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Multi-objective Production Planning Problem: Interval-valued Trapezoidal Neutrosophic and Multi-Choice Parameters

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Abstract: An intuitionistic multi-objective programming problem with interval-valued trapezoidal neutrosophic (IVTN) and multi-choice interval type has been considered in this paper. The coefficients of objective functions and parameters of the left side of the constraints are in the multi-choice environment, and the right-hand side of the constraints are in the IVTN number type. The formulated problem's multi-choice parameters were transformed into the deterministic form using the binary variable transformation technique. A procedure is defined to change the IVTN number into the deterministic form. Then, intuitionistic fuzzy programming (IFP) with two different scalarization models has been used to achieve each membership goal's highest degree and obtain a satisfactory decision-making solution. Finally, a numerical case study for production planning (PP) is explored to validate the work's efficiency and usefulness.

Keywords: Multi-objective programming, multi-choice, interval-valued trapezoidal neutrosophic number, intuitionistic fuzzy programming.

1. Introduction

In PP, a well-organized approach is used in which raw materials are transformed into an optimal quantity of the final products to maintain the performance and quality of the item. The main aim of PP is to comprehend consumer conditions and demands and improve the product design and other enhancements to meet customers' needs while achieving a desirable profit. The universal character of machinery must be used for a specific resource to determine the number of items for the specific time, product categories, labour character classification, and manufacturing process (cycle). Customers who demand resistance in the PP process and further reflection on comfort service rates and organizational benefit are influenced by the firm's profit and level of service planning. The manufacturing processes are streamlined to win the company struggle in the global marketplace. Nowadays, industry experts and analysts use optimization methods for the PP models to ensure maximum benefit with minimum unit production. Mosadegh et al. [33] addressed four criteria: idle time and overtime, employment size, inventory and scarcity, and currency preservation. Jaggi et al. [20] described the multi-objective PP problem under certain conditions for a lock industry. Ghosh and Mondal [12] discussed a production-distribution planning model and found a suitable solution using the genetic algorithm and a two-echelon supply chain. Gupta et al.(2019) [14] discussed certain and uncertain environments for a two-stage transportation problem and used fuzzy goal programming to find a compromise solution.

The experts or decision-makers fixed the mathematical problem's parameter. In real-life scenarios, the parameters are unknown, and the parameters of the optimization problem have become either random or fuzzy variables. In this study, the problem's parameters are in the form of a multi-choice or IVTN number. The multi-choice programming problem avoids the wastage of resources and chooses the best resource. Such problems arise in finance, health care, manufacturing, agriculture, transportation, engineering, military, and technology.

2. Literature Review

The most critical activity in the manufacturing process is PP. Manufacturing firms establish a development schedule at the start of each fiscal year. The ideal development schedule provides a complete picture of how many products will be manufactured during each period and the demand for each period over the fiscal year. The production schedule may be carried out regularly, monthly, yearly, or even annually, depending on the product's demand. Production scheduling is the process of allocating available production capacity over time to meet certain requirements such as delivery time, cost, supply and demand. Machine capacity planning, production management, transportation, and freight schedules are all examples of production-related issues. Over the last two decades, international competitions, technical advances, and market dynamics have impacted the manufacturing sector. The majority of PP issues are multi-objective. The researchers used the e-constraint approach to reduce a multi-objective problem to a single goal. The problem of organizing the output and distribution functions was explored by Chandra and Fisher [4]. A single plant with multi-commodity, multi-period manufacturing environments produces products processed in the plant before being shipped to consumers. Yan et al. [45] defined a strategic production-distribution model with multiple manufacturers, distribution centres, retailers, and consumers in which multiple goods are produced in a single cycle. The fuzzy multi-objective linear programming model effectively solves real-world PP problems. Nowadays, businesses seek to achieve more than one target goals to improve the PP system's consistency and response. The concept of fuzzy set was developed by Zadeh [46]. Zimmermann [49] applied the fuzzy linear programming approaches to the linear vector maximum problem.

In multi-choice programming problems, the decision-maker can consider many options for a parameter problem, but only one must be chosen to optimize the goal value. Healey [17] introduced the concept of multi-choice and considered a case study on the mixed-integer programming problem. Chang [5,6] formulated the multi-choice programming problem with binary variables and suggested a modified approach for the multi-choice objective programming model. Biswal and Acharya [2] recommended the generalized transformation technique for solving multi-choice linear programming problems in which constraints parameters are bound to certain multi-choices. Haq et al. [15] used fuzzy goal programming to solve the optimal case study's PP problem. Khan et al. [23] discussed the IVTN number and used neutrosophic and intuitionistic fuzzy programming to solve a PP problem. Haq et al. [16] discussed the neutrosophic fuzzy programming for the sustainable development goal's problem. Oluyisola et al. [34] prescribed a methodology for designing and developing a smart PP and control system and discussed PP and control challenges in manufacturing technologies for planning environment characteristics. Lohmer and Lasch [28] studied the multi-factory PP and scheduling problems; and classified the literature according to shop configuration, network structure, objectives, and solution methods. Raza and Hameed [36] worked on maintenance planning and scheduling and provided effective guidelines for future studies in the research area. Some conflicting issues such as growing economic demand, increasing energy supply, shrinking energy resources, changing climate conditions, and tightening environmental requirements pose significant challenges for planning energy systems towards cleaner production and sustainable development. Suo et al. [42] developed the ensemble energy system model for China (CN-EES model), incorporating a computable general equilibrium model and interval-parameter programming method within an energy system optimization framework.

The CN-EES model can predict energy demands under different economic-development scenarios, reflecting uncertainties derived from the long-term (2021–2050) planning period and providing optimal solutions for China's energy system transition and management. Some significant research contributions in PP are summarized in Table 1:

Table 1: Review Summary

Researchers	**Goals**	**Applied Techniques**							**Clarifications**
		GP	FP	LP	SP	GA	BL	IP	
Kanyalkar and Adil (2005) [22]	Multi	✓			✓				Dynamic Allocation, Hierarchical Planning Approach, Stochastic Programming, Goal Programming,
Leung and Chan (2009) [24]	Multi	✓							Goal Programming
Baykasoglu and Gocken (2010) [1]	Multi		✓						Tabu Search, Ranking Method, Metaheuristic Algorithm
Che and Chiang (2010) [7]	Multi					✓			Modified Pareto, Genetic Algorithm
Liu et al. (2011) [27]	Multi					✓			Genetic Algorithm, Aggregate PP
Sillekens et al. (2011) [39]	Single								Linear Approximation, Mixed Integer Linear Programming, Aggregate PP
Ramezanian et al. (2012) [35]	Single					✓			Tabu search Mixed, Two-phase Aggregate PP, Integer Linear Programming Model, Genetic Algorithm
Mortezaei et al. (2013) [32]	Multi		✓			✓			Aggregate PP
Liu and Papageorgiou (2013) [26]	Multi			✓					ε-constraint Method, Mixed-integer Linear Programming, Lexicographic Mini-max Method
Madadi and Wong [29]	Multi		✓						IBM ILOG CPLEX Optimization Studio Software, Multiobjective Fuzzy Aggregate PP
Chen and Huang (2014) [8]	Multi		✓						Aggregate PP, Parametric Programming
da Silva and Marins (2014) [9]	Multi	✓	✓						Agricultural and Logistics Phase, Fuzzy Goal Programming
Singh and Yadav (2015) [40]	Multi							✓	Multi-objective Linear Programming, Interval-valued Intuitionistic
Gholamian et al. (2016) [11]	Multi		✓						GAMS Software, Mixed-integer Non-linear Programming, Supply Chain Planning
Lin et al. (2016) [25]	Multi		✓						Multi-objective Optimization Evolutionary Algorithm, Integrated PP
Meistering and Stadtler (2019) [31]	Multi						✓		Bi-level Programming, Hierarchical PP
Zhao et al. [48]	Multi				✓				Multi-stage Stochastic Programming, Progressive Hedging Algorithm
Goli et al. (2019) [13]	Multi	✓							Goal Programming, Robust Multi-objective Multi-period Aggregate PP
Hu et al. (2020) [18]	Single				✓				Two-stage Stochastic Programming
Discussed Model	Multi							✓	IVTN number, IFP, Multi-choice

In this paper, we propose a multi-objective industrial development planning problem with a multi-choice interval type and IVTN parameter. This paper addresses the situation of multi-choices in the objective and the constraints' left-hand side. We have used general transformation methodology given by Roy and Maity [37] to achieve the crisp form for the multi-choice parameter. Moreover, the right-hand sides of the restrictions are of the IVTN form. The formulated problem's compromise solution is obtained by the IFP and compared for both models.

3. Prerequisites

This section discussed some fundamental definitions regarding the intuitionistic fuzzy (IF) number and neutrosophic fuzzy number.

Definition 3.1: [Zadeh (1965) [46]] $\tilde{A}$ of X is a fuzzy set having the form $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ that represents the membership degree with $\mu_{\tilde{A}} : X \to [0,1]$. $\tilde{A}$ on $\Re$ is convex if and if for each pair of point x_1, x_2 in X, and $\tilde{A}$ satisfies the inequality

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right\} \quad \forall\, x_1, x_2 \in X, \lambda \in [0,1]$$

Definition 3.2: [Ebrahimnejad and Verdegay, 2018 [10]; Mahajan and Gupta, 2019 [30]] $\tilde{A}^I$ in X is an IF set of ordered triples $\tilde{A}^I = \left\{\left(x, \mu_{\tilde{A}^I}(x), \upsilon_{\tilde{A}^I}(x)\right) : x \in X\right\}$, where $\mu_{\tilde{A}^I}(x) : X \to [0,1]$ and $\upsilon_{\tilde{A}^I}(x) : X \to [0,1]$ represent the membership degree and non-membership degree, such that $0 \leq \mu_{\tilde{A}^I}(x) + \upsilon_{\tilde{A}^I}(x) \leq 1, \ \forall x \in X$.

Definition 3.3: [Smarandache, 1999 [41]; Wang et al., 2010 [44]] A in X is a neutrosophic set characterized by a truth-membership $\mu_A^T(x)$, an indeterminacy membership $\sigma_A^I(x)$, and a falsity-membership $\upsilon_A^F(x)$, $\mu_A^T(x), \sigma_A^I(x), \upsilon_A^F(x) \in (0,1)$ or $[0,1], \forall x \in X$ and $0^+ \leq \sup\ \mu_A^T(x) + \sup\ \sigma_A^I(x) + \sup\ \upsilon_A^F(x) \leq 3^-$ or $0 \leq \sup\ \mu_A^T(x) + \sup\ \sigma_A^I(x) + \sup\ \upsilon_A^F(x) \leq 3$.

Definition 3.4: [Ishibuchi and Tanaka, [19]] An interval on $\Re$ is as $A = \left[a^L, a^R\right] = \left\{a : a^L \leq a \leq a^R,\ a \in \Re\right\}$, a^R is the right limit and a^L is left limit of A. Or $A = \langle a_c, a_w \rangle = \left\{a : a_c - a_w \leq a \leq a_c + a_w,\ a \in \Re\right\}$, centre and width of A is $a_c = \frac{1}{2}\left(a^R + a^L\right)$ and $a_w = \frac{1}{2}\left(a^R - a^L\right)$ respectively.

Definition 3.5: [Broumi and Smarandache, 2015] [3] An interval-valued neutrosophic (IVN) set $\tilde{A}^{IV}$ of X is as follows:

$$\tilde{A}^{IV} = \left\{\left\langle x;\ \left[\mu_k^{TL}, \mu_k^{TU}\right], \left[\sigma_k^{IL}, \sigma_k^{IU}\right], \left[\upsilon_k^{FL}, \upsilon_k^{FU}\right]\right\rangle : x \in X\right\},$$

where $\left[\mu_k^{TL}, \mu_k^{TU}\right], \left[\sigma_k^{IL}, \sigma_k^{IU}\right]$ and $\left[\upsilon_k^{FL}, \upsilon_k^{FU}\right] \subset [0,1]$ for each $x \in X$

Definition 3.6: [Broumi and Smarandache, 2015] [3] Let $\tilde{A}^{IV} = \left\{\ \left\langle x;\ \left[\mu_k^{TL}, \mu_k^{TU}\right], \left[\sigma_k^{IL}, \sigma_k^{IU}\right], \left[\upsilon_k^{FL}, \upsilon_k^{FU}\right]\ \right\rangle : x \in X\right\}$ be IVN set, then

(i) $\tilde{A}^{IV}$ is empty if $\mu_k^{TL} = \mu_k^{TU} = 0, \quad \sigma_k^{IL} = \sigma_k^{IU} = 1,\ \upsilon_k^{FL} = \upsilon_k^{FU} = 1,\ \forall\ x \in \tilde{A}$

(ii) Let $\underline{1} = \langle x;\ 1,\ 0,\ 0\rangle$ **and** $\underline{0} = \langle x;\ 0,\ 1,\ 1\rangle$.

Definition 3.7: (IVTN number) Let $\mu_{\tilde{a}}, \sigma_{\tilde{a}}, \upsilon_{\tilde{a}} \subset [0,1]$, and $a_1, a_2, a_3, a_4 \in \Re$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. Then an interval-valued trapezoidal fuzzy neutrosophic number,

$$\tilde{a} = \left\langle (a_1, a_2, a_3, a_4);\ \left[\mu_{\tilde{a}}^L, \mu_{\tilde{a}}^U\right], \left[\sigma_{\tilde{a}}^L, \sigma_{\tilde{a}}^U\right], \left[\upsilon_{\tilde{a}}^L, \upsilon_{\tilde{a}}^U\right]\right\rangle,$$

membership degrees, indeterminacy degrees and non-membership degrees are

$$\mu_{\tilde a}(x)=\begin{cases}\mu^U_{\tilde a}\left(\dfrac{x-a_1}{a_2-a_1}\right), & x\in[a_1,a_2],\\ \mu^U_{\tilde a}\ , & x\in[a_2,a_3],\\ \mu^U_{\tilde a}\left(\dfrac{x-a_4}{a_3-a_4}\right), & x\in[a_3,a_4],\\ 0, & \text{other}\end{cases}\qquad \upsilon_{\tilde a}(x)=\begin{cases}\dfrac{\upsilon^L_{\tilde a}(x-a_1)+a_2-x}{a_2-a_1}, & x\in[a_1,a_2]\\ \upsilon^L_{\tilde a}\ , & x\in[a_2,a_3]\\ \dfrac{\upsilon^L_{\tilde a}(a_4-x)+x-a_3}{a_4-a_3}, & x\in[a_3,a_4]\\ 1\ , & \text{other}\end{cases}$$

$$\sigma_{\tilde a}(x)=\begin{cases}\dfrac{a_2-x+\sigma^L_{\tilde a}(x-a_1)}{a_2-a_1}\ , & x\in[a_1,a_2]\\ \sigma^L_{\tilde a}\ , & x\in[a_2,a_3]\\ \dfrac{x-a_3+\sigma^L_{\tilde a}(a_4-x)}{a_4-x_3}, & x\in[a_3,a_4]\\ 1\ , & \text{other}\end{cases}$$

Definition 3.8: [Thamaraiselvi and Santhi, 2015 [43]] The score function for the IVN number $\tilde a=\left\langle (a_1,a_2,a_3,a_4);\ \left[\mu^L_{\tilde a},\mu^U_{\tilde a}\right],\left[\sigma^L_{\tilde a},\sigma^U_{\tilde a}\right],\left[\upsilon^L_{\tilde a},\upsilon^U_{\tilde a}\right]\ \right\rangle$ is defined as

$$S(\tilde a)=\frac{(a_1+a_2+a_3+a_4)}{16}\left[\mu_{\tilde a}(x)+\left(1-\upsilon_{\tilde a}(x)\right)+\left(1-\sigma_{\tilde a}(x)\right)\right]$$

3.1 Transformation Technique for Multi-Choice Parameter [Roy et al., 2017 [38]]

The selection procedure of multi-choice in the problem parameter should help to optimize the problem. The binary variable concepts play a vital role in selecting a choice from the problem's values. Among t numbers of possibilities, p numbers of binary variables are used, where $2^{p-1}<t\le 2^p$.

Let $t={}^pC_0+{}^pC_1+{}^pC_2+\ldots+{}^pC_d+k$ for some d satisfying $1\le d\le p,\ 0\le k<{}^pC_{d+1}$.

If $d=p$, then $k=0,$ and $k\ne 0,$ then $d<p$ in the selection procedure, ${}^pC_i$ numbers of possibilities have value zero for i binary variables among p variables in selecting a single choice from multi-choice parameters.

p binary variables $z_j^1,\ z_j^2,\ \ldots,\ z_j^p$ are taken to reduce the formula in selecting the t values of $c_j^1,\ c_j^2,\ \ldots,\ c_j^t$. We further construct p binary variable's function $f_0(z)=(z_j^1\ z_j^2\ \ldots\ z_j^p)\,c_j^1$, where $z=(z_j^1\ z_j^2\ldots\ z_j^p)$, when each $z_j^1=1$ for $j=1,\ 2,\ \ldots,\ p$. where, $f_0(z)=c_j^1$, while $z_j^1+z_j^2+\ldots+z_j^p=p$ and we adopt a function $f_1(z)=(1-z_j^1)\,z_j^2\ldots z_j^p\,c_j^2+(1-z_j^2)\,z_j^1\,z_j^3\ldots z_j^p\,c_j^3+\ldots+(1-z_j^p)\,z_j^1\ldots z_j^{p-1}\,c_j^{1+{}^pC_1}$.

If $z_j^1 + z_j^2 + ... + z_j^p = p-1,\ f_1(z)$ gives a value among the following parameters of $c_j^1, c_j^2, c_j^3 ..., c_j^{1+{}^p c_1}$. Similarly,

$$
\begin{aligned}
f_2(z) &= (1-z_j^1)(1-z_j^2)\, z_j^3 \ldots z_j^p\, c_j^{1+{}^p c_1+1} + (1-z_j^1)(1-z_j^3)\, z_j^2\, z_j^4 \ldots z_j^p\, c_j^{1+{}^p c_1+2} \\
&+ \ldots + (1-z_j^1)(1-z_j^p)\, z_j^2 \ldots z_j^{p-1}\, c_j^{1+{}^p c_1+(p-2)} + (1-z_j^2)(1-z_j^3)\, z_j^1\, z_j^4 \ldots z_j^p\, c_j^{1+{}^p c_1+(p-2)+1} \\
&+ \ldots + (1-z_j^{p-1})(1-z_j^p)\, z_j^1 \ldots z_j^{p-2}\, c_j^{1+{}^p c_1+{}^p c_2}
\end{aligned}
$$

If $z_j^1 + z_j^2 + ... + z_j^p = p-2$, the $f_2(z)$ function gives a value from the parameters $c_j^t:\ c_j^{1+{}^p c_1+1},\ c_j^{1+{}^p c_1+2},\ ..., c_j^{1+{}^p c_1+{}^p c_2}$.

Proceeding similarly, we have

$$
\begin{aligned}
f_d(z) &= (1-z_j^1)(1-z_j^2)\ldots(1-z_j^d)\, z_j^{d+1} \ldots z_j^p\, c_j^{1+{}^p c_1+{}^p c_2+\ldots+{}^p c_{d-1}+1} \\
&+ (1-z_j^1)(1-z_j^2)\ldots(1-z_j^{d-1})(1-z_j^{d+1})\, z_j^d\, z_j^{d+2} \ldots z_j^p\, c_j^{1+{}^p c_1+{}^p c_2+\ldots+{}^p c_{d-1}+2} \\
&+ \ldots + (1-z_j^{p-d+1})(1-z_j^{p-d+2})(1-z_j^p)\, z_j^1 \ldots z_j^{p-d}\, c_j^{1+{}^p c_1+{}^p c_2+\ldots+{}^p c_d}
\end{aligned}
$$

If $z_j^1 + z_j^2 + ... + z_j^p = p-d$, the $f_d(z)$ function gives a number from the

$$
C_j^{t's}:\ c_j^{1+{}^p c_1+{}^p c_2+\ldots+{}^p c_{d-1}+1},\ c_j^{1+{}^p c_1+{}^p c_2+\ldots+{}^p c_{d-1}+2},\ \ldots,\ c_j^{1+{}^p c_1+{}^p c_2+\ldots+{}^p c_{d-1}+{}^p c_d}.
$$

For $k=0$, then $f(z) = f_0(z) + f_1(z) + ... + f_d(z)$ & $f(z)$ function gives a value from the $c_j^t, \forall z$ that satisfy $p-d \le z_j^1 + z_j^2 + ... + z_j^p \le p$.

If $k \ne 0$ then $k < {}^p c_{d+1}$ and the formulated function

$$
\begin{aligned}
f_{d+1}(z) &= (1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})(1-z_j^{i_{d+1}})\, z_j^{d+2} \ldots z_j^n\, c_j^{t-k+1} \\
&+ (1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})(1-z_j^{i_{d+2}})\, z_j^{d+1}\, z_j^{d+3} \ldots z_j^p\, c_j^{t-k+2} \\
&+ \ldots + (\text{terms up} - \text{to } c_j^t)
\end{aligned}
$$

Whenever $z_j^1 + z_j^2 + ... + z_j^p = p-(d+1),\ f_{d+1}(z)$ gives a value from ${}^p c_{d+1}$ numbers and restrictions ${}^p c_{d+1} - k$ are used to reduce the possible outputs on the k number. The *k^th^* term occurred at $i_1 = i_1',\ i_2 = i_2',\ ...,\ i_{d+1} = i_{d+1}'$ so the restrictions will be

$$p-(d+1)\le z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_p}\le p;\ z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}}\ge 1, \forall\ i_1=i_1',\ i_2=i_2',\ \ldots,\ i_d=i_d';\ i_p\ge i_{d+1}>i_{d+1}';$$
$$z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}}\ge 1,\ \forall\ i_1=i_1',\ i_2=i_2',\ \ldots,\ i_{d-1}=i_{d-1}',\ i_{p-1}\ge i_d>i_d';$$
$$z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}}\ge 1,\ \forall\ i_{p-d-1}\ge i_1>i_1'$$

So, $f(z)=f_0(z)+f_1(z)+\ldots+f_d(z)+f_{d+1}(z)$ gives the general function without loss of the generality for the selection of the multi-choice parameters c_j^t, the value of $c_j^t=1$ and used the summation and product multiplication, the formula in selecting the crisp values for the multi-choice parameters are:

$$\prod_{i=1}^{p} z_j^i+\sum_{i_1}^{p}\left[(1-z_j^{i_1})\prod_{i=1,i\ne i_1}^{p} z_j^i\right]+\sum_{\substack{i_2=2\\ i_2>i_1}}^{p}\sum_{i_1=1}^{p}\left[\left[(1-z_j^{i_1})(1-z_j^{i_2})\prod_{i=1,i\ne i_1,i_2}^{p} z_j^i\right]\right]$$
$$+\ldots+\sum_{\substack{i_d=d\\ i_d>i_{(d-1)}}}^{p}\ \sum_{\substack{i_{d-1}=d-1\\ i_{d-1}>i_{d-2}}}^{p}\cdots\sum_{i_1=1}^{p}\left[(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})\prod_{i=1,i\ne i_1,\ldots,i_d}^{p} z_j^i\right]$$

where, $p-d\le z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_p}\le p,\ \forall\ i_1<i_2<\ldots.<i_p$

If $k\ne 0$, the first *k* terms will be added with the above function through the formula:

$$(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})(1-z_j^{i_{d+1}})\prod_{\substack{i=1,i\ne i_1,.\\ ..,i_d,i_{d+1}}}^{p} z_j^i+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})(1-z_j^{i_{d+2}})\prod_{\substack{i=1,i\ne i_1,.\\ ..,i_d,i_{d+2}}}^{p} z_j^i$$
$$+\ldots+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_d})(1-z_j^{i_p})\prod_{\substack{i=1,i\ne i_1,.\\ ..,i_d,i_p}}^{p} z_j^i+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_{d+1}})(1-z_j^{i_{d+2}})\prod_{\substack{i=1,i\ne i_1,.\\ ..,i_{d+1},i_{d+2}}}^{p} z_j^i$$
$$+(1-z_j^{i_1})(1-z_j^{i_2})\ldots(1-z_j^{i_{d+1}})(1-z_j^{i_{d+3}})\prod_{\substack{i=1,i\ne i_1,.\\ ..,i_{d+1}}}^{p} z_j^i+\ldots+(1-z_j^{i_{1p-(d+1)}})(1-z_j^{i_{p-(d-1)}})\ldots(1-z_j^{i_{p-1}})(1-z_j^{i_p})\prod_{\substack{i\ne i_{p-d-1},\\ i_{p-d+1},\ldots,i_p}}^{p} z_j^i.$$

If, $i_1<i_2<\ldots.<i_p$ and the k^th^ term occurs at $i_1',i_2',\ldots,i_{d+1}'$, so the restrictions are

$$p-(d+1)\le z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_p}\le p;\ z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}}\ge 1, \forall\ i_1=i_1',i_2=i_2',\ldots,i_d=i_d',\ i_p\ge i_{d+1}>i_{d+1}';$$
$$z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}}\ge 1,\ \forall\ i_1=i_1',i_2=i_2',\ldots,i_{d-1}=i_{d+1}',\ i_{p-1}\ge i_d>i_d';$$
$$z_j^{i_1}+z_j^{i_2}+\ldots+z_j^{i_{d+1}}\ge 1,\ \forall\, i_{p-d-1}\ge i_1>i_1^{'}$$

Let, $\tilde{C}_{ij}'=\sum_{g=1}^{t}(term)^g\left[C_{ij}^{g^l}(1-\lambda^{C_{ij}^g})\right]+C_{ij}^{g^u}\lambda^{C_{ij\alpha_c}^g},\ \ i=1,2,,\ldots,m\ ;\ j=1,2,\ldots,n$

where, $(\text{term})^g,\ \forall\, g=1,\ 2,\ \ldots,\ t$ are the *t* numbers in the form of binary variables. Similarly,

$$\tilde{a}'_{ij} = \sum_{g=1}^{p} (term)^g \left[a_{ij}^{g^l} (1-\lambda^{a_{ij}^g}) \right] + a_{ij}^{g^u} \lambda^{a_{ij}^g}, \; j = 1, 2, ,..., n, \; (i = 1, 2, ,..., m).$$

$$\text{and } \; \tilde{b}'_i = \sum_{g=1}^{q} (term)^g \left[b_i^{g^l} (1-\lambda^{b_i^g}) \right] + b_i^{g^u} \lambda^{b_i^g}, \; (i = 1, 2, ,..., m).$$

3.2 Conversion process for IVNN

The PP problem is expressed in terms of IVTN numbers. To demonstrate, assume the right-hand side of the multi-objective PP model constraints is IVTN numbers. The following is a step-by-step procedure shown to convert in crisp form.

Step 1: Discuss the problems in terms of IVTN numbers.
Step 2: The score function transforms the IVTN numbers problem into an interval-valued problem.
Step 3: The $\alpha-$cut approach is used to transform an interval-valued in a crisp form. For $\left[a, b\right]$

$$\alpha a + (1-\alpha) b$$

3.3 Methodologies

The multi-objective Model:

Maximize (Minimize) Z(x) $= \left\{ Z_1(x), Z_2(x), ..., Z_k(x) \right\}$

Subject to

$g(x) \le 0$

$x \in X$

In solving the multi-objective optimization challenge, each objective function is independently solved to find the best solution, ignoring the other objectives of the Model. The process will be repeated until the optimal solutions for each objective are obtained, and a payoff matrix is generated. The lower and upper bounds for each goal function are identified U_k and L_k $\forall\, k = 1, 2, ..., K$ from the payoff matrix. For the solution of a multi-objective PP problem, we used the IFP approach.

Intuitionistic Fuzzy Programming

Maximized type objective: The membership and non-membership functions are defined as follows:

$$\mu_k^M (Z_k(x)) = \begin{cases} 0, & L_k > Z_k(x) \\ \dfrac{Z_k(x) - L_k}{U_k - L_k}, & Z_k(x) \in [L_k, U_k] \\ 1, & U_k < Z_k(x) \end{cases} \quad \textbf{(1)}$$

$$\upsilon_k^{NM} (Z_k(x)) = \begin{cases} 1, & Z_k(x) < R_k \\ \dfrac{U_k - Z_k(x)}{U_k - R_k}, & Z_k(x) \in [R_k, U_k] \\ 0, & Z_k(x) > U_k \end{cases} \quad \textbf{(2)}$$

where, $R_k < L_k < U_k$

Minimized type objective: The membership and non-membership functions are defined as follows:

$$\mu_k^M(Z_k(x)) = \begin{cases} 1, & L_k > Z_k(x) \\ \dfrac{U_k - Z_k(x)}{U_k - L_k}, & Z_k(x) \in [L_k, U_k] \\ 0, & U_k < Z_k(x) \end{cases} \quad \textbf{(3)}$$

$$\upsilon_k^{NM}(Z_k(x)) = \begin{cases} 0, & Z_k(x) < L_k \\ \dfrac{Z_k(x) - L_k}{W_k - L_k}, & Z_k(x) \in [L_k, W_k] \\ 1, & Z_k(x) > W_k \end{cases} \quad (4)$$

where, $L_k < U_k \le W_k$

The above-defined Eqns [1-4] are used in solving the multi-objective optimization problem. As follows:

$Max\ \mu_k, Min\ \upsilon_k$

Subject to

for maximization problem

$$\mu_k^M = \frac{Z_k(x) - L_k}{U_k - L_k}, \upsilon_k^{NM} = \frac{U_k - Z_k(x)}{U_k - R_k},\ R_k < L_k < U_k$$

for minimization problem

$$\mu_k^M = \frac{U_k - Z_k(x)}{U_k - L_k}, \upsilon_k^{NM} = \frac{Z_k(x) - L_k}{W_k - L_k},\ L_k < U_k < W_k$$

$$\mu_k^M \le \mu_k,\ \ \upsilon_k^{NM} \ge \upsilon_k,\ \ \mu_k \ge \upsilon_k,\ \ \mu_k + \upsilon_k \le 1,$$

$$\mu_k,\ \upsilon_k \in [0,1],\ \ \forall k = 1,\ 2, ..., K$$

$$g(x) \le 0,\ \forall\ x \in X$$

Therefore, we can write

$$Max\ \sum_{k=1}^{K} \left(\mu_k - \upsilon_k\right)$$

Subject to

for maximization problem

$$\mu_k^M = \frac{Z_k(x) - L_k}{U_k - L_k}, \upsilon_k^{NM} = \frac{U_k - Z_k(x)}{U_k - R_k},\ R_k < L_k < U_k$$

for minimization problem

$$\mu_k^M = \frac{U_k - Z_k(x)}{U_k - L_k}, \upsilon_k^{NM} = \frac{Z_k(x) - L_k}{W_k - L_k},\ L_k < U_k < W_k$$

$$\mu_k^M \le \mu_k,\ \ \upsilon_k^{NM} \ge \upsilon_k,\ \mu_k \ge \upsilon_k,\ \ \mu_k + \upsilon_k \le 1,$$

$$\mu_k,\ \upsilon_k \in [0,1],\ \ \forall k = 1,\ 2, ..., K$$

$$g(x) \le 0,\ \forall\ x \in X$$

4. Mathematical Model

The PP challenge has selected various machinery types for the manufacturing process: lathe, milling machine, grinder, jigsaw, band saw, and drill press. The main objective of the production industry is to make profit so that the company runs smoothly. It is always advisable for a company to prepare a production plan based on scientific methods to get a clear direction for how the production process should be carried out. The main objective of this study is to optimize the profit, product liability, quality, and satisfaction of workers. The input information such as, the available facilities and resource information, the units of machine available for the manufacturing items, and the number of hours spent using the machine to produce the product, including production machinery is required to formulate the problem,. The PP and control model is shown in Figure 1.

Fig. 1: Model for PP and control

The following principles and drawbacks are essential for an organization's industrial planning model:

- We maximize the industry's profit, productivity, product liability, and worker satisfaction.
- The multi-item output model is taken into account.
- A single unit is running a single task at a time on a machine.
- It is not likely to have a shortage of products in the manufacturing process.
- Final products demand only.
- In any case, it cannot reach the maximum level of the machine timing.

Kamal et al. [21] have discussed the mathematical model of the industrial programming problem as follows:

$$optimize\begin{cases} Max\ \ Z_1 = \sum_{j=1}^{3} P_j x_j & \text{(Profit)} \\ Max\ \ Z_2 = \dfrac{\sum_{j=1}^{3} L_j x_j}{\sum_{j=1}^{3} x_j} & \text{(Overall Product Reliability)} \\ Max\ \ Z_3 = \sum_{j=1}^{3} Q_j x_j & \text{(Quality of the product)} \\ Max\ \ Z_4 = \sum_{j=1}^{3} W_j x_j & \text{(Worker's satisfaction)} \end{cases}$$

$$\text{constraints}\begin{cases}\sum_{j=1}^{3} m_j x_j \le M & \text{(for Milling machine)}\\ \sum_{j=1}^{3} l_j x_j \le L & \text{(for Lathe machine)}\\ \sum_{j=1}^{3} g_j x_j \le G & \text{(for Grinder machine)}\\ \sum_{j=1}^{3} s_j x_j \le S & \text{(for Jig saw machine)}\\ \sum_{j=1}^{3} d_j x_j \le D & \text{(for Drill press machine)}\\ \sum_{j=1}^{3} b_j x_j \le B & \text{(for Band saw machine)}\end{cases}$$

In real-life problems, the parameters of the optimization are commonly unknown. The coefficients of objective functions and constraints are represented in the form of interval multi-choice. There are several options in such a case, and the decision-maker is perplexed on one of the problem's criteria to choose. The multi-choice interval form deals with the problem's complexity in the Model. The right-hand sides of the constraint are in the form of an IVTN number type. Then, the industrial programming problem is as follows:

$$Max\ Z_1 = \tilde{P}_1^{MC} x_1 + \tilde{P}_2^{MC} x_2 + \tilde{P}_3^{MC} x_3, \quad Max\ Z_2 = (\tilde{L}_1^{MC} x_1 + \tilde{L}_2^{MC} x_2 + \tilde{L}_3^{MC} x_3)/(x_1 + x_2 + x_3)$$

$$Max\ Z_3 = \tilde{Q}_1^{MC} x_1 + \tilde{Q}_2^{MC} x_2 + \tilde{Q}_3^{MC} x_3, \quad Max\ Z_4 = \tilde{W}_1^{MC} x_1 + \tilde{W}_2^{MC} x_2 + \tilde{W}_3^{MC} x_3$$

Subject to the constraints

$$\tilde{m}_1^{MC} x_1 + \tilde{m}_2^{MC} x_2 \le \tilde{M}^{IVTN}, \quad \tilde{l}_1^{MC} x_1 + \tilde{l}_2^{MC} x_2 + \tilde{l}_3^{MC} x_3 \le \tilde{L}^{IVTN}$$

$$\tilde{g}_1^{MC} x_1 + \tilde{g}_3^{MC} x_3 \le \tilde{G}^{IVTN}, \quad \tilde{s}_1^{MC} x_1 + \tilde{s}_3^{MC} x_3 \le \tilde{S}^{IVTN}$$

$$\tilde{d}_2^{MC} x_2 + \tilde{d}_3^{MC} x_3 \le \tilde{D}^{IVTN}, \quad \tilde{b}_1^{MC} x_1 + \tilde{b}_2^{MC} x_2 + \tilde{b}_3^{MC} x_3 \le \tilde{B}^{IVTN}$$

$$x_1, x_2, x_3 \ge 0$$

5. Numerical Illustration

Zeleny [47] considered six types of machines, i.e. Lathe, milling machine, jig saw, band saw, drill press, grinder for the PP problem and used the deterministic parameters in the formulation.

The available capacities of each machine are in the form of an IVTN number, i.e. given in Table 2.

Table 2: Capacity of machines

Right Side of the Constraints	
Interval-valued neutrosophic form	***Interval-valued form***
$\tilde{M}^{IVTN}$ *=1200,1350,1500,1600; [0.7,0.9], [0.1,0.3], [0.5,0.7]*	*[1200,1366.67]*
$\tilde{L}^{IVTN}$ *=800,1000,1200,1400; [0.5,0.7], [0.3,0.5], [0.4,0.6]*	*[813.54,1085.71]*
$\tilde{G}^{IVTN}$ *=1650,1800,1950,2050; [0.7,1.0], [0.2,0.3], [0.2,0.5]*	*[1650,1925]*
$\tilde{S}^{IVTN}$ *=1225,1290,1340,1425; [0.3,0.7], [0.1,0.4], [0.3,0.7]*	*[1225,1297.22]*
$\tilde{D}^{IVTN}$ *=700,900,1100,1300; [0.6,0.8], [0.3,0.6], [0.2,0.4]*	*[893.93,1050]*
$\tilde{B}^{IVTN}$ *=1075,1275,1475,1675; [0.5,0.9], [0.1,0.3], [0.3,0.6]*	*[1075,1452.78]*

The coefficients of objective functions are given in Table 3.

Table 3: Coefficients of objective functions

$\tilde{P}_1^{MC}$ =*[40,50]* or *[50,60]* or *[60,70]*	$\tilde{P}_2^{MC}$ =*[90,100]* or *[100,110]* or *[110,120]*	$\tilde{P}_2^{MC}$ =*[16.5,17.5]* or *[17.5,18.5]*
$\tilde{L}_1^{MC}$ =*[0.70,0.72]* or *[0.72,0.74]* or *[0.74,0.76]*	$\tilde{L}_2^{MC}$ =*[0.81,0.85]* or *[0.85,0.89]*	$\tilde{L}_3^{MC}$ =*[0.75,0.78]* or *[0.78,0.81]* or *[0.81,0.84]* or *[0.84,0.87]*
$\tilde{Q}_1^{MC}$ =*[82,92]* or *[92,102]*	$\tilde{Q}_2^{MC}$ =*[65,75]* or *[75,85]* or *[85,95]* or *[95,105]*	$\tilde{Q}_3^{MC}$ =*[40,50]* or *[50,60]* or *[60,70]*
$\tilde{W}_1^{MC}$ =*[15,25]* or *[25,35]*	$\tilde{W}_2^{MC}$ =*[90,100]* or *[100,110]* or *[110,120]* or *[120,130]*	$\tilde{W}_3^{MC}$ =*[65,75]* or *[75,85]*

The left side's coefficients of the constraints are given in Table 4.

Table 4: Coefficients of left side of the constraints

$\tilde{m}_1^{MC}$ =*[10,12]* or *[12,14]*	$\tilde{m}_2^{MC}$ =*[15,17]* or *[17,19]* or *[19,21]*	---
$\tilde{l}_1^{MC}$ =*[3,5]* or *[5,7]*	$\tilde{l}_2^{MC}$ =*[7,9]* or *[9,11]* or *[11,13]*	$\tilde{l}_3^{MC}$ =*[6,8]* or *[8,10]*
$\tilde{g}_1^{MC}$ =*[8,10]* or *[10,12]* or *[12,14]*	$\tilde{g}_2^{MC}$ =*[13,15]* or *[15,17]*	$\tilde{g}_3^{MC}$ =*[15,17]* or *[17,19]*
$\tilde{s}_1^{MC}$ =*[4,6]* or *[6,8]*	---	$\tilde{s}_3^{MC}$ =*[12,14]* or *[14,16]* or *[16,18]*
---	$\tilde{d}_2^{MC}$ =*[10,12]* or *[12,14]*	$\tilde{d}_3^{MC}$ =*[5,7]* or *[7,9]* or *[9,11]*
$\tilde{b}_1^{MC}$ =*[9.5,11.5]* or *[11.5,13.5]*	$\tilde{b}_2^{MC}$ =*[9.5,11.5]* or *[11.5,13.5]*	$\tilde{b}_3^{MC}$ =*[4,6]* or *[6,8]* or *[8,10]* or *[10,12]*

The membership and non-membership of the intuitionistic function of the problem are as follows:

The objective membership of the problem will be

$$\mu_1(Z_1)=\begin{cases}0, & Z_1\le 6152.5\\ \dfrac{Z_1-6152.5}{12348-6152.5}, & 6152.5<Z_1\le 12348\\ 1, & Z_1>12348\end{cases}\qquad \mu_2(Z_2)=\begin{cases}0, & Z_2\le 0.72\\ \dfrac{Z_2-0.72}{0.89-0.72}, & 0.72<Z_2\le 0.\\ 1, & Z_1>0.89\end{cases}$$

$$\mu_3(Z_3)=\begin{cases}0, & Z_3\le 7035\\ \dfrac{Z_3-7035}{16006-7035}, & 7035<Z_3\le 16006\\ 1, & Z_3>16006\end{cases}\qquad \mu_4(Z_4)=\begin{cases}0, & Z_4\le 7330\\ \dfrac{Z_4-7330}{13710-7330}, & 7330<Z_4\le 13710\\ 1, & Z_4>13710\end{cases}$$

The objective non-membership of the problem will be

$$\upsilon_1(Z_1)=\begin{cases}1, & Z_1\le 6772.05\\ \dfrac{12348-Z_1}{12348-6772.05}, & 6772.05<Z_1\le 12348\\ 0, & Z_1>12348\end{cases}\qquad \upsilon_2(Z_2)=\begin{cases}1, & Z_2\le 0.75\\ \dfrac{0.89-Z_2}{0.89-0.75}, & 0.75<Z_2\le 0\\ 0, & Z_2>0.89\end{cases}$$

$$\upsilon_3(Z_3)=\begin{cases}1, & Z_3\le 7932.1\\ \dfrac{16006-Z_3}{16006-7932.1}, & 7932.1<Z_3\le 16006\\ 0, & Z_3>16006\end{cases}\qquad \upsilon_4(Z_4)=\begin{cases}1, & Z_4\le 7968\\ \dfrac{13710-Z_4}{13710-7968}, & 7968<Z_4\le 13710\\ 0, & Z_4>13710\end{cases}$$

The intuitionistic formulation of the problem is

$$Max\ \ (\mu-\upsilon)$$

Subject to constraints

$$\mu_1(Z_1)\ge\frac{Z_1-6152.5}{12348-6152.5},\ \ \upsilon_1(Z_1)\le\frac{12348-Z_1}{12348-6772.05},\ \ \mu_2(Z_2)\ge\frac{Z_2-0.72}{0.89-0.72},\ \ \upsilon_2(Z_2)\le\frac{0.89-Z_2}{0.89\text{-}0.75}$$

$$\mu_3(Z_3)\ge\frac{Z_3-7035}{16006-7035},\ \ \upsilon_3(Z_3)\le\frac{16006-Z_3}{16006-7932.1},\ \ \mu_4(Z_4)\ge\frac{Z_4-7330}{13710-7330},\ \ \upsilon_4(Z_4)\le\frac{13710-Z_4}{13710-7968}$$

$$Z_1=\tilde{P}_1^{MC}x_1+\tilde{P}_2^{MC}x_2+\tilde{P}_3^{MC}x_3,\ \ Z_2=(\tilde{L}_1^{MC}x_1+\tilde{L}_2^{MC}x_2+\tilde{L}_3^{MC}x_3)/(x_1+x_2+x_3)$$

$$Z_3=\tilde{Q}_1^{MC}x_1+\tilde{Q}_2^{MC}x_2+\tilde{Q}_3^{MC}x_3,\ \ Z_4=\tilde{W}_1^{MC}x_1+\tilde{W}_2^{MC}x_2+\tilde{W}_3^{MC}x_3$$

$$\tilde{m}_1^{MC}x_1+\tilde{m}_2^{MC}x_2\le\tilde{M}^{IVTN},\ \ \tilde{l}_1^{MC}x_1+\tilde{l}_2^{MC}x_2+\tilde{l}_3^{MC}x_3\le\tilde{L}^{IVTN}$$

$$\tilde{g}_1^{MC}x_1+\tilde{g}_3^{MC}x_3\le\tilde{G}^{IVTN},\ \ \ \tilde{s}_1^{MC}x_1+\tilde{s}_3^{MC}x_3\le\tilde{S}^{IVTN}$$

$$\tilde{d}_2^{MC}x_2+\tilde{d}_3^{MC}x_3\le\tilde{D}^{IVTN},\ \ \ \tilde{b}_1^{MC}x_1+\tilde{b}_2^{MC}x_2+\tilde{b}_3^{MC}x_3\le\tilde{B}^{IVTN}$$

$$x_1,x_2,x_3\ge 0,\ \mu_j(Z_j)+\upsilon_j(Z_j)\le 1,\forall\ \mu_j(Z_j)\ \&\ \upsilon_j(Z_j)\in[0,1]$$

$$\mu\le\mu_j(Z_j),\upsilon\ge\upsilon_j(Z_j),\mu\ge\upsilon,\ \ j=1,\ 2,\ 3,\ 4.$$

We get the following compromise solution using the IFP technique, which is given below in Table 5.

Table 5: Compromise optimal solution

Result
$F_1=11852, F_2=0.8652414, F_3=12320, F_4=13321.67, x_1=15, x_2=78, x_3=52$

The graphical representations of compromise optimal solution through IFP for both models are shown in Figure 2. The Membership and non-membership values are shown in Figure 3.

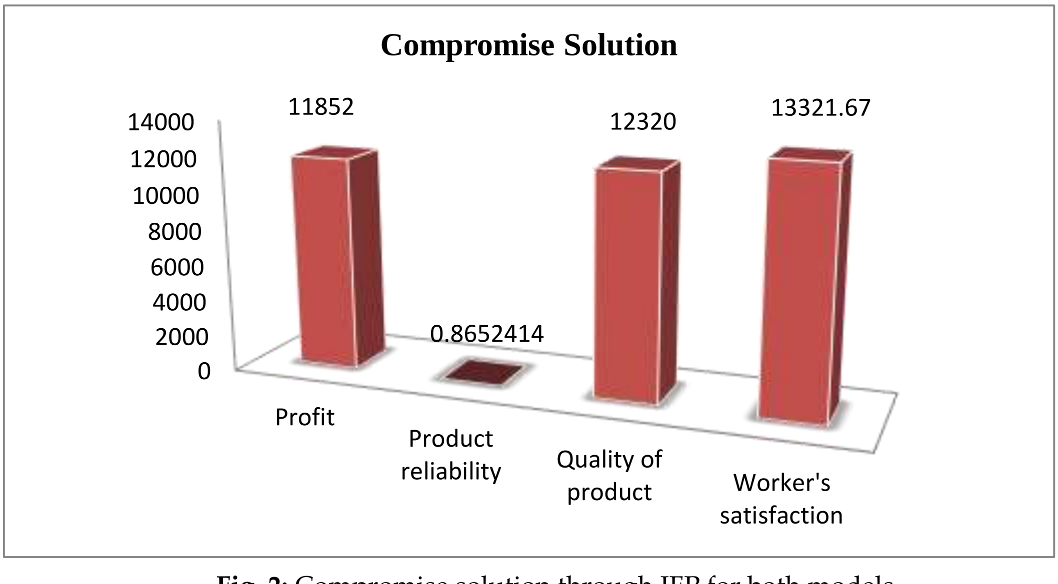

Fig. 2: Compromise solution through IFP for both models

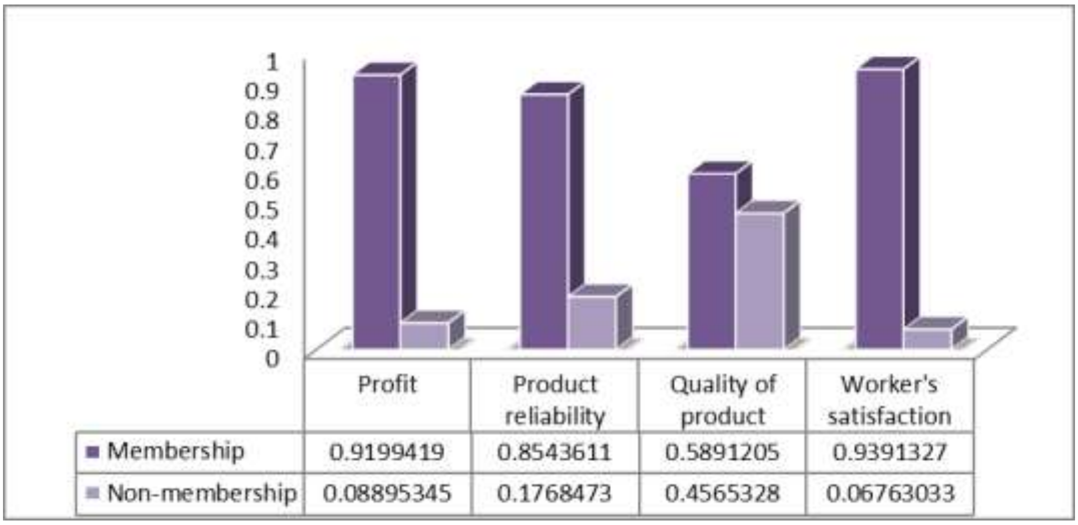

Fig. 3: Membership and non-membership value for the Model I

The comparison of the IFP with fuzzy goal programming (FGP) and bi-level fuzzy goal programming (BL-FGP) are shown in Table 6.

Table 6: Comparison IFP with BL-FGP and FGP

Model	Methods	Profit	Product Reliability	Quality of the Product	Worker's Satisfaction	Decision Variable's Values
Kamal et al. (2019) [21]	BL-FGP	5697.5	0.80734	12307	10000	$x_1=71,\ x_2=3,\ x_3=95$
	FGP	11894	0.812350	10989	11000	$x_1=42,\ x_2=59,\ x_3=54$
Discussed Model	IFP	11852	0.8652414	12320	13321.67	$x_1=15, x_2=78, x_3=52$

From the solutions of Table 6, it can be observed that discussed Model gives the more optimal solution than Kamal et al. [21] Model for profit, product reliability and worker's satisfaction, but for product quality, the BL-FGP Model has the more improved solution.

6. Motivation and Contribution

This study is motivated by an Intuitionistic programming research area with the potential to capture decision-makers. The following are the contributions of the study:

i. It serves as an additional contribution to the literature of PP.

ii. A case study is provided in which solution procedures for multi-objective multi-product problem formulation is reported.

iii. In this study, a new approach based on intuitionistic has been applied.

iv. The approach is compared with BL-FGP and FGP, and the result proves to be better.

v. The applicability of Interval-valued Neutrosophic and multi-choice parameters have also been discussed and reported.

Conclusion

The Model for the PP problem with rational expectations was explored in this article. The optimization problem is represented with multi-choice type parameters in the objective's coefficient and the left-hand side of the constraints, and it is transformed into the deterministic form using the binary variable transformation technique. Some parameters are IVTN number types that are transformed into deterministic forms using the score function. In determining the optimum quantity of products, the industrial PP problem is solved using intuitionistic fuzzy IFP. The comparison of the IFP with BL-FGP and FGP are shown in Table 6.

Furthermore, it can be observed that the discussed model gives a more compromise solution than BL-FGP and FGP for profit, product reliability and worker's satisfaction, but for product quality, the BL-FGP model has the more improved solution. This paper presents a detailed investigation into IFP approaches for solving multi-objective optimization problems in the presence of multi-choice and neutrosophic environments. It will help solve and understand the production-related problems by the IFP approach. The IFP approach will help in solving the other complex production-related problems. The complex PP challenge will be considered with several new solutions based on fuzzy logic.

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