{array}{cccc}
\lor_N & T & I & F \\
T & F & T & T \\
I & T & I & F \\
F & T & F & F \\
\end{array}
\]

using the prevalence order

\(T \succ_F \succ_I\).

In classical logic, one has:

8.11 Classical Sheffer's Stroke

\[
\begin{array}{ccc}
\mid & T & F \\
T & F & T \\
F & T & T \\
\end{array}
\]

In neutrosophic logic, one has:

8.12 Neutrosophic Sheffer's Stroke

\[
\begin{array}{ccc}
\mid_N & T & I & F \\
T & F & T & T \\
I & T & I & I \\
F & T & I & T \\
\end{array}
\]

using the prevalence order

\(T \succ_I \succ_F\).

In classical logic, one has:

8.13 Classical Implication

\[
\begin{array}{ccc}
\rightarrow & T & F \\
T & T & F \\
F & T & T \\
\end{array}
\]
In neutrosophic logic, one has:

### 8.14 Neutrosophic Implication

<table>
<thead>
<tr>
<th>$\rightarrow_N$</th>
<th>$T$</th>
<th>$I$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$I$</td>
<td>$F$</td>
</tr>
<tr>
<td>$I$</td>
<td>$T$</td>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

using the subjective preference that $I \rightarrow_N T$ is true (because in the classical implication $T$ is implied by anything), and $I \rightarrow_N F$ is false, while $I \rightarrow_N I$ is true because it is similar to the classical implications $T \rightarrow T$ and $F \rightarrow F$, which are true.

The reader is free to check different subjective preferences.

In classical logic, one has:

### 8.15 Classical Equivalence

<table>
<thead>
<tr>
<th>$\leftrightarrow$</th>
<th>$T$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

In neutrosophic logic, one has:

### 8.16 Neutrosophic Equivalence

<table>
<thead>
<tr>
<th>$\leftrightarrow_N$</th>
<th>$T$</th>
<th>$I$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$I$</td>
<td>$F$</td>
</tr>
<tr>
<td>$I$</td>
<td>$I$</td>
<td>$I$</td>
<td>$I$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$I$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

using the subjective preference that $I \leftrightarrow_N I$ is true, because it is similar to the classical equivalences that $T \rightarrow T$ and $F \rightarrow F$ are true, and also using the prevalence: $I >_e F >_e T$.

### 9 Refined Neutrosophic Literal Logic

9.1 Refined Neutrosophic Literal Conjunction Operator

Each particular case has to be treated individually.

In this paper, we present a simple example.

Let’s consider the following neutrosophic logical propositions:

- $T = \text{Tomorrow it will rain or snow.}$
  - $T$ is split into
    - $\rightarrow \text{Tomorrow it will rain.}$
    - $\rightarrow \text{Tomorrow it will snow.}$
  - $F = \text{Tomorrow it will neither rain nor snow.}$
  - $F$ is split into
    - $\rightarrow \text{Tomorrow it will not rain.}$
    - $\rightarrow \text{Tomorrow it will not snow.}$
- $I = \text{Do not know if tomorrow it will be raining, nor if it will be snowing.}$
  - $I$ is split into
    - $\rightarrow \text{Do not know if tomorrow it will be raining or not.}$
    - $\rightarrow \text{Do not know if tomorrow it will be snowing or not.}$

Then:

<table>
<thead>
<tr>
<th>$\neg_N$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$I_1$</th>
<th>$I_2$</th>
<th>$F_1$</th>
<th>$F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>$F_2$</td>
<td>$T_1$</td>
<td>$T_2$</td>
<td>$\lor T_1$</td>
<td>$\lor T_2$</td>
<td></td>
</tr>
<tr>
<td>$T_1$</td>
<td>$T_2$</td>
<td>$I_1$</td>
<td>$I_2$</td>
<td>$I_1$</td>
<td>$I_2$</td>
<td>$F_1$</td>
</tr>
<tr>
<td>$I_2$</td>
<td>$I_2$</td>
<td>$I$</td>
<td>$I$</td>
<td>$F_1$</td>
<td>$F_2$</td>
<td></td>
</tr>
<tr>
<td>$F_1$</td>
<td>$F_2$</td>
<td>$F_1$</td>
<td>$F_2$</td>
<td>$F_2$</td>
<td>$F_2$</td>
<td></td>
</tr>
</tbody>
</table>

It is clear that the negation of $T_1$ (Tomorrow it will raining) is $F_2$ (Tomorrow it will not be raining). Similarly for the negation of $T_2$, which is $F_2$.

But, the negation of $I_1$ (Do not know if tomorrow it will be raining or not) is “Do know if tomorrow it will be raining or not”, which is equivalent to “We know that tomorrow it will be raining” ($T_1$), or “We know that tomorrow it will not be raining” ($F_1$). Whence, the negation of $I_1$ is $T_1 \lor F_1$, and similarly, the negation of $I_2$ is $T_2 \lor F_2$.
9.2 Refined Neutrosophic Literal Disjunction Operator

\[
\begin{array}{cccccccc}
T_i & T_2 & T_3 & I_1 & I_2 & I_3 & F_1 & F_2 \\
T_1 & T & T & T & T & T & F & F \\
T_2 & T & T & T & T & T & F & F \\
I_1 & T & I & T & I & I & F & F \\
I_2 & I & I & I & I & I & F & F \\
F_1 & F & F & F & F & F & F & F \\
F_2 & F & F & F & F & F & F & F \\
\end{array}
\]

With respect to the neutrosophic disjunction, \( T_i \) prevail in front of \( F_i \), which prevail in front of \( I_i \), or
\[ T_i \sqsupset F_i \sqsupset I_i \]
for all \( i, j, k \in \{1, 2\} \).
For example, \( T_1 \lor T_2 = T \), but
\[ F_1 \lor F_2 \notin \{T, I, F\} \cup \{T_1, T_2, I_1, I_2, F_1, F_2\}. \]

10 The Refinement Neutrosophic Literal Space

The Refinement Neutrosophic Literal Space \( \{T_1, T_2, I_1, I_2, F_1, F_2\} \) is not closed under neutrosophic negation, neutrosophic conjunction, and neutrosophic disjunction.

The reader can check the closeness under other neutrosophic literal operations.

A neutrosophic refined literal space
\[ S_N = \{T_1, T_2, I_1, I_2, F_1, F_2, ...\} \]
where \( p, r, s \) are integers \( \geq 1 \), is said to be closed
under a given neutrosophic operator \( \theta_{N^*} \), if for any elements \( X, Y \in S_N \) one has \( X \theta_{N^*} Y \in S_N \).

Let’s denote the extension of \( S_N \) with respect to a single \( \theta_{N^*} \) by:
\[ S^N_{N^*} = (S_N, \theta_{N^*}). \]
If \( S_N \) is not closed with respect to the given neutrosophic operator \( \theta_{N^*} \), then \( S^N_{N^*} \neq S_N \), and we extend \( S_N \) by adding in the new elements resulted from the operation \( X \theta_{N^*} Y \), let’s denote them by \( A_1, A_2, ... A_m \).

Therefore,
\[ S^N_{N^*} = S_N \cup \{A_1, A_2, ... A_m\}. \]
\( S^N_{N^*} \) encloses \( S_N \).

Similarly, we can define the closeness of the neutrosophic refined literal space \( S_N \) with respect to the two or more neutrosophic operators \( \theta_{1_{N^*}}, \theta_{2_{N^*}}, ..., \theta_{w_{N^*}} \) for \( w \geq 2 \).
\[ S_N \] is closed under \( \theta_{1_{N^*}}, \theta_{2_{N^*}}, ..., \theta_{w_{N^*}} \) if for any \( X, Y \in S_N \) and for any \( i \in \{1, 2, ..., w\} \) one has \( X \theta_{N^*} Y \in S_N \).

If \( S_N \) is not closed under these neutrosophic operators, one can extend it as previously.

Let’s consider \( S^N_{N^*} = (S_N, \theta_{1_{N^*}}, \theta_{2_{N^*}}, ..., \theta_{w_{N^*}}) \) which is \( S_N \) closed with respect to all neutrosophic operators \( \theta_{1_{N^*}}, \theta_{2_{N^*}}, ..., \theta_{w_{N^*}} \), then \( S^N_{N^*} \) encloses \( S_N \).

Conclusion

We have defined for the first time three neutrosophic actions and their properties. We have introduced the prevalence order on \( \{T, I, F\} \) with respect to a given neutrosophic operator \( \theta \), the refinement of neutrosophic entities \( <A>_N \), \( <\text{neut}A>_N \), and \( <\text{anti}A>_N \), and the neutrosophic literal logical operators, the refined literal logical operators, as well as the refinement neutrosophic literal space.

References


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