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More on neutrosophic soft rough sets and its modification

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Abstract. This paper aims to introduce and discuss anew mathematical tool for dealing with uncertainties, which is a combination of neutrosophic sets, soft sets and rough sets, namely neutrosophic soft rough set model. Also, its modification is introduced. Some of their properties are studied and supported with proved propositions and many counter examples. Some of rough relations are redefined as a neutrosophic soft rough relations. Comparisons among traditional rough model, suggested neutrosophic soft rough model and its modification, by using their properties and accuracy measures are introduced. Finally, we illustrate that, classical rough set model can be viewed as a special case of suggested models in this paper.

Keywords: Neutrosophic set, soft set, rough set approximations, neutrosophic soft set, neutrosophic soft rough set approximations.

1 Introduction

In recent years, many theories based on uncertainty have been proposed, such as fuzzy set theory [36], intuitionistic fuzzy set theory [5], vague set theory [10] and interval-valued fuzzy set theory [11].

In 1982, Pawlak [22] initiated his rough set model, based on equivalence relations, as a new approach towards soft computing finding a wide application. Rough set model has been developed, in many papers, as a generalization models. These models based on reflexive relation, symmetric relation, preference relation, tolerance relation, any relation, coverings, different neighborhood operators, using uncertain function, etc. [12, 15, 16, 24, 25, 29, 32-34, 37]. Also, many papers, recently, have been appeared to apply it in many real life applications such as [2, 3, 7, 17, 27, 28, 30, 35].

In 1995, Smarandache, started his study of the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise data. Neutrosophic logic is a generalization of intuitionistic fuzzy logic. In neutrosophic logic a proposition is $t\%$ true, $i\%$ indeterminate, and $f\%$ false. For example, let's analyze the following proposition: Let $x(0.6, 0.4, 0.3)$ belongs to A means, with probability of 60% (x in A), with probability of 30% (x not in A)and with probability of 40% (undecidable).

Soft set theory [21], proposed by Molodtsov in 1999, is also a mathematical tool for dealing with uncertainties. Recently, traditional soft model has been developed and applied in some decision making problems in many papers such as [1, 4, 6, 8, 13, 14, 18, 19, 31].

In 2011, Feng et al. [9] introduced the soft rough set model and proved its properties. In 2013, Maji [20] introduced neutrosophic soft set.

In this paper, we introduce a combination of neutrosophic sets, soft sets and rough sets, called neutrosophic soft rough set model. Also, a modification of it is introduced. Basic properties and concepts of suggested models are deduced. We compare between traditional rough model and proposed models to illustrate that traditional rough model is a special case of these proposed models.

2 Preliminaries

In this section we recall some definitions and properties regarding rough set, neutrosophic set, soft set and neutrosophic soft set theories required in this paper.

The following definitions and proposition are given in [22], as follows

Definition 2.1 An equivalence class of an element $x \in U$, determined by the equivalence relation E is

$$[x]_E = \{x' \in U : E(x) = E(x')\} .$$

Definition 2.2 Lower, upper and boundary approximations of a subset $X \subseteq U$ are defined as

$$\underline{E}(X) = \cup\{[x]_E : [x]_E \subseteq X\},$$

$$\overline{E}(X) = \cup\{[x]_E : [x]_E \cap X \neq \varphi\},$$

$$BND_E(X) = \overline{E}(X) - \underline{E}(X).$$

Definition 2.3 Pawlak determined the degree of crispness of any subset $X \subseteq U$ by a mathematical tool, named the accuracy measure of it, which is defined as

$$\alpha_E(X) = \underline{E}(X) / \overline{E}(X), \overline{E}(X) \neq \varnothing.$$

Properties of Pawlak's approximations are listed in the following proposition.

Proposition 2.1 Let (U, E) be a Pawlak approximation space and let $X, Y \subseteq U$. Then,

(a) $\underline{E}(X) \subseteq X \subseteq \overline{E}(X)$.

(b) $\underline{E}(\phi) = \phi = \overline{E}(\phi)$ and $\underline{E}(U) = U = \overline{E}(U)$.

(c) $\overline{E}(X \cup Y) = \overline{E}(X) \cup \overline{E}(Y)$.

(d) $\underline{E}(X \cap Y) = \underline{E}(X) \cap \underline{E}(Y)$.

(e) $X \subseteq Y$, then $\underline{E}(X) \subseteq \underline{E}(Y)$ and $\overline{E}(X) \subseteq \overline{E}(Y)$.

(f) $\underline{E}(X \cup Y) \supseteq \underline{E}(X) \cup \underline{E}(Y)$.

(g) $\overline{E}(X \cap Y) \subseteq \overline{E}(X) \cap \overline{E}(Y)$.

(h) $\underline{E}(X^c) = [\overline{E}(X)]^c$, X^c is the complement of X.

(i) $\overline{E}(X^c) = [\underline{E}(X)]^c$.

(j) $\underline{E}(\underline{E}(X)) = \overline{E}(\underline{E}(X)) = \underline{E}(X)$.

(k) $\overline{E}(\overline{E}(X)) = \underline{E}(\overline{E}(X)) = \overline{E}(X)$.

Definition 2.4 [23] An information system is a quadruple $IS = (U, A, V, f)$, where U is a non-empty finite set of objects, A is a non-empty finite set of attributes, $V = \cup\{V_e, e \in A\}$, V_e is the value set of attribute e, an $f : U \times A \to V$, is called an information (knowledge) function.

Definition 2.5 [21] Let U be an initial universe set, E be a set of parameters, $A \subseteq E$ and let $P(U)$ denotes the power set of U. Then, a pair $S = (F, A)$ is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of S.

Definition 2.6 [26] A neutrosophic set A on the universe of discourse U is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}, where$$

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+, where$$

$$T, I, F : U \to]^-0, 1^+[.$$

Definition 2.7 [20] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$, and let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by

$$F : A \to P(U).$$

3 Neutrosophic soft lower and upper concepts and their properties

In this section, neutrosophic soft rough lower and upper approximations are introduced and their properties are deduced and proved. Moreover, many counter examples are obtained.

For more illustration the meaning of neutrosophic soft set, we consider the following example

Example 3.1 Let U be a set of cars under consideration and E is the set of parameters (or qualities). Each parameter is a generalized neutrosophic word or sentence involving generalized neutrosophic words. Consider E = {beautiful, cheap, expensive, wide, modern,in good repair, costly, comfortable}. In this case, to define a neutrosophic soft set means to point out beautiful cars, cheap cars and so on. Suppose that, there are five cars in the universe U, given by, $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$, where each e_i is a specific criterion for cars: e_1 stands for (beautiful), e_2 stands for (cheap), e_3 stands for (modern), e_4 stands for (comfortable). Suppose that,

F(beautiful) =

$\{\langle h_1, 0.6, 0.6, 0.2\rangle, \langle h_2, 0.4, 0.6, 0.6\rangle, \langle h_3, 0.6, 0.4, 0.2\rangle, \langle h_4, 0.6, 0.3, 0.3\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}$

F(cheap) =

$\{\langle h_1, 0.8, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.2, 0.4\rangle, \langle h_3, 0.8, 0.1, 0.3\rangle, \langle h_4, 0.8, 0.2, 0.2\rangle, \langle h_5, 0.8, 0.3, 0.2\rangle\}$,

F(modern) =

$\{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.4, 0.3\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}$,

F(comfortable)=

$\{\langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.6, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.5, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\}$.

In order to store a neutrosophic soft set in a computer, we could represent it in the form of a table as shown in Table 1 (corresponding to the neutrosophic soft set in Example 3.1). In this table, the entries are c_{ij} corresponding to the car h_i and the parameter e_j, where c_{ij} = (true membership value of h_i, indeterminacy-membership value of h_i, falsity membership value of h_i) in $F(e_j)$. Table 1, represents the neutrosophic soft set (F, A) as follows

U	e_1	e_2	e_3	e_4
h_1	(0.6, 0.6, 0.2)	(0.8, 0.4, 0.3)	(0.7, 0.4, 0.3)	(0.8, 0.6, 0.4)
h_2	(0.4, 0.6, 0.6)	(0.6, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.6, 0.6)
h_3	(0.6, 0.4, 0.2)	(0.8, 0.1, 0.3)	(0.7, 0.2, 0.5)	(0.7, 0.6, 0.4)
h_4	(0.6, 0.3, 0.3)	(0.8, 0.2, 0.2)	(0.5, 0.2, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.2)	(0.7, 0.3, 0.4)	(0.9, 0.5, 0.7)

Table1: Tabular representation of (F, A) of Example 3.1.

Definition 3.1 Let (G, A) be a neutrosophic soft set on a universe U. For any element $h \in U$, a neutrosophic right neighborhood, with respect to $e \in A$ is defined as follows

$$h_e = \{h_i \in U : T_e(h_i) \geq T_e(h), I_e(h_i) \geq I_e(h), F_e(h_i) \leq F_e(h)\}.$$

Definition 3.2 Let (G, A) be a neutrosophic soft set on a universe U. For any element $h \in U$, a neutrosophic right neighborhood, with respect to all parameters A is defined as follows

$$h]_A = \cap\{h_{e_i} : e_i \in A\}.$$

For more illustration of Definitions 3.1 and 3.2, the following example is introduced.

Example 3.2 According Example 3.1, we can deduce the following results:

$h_{1e_1} = h_{1e_2} = h_{1e_3} = h_{1e_4} = \{h_1\}$, $h_{2e_1} = h_{2e_3} = \{h_1, h_2\}$, $h_{2e_2} = \{h_1, h_2, h_4, h_5\}$, $h_{2e_4} = \{h_1, h_2, h_3\}$, $h_{3e_1} = h_{3e_4} = \{h_1, h_3\}$, $h_{3e_2} = \{h_1, h_3, h_4, h_5\}$, $h_{3e_3} = \{h_1, h_3, h_5\}$, $h_{4e_1} = \{h_1, h_3, h_4\}$, $h_{4e_2} = \{h_4, h_5\}$, $h_{4e_3} = U$, $h_{4e_4} = \{h_1, h_2, h_3, h_4\}$, $h_{5e_1} = h_{5e_2} = h_{5e_4} = \{h_5\}$, $h_{5e_3} = \{h_1, h_5\}$. It follows that, $h_1]_A = \{h_1\}$, $h_2]_A = \{h_1, h_2\}$, $h_3]_A = \{h_1, h_3\}$, $h_4]_A = \{h_4\}$ and $h_5]_A = \{h_5\}$.

Proposition 3.1 Let (G, A) be a neutrosophic soft set on a universe U, ξ is the family of all neutrosophic right neighborhoods on it, and let

$R_e : U \to \xi, R_e(h) = h_e$. Then,

(a) R_e is reflexive relation.

(b) R_e is transitive relation.

(c) R_e may be not symmetric relation.

Proof Let

$\langle h_1, T_e(h_1), I_e(h_1), F_e(h_1)\rangle$, $\langle h_2, T_e(h_2), I_e(h_2), F_e(h_2)\rangle$ and $\langle h_3, T_e(h_3), I_e(h_3), F_e(h_3)\rangle \in G(A)$. Then,

(a) Obviously,

$T_e(h_1) = T_e(h_1)$, $I_e(h_1) = I_e(h_1)$ and $F_e(h_1) = F_e(h_1)$. Hence, for every $e \in A$, $h_1 \in h_{1e}$. Then $h_1 \; R_e \; h_1$ and then R_e is reflexive relation.

(b) Let $h_1 \; R_e \; h_2$ and $h_2 \; R_e \; h_3$. Then, $h_2 \in h_{1e}$ and $h_3 \in h_{2e}$. Hence, $T_e(h_2) \geq T_e(h_1)$, $I_e(h_2) \geq I_e(h_1)$, $F_e(h_2) \leq F_e(h_1)$, $T_e(h_3) \geq T_e(h_2)$, $I_e(h_3) \geq I_e(h_2)$ and $F_e(h_3) \leq F_e(h_2)$. Consequently, we have $T_e(h_3) \geq T_e(h_1)$, $I_e(h_3) \geq I_e(h_1)$ and $F_e(h_3) \leq F_e(h_1)$. It follows that, $h_3 \in$

h_{1e}. Then $h_1 \; R_e \; h_3$ and then R_e is transitive relation.

The following example proves (c), of Proposition 3.1.

Example 3.3 From Example 3.2, we have,

$h_{1e_1} = \{h_1\}$ and $h_{3e_1} = \{h_1, h_3\}$. Hence, $(h_3, h_1) \in R_{e_1}$ but $(h_1, h_3) \notin R_{e_1}$. Then, R_e isn't symmetric relation.

Neutrosophic soft lower and upper approximations are defined as follows

Definition 3.3. Let (G, A) be a neutrosophic soft set on U. Then, neutrosophic soft lower, upper and boundary approximations of $X \subseteq U$, respectively, are

$$\underline{NR}X = \cup\{h]_A : h \in U, h]_A \subseteq X\},$$

$$\overline{NR}X = \cup\{h]_A : h \in U, h]_A \cap X \neq \varnothing\},$$

$$b_{NR}X = \overline{NR}X - \underline{NR}X.$$

Properties of neutrosophic soft rough set approximations are introduced in the following proposition.

Proposition 3.2 Let (G, A) be a neutrosophic soft set on U, and let $X, Z \subseteq U$. Then the following properties hold

(a) $\underline{NR}X \subseteq X \subseteq \overline{NR}X$.

(b) $\underline{NR}\varnothing = \overline{NR}\varnothing = \varnothing$.

(c) $\underline{NR}U = \overline{NR}U = U$.

(d) $X \subseteq Z \Rightarrow \underline{NR}X \subseteq \underline{NR}Z$.

(e) $X \subseteq Z \Rightarrow \overline{NR}X \subseteq \overline{NR}Z$.

(f) $\underline{NR}(X \cap Z) = \underline{NR}X \cap \underline{NR}Z$.

(g) $\underline{NR}(X \cup Z) \supseteq \underline{NR}X \cup \underline{NR}Z$.

(h) $\overline{NR}(X \cap Z) \subseteq \overline{NR}X \cap \overline{NR}Z$.

(i) $\overline{NR}(X \cup Z) = \overline{NR}X \cup \overline{NR}Z$.

Proof

(a) From Definition 3.3, obviously, we can deduce that, $\underline{NR}X \subseteq X$. Also, let $h \in X$, but R_e, defined in Proposition 3.1, is reflexive relation. Then, for all $e \in A$, there exists h_e such that, $h \in h_e$, and then $h \in h]_A$. So $h]_A \cap X \neq \varnothing$. Hence, $h \in \overline{NR}X$. Therefor $\underline{NR}X \subseteq X \subseteq \overline{NR}X$.

(b) Proof of (b), follows directly, from Definition 3.3 and Property (a).

(c) From Property (a), we have $U \subseteq \overline{NR}U$, but U is the universe set, then $\overline{NR}U = U$. Also, from Definition 3.3, we have $\underline{NR}U = \cup\{h]_A : h]_A \subseteq U\}$, but for all $h \in U$, we have $h \in h]_A \subseteq U$. Hence, $\underline{NR}U = U$. Therefor $\underline{NR}U = \overline{NR}U = U$.

(d) Let $X \subseteq Z$ and $p \in \underline{NR}X$. Then, there exists $h]_A$ such that, $p \in h]_A \subseteq X$. But $X \subseteq Z$, then $p \in h]_A \subseteq Z$. Hence, $p \in \underline{NR}Z$. Therefor $\underline{NR}X \subseteq \underline{NR}Z$.

(e) Let $X \subseteq Z$ and $p \in \overline{NR}X$. Then, there exists $h]_A$ such that, $p \in h]_A$, $h]_A \cap X \neq \varnothing$. But $X \subseteq Z$, then $h]_A \cap Z \neq \varnothing$. Hence, $p \in \overline{NR}Z$. Therefor $\overline{NR}X \subseteq \overline{NR}Z$.

(f) Let $p \in \underline{NR}(X \cap Z) = \cup\{h]_A : h]_A \subseteq (X \cap Z)\}$. So, there exists $h]_A$ such that, $p \in h]_A \subseteq (X \cap Z)$, then $p \in h]_A \subseteq X$ and $p \in h]_A \subseteq Z$. Consequently, $p \in \underline{NR}X$ and $p \in \underline{NR}Z$, then $p \in \underline{NR}X \cap \underline{NR}Z$. Thus, $\underline{NR}(X \cap Z) \subseteq \underline{NR}X \cap \underline{NR}Z$. Conversely, let $p \in \underline{NR}X \cap \underline{NR}Z$. Hence $p \in \underline{NR}X$ and $p \in \underline{NR}Z$. Then there exists $h]_A$ such that, $p \in h]_A \subseteq X$ and $p \in h]_A \subseteq Z$, then $p \in h]_A \subseteq (X \cap Z)$. Consequently, $p \in \underline{NR}(X \cap Z)$, it follows that $\underline{NR}X$

$\cap\ \underline{NR}Z \subseteq \underline{NR}(X\cap Z)$. Therefor $\underline{NR}(X\cap Z) = \underline{NR}X \cap \underline{NR}Z$.

(g) Let $p \notin \underline{NR}(X\cup Z) = \cup\{h]_A : h]_A \subseteq X\cup Z\}$. So, for all $h]_A$, such that $p\in h]_A$, we have $h]_A \not\subseteq X\cup Z$, then for all $h]_A$ containing p, we have $h]_A \not\subseteq X$ and $h]_A \not\subseteq Z$. Consequently, $p\notin \underline{NR}X$ and $p\notin \underline{NR}Z$, then $p\notin \underline{NR}X \cup \underline{NR}Z$. Therefor $\underline{NR}(X\cup Z) \supseteq \underline{NR}X \cup \underline{NR}Z$.

(h) Let $p\in \overline{NR}(X\cap Z) = \cup\{h]_A : h]_A \cap (X\cap Z)\neq\emptyset\}$. So, there exists $h]_A$ such that, $p\in h]_A$ and $h]_A \cap (X\cap Z) \neq \emptyset$, then $h]_A \cap X \neq \emptyset$ and $h]_A \cap Z \neq \emptyset$. Consequently, $p\in \overline{NR}X$ and $p\in \overline{NR}Z$, then $p\in \overline{NR}X \cap \overline{NR}Z$. Therefor $\overline{NR}(X\cap Z) \subseteq \overline{NR}X \cap \overline{NR}Z$.

(i) Let $p\notin \overline{NR}(X\cup Z) = \cup\{h]_A : h]_A \cap (X\cup Z)\neq\emptyset\}$. So, for all $h]_A$ containing p, we have $h]_A \cap (X\cup Z) = \emptyset$, then for all $h]_A$ containing p, we have $h]_A \cap X = \emptyset$ and $h]_A \cap Z = \emptyset$. Consequently, $p\notin \overline{NR}X$ and $p\notin \overline{NR}Z$, then $p\notin \overline{NR}X \cup \overline{NR}Z$. Then, $\overline{NR}(X\cup Z) \supseteq \overline{NR}X \cup \overline{NR}Z$. Conversely, let $p\in \overline{NR}(X\cup Z)$. Then, there exists $h]_A$ such that, $p\in h]_A$ and $h]_A \cap (X\cup Z) \neq \emptyset$, it follows that, $h]_A \cap X \neq \emptyset$ or $h]_A \cap Z \neq \emptyset$. Consequently, $p\in \overline{NR}X$ or $p\in \overline{NR}Z$, hence, $p\in \overline{NR}X \cup \overline{NR}Z$, then $\overline{NR}X \cup \overline{NR}Z \supseteq \overline{NR}(X\cup Z)$. Therefor $\overline{NR}X \cup \overline{NR}Z = \overline{NR}(X\cup Z)$.

The following example illustrates that, containments of Property (a), may be proper.

Example 3.4 From Example 3.1, If $X=\{h_1\}$, then $\underline{NR}X = \{h_1\}$ and $\overline{NR}X = \{h_1,h_2,h_3\}$. Hence, $\underline{NR}X \neq X$ and $X \neq \overline{NR}X$.

The following example illustrates that, containments of Properties (d) and (e), may be proper.

Example 3.5 From Example 3.1, If $X = \{h_2\}$ and $Z=\{h_2,h_4\}$, then $\underline{NR}X = \emptyset$, $\underline{NR}Z = \{h_4\}$, $\overline{NR}X = \{h_1,h_2\}$ and $\overline{NR}Z = \{h_1,h_2,h_4\}$. Hence, $\underline{NR}X \neq \underline{NR}Z$ and $\overline{NR}X \neq \overline{NR}Z$.

The following example illustrates that, a containment of Property (g), may be proper.

Example 3.6 From Example 3.1, If $X = \{h_1\}$ and $Z=\{h_2\}$, then $\underline{NR}X = \{h_1\}$, $\underline{NR}Z = \emptyset$ and $\underline{NR}(X\cup Z) = \{h_1,h_2\}$. Therefor $\underline{NR}(X\cup Z) \neq \underline{NR}X \cup \underline{NR}Z$.

The following example illustrates that, a containment of Property (h), may be proper.

Example 3.7 From Example 3.1, If $X = \{h_1,h_4\}$ and $Z=\{h_2,h_4\}$, then $\overline{NR}X = \{h_1,h_2,h_3,h_4\}$, $\overline{NR}Z = \{h_1,h_2,h_4\}$ and $\overline{NR}(X\cap Z) = \{h_4\}$. Therefor $\overline{NR}(X\cap Z) \neq \overline{NR}X \cap \overline{NR}Z$.

Proposition 3.3 Let (G,A) be a neutrosophic soft set on a unverse U, and let $X,Z\subseteq U$. Then the following properties hold.

(a) $\underline{NR}\ \underline{NR}X = \underline{NR}X$.

(b) $\underline{NR}\ \overline{NR}X = \overline{NR}X$.

Proof

(a) Let $W = \underline{NR}X$ and $p\in W = \cup\{h]_A : h]_A \subseteq X\}$. Then, there exists some $h]_A$ containing p, such that $h]_A \subseteq W$. So, $p\in \underline{NR}W$. Hence, $W \subseteq \underline{NR}W$.

Thus, $\underline{NR}X \subseteq \underline{NR}\ \underline{NR}X$. Also, from Property (a), of Proposition 3.2, we have $\underline{NR}X \subseteq X$ and by using Property (d), of Proposition 3.2, we get $\underline{NR}\ \underline{NR}X \subseteq \underline{NR}X$. Therefor $\underline{NR}\ \underline{NR}X = \underline{NR}X$.

(b) Let $W = \overline{NR}X$, by using Property (a), of Proposition 3.2, we have $\underline{NR}W \subseteq W$. Conversely, let $p \in W = \cup\{h]_A : h]_A \cap X \neq \varnothing\}$, hence there exists $h]_A$ containing p such that, $p \in h]_A \subseteq W$, it follows that, $p \in \underline{NR}W$. Consequently, $W \subseteq \underline{NR}W$, then $W = \underline{NR}W$, but $W = \overline{NR}X$. Thus, $\underline{NR}\overline{NR}X = \overline{NR}X$.

Proposition 3.4 Let (G, A) be a neutrosophic soft set on U, and let $X, Z \subseteq U$. Then, the following properties don't hold

(a) $\overline{NR}\ \overline{NR}X = \overline{NR}X$.

(b) $\overline{NR}\ \underline{NR}X = \underline{NR}X$.

(c) $\underline{NR}X^c = [\overline{NR}X]^c$.

(d) $\overline{NR}X^c = [\underline{NR}X]^c$.

(e) $\underline{NR}(X-Z) = \underline{NR}X - \underline{NR}Z.$

(f) $\overline{NR}(X-Z) = \overline{NR}X - \overline{NR}Z.$

The following example proves (a) of Proposition 3.4.

Example 3.8 From Example 3.1, If $X = \{h_2\}$, then $\overline{NR}X = \{h_1, h_2\}$ and $\overline{NR}\ \overline{NR}X = \{h_1, h_2, h_3\}$. Hence, $\overline{NR}\ \overline{NR}X \neq \overline{NR}X$.

The following example proves (b) of Proposition 3.4.

Example 3.9 From Example 3.1, If $X = \{h_1\}$, then $\underline{NR}X = \{h_1\}$ and $\overline{NR}\ \underline{NR}X = \{h_1, h_2, h_3\}$. Hence, $\overline{NR}\ \underline{NR}X \neq \overline{NR}X$.

The following example proves (c) of Proposition 3.4.

Example 3.10 From Example 3.1, If $X = \{h_2\}$, then $\underline{NR}X^c = \{h_1, h_3, h_4, h_5\}$ and $[\overline{NR}X]^c = \{h_3, h_4, h_5\}$. Therefor $\underline{NR}X^c \neq [\overline{NR}X]^c$.

The following example proves (d) of Proposition 3.4.

Example 3.11 From Example 3.1, If $X = \{h_1, h_3, h_4, h_5\}$, then $\overline{NR}X^c = \{h_1, h_2\}$ and $[\underline{NR}X]^c = \{h_2\}$. Therefor $\overline{NR}X^c \neq [\underline{NR}X]^c$.

The following example proves (e), (f) of Proposition 3.4.

Example 3.12 From Example 3.1, If $X = \{h_1, h_2\}$ and $Z = \{h_1, h_3\}$, then $\underline{NR}X = \{h_1, h_2\}$, $\underline{NR}Z = \{h_1, h_3\}$, $\underline{NR}(X-Z) = \varnothing$, $\overline{NR}X = \{h_1, h_2, h_3\}$, $\overline{NR}Z = \{h_1, h_2, h_3\}$, $\overline{NR}(X-Z) = \{h_1, h_2\}$. Therefor $\underline{NR}(X-Z) \neq \underline{NR}X - \underline{NR}Z$ and $\overline{NR}(X-Z) \neq \overline{NR}X - \overline{NR}Z$.

4 Modification of suggested neutrosophic soft rough set approximations

In this section, we introduce a modification of suggested neutrosophic soft rough set approximations, introduced in Section 3. Some basic properties are introduced and proved. Finally, a comparison among traditional rough set model, suggested neutrosophic soft rough set model and its modification, by using their properties.

Modified neutrosophic soft lower and upper approximations are defined as follows

Definition 4.1 Let (G, A) be a neutrosophic soft set on U. Then, modified neutrosophic soft lower, upper and boundary approximations of $X \subseteq U$, respectively, are

$$N_R X = \cup\{h]_A : h \in U, h]_A \subseteq X\},$$

$$N^R X = [N_R X^c]^c,$$

$$b_{NR} X = N^R X - N_R X.$$

Modified neutrosophic soft lower and upper approximations properties are introduced in the following proposition.

Proposition 4.1 Let (G, A) be a neutrosophic soft set on

U, and let $X, Z \subseteq U$. Then the following properties hold

(a) $N_R X \subseteq X \subseteq N^R X$.

(b) $N_R \varnothing = N^R \varnothing = \varnothing$.

(c) $N_R U = N^R U = U$.

(d) $X \subseteq Z \Rightarrow N_R X \subseteq N_R Z$.

(e) $X \subseteq Z \Rightarrow N^R X \subseteq N^R Z$.

(f) $N_R (X \cap Z) = N_R X \cap N_R Z$.

(g) $N_R (X \cup Z) \supseteq N_R X \cup N_R Z$.

(h) $N^R (X \cap Z) \subseteq N^R X \cap N^R Z$.

(i) $N^R (X \cup Z) = N^R X \cup N^R Z$.

(j) $N_R \; N_R X = N_R X$.

(k) $N^R \; N^R X = N^R X$.

(l) $N_R X^c = [N^R X]^c$.

(m) $N^R X^c = [N_R X]^c$.

Proof

Properties (a)-(i) are proved at the same way as Proposition 3.2.

(j) Let

$W = N_R X$ and $p \in W = \cup\{h]_A : h]_A \subseteq X\}$.

Then, there exists some $h]_A$ containing p, such that $h]_A \subseteq W$. So, $p \in N_R W$. Hence, $W \subseteq N_R W$. Thus, $N_R X \subseteq N_R \; N_R X$. Also, from Property (a), of Proposition 3.2, we have $N_R X \subseteq X$ and by using Property (d), of Proposition 3.2, we can deduce that $N_R \; N_R X \subseteq N_R X$. Therefor $N_R \; N_R X = N_R X$.

(k) $N^R \; N^R X = N^R [N_R X^c]^c = [N_R ([N_R X^c]^c)^c]^c = [N_R \; N_R X^c]^c$, from Property (j) of Proposition 4.1, we can deduce that $N_R \; N_R X^c = N_R X$. Then $[N_R \; N_R X^c]^c = [N_R X^c]^c$, from Definition 4.1, we have $[N_R X^c]^c = N^R X$. Therefor $N^R \; N^R X = N^R X$.

Properties (l), (m) can be proved, directly, by using Definition 4.1.

The following example illustrates that, containments of Property (a), may be proper.

Example 4.1 From Example 3.1, If $X = \{h_1\}$, then $N_R X = \{h_1\}$ and $N^R X = \{h_1, h_2, h_3\}$. Hence, $N_R X \neq X$ and $X \neq N^R X$.

The following example illustrates that, containments of Properties (d) and (e), may be proper.

Example 4.2 From Example 3.1, If $X = \{h_2\}$ and $Z = \{h_2, h_4\}$, then $N_R X = \varnothing$, $N_R Z = \{h_4\}$, $N^R X = \{h_2\}$ and $N^R Z = \{h_2, h_4\}$. Hence, $N_R X \neq N_R Z$ and $N^R X \neq N^R Z$.

The following example illustrates that, a containment of Property (g), may be proper.

Example 4.3 From Example 3.1, If $X = \{h_1\}$ and $Z = \{h_2\}$, then $N_R Z = \varnothing$, $N_R X = \{h_1\}$ and $N_R (X \cup Z) = \{h_1, h_2\}$. Therefor $N_R (X \cup Z) \neq N_R X \cup N_R Z$.

The following example illustrates that, a containment of Property (h), may be proper.

Example 4.4 From Example 3.1, If $X = \{h_1, h_4\}$ and $Z = \{h_2, h_4\}$, then $N^R X = \{h_1, h_2, h_3, h_4\}$, $N^R Z = \{h_2, h_4\}$ and $N^R (X \cap Z) = \{h_4\}$. Therefor

$N^R(X \cap Z) \neq N^R X \cap N^R Z$.

Proposition 4.2 Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. Then, the following properties don't hold

(a) $N_R \ N^R X = N^R X$.

(b) $N^R \ N_R X = N_R X$.

(c) $N_R(X - Z) = N_R X - N_R Z.$

(d) $N^R(X - Z) = N^R X - N^R Z.$

The following example proves (a) of Proposition 4.2.

Example 4.5 From Example 3.1, If $X = \{h_2\}$, then $N^R X = \{h_2\}$ and $N_R \ N^R X = \varnothing$. Hence, $N_R \ N^R X \neq N^R X$.

The following example proves (b) of Proposition 4.2.

Example 4.6 From Example 3.1, If $X = \{h_1\}$, then $N_R X = \{h_1\}$ and $N^R \ N_R X = \{h_1, h_2, h_3\}$. Hence, $N^R \ N_R X \neq N^R X$.

The following example proves (c), (d) of Proposition 4.2.

Example 4.7 From Example 3.1, If $X = \{h_1, h_2\}$ and $Z = \{h_1, h_3\}$, then $N_R X = \{h_1, h_2\}$, $N_R Z = \{h_1, h_3\}$, $N_R(X - Z) = \varnothing$, $N^R X = \{h_1, h_2, h_3\}$, $N^R Z = \{h_1, h_2, h_3\}$, $N^R(X - Z) = \{h_2\}$. Therefor $N_R(X - Z) \neq N_R X - N_R Z$ and $N^R(X - Z) \neq N^R X - N^R Z$.

Remark 4.1 A comparison among traditional rough model, suggested neutrosophic soft rough model and its modification, by using their properties, is concluded in Table 2, where traditional rough are symboled by (T), neutrosophic soft rough by(N), its modification by (M) and (*) means that, this property is satisfied, as follows

Rough properties	T	N	M
$\underline{E}(\varnothing) = \overline{E}(\varnothing) = \varnothing$	*	*	*
$\underline{E}(U) = \overline{E}(U) = U$	*	*	*
$\underline{E}(X) \subseteq X \subseteq \overline{E}(X)$	*	*	*
$\overline{E}(X \cup Y) = \overline{E}(X) \cup \overline{E}(Y)$	*	*	*
$\underline{E}(X \cap Y) = \underline{E}(X) \cap \underline{E}(Y)$	*	*	*
$\overline{E}(X \cap Y) \subseteq \overline{E}(X) \cap \overline{E}(Y)$	*	*	*
$\underline{E}(X \cup Y) \supseteq \underline{E}(X) \cup \underline{E}(Y)$	*	*	*
$\underline{E}(X^c) = [\overline{E}(X)]^c$	*		*
$\overline{E}(X^c) = [\underline{E}(X)]^c$	*		*
$X \subseteq Y \rightarrow \underline{E}(X) \subseteq \underline{E}(Y)$	*	*	*
$X \subseteq Y \rightarrow \overline{E}(X) \subseteq \overline{E}(Y)$	*	*	*
$\underline{E}(\underline{E}(X)) = \underline{E}(X)$	*	*	*
$\underline{E}(\overline{E}(X)) = \overline{E}(X)$	*	*	
$\overline{E}(\underline{E}(X)) = \underline{E}(X)$	*		
$\overline{E}(\overline{E}(X)) = \overline{E}(X)$	*		*

Table 2: Comparison among traditional rough and suggested models, by using their properties.

To compare between suggested neutrosophic soft upper approximation and its modification, the following proposition is introduced.

Proposition 4.3 Let (G, A) be a neutrosophic soft set on a unverse U. For any considered set $X \subseteq U$, the following property holds

$$N^R X \subseteq \overline{NRX}$$

Proof Obvious.

The following example illustrates that a containment relationship between suggested neutrosophic soft upper and its modification, may be proper.

Example 4.7 According to Example 3.1, Table 3 can be created as follows

X	$N^R X$	$\overline{NR}X$
$\{h_2\}$	$\{h_2\}$	$\{h_1,h_2\}$
$\{h_3\}$	$\{h_3\}$	$\{h_1,h_3\}$
$\{h_2,h_3\}$	$\{h_2,h_3\}$	$\{h_1,h_2,h_3\}$
$\{h_2,h_4\}$	$\{h_2,h_4\}$	$\{h_1,h_2,h_4\}$
$\{h_2,h_5\}$	$\{h_2,h_5\}$	$\{h_1,h_2,h_5\}$
$\{h_3,h_4\}$	$\{h_3,h_4\}$	$\{h_1,h_3,h_4\}$
$\{h_3,h_5\}$	$\{h_3,h_5\}$	$\{h_1,h_3,h_5\}$
$\{h_2,h_3,h_4\}$	$\{h_2,h_3,h_4\}$	$\{h_1,h_2,h_3,h_4\}$
$\{h_2,h_3,h_5\}$	$\{h_2,h_3,h_5\}$	$\{h_1,h_2,h_3,h_5\}$
$\{h_2,h_4,h_5\}$	$\{h_2,h_4,h_5\}$	$\{h_1,h_2,h_4,h_5\}$
$\{h_3,h_4,h_5\}$	$\{h_3,h_4,h_5\}$	$\{h_1,h_3,h_4,h_5\}$
$\{h_2,h_3,h_4,h_5\}$	$\{h_2,h_3,h_4,h_5\}$	U

Table 3: Comparison between suggested upper approximation and its modification.

From Table 3, we can deduce that, for any considered set X, the modified upper approximation is decreased. It follows that its boundary region is decreased.

5 Neutrosophic soft rough concepts and their modification

In this section, some of neutrosophic soft rough concepts are defined as a generalization of rough concepts. Their modification are introduced and compare with them.

Neutrosophic soft rough NR-definability and N_R-definability of any subset $X \subseteq U$, is defined as follows

Definition 5.1. Let (G, A) be a neutrosophic soft set on U, and let $X \subseteq U$. A subset $X \subseteq U$, is called

(a) NR-definable, if $\underline{NR}X = \overline{NR}X = X$.

(b) N_R-definable, if $N_R X = N^R X = X$.

(c) Internally NR-definable, if $\underline{NR}X = X$ and $\overline{NR}X \neq X$.

(d) Internally N_R-definable, if $N_R X = X$ and $N^R X \neq X$.

(e) Externally NR-definable, if $\underline{NR}X \neq X$ and $\overline{NR}X = X$.

(f) Externally N_R-definable, if $N_R X \neq X$ and $N^R X = X$.

(g) NR-rough, if $\underline{NR}X \neq X$ and $\overline{NR}X \neq X$.

(h) N_R-rough, if $N_R X \neq X$ and $N^R X \neq X$.

Proposition 5.1 Let (G, A) be a neutrosophic soft set on U. For any considered set $X \subseteq U$, the following properties hold

(a) X is NR-definable set $\rightarrow$ X is N_R-definable set.

(b) X is externally NR-definable set $\rightarrow$ X is externally N_R-definable set.

(c) X is N_R-rough set $\rightarrow$ X is NR-rough set.

Proof Obvious.

The following example proves that the inverse of Proposition 5.1, does not hold.

Example 5.1 According to Example 3.1, Table 4 can be created, where (Ex) means externally definable and (R) means rough as follows

Ex- NR	Ex- N_R	N_R -R	NR -R
	$\{h_2\}$		$\{h_2\}$
	$\{h_3\}$		$\{h_3\}$
	$\{h_2,h_3\}$		$\{h_2,h_3\}$
	$\{h_2,h_4\}$		$\{h_2,h_4\}$
	$\{h_2,h_5\}$		$\{h_2,h_5\}$
	$\{h_3,h_4\}$		$\{h_3,h_4\}$
	$\{h_3,h_5\}$		$\{h_3,h_5\}$
	$\{h_2,h_3,h_4\}$		$\{h_2,h_3,h_4\}$
	$\{h_2,h_3,h_5\}$		$\{h_2,h_3,h_5\}$
	$\{h_2,h_4,h_5\}$		$\{h_2,h_4,h_5\}$
	$\{h_3,h_4,h_5\}$		$\{h_3,h_4,h_5\}$
	$\{h_2,h_3,h_4,h_5\}$		$\{h_2,h_3,h_4,h_5\}$

Table 4: Comparison between NR -definabilityand its modification.

From Table 4, it is clear that, by using a modified suggested upper approximation, any considered set has a big chance to change from NR-rough set to externally N_R -definable set. The reason of this is that its suggested modified upper approximation is decreased in some degrees.

In the following definition neutrosophic soft rough membership relations and their modifications are defined.

Definition 5.2 Let (G,A) be a neutrosophic soft set on U , and let $x \in U$, $X \subseteq U$. Then

$$x \underline{\in}_{NR} X, if \quad x \in \underline{NR}X,$$

$$x \overline{\in}_{NR} X, if \quad x \in \overline{NR}X,$$

$$x \overline{\in}_{N_R} X, if \quad x \in N^R X.$$

Proposition 5.2 Let (G,A) be a neutrosophic soft set on a unverse U , and let $x \in U$, $X \subseteq U$. Then,

$$x \underline{\in}_{NR} X \rightarrow x \in X \rightarrow x \overline{\in}_{N_R} X \rightarrow x \overline{\in}_{NR} X.$$

Proof From Propositions 3.2, 4.1 and 4.3, we can deduce that

$\underline{NR}X \subseteq X \subseteq N^R X \subseteq \overline{NR}X$. Then, by using Definition 5.2, we get the proof, directly.

The following example illustrates that, the inverse of Proposition 5.2, doesn't hold.

Example 5.2 In Example 3.1, if

$X = \{h_1\}$ and $Z = \{h_2\}$, then $\underline{NR}Z = \varnothing$, $\overline{NR}Z = \{h_1,h_2\}$, $N^R Z = \{h_2\}$ and $N^R X = \{h_1,h_2,h_3\}$. Hence, $h_2 \underline{\notin}_{NR} Z$, although, $h_2 \in Z$, $h_1 \overline{\notin}_{N_R} Z$, although, $h_1 \overline{\in}_{NR} Z$ and $h_3 \notin X$, although, $h_3 \overline{\in}_{N_R} X$.

In the following definition neutrosophic soft rough inclusion relations and their modifications are defined.

Definition 5.3 Let (G,A) be a neutrosophic soft set on U , and let $X, Z \subseteq U$. Then

$$X \underset{\rightarrow}{\subset}_{NR} Z, if \ \underline{NR}X \subseteq \underline{NR}Z,$$

$$X \overset{\rightarrow}{\subset}_{NR} Z, if \quad \overline{NR}X \subseteq \overline{NR}Z,$$

$$X \overset{\rightarrow}{\subset}_{N_R} Z, if \quad N^R X \subseteq N^R Z.$$

In the following definition neutrosophic soft rough equality relations and their modifications are defined.

Definition 5.4 Let (G, A) be a neutrosophic soft set on a unverse U , and let $X, Z \subseteq U$. Then

$$X \underset{\rightarrow NR}{=} Z, if \quad \underline{NR}X = \underline{NR}Z,$$

$$X \overset{\rightarrow}{=}_{NR} Z, if \quad \overline{NR}X = \overline{NR}Z,$$

$$X \overset{\rightarrow}{=}_{N_R} Z, if \quad N^R X = N^R Z,$$

$$X \approx_{NR} Z, if \quad X \underset{\rightarrow NR}{=} Z \quad and \quad X \overset{\rightarrow}{=}_{NR} Z,$$

$$X \approx_{N_R} Z, if \quad X \underset{\rightarrow N_R}{=} Z \quad and \quad X \overset{\rightarrow}{=}_{N_R} Z.$$

The following examples illustrate Definition 5.4.

Example 5.3 In Example 3.1, if

$X_1 = \{h_2\}$, $X_2 = \{h_3\}$, $X_3 = \{h_1, h_2\}$ and $X_4 = \{h_1, h_3\}$. Then, $\underline{NR}X_1 = \underline{NR}X_2 = \varnothing$ and $\overline{NR}X_3 = \overline{NR}X_4 = N^R X_3 = N^R X_4 = \{h_1, h_2, h_3\}$. Consequently, $X_1 \underset{\rightarrow NR}{=} X_2$, $X_3 \overset{\rightarrow}{=}_{NR} X_4$ and $X_3 \overset{\rightarrow}{=}_{N_R} X_4$.

Example 5.4 According to Example 3.1, if $A' = \{e_1, e_3\}$. Tabular representation of Neutrosophic soft set (G, A') can be seen in Table 5, as follows

A'	h_1	h_2	h_3	h_4	h_5
e_1	(.6, .6, .2)	(.4, .6, .6)	(.6, .4, .2)	(.6, .3, .3)	(.8, .2, .3)
e_3	(.7, .4, .3)	(.6, .4, .3)	(.7, .2, .5)	(.5, .2, .6)	(.7, .3, .4)

Table 5: Tabular representation of neutrosophic soft set in Example 5.4.

It follows that,

$h_1]_{A'} = \{h_1\}$, $h_2]_{A'} = \{h_1, h_2\}$, $h_3]_{A'} = \{h_1, h_3\}$, $h_4]_{A'} = \{h_1, h_3, h_4\}$ and $h_5]_{A'} = \{h_5\}$.

If we take $X_1 = \{h_3\}$ and $X_2 = \{h_3, h_4\}$, then $\underline{NR}X_1 = \underline{NR}X_2 = \varnothing$ and $\overline{NR}X_1 = \overline{NR}X_2 = \{h_1, h_3, h_4\}$ and $N^R X_1 = N^R X_2 = \{h_3, h_4\}$.

Therefor $X_1 \approx_{NR} X_2$ and $X_5 \approx_{N_R} X_2$.

Proposition 5.3 Let (G, A) be a neutrosophic soft set on a unverse U , $X, Z \subseteq U$ and let $I \in \{NR, N_R\}$.

Then,

(a) $X \underset{\rightarrow NR}{=} \underline{NR}X$.

(b) $X \underset{\rightarrow N_R}{=} N_R X$.

(c) $X \overset{\rightarrow}{=}_{N_R} N^R X$.

(d) $X = Y \rightarrow X \approx_I Z$.

(e) $X \subseteq Z \rightarrow X \underset{\rightarrow I}{\subset} Z$ and $X \overset{\rightarrow}{\subset}_I Z.$

(f) $X \subseteq Z$, $Z \underset{\rightarrow I}{=} \varnothing \rightarrow X \underset{\rightarrow I}{=} \varnothing$.

(g) $X \subseteq Z$, $X \underset{\rightarrow I}{=} U \rightarrow Z \underset{\rightarrow I}{=} U$.

(h) $X \subseteq Z$, $Z \overset{\rightarrow}{=}_I \varnothing \rightarrow X \overset{\rightarrow}{=}_I \varnothing$.

(i) $X \subseteq Z$, $X \overset{\rightarrow}{=}_I U \rightarrow Z \overset{\rightarrow}{=}_I U$.

Proof From Propositions 3.2, 3.3 and 4.1, we get the proof, directly.

We can determine the degree of neutrosophic soft NR-definability and N_R -definability of $X \subseteq U$, by using their accuracy measures denoted by $C_{NR}X$ and $C_{N_R}X$, respectively, which are defined as follows

Definition 5.5 Let (G, A) be a neutrosophic soft set on U and let $X \subseteq U$. Then,

$$C_{NR}X = \frac{\underline{\underline{NR}}X}{\overline{\overline{NR}}X}, \quad X \neq \phi,$$

$$C_{N_R}X = \frac{\underline{N}_R X}{\overline{N}^R X}, \quad X \neq \phi.$$

Proposition 5.4 Let (G, A) be a neutrosophic soft set on U and let $X \subseteq U$, the following statements are satisfied

(a) $0 \leq C_{NR}(X) \leq C_{N_R}(X) \leq 1.$

(b) X is NR-definable, if and only if, $C_{NR}(X) = 1.$

(c) X is N_R -definable, if and only if, $C_{N_R} = 1.$

Proof From Definitions 3.3, 4.1, 5.1 and 5.5, we get the proof, directly.

A comparison between suggested neutrosophic soft rough model and its modification, by using their accuracy measures, is concluded in Table 6.

Example 5.5 From Example 3.1, we can create Table 6, as follows

Accuracy measures / Sets	C_{NR}	C_{N_R}
$\{h_3, h_4\}$	0.33	0.50
$\{h_2, h_4\}$	0.33	0.50
$\{h_2, h_5\}$	0.33	0.50
$\{h_2, h_4, h_5\}$	0.50	0.67
$\{h_3, h_4, h_5\}$	0.50	0.67
$\{h_2, h_3, h_4, h_5\}$	0.40	0.50

Table 6: Comparison between suggested neutrosophic soft rough model and its modification, by using their accuracy measures.

From Table 6, by using suggested modified approximations, the degree of definability of all these subsets is increased. It means that, when we use suggested modified approximations, we notice that, for any considered neutrosophic soft rough set, its boundary region is decreased. It leads to more accurate results of any real life application.

Remark 5.1 Let (G, A) be a neutrosophic soft set on a unverse U, and let $h \in U$, $X \subseteq U$. If we consider the following case : If

$T_e(h_i) > 0.5$, then $e(h) = 1$, otherwise, $e(h) = 0$.

Hence, neutrosophic right neighborhood of an element h is replaced by the following equivalence class $[h] = \{h_i \in U: e(h_i) = e(h), \ e \in A\}$. It follows that, neutrosophic soft rough set approximations will be returned to Pawlak's rough set approximations. Consequently, all properties of traditional rough set approximations will be satisfied. Hence, Pawlak's approach to rough sets is a special case of the proposed approaches in this paper.

Conclusion

The difference in neutrosophic logic is that there is a component of indeterminate I, which means, for example in decision making and control theory, that we have ($I\%$) hesitating to take a decision. It follows that proposed models, in this paper, are more realistic than Pawlak's model. Pawlak's approach to rough sets can be viewed as a special case of neutrosophic soft approach to rough sets. Our future work, aims to apply them in solving many practical problems in medical science.

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