Multi-level linear programming problem with neutrosophic numbers: A goal programming strategy

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Abstract: In the paper, we propose an alternative strategy for multi-level linear programming (MLP) problem with neutrosophic numbers through goal programming strategy. Multi-level linear programming problem consists of *k* levels where there is an upper level at the first level and multiple lower levels at the second level with one objective function at every level. Here, the objective functions of the level decision makers and constraints are described by linear functions with neutrosophic numbers of the form [u + v*I*], where u, v are real numbers and *I* signifies the indeterminacy. At the beginning, the neutrosophic numbers are transformed into interval numbers and consequently, the original problem transforms into MLP problem with interval numbers. Then we compute the target interval of the objective functions via interval programming procedure and formulate the goal achieving functions. Due to potentially conflicting objectives of *k* decision makers, we consider a possible relaxation on the decision variables under the control of each level in order to avoid decision deadlock. Thereafter, we develop three new goal programming models for MLP problem with neutrosophic numbers. Finally, an example is solved to exhibit the applicability, feasibility and simplicity of the proposed strategy.

Keywords: neutrosophic numbers; interval numbers; multi-level linear programming; goal programming

1. Introduction

Multi-level programming (MLP) programming problem consists of multi-levels with single objective function at each level where each level decision maker (DM) tries to get maximum benefit over a common feasible region. In the paper, we consider an MLP problem with neutrosophic numbers information where the objective functions and common constraints are linear functions and each DM independently controls a set of decision variables. In 1988, Anandalingam [1] proposed Stackelberg solution concept for MLP problem in crisp environment and extended the concept to solve decentralized bi-level programming problem.

Goal programming (GP) [2, 3, 4, 5, 6, 7, 8] is one of the popular mathematical tools for solving multi-objective mathematical programming problems with multiple and conflicting objectives to obtain optimal compromise solutions. In 1991, Inuiguchi and Kume [9] incorporated the notion of interval GP.

In 1998, Smarandache [10] incorporated a novel concept called neutrosophic set to tackle with inconsistent, incomplete, indeterminate information where indeterminacy is an independent and important factor. Roy and Das [11] developed a computational algorithm for solving multi-objective linear programming problem by utilizing neutrosophic optimization technique. Das and Roy [12] used neutrosophic optimization method for obtaining optimal solution for multi-objective non linear programming problem. Hezam et al. [13] used first order Taylor polynomial series approximation method for neutrosophic multi-objective programming problem. Abdel-Baset et al. [14] developed two models for neutrosophic goal programming problems and applied the concept to industrial design problem. In 2016, Pramanik [15] proposed three novel neutrosophic GP models for optimization problem by minimizing indeterminacy membership functions for practical neutrosophic optimization. Pramanik [16] also proposed the framework of neutrosophic linear goal programming for multi-objective optimization with uncertainty and indeterminacy simulteneously.

Smarandache [17, 18] introduced the concept of neutrosophic number and presented its fundamental properties. Jiang and Ye [19] presented a general neutrosophic number optimization model for solving optimal design of truss structures. Deli and Şubaş [20] developed a ranking method for single valued neutrosophic numbers and applied the concept to solve a multi-attribute decision making problem. Ye [21] discussed a neutrosophic number linear programming technique for neutrosophic number optimization problems where objective functions and constraints are described by neutrosophic numbers. Ye et al. [22] presented general solutions of neutrosophic number non-linear optimization models for unconstrained and constrained problems.

In 2018, Pramanik and Banerjee [23] discussed a solution methodology for single-objective linear programming problem where the coefficients of objective functions and the constraints are neutrosophic numbers. Pramanik and Banerjee [24] also studied GP technique for multi-objective linear programming problem with neutrosophic coefficients. Recently, Pramanik and Dey [25] proposed novel GP models for solving bi-level programming problem with neutrosophic numbers by minimizing deviational variables. In this paper, we extend the concept of Pramanik and Dey [25] to solve MLP problem with neutrosophic numbers based on GP strategy.

We organize the paper in the following way. In section 2, some definitions concerning interval numbers, neutrosophic numbers and their essential properties are given. In section 3, we present the mathematical formulation of MLP problem described by neutrosophic numbers. In section 4, the GP strategies for MLP problem with neutrosophic numbers is discussed by considering upper (superior) and lower (inferior) preference bounds on the decision vectors of the level DMs. In section 5, an application of the developed strategy for MLP problem is demonstrated. Finally, conclusion with some future scope of research is provided in the last section.

2. Preliminaries

In the section, we provide some basic definitions regarding interval numbers, neutrosophic numbers.

2.1 Interval number [26]

An interval number is defined by $P = [P^L, P^U] = \{p: P^L \le p \le P^U, p \in \Re\}$, where P^L, P^U are left and right limit of the interval P on the real line $\Re$.

Definition 2.1: Let γ (*P*) and δ (*P*) be the midpoint and the width of an interval number, respectively.

Then, $\gamma(P) = \frac{1}{2}[P^L + P^U]$ and $\delta(P) = [P^U - P^L]$

The scalar multiplication of *P* by μ is defined as given below.

$$\mu P = \begin{cases} [\mu P^L, \mu P^U], \mu \geq 0, \\ [\mu P^U, \mu P^L], \mu \leq 0 \end{cases}$$

The absolute value of *P* is defined as given below.

$$|P| = \begin{cases} [P^L, P^U], P^L \geq 0, \\ [0, \max\{-P^L, P^U\}], P^L < 0 < P^U \\ [-P^U, -P^L], P^U \leq 0 \end{cases}$$

The binary operation * between $P_1 = [P_1^L, P_1^U]$ and $P_2 = [P_2^L, P_2^U]$ is defined as follows:

$P_1 * P_2 = \{p_1 * p_2: P_1^L \leq p_1 \leq P_1^U, P_2^L \leq p_2 \leq P_2^U, p_1, p_2 \in \Re\}$.

2.2 Neutrosophic number [17, 18]

A neutrosophic number is represented by E = m + nI, where m, n are real numbers where m is determinate part and nI is indeterminate part and $I \in [I^L, I^U]$ represents indeterminacy.

Therefore, $E = [m + nI^L, m + nI^U] = [E^L, E^U]$, (say)

Example: Suppose a neutrosophic number $E = 2 + 3I$, where 2 is determinate part and $3I$ is indeterminate part. Here, we take $I \in [0.2, 0.7]$. Then, E becomes an interval number of the form N = [2.6, 4.1].

Now, we define some properties regarding neutrosophic numbers as follows:

Suppose that $E_1 = [m_1 + n_1 I_1] = [m_1 + n_1 I_1^L, m_1 + n_1 I_1^U] = [E_1^L, E_1^U]$ and $E_2 = [m_2 + n_2 I_2] = [m_2 + n_2 I_2^L, m_2 + n_2 I_2^U] = [E_2^L, E_2^U]$ be two neutrosophic numbers where $I_1 \in [I_1^L, I_1^U]$, $I_2 \in [I_2^L, I_2^U]$, then

(i). $E_1 + E_2 = [E_1^L + E_2^L, E_1^U + E_2^U]$,

(ii). $E_1 - E_2 = [E_1^L - E_2^U, E_1^U - E_2^L]$,

(iii). $E_1 \times E_2 = [\text{Min}\{E_1^L \times E_2^L, E_1^L \times E_2^U, E_1^U \times E_2^L, E_1^U \times E_2^U\}, \text{Max}\{E_1^L \times E_2^L, E_1^L \times E_2^U, E_1^U \times E_2^L, E_1^U \times E_2^U\}]$

(iv). $E_1 / E_2 = [\text{Min}\{E_1^L / E_2^L, E_1^L / E_2^U, E_1^U / E_2^L, E_1^U / E_2^U\}, \text{Max}\{E_1^L / E_2^L, E_1^L / E_2^U, E_1^U / E_2^L, E_1^U / E_2^U\}]$, if $0 \notin E_2$.

3. Formulation of MLP problem for minimization-type objective function with neutrosophic numbers

Mathematically, an MLP problem with neutrosophic numbers for minimization-type objective function at every level can be formulated as given below.

$$\underset{x_1}{Min}\, Z_1(x) = [A_{11} + B_{11}I_{11}]\, x_1 + [A_{12} + B_{12} I_{12}]\, x_2 + \ldots + [A_{1k} + B_{1k} I_{1k}]\, x_k + [G_1 + H_1 I_1] \quad (1)$$

$$\underset{x_2}{Min}\, Z_2(x) = [A_{21} + B_{21}I_{21}]\, x_1 + [A_{22} + B_{22} I_{22}]\, x_2 + \ldots + [A_{2k} + B_{2k} I_{2k}]\, x_k + [G_2 + H_2 I_2] \quad (2)$$

$$\underset{x_k}{Min}\, Z_k(x) = [A_{k1} + B_{k1}I_{k1}]\, x_1 + [A_{k2} + B_{k2} I_{12}]\, x_2 + \ldots + [A_{kk} + B_{kk} I_{kk}]\, x_k + [G_k + H_k I_k] \quad (3)$$

Subject to

$$x \in X = \{x = (x_1, x_2, \ldots, x_k) \in R^N \mid [C_1 + D_1 I_1^{/}]\, x_1 + [C_2 + D_2 I_2^{/}]\, x_2 + \ldots + [C_k + D_k I_k^{/}]\, x_k \leq \rho + \sigma I^{/}, x \geq 0\}. \quad (4)$$

Here, $x_i = (x_{i1}, x_{i2}, \ldots, x_{iN_i})^T$: Decision vector under the control of i-th level DM, i = 1, 2, .., *k*. A_{i1}, B_{i1} (i = 1, 2, ..., *k*) are N_1- dimension row vectors; A_{i2}, B_{i2} (i = 1, 2, ..., *k*) are N_2- dimension row vectors;

and similarly, A_{ik}, B_{ik} (i = 1, 2, ..., k) are N_k- dimension row vectors where N = $N_1 + N_2 + ... + N_k$; and G_i, H_i (i = 1, 2, ..., k) are constants. C_i, D_i (i = 1, 2, ..., k) are $M \times N_i$ (i = 1, 2, ..., k) constant matrix and ρ , σ are M dimensional constant column matrix. X ($\neq \Phi$) is considered compact and convex in R^N. Also, we have $I_{ij} \in [I_{ij}^L, I_{ij}^U]$, i = 1, 2, ..., k; j = 1, 2, ..., k; $I_i \in [I_i^L, I_i^U]$, $I_i^{/} \in [I_i^{/L}, I_i^{/U}]$, i = 1, 2, ..., k. Representation of an MLP problem is shown in Figure 1 as follows.

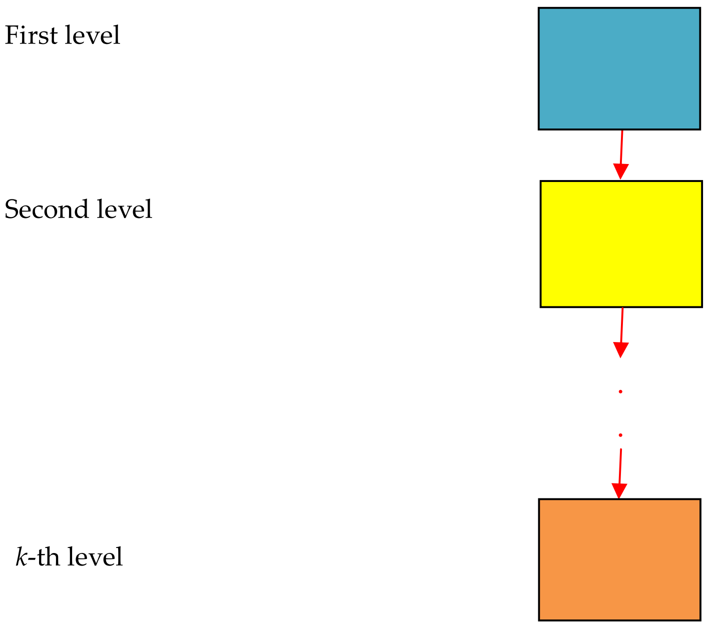

Figure 1. Depiction of an MLP problem

4. Goal programming strategy for solving MLP problem involving neutrosophic numbers

The MLP problem with neutrosophic numbers that is defined in Section 3 can be restated as follows:

First level:

$$\underset{x_1}{Min}\; Z_1(x) = [A_{11} + B_{11}I_{11}]\, x_1 + [A_{12} + B_{12}\, I_{12}]\, x_2 + ... + [A_{1k} + B_{1k}\, I_{1k}]\, x_k + [G_1 + H_1 I_1]$$

$$= \{[A_{11} + B_{11}\, I_{11}^L]\, x_1 + [A_{12} + B_{12}\, I_{12}^L]\, x_2 + ... + [A_{1k} + B_{1k}\, I_{1k}^L]\, x_k + [G_1 + H_1\, I_1^L],\ [A_{11} + B_{11}\, I_{11}^U]\, x_1 + [A_{12} + B_{12}\, I_{12}^U]\, x_2 + ... + [A_{1k} + B_{1k}\, I_{1k}^U]\, x_k + [G_1 + H_1\, I_1^U]\} = [S_1^L(x), S_1^U(x)] \text{ (say)}; \quad (5)$$

Second level:

$$\underset{x_2}{Min}\; Z_2(x) = [A_{21} + B_{21}I_{21}]\, x_1 + [A_{22} + B_{22}\, I_{22}]\, x_2 + ... + [A_{2k} + B_{2k}\, I_{2k}]\, x_k + [G_2 + H_2\, I_2]$$

$$= \{[A_{21} + B_{21}\, I_{21}^L]\, x_1 + [A_{22} + B_{22}\, I_{22}^L]\, x_2 + ... + [A_{2k} + B_{2k}\, I_{2k}^L]\, x_k + [G_2 + H_2\, I_2^L],\ [A_{21} + B_{21}\, I_{21}^U]\, x_1 + [A_{22} + B_{22}\, I_{22}^U]\, x_2 + ... + [A_{2k} + B_{2k}\, I_{2k}^U]\, x_k + [G_2 + H_2\, I_2^U]\} = [S_2^L(x), S_2^U(x)] \text{ (say)}; \quad (6)$$

and similarly, for

k- th level:

$$\underset{x_k}{Min}\; Z_k(x) = [A_{k1} + B_{k1}I_{k1}]\, x_1 + [A_{k2} + B_{k2}\, I_{k2}]\, x_2 + ... + [A_{kk} + B_{kk}\, I_{kk}]\, x_k + [G_k + H_k\, I_k]$$

$$= \{[A_{k1} + B_{k1}\, I_{k1}^L]\, x_1 + [A_{k2} + B_{k2}\, I_{k2}^L]\, x_2 + ... + [A_{kk} + B_{kk}\, I_{kk}^L]\, x_k + [G_k + H_k\, I_k^L],\ [A_{k1} + B_{k1}\, I_{k1}^U]\, x_1 + [A_{k2} + B_{k2}\, I_{k2}^U]\, x_2 + ... + [A_{kk} + B_{kk}\, I_{kk}^U]\, x_k + [G_k + H_k\, I_k^U]\} = [S_k^L(x), S_k^U(x)] \text{ (say)}; \quad (7)$$

and the system constrains reduce to

$$[C_1 + D_1\, I_1^{/}]\, x_1 + [C_2 + D_2\;\; I_2^{/}]\, x_2 + ...+\;\; [C_k + D_k\, I_k^{/}]\, x_k \geq \rho + \sigma\, I^{/}$$

$\Rightarrow$ {[$C_1 + D_1 I_1^{/L}$] x_1 + [$C_2 + D_2 I_2^{/L}$] x_2 + ...+ [$C_k + D_k I_k^{/L}$] x_k, {[$C_1 + D_1 I_1^{/U}$] x_1 + [$C_2 + D_2 I_2^{/U}$] x_2 + ...+ [C_k + $D_k I_k^{/U}$] x $\geq$ [$\rho + \sigma\ I^{/L}$, $\rho + \sigma\ I^{/U}$] = [R^L, R^U] (say)

$\Rightarrow [W^L(x), W^U(x)] \geq [R^L, R^U]$. (8)

Proposition 1. [27]

If $\sum_{j=1}^{n}[\alpha_1^j, \alpha_2^j]z_j \geq [q_1, q_2]$, then $\sum_{j=1}^{n}[\alpha_2^j]z_j \geq q_1$, $\sum_{j=1}^{n}[\alpha_1^j]z_j \geq q_2$ are the maximum and minimum value range inequalities for the constraint condition, respectively.

According to the proposition 1 of Shaocheng [27], the interval inequality of the system constraints (8) transform to the following inequalities as follows:

[$C_1 + D_1 I_1^{/L}$] x_1 + [$C_2 + D_2 I_2^{/L}$] $x_2 \geq R^U$, [$C_1 + D_1 I_1^{/U}$] x_1 + [$C_2 + D_2 I_2^{/U}$] $x_2 \geq R^L$, $x_i \geq 0$, i = 1, 2,

i.e. $W^L(x) \geq R^U$, $W^U(x) \geq R^L$, $x \geq 0$.

Hence, the minimization-type MLP problem can be re-formulated as follows:

First level: $\underset{x_1}{Min}\ Z_1(x) = [S_1^L(x), S_1^U(x)]$,

Second level: $\underset{x_2}{Min}\ Z_2(x) = [S_2^L(x), S_2^U(x)]$,

.

.

k-th level: $\underset{x_k}{Min}\ Z_k(x) = [S_k^L(x), S_k^U(x)]$,

Subject to

$[W^L(x), W^U(x)] \geq [R^L, R^U]$, $x \geq 0$. (9)

For getting the best optimal solution of Z_i, (i = 1, 2, ..., k), the following problem is solved owing to Ramadan [28] as follows:

$\underset{x \in X}{Min}\ Z_i(x) = S_i^L(x)$, i = 1, 2, ..., k

Subject to

$W^U(x) \geq R^L$, $x \geq 0$, i = 1, 2, ..., k. (10)

We solve the Eq. (10) and let $x_i^B = (x_{i1}^B, x_{i2}^B, ..., x_{iN_1}^B, x_{iN_{1+1}}^B, ..., x_{iN}^B)$, (i = 1, 2, ..., k) be the individual best solution of i-th level DM and $S_i^L(x_i^B)$, (i = 1, 2, ..., k) be the individual best objective value of i-th level DM, (i = 1, 2, ..., k).

For obtaining the worst optimal solution of Z_i, (i = 1, 2, ..., k), we solve the following problem due to Ramadan [28] as given below.

$\underset{x \in X}{Min}\ Z_i(x) = S_i^U(x)$, i = 1, 2, ..., k

Subject to

$W^L(x) \geq R^U$, $x \geq 0$. (11)

Let $x_i^* = (x_{i1}^*, x_{i2}^*, ..., x_{iN_1}^*, x_{iN_{1+1}}^*, ..., x_{iN}^*)$, (i = 1, 2, ..., k) be the individual worst solution of i-th level DM subject to the given constraints and $S_i^U(x_i^*)$, (i = 1, 2, ..., k) be the individual worst objective value of i-th level DM, (i = 1, 2, ..., k).

Therefore, [$S_i^L(x_i^B)$, $S_i^U(x_i^*)$] be the optimal value of i-th level DM, (i = 1, 2, ..., k) in the interval form. Let [T_i^+, U_i^+] be the target interval of i-th objective functions set by level DMs.

The target level of i-th objective function can be formulated as follows:

$S_i^U(x) \geq T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) \leq U_i^+$, (i = 1, 2, ..., k).

Hence, the goal achievement functions are formulated as follows:

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

where d_i^U, d_i^L, (i = 1, 2, ..., k) are deviational variables.

In a large hierarchical organization, the individual benefit of the level DMs are not same, cooperation between k level DMs is necessary to arrive at a compromise optimal solution.

Suppose that $x_i^B = (\mathrm{x}_{i1}^{\mathrm{B}}, \mathrm{x}_{i2}^{\mathrm{B}}, ..., \mathrm{x}_{iN_i}^{\mathrm{B}}, \mathrm{x}_{iN_i+1}^{\mathrm{B}}, ..., \mathrm{x}_{iN}^{\mathrm{B}})$, (i = 1, 2, ..., k) be the individual best solution of i-th level DM. Suppose ($x_i^B - \eta_i$) and ($x_i^B + \tau_i$), (i = 1, 2, ..., k) be the lower and upper bounds of decision vector provided by i-th level DM where η_i and τ_i, (i = 1, 2, ..., k) are the negative and positive tolerance variables which are not essentially equal [25, 29-41].

Now by considering the preference bounds of the decision variables, we propose three alternative GP models for MLP problem with neutrosophic numbers as follows:

GP Model I.

Min $\sum_{i=1}^{k} (d_i^U + d_i^L)$

Subject to

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

$W^L(x) \geq R^U$, $W^U(x) \geq R^L$,

$(x_i^B - \eta_i) \leq x_i \leq (x_i^B + \tau_i)$, (i = 1, 2, ..., k)

d_i^L, d_i^U, $x \geq 0$, (i = 1, 2).

GP Model II.

Min $\sum_{i=1}^{k} (w_i^U d_i^U + w_i^L d_i^L)$

Subject to

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

$W^L(x) \geq R^U$, $W^U(x) \geq R^L$,

$(x_i^B - \eta_i) \leq x_i \leq (x_i^B + \tau_i)$, (i = 1, 2, ..., k)

$w_i^U \geq 0$, $w_i^L \geq 0$, (1, 2, ..., k)

d_i^L, d_i^U, $x \geq 0$, (1, 2, ..., k).

GP Model III.

Min ψ

Subject to

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

$W^L(x) \geq R^U$, $W^U(x) \geq R^L$,

$(x_i^B - \eta_i) \leq x_i \leq (x_i^B + \tau_i)$, (i = 1, 2, ..., k)

$\psi \geq d_i^U, \psi \geq d_i^L$, (i = 1, 2)

$d_i^L, d_i^U, x \geq 0$, (1, 2, ..., k).

A flowchart of the proposed strategy for MLP problem with neutrosophic coefficients is shown in Figure 2.

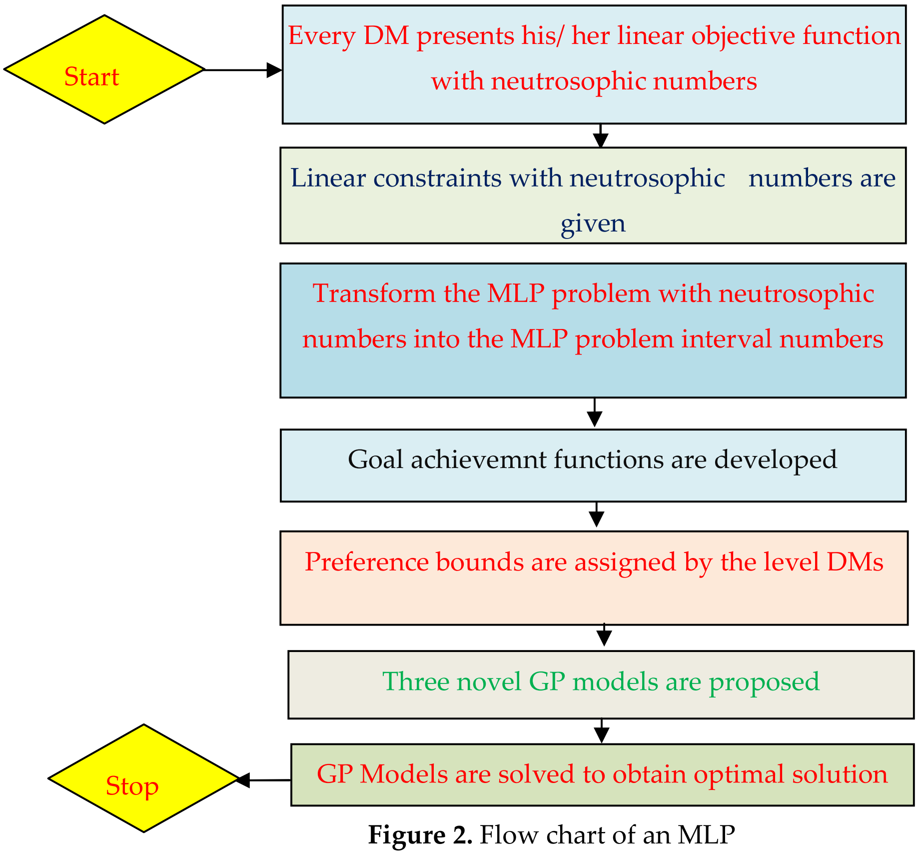

Figure 2. Flow chart of an MLP

5. Numerical Example

We consider the following MLP problem with neutrosophic numbers to demonstrate the proposed GP procedure. Without any loss of generality we consider $I \in [0, 1]$.

First level:

$\underset{x_1}{Min}\ Z_1(x) = [11 + 2I]\, x_1 + [7 + 3I]\, x_2 + [3 + I]\, x_3,$

Second level:

$\underset{x_2}{Min}\ Z_2(x) = [1 + 2I]\, x_1 + [2 + I]\, x_2 + [2 + 3I]\, x_3 + [4 + I],$

Third level:

$\underset{x_3}{Min}\ Z_3(x) = [1 + 2I]\, x_1 + [2 + I]\, x_2 + 0.5\, x_3 + [5 + I],$

Subject to

$[3 + 2I]\, x_1 + [1 + I]\, x_2 + [1 + 2I]\, x_3 \geq [5 + 2I],$

$[4 + I]\, x_1 + [2 + 3I]\, x_2 - [2 + I]\, x_3 \geq [4 + 3I],$

$[1 + I]\, x_1 + [2 + 2I]\, x_2 + [2 + I]\, x_3 \geq [3 + 2I],$

$x_1, x_2, x_3 \geq 0.$

Using interval programming technique, the transformed problem of first level DM can be presented as follows (see Table 1):

Table 1. First level DM's problem for best and worst solutions

First level DM's problem to obtain best solution	First level DM's problem to obtain worst solution
Min $S_1^L(x) = 11x_1 + 7x_2 + 7x_3$	Min $S_1^U(x) = 13x_1 + 10x_2 + 4x_3$
Subject to	Subject to
$5x_1 + 2x_2 + 3x_3 \geq 5$,	$3x_1 + x_2 + x_3 \geq 7$,
$5x_1 + 5x_2 - 3x_3 \geq 4$,	$4x_1 + 2x_2 - 2x_3 \geq 7$,
$2x_1 + 4x_2 + 3x_3 \geq 3$,	$x_1 + 2x_2 + 2x_3 \geq 5$,
$x_1, x_2, x_3 \geq 0$.	$x_1, x_2, x_3 \geq 0$.

The best and worst solutions of First level DM are calculated as follows (see Table 2)**:**

Table 2. First level DM's best and worst solutions

The best solution	The worst solution
S_1^B = 10.536 at (0.78, 0.171, 0.252)	S_1^* = 34.3 at (1.8, 0.75, 0.85)

The transformed problem of second level DM can be presented as follows (see Table 3):

Table 3. Second level DM's problem for best and worst solutions

Second level DM's problem to get best solution	Second level DM's problem to get worst solution
Min $S_1^L(x) = x_1 + 2x_2 + 2x_3 + 4$	Min $S_2^U(x) = 3x_1 + 3x_2 + 5x_3 + 5$
Subject to	Subject to
$5x_1 + 2x_2 + 3x_3 \geq 5$,	$3x_1 + x_2 + x_3 \geq 7$,
$5x_1 + 5x_2 - 3x_3 \geq 4$,	$4x_1 + 2x_2 - 2x_3 \geq 7$,
$2x_1 + 4x_2 + 3x_3 \geq 3$,	$x_1 + 2x_2 + 2x_3 \geq 5$,
$x_1, x_2, x_3 \geq 0$.	$x_1, x_2, x_3 \geq 0$.

The best and worst solutions of second level DM are determined as given below (see Table 4)

Table 4. Second level DM's best and worst solutions

The best solution	The worst solution
S_2^B = 5.5 at (0.875, 0.312, 0)	S_2^* = 15.2 at (1.8, 1.6, 0)

Similarly, the transformed problem of third level DM can be shown as follows (see Table 5):

Table 5. Third level DM's problem for best and worst solutions

Third level DM's problem to get best solution	Third level DM's problem to get worst solution
Min $S_3^L(x) = x_1 + 2x_2 + 0.5x_3 + 5$	Min $S_3^U(x) = 3x_1 + 3x_2 + 0.5x_3 + 6$
Subject to	Subject to
$5x_1 + 2x_2 + 3x_3 \geq 5$,	$3x_1 + x_2 + x_3 \geq 7$,
$5x_1 + 5x_2 - 3x_3 \geq 4$,	$4x_1 + 2x_2 - 2x_3 \geq 7$,
$2x_1 + 4x_2 + 3x_3 \geq 3$,	$x_1 + 2x_2 + 2x_3 \geq 5$,
$x_1, x_2, x_3 \geq 0$.	$x_1, x_2, x_3 \geq 0$.

The best and worst solutions of third level DM are computed as given below (see Table 6)

Table 6. Third level DM DM's best and worst solutions

The best solution	The worst solution
$S_3^B = 6.167$ at (1, 0, 0.333)	$S_3^* = 13.85$ at (2.4, 0, 1.3)

The objective function of first level DM with specified targets can be presented as follows:

$11x_1 + 7x_2 + 3x_3 \leq 35$, $13x_1 + 10x_2 + 4x_3 \geq 11$,

The goal achievement functions of first level DM with specified targets can be presented as follows:

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35$, $-13x_1 - 10x_2 - 4x_3 + d_1^U = -11$,

The objective function of second level DM with specified targets can be presented as follows:

$x_1 + 2x_2 + 2x_3 \leq 16$, $3x_1 + 3x_2 + 5x_3 + 5 \geq 6$,

Also, the goal achievement functions of LDM with specified targets can be developed as follows:

$x_1 + 2x_2 + 2x_3 + d_2^L = 16$, $-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6$,

Similarly, the objective function of third level DM with specified targets can be presented as follows:

$x_1 + 2x_2 + 0.5x_3 + 5 \leq 14$, $3x_1 + 3x_2 + 0.5x_3 + 6 \geq 7$,

Also, the goal achievement functions of third level DM with specified targets can be established as follows:

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14$, $-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7$,

Let, the first level DM assigns preference bounds on the decision variable x_1 as $0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8$, the second level DM offers preference bounds on the decision variable x_2 as $0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5$, and the third level DM provides preference bounds on the decision variable x_3 as $0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5$, in order to get optimal compromise solution.

Therefore, the GP models for MLP problem involving neutrosophic coefficients can be developed as follows:

GP Model I.

Min ($d_1^L + d_1^U + d_2^L + d_2^U + d_3^L + d_3^U$)

Subject to

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35,$

$-13x_1 - 10x_2 - 4x_3 + d_1^U = -11,$

$x_1 + 2x_2 + 2x_3 + d_2^L = 16,$

$-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6,$

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14,$

$-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7,$

$5x_1 + 2x_2 + 3x_3 \geq 5,$

$5x_1 + 5x_2 - 3x_3 \geq 4,$

$2x_1 + 4x_2 + 3x_3 \geq 3,$

$3x_1 + x_2 + x_3 \geq 7,$

$4x_1 + 2x_2 - 2x_3 \geq 7,$

$x_1 + 2x_2 + 2x_3 \geq 5,$

$0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8,$

$0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5,$

$0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5$

$d_i^L, d_i^U \geq 0$, (i = 1, 2, 3)

$x_1, x_2, x_3 \geq 0.$

GP Model II.

Min $\frac{1}{6}(d_1^L + d_1^U + d_2^L + d_2^U + d_3^L + d_3^U)$

Subject to

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35,$

$-13x_1 - 10x_2 - 4x_3 + d_1^U = -11,$

$x_1 + 2x_2 + 2x_3 + d_2^L = 16,$

$-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6,$

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14,$

$-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7,$

$5x_1 + 2x_2 + 3x_3 \geq 5,$

$5x_1 + 5x_2 - 3x_3 \geq 4,$

$2x_1 + 4x_2 + 3x_3 \geq 3,$

$3x_1 + x_2 + x_3 \geq 7,$

$4x_1 + 2x_2 - 2x_3 \geq 7,$

$x_1 + 2x_2 + 2x_3 \geq 5,$

$0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8,$

$0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5,$

$0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5$

$d_i^L, d_i^U \geq 0$, (i = 1, 2, 3)

$x_1, x_2, x_3 \geq 0.$

GP Model III.

Min ψ

Subject to

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35,$

$-13x_1 - 10x_2 - 4x_3 + d_1^U = -11,$

$x_1 + 2x_2 + 2x_3 + d_2^L = 16,$

$-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6,$

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14,$

$-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7,$

$5x_1 + 2x_2 + 3x_3 \geq 5,$

$5x_1 + 5x_2 - 3x_3 \geq 4,$

$2x_1 + 4x_2 + 3x_3 \geq 3,$

$3x_1 + x_2 + x_3 \geq 7,$

$4x_1 + 2x_2 - 2x_3 \geq 7,$

$x_1 + 2x_2 + 2x_3 \geq 5,$

$0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8,$

$0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5,$

$0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5,$

$\psi \geq D_i^L, \psi \geq D_i^U$, (i = 1, 2, 3)

$d_i^L, d_i^U \geq 0$, (i = 1, 2, 3)

$x_1, x_2, x_3 \geq 0.$

The solutions of the developed GP models are shown in the Table 7 as follows:

Table 7. The solutions of the MLP problem involving neutrosophic numbers

GP Model	Solution point (x_1, x_2, x_3)	Objective values Z_1	Z_2	Z_3
GP Model I	**(1.58, 1.3, 0.96)**	**(29.36, 37.38)**	**(10.10, 18.44)**	**(9.66, 15.12)**
GP Model II	**(1.58, 1.3, 0.96)**	**(29.36, 37.38)**	**(10.10, 18.44)**	**(9.66, 15.12)**
GP Model III	**(1.58, 1.3, 0.96)**	**(29.36, 37.38)**	**(10.10, 18.44)**	**(9.66, 15.12)**

Note: It is observed that the three GP models produce the same optimal compromise solution set.

6. Conclusion

In the paper, we have proposed three new goal programming models for multi-level linear programming problem where objective and constraints are linear functions with neutrosophic coefficients. By applying interval programming procedure, we transform the multi-level linear programming problem into interval programming problem. Then, we determine best and worst solutions for all k -level decision makers and establish the goal achievement functions. We consider preference upper and lower bounds on the decision variables under the control of all k - level decision makers in order to achieve optimal compromise solution of the multi-level system. Finally, goal programming models are proposed to solve multi-level linear programming problem by minimizing deviational variables. A multi-level linear programming under neutrosophic numbers environment is finally solved to show the applicability and feasibility of the proposed GP strategy.

In future, we hope to utilize the proposed GP strategy to solve multi-objective decentralized bi-level linear programming, multi-objective decentralized multi-level linear programming problems, and other real world decision-making problems with neutrosophic numbers information.

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