



# Multi-valued Neutrosophic Sets and its Application in Multi-criteria Decision-making Problems

Juan-juan Peng<sup>1,2</sup> and Jian-qiang Wang<sup>1\*</sup>

<sup>1</sup>School of Business, Central South University, Changsha 410083, China. E-mail: jqwang@csu.edu.cn

<sup>2</sup>School of Economics and Management, Hubei University of Automotive Technology, Shiyan 442002, China

\*Corresponding author e-mail address: jqwang@csu.edu.cn

**Abstract.** In recent years, hesitant fuzzy sets and neutrosophic sets have aroused the interest of researchers and have been widely applied to multi-criteria decision-making problems. The operations of multi-valued neutrosophic sets are introduced and a comparison method is developed based on related research of hesitant fuzzy sets and intuitionistic fuzzy sets in this paper. Furthermore,

some multi-valued neutrosophic number aggregation operators are proposed and the desirable properties are discussed as well. Finally, an approach for multi-criteria decision-making problems was explored applying the aggregation operators. In addition, an example was provided to illustrate the concrete application of the proposed method.

**Keywords:** Multi-valued neutrosophic sets; multi-criteria decision-making; aggregation operators

## 1. Introduction

Atanassov introduced intuitionistic fuzzy sets (AIFSs) [1-4], which an extension of Zadeh's fuzzy sets (FSs) [5]. As for the present, AIFS has been widely applied in solving multi-criteria decision-making (MCDM) problems [6-10], neural networks [11, 12], medical diagnosis [13], color region extraction [14, 15], market prediction [16]. Then, AIFS was extended to the interval-valued intuitionistic fuzzy sets (AIVIFSs) [17]. AIFS took into account membership degree, non-membership degree and degree of hesitation simultaneously. So it is more flexible and practical in addressing the fuzziness and uncertainty than the traditional FSs. Moreover, in some actual cases, the membership degree, non-membership degree and hesitation degree of an element in AIFS may not be only

one specific number. To handle the situations that people are hesitant in expressing their preference over objects in a decision-making process, hesitant fuzzy sets (HFSs) were introduced by Torra [18] and Narukawa [19]. Then generalized HFSs and dual hesitant fuzzy sets (DHFSs) were developed by Qian and Wang [20] and Zhu et al. [21] respectively.

Although the FS theory has been developed and generalized, it can not deal with all sorts of uncertainties in different real physical problems. Some types of uncertainties such as the indeterminate information and inconsistent information can not be handled. For example, when we ask the opinion of an expert about certain statement, he or she may say that the possibility that the statement is true is 0.6, the statement is false is 0.3 and the

degree that he or she is not sure is 0.2 [22]. This issue is beyond the field of the FSs and AIFSs. Therefore, some new theories are required.

Florentin Smarandache coined neutrosophic logic and neutrosophic sets (NSs) in 1995 [23, 24]. A NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies in  $]0^-, 1^+[$ , the non-standard unit interval [25]. Obviously, it is the extension to the standard interval  $[0, 1]$  as in the AIFS. And the uncertainty present here, i.e. indeterminacy factor, is independent of truth and falsity values while the incorporated uncertainty is dependent of the degree of belongingness and degree of non belongingness in AIFSs [26]. So for the aforementioned example, it can be expressed as  $x(0.6, 0.3, 0.2)$  in the form of NS.

However, without being specified, it is difficult to apply in the real applications. Hence, a single-valued neutrosophic sets (SVNSs) was proposed, which is an instance of the NSs [22, 26]. Furthermore, the information energy of SVNSs, correlation and correlation coefficient of SVNSs as well as a decision-making method based on SVNSs were presented [27]. In addition, Ye also introduced the concept of simplified neutrosophic sets (SNSs), which can be described by three real numbers in the real unit interval  $[0,1]$ , and proposed a MCDM using aggregation operators for SNSs [28]. Majumdar et al. introduced a measure of entropy of a SNS [26]. Wang et al.

and Lupiáñez proposed the concept of interval-valued neutrosophic sets (IVNS) and gave the set-theoretic operators of IVNS [29, 30]. Furthermore, Ye proposed the similarity measures between SVNS and INs based on the relationship between similarity measures and distances [31, 32].

However, in some cases, the operations of SNSs in Ref. [28] might be irrational. For instance, the sum of any element and the maximum value should be equal to the maximum one, while it does not hold with the operations in Ref. [28]. Furthermore, decision-makers also hesitant to express their evaluation values for each membership in SNS. For instance, in the example given above, if decision-maker think that the possibility that statement is true is 0.6 or 0.7, the statement is false is 0.2 or 0.3 and the degree that he or she is not sure is 0.1 or 0.2. Then how to handle these circumstances with SVNS is also a problem. At the same time, if the operations and comparison method of SVNSs are extended to multi-valued in SVNS, then there exist shortcomings else as we discussed earlier. Therefore, the definition of multi-valued neutrosophic sets (MVNSs) and its operations along with comparison approach between multi-valued neutrosophic numbers (MVNNs), and aggregation operators for MVNS are defined in this paper. Thus, a MCDM method is established based on the proposed operators, an illustrative example is given to demonstrate the application of the proposed method.

The rest of paper is organized as follows. Section 2 briefly introduces the concepts and operations of NSs and SNSs. The definition of MVNS along with its operations and comparison approach for MVNSs is defined on the basis of AIFS and HFSs in Section 3. Aggregation operators MVNNs are given and a MCDM method is developed in Section 4. In Section 5, an illustrative example is presented to illustrate the proposed method and the comparative analysis and discussion were given. Finally, Section 6 concludes the paper.

## 2. Preliminaries

In this section, definitions and operations of NSs and SNSs are introduced, which will be utilized in the rest of the paper.

**Definition 1** [25]. Let  $X$  be a space of points (objects), with a generic element in  $X$  denoted by  $x$ . A NS  $A$  in  $X$  is characterized by a truth-membership function  $T_A(x)$ , a indeterminacy-membership function  $I_A(x)$  and a falsity-membership function  $F_A(x)$ .  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$  are real standard or nonstandard subsets of  $]0^-, 1^+[$ , that is,  $T_A(x) : X \rightarrow ]0^-, 1^+[$ ,  $I_A(x) : X \rightarrow ]0^-, 1^+[$ , and  $F_A(x) : X \rightarrow ]0^-, 1^+[$ . There is no restriction on the sum of  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$ , so  $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ .

**Definition 2** [25]. A NS  $A$  is contained in the other NS  $B$ , denoted as  $A \subseteq B$ , if and only if  $\inf T_A(x) \leq \inf T_B(x)$ ,

$$\begin{aligned} \sup T_A(x) &\leq \sup T_B(x), \inf I_A(x) \leq \inf I_B(x), \\ \sup I_A(x) &\leq \sup I_B(x), \inf F_A(x) \leq \inf F_B(x) \text{ and} \\ \sup F_A(x) &\leq \sup F_B(x) \text{ for } x \in X. \end{aligned}$$

Since it is difficult to apply NSs to practical problems, Ye reduced NSs of nonstandard intervals into a kind of SNSs of standard intervals that will preserve the operations of the NSs [26].

**Definition 3** [28]. Let  $X$  be a space of points (objects), with a generic element in  $X$  denoted by  $x$ . A NS  $A$  in  $X$  is characterized by  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$ , which are singleton subintervals/subsets in the real standard  $[0, 1]$ , that is  $T_A(x) : X \rightarrow [0, 1]$ ,  $I_A(x) : X \rightarrow [0, 1]$ , and  $F_A(x) : X \rightarrow [0, 1]$ . Then, a simplification of  $A$  is denoted by

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \},$$

which is called a SNS. It is a subclass of NSs.

The operational relations of SNSs are also defined in Ref. [28].

**Definition 4** [31]. Let  $A$  and  $B$  are two SNSs. For any  $x \in X$ ,

$$(1) A + B = \left\langle \begin{aligned} &T_A(x) + T_B(x) - T_A(x) \cdot T_B(x), \\ &I_A(x) + I_B(x) - I_A(x) \cdot I_B(x), \\ &F_A(x) + F_B(x) - F_A(x) \cdot F_B(x) \end{aligned} \right\rangle,$$

$$(2) A \cdot B = \langle T_A(x) \cdot T_B(x), I_A(x) \cdot I_B(x), F_A(x) \cdot F_B(x) \rangle,$$

$$(3) \lambda \cdot A = \langle 1 - (1 - T_A(x))^\lambda, 1 - (1 - I_A(x))^\lambda, 1 - (1 - F_A(x))^\lambda \rangle, \lambda > 0,$$

$$(4) A^\lambda = \langle T_A^\lambda(x), I_A^\lambda(x), F_A^\lambda(x) \rangle, \lambda > 0.$$

It has some limitations in Definition 9.

(1) In some situations, the operations, such as  $A+B$  and  $A \cdot B$ , as given in Definition 9, might be irrational. This is shown in the example below.

Let  $a = \langle 0.5, 0.5, 0.5 \rangle$ ,  $a^* = \langle 1, 0, 0 \rangle$  be two simplified neutrosophic numbers (SNNs). Obviously,  $a^* = \langle 1, 0, 0 \rangle$  is the maximum of the SNS. It is notorious that the sum of any number and the maximum number should be equal to the maximum one. However, according to the equation (1) in Definition 9,  $a + b = \langle 1, 0.5, 0.5 \rangle \neq b$ . Hence, the equation (1) does not hold. So does the other equations in Definition 9. It shows that the operations above are incorrect.

(2) The correlation coefficient for SNSs in Ref. [27] on basis of the operations does not satisfy in some special cases.

Let  $a_1 = \langle 0.8, 0, 0 \rangle$  and  $a_2 = \langle 0.7, 0, 0 \rangle$  be two SNSs, and  $a^* = \langle 1, 0, 0 \rangle$  be the maximum of the SNS. According to the MCDM based on the correlation coefficient for SNSs under the simplified neutrosophic environment in Ref. [29], we can obtain the result  $W_1(a_1, a^*) = W_2(a_2, a^*) = 1$ . We cannot distinguish the best one. However, it is clear that the alternative  $a_1$  is superior to alternative  $a_2$ .

(3) In addition, the similarity measure for SNSs in Ref. [32] on basis of the operations does not satisfy in special cases.

Let  $a_1 = \langle 0.1, 0, 0 \rangle$ ,  $a_2 = \langle 0.9, 0, 0 \rangle$  be two SNSs, and  $a^* = \langle 1, 0, 0 \rangle$  be the maximum of the SNS. According to the decision making method based on the cosine similarity measure for SNSs under the simplified neutrosophic environment in Ref. [28], we can obtain the result  $S_1(a_1, a^*) = S_2(a_2, a^*) = 1$ , that is, the alternative  $a_1$  is equal to alternative  $a_2$ . We cannot distinguish the best one else. However,  $T_{a_2}(x) > T_{a_1}(x)$ ,  $I_{a_2}(x) > I_{a_1}(x)$  and  $F_{a_2}(x) > F_{a_1}(x)$ , it is clear that the alternative  $a_2$  is superior to alternative  $a_1$ .

(4) If  $I_A = I_B$ , then  $A$  and  $B$  are reduced to two AIFNs. However, above operations are not in accordance with the laws for two AIFSs in [4,6-10,30].

### 3. Multi-valued neutrosophic sets and their operations

In this section, MVNSs is defined, and its operations based on AIFSs [4,6-10,30] are developed as well.

**Definition 5.** Let  $X$  be a space of points (objects), with a generic element in  $X$  denoted by  $x$ . A *MVNS*  $A$  in  $X$  is characterized by three functions  $\tilde{T}_A(x)$ ,  $\tilde{I}_A(x)$  and  $\tilde{F}_A(x)$  in the form of subset of  $[0, 1]$ , which can be denoted as follows:

$$A = \{ \langle x, \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x) \rangle \mid x \in X \}$$

Where  $\tilde{T}_A(x)$ ,  $\tilde{I}_A(x)$ , and  $\tilde{F}_A(x)$  are three sets of some values in  $[0,1]$ , denoting the truth-membership degree, indeterminacy-membership function and falsity-membership degree respectively, with the conditions:

$$0 \leq \gamma, \eta, \xi \leq 1, 0 \leq \gamma^+ + \eta^+ + \xi^+ \leq 3, \text{ where}$$

$$\gamma \in \tilde{T}_A(x), \eta \in \tilde{I}_A(x), \xi \in \tilde{F}_A(x), \text{ and } \gamma^+ = \sup \tilde{T}_A(x),$$

$$\eta^+ = \sup \tilde{I}_A(x) \text{ and } \xi^+ = \sup \tilde{F}_A(x). \tilde{T}_A(x), \tilde{I}_A(x), \text{ and}$$

$$\tilde{F}_A(x) \text{ is set of crisp values between zero and one. For}$$

convenience, we call  $A = \{ \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle \}$  the multi-valued neutrosophic number (MVNN). Apparently, MVNSs are an extension of NSs.

Especially, if  $\tilde{T}_A, \tilde{I}_A$  and  $\tilde{F}_A$  have only one value  $\gamma, \eta$  and  $\xi$ , respectively, and  $0 \leq \gamma + \eta + \xi \leq 3$ , then the MVNSs are reduced to SNS; If  $\tilde{I}_A = \emptyset$ , then the MVNSs are reduced to DHFSs ;If  $\tilde{I}_A = \tilde{F}_A = \emptyset$ , then the MVNSs are reduced to HFSs. Thus the MVNSs are an extension of these sets above.

The operational relations of MVNSs are also defined as follows.

**Definition 6.** The complement of a MVNS

$$A = \{ \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle \}$$

is denoted by  $A^C$  and is defined by

$$A^C = \{ \langle \bigcup_{\gamma \in \tilde{T}_A} \{1 - \gamma\}, \bigcup_{\eta \in \tilde{I}_A} \{1 - \eta\}, \bigcup_{\xi \in \tilde{F}_A} \{1 - \xi\} \rangle \}.$$

**Definition 7.** The MVNS  $A = \{ \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle \}$  is

$$\text{contained in the other MVNS } B = \{ \langle \tilde{T}_B, \tilde{I}_B, \tilde{F}_B \rangle \},$$

$$A \subseteq B \text{ if and only if } \gamma_A^- \leq \gamma_B^+, \eta_A^- \geq \eta_B^+ \text{ and } \xi_A^- \geq \xi_B^+.$$

Where  $\gamma_A^- = \inf \tilde{T}_A, \gamma_A^+ = \sup \tilde{T}_A, \eta_A^- = \inf \tilde{I}_A,$

$$\eta_A^+ = \sup \tilde{I}_A \text{ and } \xi_A^- = \inf \tilde{F}_A, \xi_A^+ = \sup \tilde{F}_A.$$

**Definition 8.** Let  $A = \{ \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle \},$

$B = \{ \langle \tilde{T}_B, \tilde{I}_B, \tilde{F}_B \rangle \}$  be two MVNNs, and  $\lambda > 0$ . The operations for MVNNs are defined as follows.

$$(1) \lambda A = \left\langle \begin{matrix} \bigcup_{\gamma_A \in \tilde{T}_A} \{1 - (1 - \gamma_A)^\lambda\}, \\ \bigcup_{\eta_A \in \tilde{I}_A} \{(\eta_A)^\lambda\}, \\ \bigcup_{\xi_A \in \tilde{F}_A} \{(\xi_A)^\lambda\} \end{matrix} \right\rangle;$$

$$(2) A^\lambda = \left\langle \begin{matrix} \bigcup_{\gamma_A \in \tilde{T}_A} \{(\gamma_A)^\lambda\}, \\ \bigcup_{\eta_A \in \tilde{I}_A} \{1 - (1 - \eta_A)^\lambda\}, \\ \bigcup_{\xi_A \in \tilde{F}_A} \{1 - (1 - \xi_A)^\lambda\} \end{matrix} \right\rangle;$$

$$(3) A + B = \left\langle \begin{matrix} \bigcup_{\gamma_A \in \tilde{T}_A, \gamma_B \in \tilde{T}_B} \{\gamma_A + \gamma_B - \gamma_A \cdot \gamma_B\}, \\ \bigcup_{\eta_A \in \tilde{I}_A, \eta_B \in \tilde{I}_B} \{\eta_A \cdot \eta_B\}, \\ \bigcup_{\xi_A \in \tilde{F}_A, \xi_B \in \tilde{F}_B} \{\xi_A \cdot \xi_B\} \end{matrix} \right\rangle;$$

$$(4) A \cdot B = \left\langle \begin{matrix} \bigcup_{\gamma_A \in \tilde{T}_A, \gamma_B \in \tilde{T}_B} \{\gamma_A \cdot \gamma_B\}, \\ \bigcup_{\eta_A \in \tilde{I}_A, \eta_B \in \tilde{I}_B} \{\eta_A + \eta_B - \eta_A \cdot \eta_B\}, \\ \bigcup_{\xi_A \in \tilde{F}_A, \xi_B \in \tilde{F}_B} \{\xi_A + \xi_B - \xi_A \cdot \xi_B\} \end{matrix} \right\rangle.$$

Apparently, if there is only one specified number in  $\tilde{T}_A, \tilde{I}_A$  and  $\tilde{F}_A$ , then the operations in Definition 8 are reduced to the operations for MVNNs as follows:

$$(5) \lambda A = \{ \langle 1 - (1 - T_A)^\lambda, (I_A)^\lambda, (F_A)^\lambda \rangle \};$$

$$(6) A^\lambda = \{ \langle (T_A)^\lambda, 1 - (1 - I_A)^\lambda, 1 - (1 - F_A)^\lambda \rangle \};$$

$$(7) A + B = \{ \langle T_A + T_B - T_A \cdot T_B, I_A \cdot I_B, F_A \cdot F_B \rangle \};$$

$$(8) A \cdot B = \{ \langle T_A \cdot T_B, I_A + I_B - I_A \cdot I_B, F_A + F_B - F_A \cdot F_B \rangle \}.$$

Note that the operations for MVNNs are coincides with operations of AIFSs in Ref. [7,34].

**Theorem 1.** Let  $A = \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle, B = \langle \tilde{T}_B, \tilde{I}_B, \tilde{F}_B \rangle, C = \langle \tilde{T}_C, \tilde{I}_C, \tilde{F}_C \rangle$  be three MVNNs, then the following equations are true.

- (1)  $A + B = B + A,$
- (2)  $A \cdot B = B \cdot A,$
- (3)  $\lambda(A + B) = \lambda A + \lambda B, \lambda > 0,$
- (4)  $(A \cdot B)^\lambda = A^\lambda + B^\lambda, \lambda > 0,$
- (5)  $\lambda_1 A + \lambda_2 A = (\lambda_1 + \lambda_2)A, \lambda_1 > 0, \lambda_2 > 0,$
- (6)  $A^{\lambda_1} \cdot A^{\lambda_2} = A^{(\lambda_1 + \lambda_2)}, \lambda_1 > 0, \lambda_2 > 0,$
- (7)  $(A + B) + C = A + (B + C),$
- (8)  $(A \cdot B) \cdot C = A \cdot (B \cdot C).$

**3.2 Comparison rules**

Based on the score function and accuracy function of AIFS [35-38], the score function, accuracy function and certainty function of a MVNN are defined in the following.

**Definition 9.** Let  $A = \langle \tilde{T}_A, \tilde{I}_A, \tilde{F}_A \rangle$  be a MVNN, and then score function  $s(A)$ , accuracy function  $a(A)$  and certainty function  $c(A)$  of an MVNN are defined as follows:

$$(1) \quad s(A) = \frac{1}{l_{\tilde{T}_A} \cdot l_{\tilde{I}_A} \cdot l_{\tilde{F}_A}} \times \sum_{\gamma_i \in \tilde{T}_A, \eta_j \in \tilde{I}_A, \xi_k \in \tilde{F}_A} (\gamma_i + 1 - \eta_j + 1 - \xi_k) / 3 ;$$

$$(2) \quad a(A) = \frac{1}{l_{\tilde{T}_A} \cdot l_{\tilde{F}_A}} \sum_{\gamma_i \in \tilde{T}_A, \xi_k \in \tilde{F}_A} (\gamma_i - \xi_k) ;$$

$$(3) \quad c(A) = \frac{1}{l_{\tilde{T}_A}} \sum_{\gamma_i \in \tilde{T}_A} \gamma_i .$$

Where  $\gamma_i \in \tilde{T}_A, \eta_j \in \tilde{I}_A, \xi_k \in \tilde{F}_A, l_{\tilde{T}_A}, l_{\tilde{I}_A}$  and  $l_{\tilde{F}_A}$  denotes the element numbers in  $\tilde{T}_A, \tilde{I}_A$  and  $\tilde{F}_A$ , respectively.

The score function is an important index in ranking the MVNNs. For a MVNN  $A$ , the truth-membership  $\tilde{T}_A$  is bigger, the MVNN is greater. And the indeterminacy-membership  $\tilde{I}_A$  is less, the MVNN is greater. Similarly, the false-membership  $\tilde{F}_A$  is smaller, the MVNN is greater. For the accuracy function, if the difference between truth and falsity is bigger, then the statement is more affirmative. That is, the larger the values of  $\tilde{T}_A, \tilde{I}_A$  and  $\tilde{F}_A$ , the more the accuracy of the MVNN. As to the certainty function, the value of truth- membership  $\tilde{T}_A$  is bigger, it means more certainty of the MVNSN.

On the basis of Definition 9, the method to compare MVNNs can be defined as follows.

**Definition 10.** Let  $A$  and  $B$  be two MVNNs. The comparison methods can be defined as follows:

- (1) If  $s(A) > s(B)$ , then  $A$  is greater than  $B$ , that is,  $A$  is superior to  $B$ , denoted by  $A \succ B$ .
- (2) If  $s(A) = s(B)$  and  $a(A) > a(B)$ , then  $A$  is greater than  $B$ , that is,  $A$  is superior to  $B$ , denoted by  $A \succ B$ .
- (3) If  $s(A) = s(B)$ ,  $a(A) = a(B)$  and  $c(A) > c(B)$ , then  $A$  is greater than  $B$ , that is,  $A$  is superior to  $B$ , denoted by  $A \succ B$ .

(4) If  $s(A) = s(B)$ ,  $a(A) = a(B)$  and  $c(A) = c(B)$ ,

then  $A$  is equal to  $B$ , that is,  $A$  is indifferent to  $B$ , denoted by  $A \sim B$ .

**4. Aggregation operators of MVNNs and their application to multi-criteria decision-making problems**

In this section, applying the MVNSs operations, we present aggregation operators for MVNNs and propose a method for MCDM by utilizing the aggregation operators.

**4.1 MVNN aggregation operators**

**Definition 11.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be

a collection of MVNNs, and let

$$MVNNWA : MVNN^n \rightarrow MVNN,$$

$$MVNNWA (A_1, A_2, \dots, A_n) = w_1 A_1 + w_2 A_2 + \dots + w_n A_n = \sum_{j=1}^n w_j A_j, \tag{1}$$

then  $MVNNWA$  is called the multi-valued neutrosophic number weighted averaging operator of dimension  $n$ ,

where  $W = (w_1, w_2, \dots, w_n)$  is the weight vector of  $A_j$  ( $j = 1, 2, \dots, n$ ), with  $w_j \geq 0$  ( $j = 1, 2, \dots, n$ ) and

$$\sum_{j=1}^n w_j = 1.$$

**Theorem 2.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be a collection of MVNNs,  $W = (w_1, w_2, \dots, w_n)$  be the weight vector of  $A_j$  ( $j = 1, 2, \dots, n$ ), with  $w_j \geq 0$  ( $j = 1, 2, \dots, n$ ) and  $\sum_{j=1}^n w_j = 1$ , then their aggregated

result using the  $MVNNWA$  operator is also an MVNN, and

$$MVNNWA (A_1, A_2, \dots, A_n) = \left\langle \bigcup_{\gamma_j \in \tilde{T}_{A_j}} \left\{ 1 - \prod_{j=1}^n (1 - \gamma_j)^{w_j} \right\}, \bigcup_{\eta_j \in \tilde{I}_{A_j}} \left\{ \prod_{j=1}^n \eta_j^{w_j} \right\}, \bigcup_{\xi_j \in \tilde{F}_{A_j}} \left\{ \prod_{j=1}^n \xi_j^{w_j} \right\} \right\rangle \tag{2}$$

Where  $W = (w_1, w_2, \dots, w_n)$  is the vector of

$$A_j (j = 1, 2, \dots, n), w_j \in [0, 1] \text{ and } \sum_{j=1}^n w_j = 1.$$

It is obvious that the  $MVNNWA$  operator has the following properties.

(1) (Idempotency): Let  $A_j (j = 1, 2, \dots, n)$  be a

collection of MVNNs. If all  $A_j (j = 1, 2, \dots, n)$  are equal,

i.e.,  $A_j = A$ , for all  $j \in \{1, 2, \dots, n\}$ , then

$$MVNNWA (A_1, A_2, \dots, A_n) = A.$$

(2) (Boundedness): If  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ )

is a collection of MVNNs and

$$A^- = \left\langle \min_j T_{A_j}, \max_j I_{A_j}, \max_j F_{A_j} \right\rangle,$$

$$A^+ = \left\langle \max_j T_{A_j}, \min_j I_{A_j}, \min_j F_{A_j} \right\rangle, \text{ for all}$$

$j \in \{1, 2, \dots, n\}$ , then

$$A^- \subseteq MVNNWA (A_1, A_2, \dots, A_n) \subseteq A^+.$$

(3) (Monotonicity): Let  $A_j (j = 1, 2, \dots, n)$  a collection of

MVNNs. If  $A_j \subseteq A_j^*$ , for  $j \in \{1, 2, \dots, n\}$ , then

$$SNNWA_w (A_1, A_2, \dots, A_n) \subseteq SNNWA_w (A_1^*, A_2^*, \dots, A_n^*).$$

**Definition 12.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be

a collection of MVNNs, and let

MVNNWG :  $MVNN^n \rightarrow MVNN$  :

$$MVNNWG_w(A_1, A_2, \dots, A_n) = \prod_{j=1}^n A_j^{w_j} \quad (3)$$

then MVNNWG is called an multi-valued neutrosophic number weighted geometric operator of dimension  $n$ ,

where  $W = (w_1, w_2, \dots, w_n)$  is the weight vector of

$A_j$  ( $j = 1, 2, \dots, n$ ), with  $w_j \geq 0$  ( $j = 1, 2, \dots, n$ ) and

$$\sum_{j=1}^n w_j = 1.$$

**Theorem 3.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be a collection of MVNNs, we have the following result:

$$MVNNWG_w(A_1, A_2, \dots, A_n) = \left\langle \begin{matrix} \bigcup_{\gamma_j \in \tilde{T}_{A_j}} \left\{ \prod_{j=1}^n (\gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{A_j}} \left\{ - \prod_{j=1}^n (1 - \eta_j)^{w_j} \right\}, \\ \bigcup_{\xi_j \in \tilde{F}_{A_j}} \left\{ 1 - \prod_{j=1}^n (1 - \xi_j)^{w_j} \right\} \end{matrix} \right\rangle, \quad (4)$$

where  $W = (w_1, w_2, \dots, w_n)$  is the vector of

$A_j$  ( $j = 1, 2, \dots, n$ ),  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ .

**Definition 13.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be

a collection of MVNNs, and let

MVNNOWA :  $MVNN^n \rightarrow MVNN$  :

$MVNNOWA(A_1, A_2, \dots, A_n) =$

$$w_1 A_{\sigma(1)} + w_2 A_{\sigma(2)} + \dots + w_n A_{\sigma(n)} = \sum_{j=1}^n w_j A_{\sigma(j)} \quad (5)$$

then MVNNOWA is called the multi-valued neutrosophic number ordered weighted averaging operator of dimension

$n$ , where  $A_{\sigma(j)}$  is the  $j$ -th largest value.

$W = (w_1, w_2, \dots, w_n)$  is the weight vector of  $A_j$

( $j = 1, 2, \dots, n$ ), with  $w_j \geq 0$  ( $j = 1, 2, \dots, n$ ) and

$$\sum_{j=1}^n w_j = 1.$$

**Theorem 4.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be a

collection of MVNNs,  $W = (w_1, w_2, \dots, w_n)$  be the weight

vector of  $A_j$  ( $j = 1, 2, \dots, n$ ), with  $w_j \geq 0$

( $j = 1, 2, \dots, n$ ) and  $\sum_{j=1}^n w_j = 1$ , then their aggregated

result using the MVNNOWA operator is also an MVNN,

and

$$MVNNOWA_w(A_1, A_2, \dots, A_n) = \left\langle \begin{matrix} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^n (1 - \gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^n \eta_j^{w_j} \right\}, \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^n \xi_j^{w_j} \right\} \end{matrix} \right\rangle, \quad (6)$$

where  $A_{\sigma(j)}$  is the  $j$ -th largest value according to the total

order:  $A_{\sigma(1)} \geq A_{\sigma(2)} \geq \dots \geq A_{\sigma(n)}$ .

**Definition 14.** Let  $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be

a collection of MVNNs, and let

MVNNOWG :  $MVNN^n \rightarrow MVNN$  :



$$MVNNOWG(A_1, A_2, \dots, A_n) = \prod_{j=1}^n A_{\sigma(j)}^{w_j} \quad (7)$$

then MVNNOWG is called an multi-valued neutrosophic number ordered weighted geometric operator of dimension  $n$ , where  $A_{\sigma(j)}$  is the  $j$ -th largest value and

$W = (w_1, w_2, \dots, w_n)$  is the weight vector of

$A_j (j = 1, 2, \dots, n)$ , with  $w_j \geq 0 (j = 1, 2, \dots, n)$  and

$$\sum_{j=1}^n w_j = 1.$$

**Theorem 5.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle (j = 1, 2, \dots, n)$  be a collection of MVNNs, we have the following result:

$$MVNNOWG_w(A_1, A_2, \dots, A_n) = \left\langle \begin{matrix} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^n (\gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^n (1 - \eta_j)^{w_j} \right\}, \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^n (1 - \xi_j)^{w_j} \right\} \end{matrix} \right\rangle \quad (8)$$

where  $W = (w_1, w_2, \dots, w_n)$  is the vector of

$A_j (j = 1, 2, \dots, n)$ ,  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ , and

$A_{\sigma(j)}$  is the  $j$ -th largest value according to the total

order:  $A_{\sigma(1)} \geq A_{\sigma(2)} \geq \dots \geq A_{\sigma(n)}$ .

**Definition 15.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle (j = 1, 2, \dots, n)$  be a collection of MVNNs, and let

MVNNHOWA :  $MVNN^n \rightarrow MVNN$  :

$$MVNNHOWA(A_1, A_2, \dots, A_n) = w_1 \dot{A}_{\sigma(1)} + w_2 \dot{A}_{\sigma(2)} + \dots + w_n \dot{A}_{\sigma(n)} = \sum_{j=1}^n w_j \dot{A}_{\sigma(j)} \quad (9)$$

then MVNNHOWA is called the multi-valued neutrosophic number hybrid ordered weighted averaging operator of dimension  $n$ , where  $\dot{A}_{\sigma(j)}$  is the  $j$ -th largest of the weighted value

$\dot{A}_j (\dot{A}_j = m w_j A_j, j = 1, 2, \dots, n)$ ,  $W = (w_1, w_2, \dots, w_n)$  is

the weight vector of  $A_j (j = 1, 2, \dots, n)$ , with

$w_j \geq 0 (j = 1, 2, \dots, n)$  and  $\sum_{j=1}^n w_j = 1$ , and  $n$  is the

balancing coefficient.

**Theorem 6.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle (j = 1, 2, \dots, n)$  be a collection of MVNNs,  $W = (w_1, w_2, \dots, w_n)$  be the weight vector of  $A_j (j = 1, 2, \dots, n)$ , with  $w_j \geq 0$

$(j = 1, 2, \dots, n)$  and  $\sum_{j=1}^n w_j = 1$ , then their aggregated

result using the SNNHOWA operator is also a MVNN, and

$$MVNNHOWA_w(A_1, A_2, \dots, A_n) = \left\langle \begin{matrix} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^n (1 - \gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^n \eta_j^{w_j} \right\}, \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^n \xi_j^{w_j} \right\} \end{matrix} \right\rangle \quad (10)$$

where  $\dot{A}_{\sigma(j)}$  is the  $j$ -th largest of the weighted value

$\dot{A}_j (\dot{A}_j = m w_j A_j, j = 1, 2, \dots, n)$ .

**Definition 16.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle (j = 1, 2, \dots, n)$  be

a collection of MVNNs, and let

$$MVNNHOWG : MVNN^n \rightarrow MVNN$$

$$MVNNHOWG (A_1, A_2, \dots, A_n) = \prod_{j=1}^n \dot{A}_{\sigma(j)}^{w_j} \quad (11)$$

then MVNNHOWG is called the multi-valued

neutrosophic number hybrid ordered weighted geometric

operator of dimension  $n$ , where  $\dot{A}_{\sigma(j)}$  is the  $j$ -th largest of

the weighted value

$$\dot{A}_j (\dot{A}_j = n w_j A_j, j = 1, 2, \dots, n), W = (w_1, w_2, \dots, w_n)$$

is the weight vector of  $A_j$  ( $j = 1, 2, \dots, n$ ), with

$$w_j \geq 0 \quad (j = 1, 2, \dots, n) \text{ and } \sum_{j=1}^n w_j = 1, \text{ and } n \text{ is the}$$

balancing coefficient.

**Theorem 7.** Let  $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$  ( $j = 1, 2, \dots, n$ ) be a

collection of MVNNs,  $W = (w_1, w_2, \dots, w_n)$  be the weight

vector of  $A_j$  ( $j = 1, 2, \dots, n$ ), with  $w_j \geq 0$

$$(j = 1, 2, \dots, n) \text{ and } \sum_{j=1}^n w_j = 1,$$

$$MVNNHOWG (A_1, A_2, \dots, A_n) =$$

$$\left\langle \begin{array}{l} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^n (\gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^n (1 - \eta_j)^{w_j} \right\}, \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^n (1 - \xi_j)^{w_j} \right\} \end{array} \right\rangle \quad (12)$$

Where  $\dot{A}_{\sigma(j)}$  is the  $j$ -th largest of the weighted value

$$\dot{A}_j (\dot{A}_j = A_j^{n w_j}, j = 1, 2, \dots, n).$$

Similarly, it can be proved that the mentioned operators have the same properties as the MVNNWA operator.

#### 4.2 Multi-criteria decision-making method based on the MVNN aggregation operators

Assume there are  $n$  alternatives  $A = \{a_1, a_2, \dots, a_n\}$

and  $m$  criteria  $C = \{c_1, c_2, \dots, c_m\}$ , whose criterion

weight vector is  $w = (w_1, w_2, \dots, w_m)$ , where  $w_j \geq 0$

$$(j = 1, 2, \dots, m), \sum_{j=1}^m w_j = 1.$$

Let  $R = (a_{ij})_{n \times m}$  be the

simplified neutrosophic decision matrix, where

$a_{ij} = \langle T_{a_{ij}}, I_{a_{ij}}, F_{a_{ij}} \rangle$  is a criterion value, denoted by

MVNN, where  $T_{a_{ij}}$  indicates the truth-membership

function that the alternative  $a_i$  satisfies the criterion  $c_j$ ,

$I_{a_{ij}}$  indicates the indeterminacy-membership function that

the alternative  $a_i$  satisfies the criterion  $c_j$  and  $F_{a_{ij}}$

indicates the falsity-membership function that the

alternative  $a_i$  satisfies the criterion  $c_j$ .

In the following, a procedure to rank and select the most

desirable alternative(s) is given.

**Step 1:** Aggregate the MVNNs.

Utilize the MVNNA operator or the MVNNAWG operator or MVNNHOWA operator or the MVNNHOWG to aggregate MVNNs and we can get the individual value of the alternative  $a_i$  ( $i = 1, 2, \dots, n, j = 1, 2, \dots, m$ ).

$$x_i = MVNNA_w(a_{i1}, a_i, \dots, a_{im}), \text{ or}$$

$$x_i = MVNNAWG_w(a_{i1}, a_i, \dots, a_{im}), \text{ or}$$

$$x_i = MVNNOWA_w(a_{i1}, a_i, \dots, a_{im}), \text{ or}$$

$$x_i = MVNNOWA_w(a_{i1}, a_i, \dots, a_{im}), \text{ or}$$

$$x_i = MVNNHOWA_w(a_{i1}, a_i, \dots, a_{im}), \text{ or}$$

$$x_i = MVNNHOWG_w(a_{i1}, a_i, \dots, a_{im}).$$

**Step 2:** Calculate the score function value  $s(y_i)$ , accuracy function value  $a(y_i)$  and certainty function value  $c(y_i)$  of  $y_i$  ( $i = 1, 2, \dots, m$ ) by Definition 9.

**Step 3:** Rank the alternatives. According to Definition 10, we could get the priority of the alternatives  $a_i$  ( $i = 1, 2, \dots, m$ ) and choose the best one.

**5. Illustrative example**

In this section, an example for the multi-criteria decision making problem of alternatives is used as the demonstration of the application of the proposed decision making method, as well as the effectiveness of the proposed method.

Let us consider the decision making problem adapted from Ref. [28]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money:

- (1)  $A_1$  is a car company;
- (2)  $A_2$  is a food company;
- (3)  $A_3$  is a computer company;
- (4)  $A_4$  is an arms company.

The investment company must take a decision according to the following three criteria:

- (1)  $C_1$  is the risk analysis;
- (2)  $C_2$  is the growth analysis;
- (3)  $C_3$  is the environmental impact analysis, where  $C_1$

and  $C_2$  are benefit criteria, and  $C_3$  is a cost criterion. The weight vector of the criteria is given by  $W = (0.35, 0.25, 0.4)$ . The four possible alternatives are to be evaluated under the above three criteria by the form of MVNNs, as shown in the following simplified neutrosophic decision matrix  $D$ :

$$D = \begin{bmatrix} \langle\{0.4, 0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.4\}, \{0.2, 0.3\}, \{0.3\}\rangle & \langle\{0.2\}, \{0.2\}, \{0.5\}\rangle \\ \langle\{0.6\}, \{0.1, 0.2\}, \{0.2\}\rangle & \langle\{0.6\}, \{0.1\}, \{0.2\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.1, 0.2\}\rangle \\ \langle\{0.3, 0.4\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2\}, \{0.3\}\rangle & \langle\{0.5\}, \{0.2, 0.3\}, \{0.2\}\rangle \\ \langle\{0.7\}, \{0.1, 0.2\}, \{0.1\}\rangle & \langle\{0.6\}, \{0.1\}, \{0.2\}\rangle & \langle\{0.4\}, \{0.3\}, \{0.2\}\rangle \end{bmatrix}$$

The procedures of decision making based on MVNS are shown as following.

**Step 1:** Aggregate the MVNNs.

Utilize the MVNNWA operator or the MVNNWG operator to aggregate MVNNs of each decision maker, and we can get the individual value of the alternative  $a_i$  ( $i = 1, 2, \dots, n, j = 1, 2, \dots, m$ ).

By using MVNNWA operator, the alternatives matrix  $A_{WA}$  can be obtained:

$$A_{WA} = \begin{bmatrix} \{0.327, 0.368\}, \{0.200, 0.221\}, \{0.368\} \\ \{0.563\}, \{0.132, 0.168\}, \{0.152, 0.200\} \\ \{0.438, 0.467\}, \{0.200, 0.235\}, \{0.255\} \\ \{0.574\}, \{0.155, 0.198\}, \{0.157\} \end{bmatrix}.$$

With MVNNWG operator, the alternatives matrix  $A_{WG}$  is as follows:

$$A_{WG} = \begin{bmatrix} \{0.303, 0.328\}, \{0.200, 0.226\}, \{0.388\} \\ \{0.558\}, \{0.141, 0.176\}, \{0.161, 0.200\} \\ \{0.418, 0.462\}, \{0.200, 0.242\}, \{0.262\} \\ \{0.538\}, \{0.186, 0.219\}, \{0.166\} \end{bmatrix}.$$

**Step 2:** Calculate the score function value, accuracy function value and certainty function value.

To the alternatives matrix  $A_{WA}$ , by using Definition 9, then we have:

$$s_{A_{WA}} = (0.590, 0.746, 0.660, 0.747).$$

Apparently, there is no need to compute accuracy function value and certainty function value.

To the alternatives matrix  $A_{WG}$ , by using Definition 10, the function matrix of  $A_{WG}$  is as follows:

$$s_{A_{WG}} = (0.571, 0.739, 0.653, 0.723).$$

Apparently, there is no need to compute accuracy function value and certainty function value else.

**Step 3:** Get the priority of the alternatives and choose the best one.

According to Definition 10 and results in step 2, for  $A_{WA}$ , we have  $a_4 \succ a_2 \succ a_3 \succ a_1$ . Obviously, the best alternative is  $a_4$ . for  $A_{WG}$ , we have  $a_2 \succ a_4 \succ a_3 \succ a_1$ . Obviously, the best alternative is  $a_2$ .

Similarly, if the other two aggregation operators are utilized, then the results can be founded in Table 1.

From the results in Table 1, we can see that if the *MVNNOWA* and *MVNNHOWA* are utilized in Step 1, then we can obtain the results:  $a_4 \succ a_2 \succ a_3 \succ a_1$ . The best one is  $a_4$  while the worst is  $a_1$ . If the *MVNNOWG* and *MVNNHOWG* operators are used, then the final ranking is  $a_2 \succ a_4 \succ a_3 \succ a_1$ , the best one is  $a_2$  while the worst one is  $a_1$ .

In most cases, the different aggregation operator may lead to different rankings. However, all weighted average operators and all geometry operators also lead to the same rankings respectively. So we have two ranks of four alternatives and the best one is always the  $A_4$  or  $A_2$ , the worst one is always the  $A_1$ . At the same time, decision-makers can choose different aggregation operator according to their preference.

Table 1: The rankings as aggregation operator changes

Operators	The final ranking	The best alternative(s)	The worst alternative(s)
<i>MVNNWA</i>	$a_4 \succ a_2 \succ a_3 \succ a_1$	$a_4$	$a_1$
<i>MVNNWG</i>	$a_2 \succ a_4 \succ a_3 \succ a_1$	$a_2$	$a_1$
<i>MVNNHOWA</i>	$a_4 \succ a_2 \succ a_3 \succ a_1$	$a_4$	$a_1$
<i>MVNNOWG</i>	$a_2 \succ a_4 \succ a_3 \succ a_1$	$a_2$	$a_1$
<i>MVNNHOWA</i> A	$a_4 \succ a_2 \succ a_3 \succ a_1$	$a_4$	$a_1$
<i>MVNNHOWG</i> G	$a_2 \succ a_4 \succ a_3 \succ a_1$	$a_2$	$a_1$

**6. Conclusion**

MVNSs can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications. However, as a new branch of NSs, there is not enough research about MVNSs. Especially, the existing literature does not put forward the aggregation operators and MCDM method for MVNSs. Based on the related research achievements in AIFs, the operations of MVNSs

were defined. And the approach to solve MCDM problem with MVNNs was proposed. In addition, the aggregation operators of *MVNNWA*, *MVNNWG*, *MVNNOWA*, *MVNNOWG*, *MVNNHOWA* and *MVNNHOWG* were given. Thus, a MCDM method is established based on the proposed operators. Utilizing the comparison approach, the ranking order of all alternatives can be determined and the best one can be easily identified as well. An illustrative example demonstrates the application of the proposed decision making method, and the calculation is simple. In the further study, we will continue to investigate the related comparison method for MVNSs.

**Acknowledgement**

This work was supported by Humanities and Social Sciences Foundation of Ministry of Education of China.

**References**

- [1] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20**(1986)87-96.
- [2] K. Atanassov, *Intuitionistic fuzzy sets*, Springer, Heidelberg, 1999.
- [3] K. Atanassov, Two theorems for Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **110**(2000) 267-269.
- [4] K. Atanassov, New operations defined over the intuitionistic fuzzy sets, *Fuzzy Sets and Systems* 61 (1994) 137–142.
- [5] L. A. Zadeh, Fuzzy sets, *Information Control* **8**(1965)338-356.
- [6] H. W. Liu, G. J. Wang, Multi-criteria methods based on intuitionistic fuzzy sets, *European Journal Operational Research* **179**(2007)220-233.
- [7] P. Zhi, Z. Li, A novel approach to multi-attribute decision making based on intuitionistic fuzzy sets, *Expert Systems with Applications* **39**(2012) 2560-2566.
- [8] Y. T. Chen, A outcome-oriented approach to multicriteria decision analysis with intuitionistic fuzzy

- optimistic/pessimistic operators, Expert Systems with Applications **37**(2010) 7762-7774.
- [9] S. Z. Zeng, W. H. Su, Intuitionistic fuzzy ordered weighted distance operator, Knowledge-based Systems **24**(2011) 1224-1232.
- [10] Z. S. Xu, Intuitionistic fuzzy multiattribute decision making: an interactive method, IEEE Transactions on Fuzzy Systems **20** (2012) 514-525.
- [11] L. Li, J. Yang, W. Wu, Intuitionistic fuzzy hopfield neural network and its stability, Expert Systems Applications **129**(2005) 589-597.
- [12] S. Sotirov, E. Sotirova, D. Orozova, Neural network for defining intuitionistic fuzzy sets in e-learning, NIFS **15**(2009) 33-36.
- [13] T. K. Shinoj, J. J. Sunil, Intuitionistic fuzzy multisets and its application in medical diagnosis, International Journal of Mathematical and Computational Sciences **6**(2012) 34-37.
- [14] T. Chaira, Intuitionistic fuzzy set approach for color region extraction, Journal of Scientific & Industrial Research **69**(2010) 426-432.
- [15] T. Chaira, A novel intuitionistic fuzzy C means clustering algorithm and its application to medical images, Applied Soft Computing **11**(2011) 1711-1717.
- [16] B. P. Joshi, S. Kumar, Fuzzy time series model based on intuitionistic fuzzy sets for empirical research in stock market, International Journal of Applied Evolutionary Computation **3**(2012) 71-84.
- [17] K. T. Atanassov, G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems **31** (1989), 343-349.
- [18] V. Torra, Hesitant fuzzy sets, International Journal of Intelligent Systems **25** (2010) 529-539.
- [19] V. Torra, Y. Narukawa, On hesitant fuzzy sets and decision, The 18th IEEE International Conference on Fuzzy Systems, Jeju Island, Korea, 2009, 1378-1382.
- [20] G. Qian, H. Wang, X.Q. Feng, [Generalized hesitant fuzzy sets and their application in decision support system](#), Knowledge-Based Systems **37**(2013) 357-365.
- [21] B. Zhu, Z.S. Xu, M.M. Xia, Dual Hesitant Fuzzy Sets, Journal of Applied Mathematics **2012** (2012), Article ID 879629, 13.  
<http://www.hindawi.com/journals/jam/2012/879629/...>
- [22] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman, Single valued neutrosophic sets, Multispace and Multistructure **4**(2010) 410-413.
- [23] F. Smarandache, A unifying field in logics. Neutrosophy:Neutrosophic probability, set and logic, Rehoboth: American Research Press, 1999.
- [24] F. Smarandache, A Unifying Field in Logics Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, 2003.
- [25] U. Riviuccio, Neutrosophic logics: prospects and problems, Fuzzy sets and systems **159**(2008) 1860-1868.
- [26] P. Majumdar, S. K. Samant, On similarity and entropy of neutrosophic sets, Journal of Intelligent and fuzzy Systems, **2013**, [Doi:10.3233/IFS-130810](#).
- [27] J. Ye, Multicriteria decision-making method using the correlation coefficient under single-value neutrosophic environment, International Journal of General Systems **42**(4) (2013) 386-394.
- [28] J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets, Journal of Intelligent and fuzzy Systems, **2013**, [Doi:10.3233/IFS-130916](#).
- [29] H. Wang, F. Smarandache, Y.Q. Zhang and R. Sunderraman, Interval neutrosophic sets and logic: Theory and applications in computing, Hexis, Phoenix, AZ, 2005.
- [30] F. G. Lupaşcu, Interval neutrosophic sets and topology, Kybernetes **38**(3-4) (2009) 621-624.
- [31] J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems, Applied mathematical modeling, **2013**, [Doi:http://dx.doi.org/10.1016/j.apm.2013.07.020](#).
- [32] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, Journal of Intelligent and fuzzy Systems, **2013**, [Doi:10.3233/IFS-120724](#).
- [33] G. Beliakov, H. Bustince, D.P. Goswami, U.K. Mukherjee, N.R. Pal, On averaging operators for Atanassov's intuitionistic fuzzy sets, Information Sciences **181** (2011)

1116-1124.

- [34] Z.S. Xu, Choquet integrals of weighted intuitionistic fuzzy information, *Information Sciences* 180 (2010) 726–736.
- [35] Z.S. Xu, Intuitionistic fuzzy aggregation operations, *IEEE Transactions on Fuzzy Systems* 15 (2007) 1179–1187.
- [36] Z.S. Xu, Dependent uncertain ordered weighted aggregation operators, *Information Fusion* 9(2) (2008) 310-316.
- [37] R. Yager, OWA aggregation of intuitionistic fuzzy sets, *International Journal of General Systems* 38 (2009) 617–641.

Received: September 26, 2015. Accepted: November 30, 2015