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Theory and Applications of Fermatean Neutrosophic Graphs

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Abstract: Yager et. al. defined a q-rung orthopair fuzzy sets as a new general class of Pythagorean fuzzy set in which the sum of the qth power of the support for and support against is bonded by one. Tapan et. al. extended the concept of intuitionistic fuzzy sets as 3-rung orthopair fuzzy sets or Fermatean fuzzy sets (FFSs). Later C. Antony et. al. introduced the concept of Fermatean Neutrosophic Sets. In this work, we define Fermatean neutrosophic graphs and present some operations on Fermatean neutrosophic graphs. Further, we introduce the concepts of regular Fermatean neutrosophic graphs, strong Fermatean neutrosophic graphs, Cartesian, Composition, Lexicographic product of Fermatean neutrosophic graphs. Finally, we give the applications of Fermatean neutrosophic graphs.

Keywords: Pythagorean Fuzzy sets, Fermatean Fuzzy sets, Fermatean Neutrosophic sets, Fermatean Neutrosophic graphs

1. Introduction

Mohamed [1, 2] introduced the concept of strong interval-valued Pythagorean fuzzy graphs and established some algebraic operations. Sangeetha et al. [3] defined the concept of Pythagorean Fuzzy Digraph (PyFDG), and PyFDG's score function in addition they proposed an algorithm for Pythagorean shortest path in package delivery robots. Peng et al. [4] introduced the concept of interval-valued Pythagorean fuzzy sets (IVPFSs) which is a generalization of Pythagorean Fuzzy Set (PFS) and interval-valued intuitionistic fuzzy set. Mohanta et al. [5] introduced the idea of Dombi picture fuzzy graph and develope some dombi picture graph operations. Akram et al. [6] proposed a new generalization of fuzzy graph, called Simplified Interval-Valued Pythagorean Fuzzy Graph (SIVPFGs), to describe uncertain information in graph theory. Then, they developed a series of operations on two SIVPFGs and investigated their properties and introduced new multi-agent decision-making approach based on SIVPFG. By integrating the concepts Pythagorean Neutrosophic

Fuzzy Graph (PNDFG) and Dombi operator, Ajay et al. [7] defined a new concept Pythagorean Neutrosophic Graphs by applying the concepts of Pythagorean Neutrosophic Set to fuzzy graph and defined some of its basic definitions and properties. Ajay et al. [8, 9] proposed Pythagorean Neutrosophic fuzzy graphs using Dombi operator called Pythagorean Neutrosophic Dombi Fuzzy Graphs and solved a decision-making problem involving the selection of the best money-transfer applications. Recently, they developed a new Multi Criteria Decision Making (MCDM) method using the Pythagorean Neutrosophic graphs. Jun et al. [10] introduced Neutrosophic Cubic Sets as the combination of cubic sets with Neutrosophic sets. They also defined different operations of such sets. Muhammad et al. [11] applied Cubic Neutrosophic Set concept on graphs and introduced the notion of Cubic Neutrosophic Graphs.

Senapati et al. [12, 13] proposed a new concept known as the Fermatean fuzzy set, in which the restrictions are that the total of the third powers of the membership grades and non-membership grades be less than one. By expanding the spatial extent of membership and non-membership grade, FFSs have a greater potential to support uncertain information. Later, they develop some Fermatean Fuzzy Sets operations. An extensively study of Fermatean Fuzzy Set and its applications is illustrated in [14 - 30]. Thamizhendhi et al. [31] defined the concept of Fermatean Fuzzy Hyper- Graphs (FFHGs) and developed some definition and properties. Operations on single valued Neutrosophic graphs are studied in [32] Further, the operations on Neutrosophic vague graphs are discussed in [33]. In [34], the authors extensively studied about the concept of single valued Neutrosophic graphs. Moreover, in [35], bipolar single valued Neutrosophic graphs are investigated with its related properties. R. Sundareswaran et. al. introduced and studied the vulnerability parameters in Neutrosophic environment in [36, 37].

Recently, Antony and Jansi [38] proposed a new emerging concept of Fermatean neutrosophic by blending the concept of Neutrosophic sets and Fermatean fuzzy sets. By employing the concept of Fermatean Neutrosophic Sets (FNSs), this paper presents the Fermatean neutrosophic graphs. Motivated by the above-mentioned works, to the best of the authors' knowledge, there is no work reported on the concepts of Fermatean neutrosophic graphs with the application. The major contributions in this work are explained as follows:

1) The notions of Fermatean Neutrosophic Graphs (FNGs) are introduced. This study makes the first attempt in the literature about the concept in Fermatean Neutrosophic graphs.
2) The importance of this new class of graphs and distinguishing this class with other existing classes are studied.
3) In addition, the complete and strong FNG are defined. The operations like a Cartesian product, lexicographic product, composition, union and the join of FNGs with their properties are discussed.
4) The optimum selection of a power plant among various power plants are identified by using FNG is made.

The layout of this article is arranged systematically as follows: Section 2 provides some basic concepts Pythagorean Fuzzy Sets (PFS), Fermatean Fuzzy Set (FFS), Pythagorean Neutrosophic Set (PNS), Fermatean Neutrosophic Set (FNS) and Pythagorean Neutrosophic Fuzzy Graph (PNFG) and we present the geometrical interpretation of Fermatean Neutrosophic Set and illustrated in subsection 2.1. In section 3, we introduce a new class of Neutrosophic graphs called Fermatean Neutrosophic Graphs with an illustration. In Section 4, we present the idea of Size and Types of degrees in Fermatean Neutrosophic Graphs. Finally, we discuss different types of Fermatean Neutrosophic Graphs in Section 5. The conclusion of this research work is summarized in the last Section.

2. Preliminaries

In this section, we provide the basic concepts and definitions in of PFS, PFN, FFS, FNS, FFR, PFR and PNFG. In 1999, Smarandache, F. introduced the following definition for Neutrosophic Sets [NS].

Definition 2.1 [39]
A **fuzzy set** (class) A in X is characterized by a membership (characteristic) function $\mathrm{f_A(x)}$ which associates with each point in X a real number in the interval $[0, 1]$, with the value of $\mathrm{f_A(x)}$ at x representing the "grade of membership" of x in A.

Definition 2.2 [40]
Let X be a non-empty set. An **intuitionistic fuzzy set** A in X is an object having the form $\mathrm{A} = \{\langle \mathrm{x}, \mu_\mathrm{A}(\mathrm{x}), \nu_\mathrm{A}(\mathrm{x})\rangle : \mathrm{x} \in \mathrm{X}\}$ where the functions $\mu_\mathrm{A}(\mathrm{x}), \nu_\mathrm{A}(\mathrm{x}) : \mathrm{X} \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $\mathrm{x} \in \mathrm{X}$ to the set A and for every element $\mathrm{x} \in \mathrm{X}, 0 \leq \mu_\mathrm{A}(\mathrm{x}) + \nu_\mathrm{A}(\mathrm{x}) \leq 1$.

Definition 2.3 [41]
Let X be the universe. A **Neutrosophic set** (NS) A in X is characterized by a truth membership function $\mathrm{T_A}$, an indeterminacy membership function $\mathrm{I_A}$, and a falsity membership function $\mathrm{F_A}$ where $\mathrm{T_A}, \mathrm{I_A}$ and $\mathrm{F_A}$ are real standard elements of [0,1]. It can be written as $\mathrm{A} = \{\langle \mathrm{x}, (\mathrm{T_A(x)}, \mathrm{I_A(x)}, \mathrm{F_A(x)}\rangle : \mathrm{x} \in \mathrm{X}, \mathrm{T_A}, \mathrm{I_A}, \mathrm{F_A} \in]0^-, 1^+[\}$. There is no restriction on the sum of $\mathrm{T_A(x)}, \mathrm{I_A(x)}$ and $\mathrm{F_A(x)}$ and so $0^- \leq \mathrm{T_A(x)} + \mathrm{I_A(x)} + \mathrm{F_A(x)} \leq 3^+$.

Definition 2.4 [42]
A **Pythagorean fuzzy set** (PFS) A on a universe of discourse X is a structure having the form as

$$\mathrm{A} = \{\langle x, T_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A(x): X \to [0,1]$ indicates the degree of membership and $F_A(x): X \to [0,1]$ indicates the degree of non-membership of every element $x \in X$ to the set A, respectively, with the constraints: $0 \leq (T_A(x))^2 + (F_A(x))^2 \leq 1$.

Definition 2.5 [7]
A **Pythagorean neutrosophic set** ($\mathbb{PN} - set$) A on a universe of discourse X is a structure having the form as

$$\mathrm{A} = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A(x): X \to [0,1]$ indicates the degree of membership, $I_A(x): X \to [0,1]$ indicates the degree of indeterminacy-membership, and $F_A(x): X \to [0,1]$ indicates the degree of non-membership of every element $x \in X$ to the set A, respectively, with the constraints: $0 \le (T_A(x))^2 + (F_A(x))^2 \le 1$ and $0 \le (I_A(x))^2 \le 1$ then $0 \le (T_A(x))^2 + (I_A(x))^2 + (T_A(x))^2 \le 2$.

Here, $T_A(x)$ and $F_A(x)$ are dependent component and $I(x)$ is independent component.

Definition 2.6 [12, 13]

A **Fermatean fuzzy set** ($\mathbb{FF}-set$) A on a universe of discourse X is a structure having the form as:

$$A = \{\langle x, T_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A(x): X \to [0,1]$ indicates the degree of membership, and $F_A(x): X \to [0,1]$ indicates the degree of non-membership of the element $x \in X$ to the set A, respectively, with the constraints :

$$0 \le (T_A(x))^3 + (F_A(x))^3 \le 1$$

Antony et al. [36] proposed the concept of Fermatean neutrosophic set considering more possible types of uncertainty including the measure of neutral membership. These are defined below

Definition 2.7 [36]

Fermatean neutrosophic set ($\mathbb{FN}-set$) A on a universe of discourse X is a structure having the form as :

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

where $T_A(x): X \to [0,1]$ indicates the degree of membership, $I_A(x): X \to [0,1]$ indicates the degree of indeterminacy-membership, and $F_A(x): X \to [0,1]$ indicates the degree of non-membership of the element $x \in X$ to the set A, respectively, with the constraints : $0 \le (T_A(x))^3 + (F_A(x))^3 \le 1$ and $0 \le (I_A(x))^3 \le 1$ then $0 \le (T_A(x))^3 + (I_A(x))^3 + (T_A(x))^3 \le 2 \quad \forall x \in X$.

Here, $T_A(x)$ and $F_A(x)$ are dependent component and $I_A(x)$ is independent component.

Definition 2.8 [43]

Let $G = (V, E)$ be **a graph which** is an ordered pair a set of vertices (nodes or points) and a set of edges (links or lines), which an edge is associated with two distinct vertices.

Definition 2.9 [44, 45]

Any fuzzy relation $\mu: S \times S \to [0,1]$ can be regarded as defining a weighted graph, or **fuzzy graph**, where the arc $(x, y) \in S \times S$, for all x, y in S has weight $\mu(X, Y) \in [0,1]$.

Definition 2.10 [46]

An **intuitionistic fuzzy graph** is defined as $G = (V, E, \mu, \nu)$, where

(i) $V = \{v_1, v_2, v_3, \dots v_n\}$ (non-empty set) such that $\mu_1 : V \longrightarrow [0,1]$, $\nu_1: V \longrightarrow [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$ for every $v_i \in V, i = 1,2 \dots, n$

(ii) $E \subset V \times V$ where $\mu_2 : V \times V \to [0,1]$and $\nu_2 : V \times V \to [0,1]$ are such that $\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\}$, $\nu_2(v_i, v_j) \leq \max\{\nu_1(v_i), \nu_1(v_j)\}$ and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, $0 \leq \mu_2(v_i, v_j), \nu_2(v_i v_j), \pi(v_i, v_j) \leq 1$ where $\pi(v_i, v_j) = 1 - \mu_2(v_i, v_j) - \nu_2(v_i, v_j)$ for every $(v_i, v_j) \in E, i = 1,2, \dots, n$

Definition 2.11 [47]

A **Neutrosophic graph** is of the form $G^* = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$ & $\mu = (T_2, I_2, F_2)$

(i) $V = \{v_1, v_2, v_3, \dots v_n\}$ such that $T_1 : V \longrightarrow [0,1]$, $I_1 : V \longrightarrow [0,1]$ and $F_1 : V \to [0,1]$ denote the degree of truth-membership function, indeterminacy –membership function and falsity-membership function of the vertex $v_1 \in V$ respectively and $0 \leq T_i(v) + I_i(v) + F_i(v) \leq 3, \forall\, v_i \in V$ ($i = 1,2,3, \dots n$).

(ii) $T_2 : V \times V \longrightarrow [0,1]$, $I_2 : V \times V \longrightarrow [0,1]$ and $F_2 : V \times V \to [0,1]$ where $T_2(v_i, v_j), I_2(v_i, v_j)$ and $F_2(v_i, v_j)$ denote the degree of truth-membership function , indeterminacy –membership function and falsity-membership function of the edge (v_i, v_j) respectively such that for every edge (v_i, v_j),

$$T_2(v_i, v_j) \leq \min\{T_1(v_i), T_1(v_j)\},$$
$$I_2(v_i, v_j) \leq \min\{I_1(v_i), I_1(v_j)\},$$
$$F_2(v_i, v_j) \leq \max\{F_1(v_i), F_1(v_j)\},$$

and $T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$.

Definition 2. 12 [1, 2]

A **Pythagorean Fuzzy Graph** on a universal set X is a pair $\mathbb{G}=(\mathcal{P}, \mathcal{Q})$ where $\mathcal{P}$ is Pythagorean fuzzy set on X and $\mathcal{Q}$ is a pythagorean fuzzy relation on X such that:

$$\begin{cases} T_{\mathcal{Q}}(u,v) \leq \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\} \\ F_{\mathcal{Q}}(u,v) \geq \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\} \end{cases}$$

and $0 \leq T_{\mathcal{Q}}^2(u,v) + F_{\mathcal{Q}}^2(u,v) \leq 1$ $for\ all\ u, v \in X$, where , $T_{\mathcal{Q}} : X \times X \to [0,1]$. $F_{\mathcal{Q}} : X \times X \to [0,1]$ indicates degree of membership, and degree of non-membership of $\mathcal{Q}$, correspondingly.

Definition 2. 13 [31]

A **Fermatean fuzzy Graph** (FFG) on a universal set X is a pair $\mathbb{G}=(\mathcal{P}, \mathcal{Q})$ where $\mathcal{P}$ is Fermatean fuzzy set on X and $\mathcal{Q}$ is a Fermatean fuzzy relation on X such that :

$$\begin{cases} T_{\mathcal{Q}}(u,v) \leq \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\} \\ F_{\mathcal{Q}}(u,v) \geq \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\} \end{cases}$$

and $0 \leq T_{\mathcal{Q}}^3(u,v) + F_{\mathcal{Q}}^3(u,v) \leq 1$ $for\ all\ u, v \in X$, where , $T_{\mathcal{Q}} : X \times X \to [0,1]$, $F_{\mathcal{Q}} : X \times X \to [0,1]$ indicates degree of membership and degree of non-membership of $\mathcal{Q}$, correspondingly. Here $\mathcal{P}$ is the Fermatean fuzzy vertex set of $\mathbb{G}$ and $\mathcal{Q}$ is the Fermatean fuzzy edge set of $\mathbb{G}$.

Definition 2. 14 [7]

Pythagorean Neutrosophic Fuzzy Graph (PNFG) is of the form $G^* = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$ & $\mu = (T_2, I_2, F_2)$

(i) $V = \{v_1, v_2, v_3, \dots v_n\}$ such that $T_1 : V \longrightarrow [0,1]$, $I_1 : V \longrightarrow [0,1]$ and $F_1 : V \rightarrow [0,1]$ denote the degree of truth-membership function , indeterminacy –membership function and falsity-membership function of the vertex $v_1 \in V$ respectively and $0 \leq T_i(v)^2 + I_i(v)^2 + F_i(v)^2 \leq 2, \forall\, v_i \in V$ ($i = 1,2\,,3, \dots n$).

(ii) $T_2 : V \times V \longrightarrow [0,1]$, $I_2 : V \times V \longrightarrow [0,1]$ and $F_2 : V \times V \rightarrow [0,1]$ where $T_2(v_i, v_j), I_2(v_i, v_j)$ and $F_2(v_i, v_j)$ denote the degree of truth-membership function , indeterminacy –membership function and falsity-membership function of the edge (v_i, v_j) respectively such that for every edge (v_i, v_j),

$$T_2(v_i, v_j) \leq \min\{\, T_1(v_i), T_1(v_j)\}\,,$$
$$I_2(v_i, v_j) \leq \min\{\, I_1(v_i), I_1(v_j)\}\,,$$
$$F_2(v_i, v_j) \leq \max\{F_1(v_i), F_1(v_j)\}\,,$$

and $T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$.

2.1 Merits and De-merits of uncertainty sets

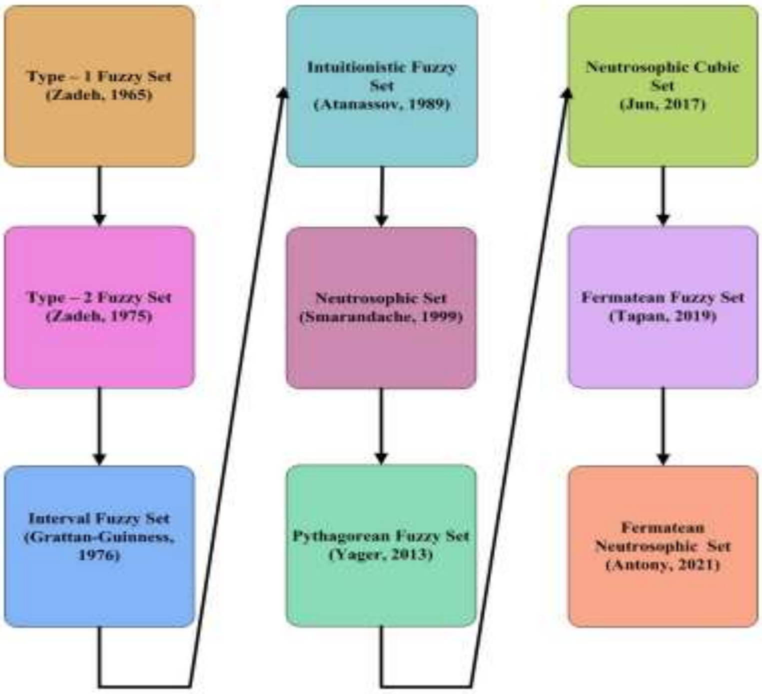
Several researchers have been introduced different kinds of sets based on the uncertainty situations. Each time, a new set is introduced, it gives an information about the limitations and advantages of the new set with a comparison of an existing one. In this section, we have listed out such discussions.

Sets	Advantages	Limitations
Fuzzy - Zadeh (1965)	Problems with uncertainty can be solved by fuzzy sets with membership values.	Decision makers can be used only membership degree $0 \leq \mu \leq 1$.
Intuitionistic Fuzzy – Atanassov (1986)	The concept of fuzzy sets is inconclusive because the exclusion of non-membership function. The IFS incorporates both membership function, μ and nonmembership function, ν with hesitation margin, π (that is, neither membership nor nonmembership functions), such that $\mu + \nu \leq 1\, and\, \mu + \nu + \pi = 1$.	Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief system. For example, when we ask the opinion of an expert about certain statement, he or she may that the possibility that the statement is true is 0.6 and the statement is false is 0.5 and the degree that he or she is not sure is 0.1
Neutrosophic – Smarandache(2019)	In Neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy membership and falsity-membership are independent. Neutrosophy was introduced by Smarandache in 1995. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra".	A Neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function $F_A . T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of]0- ,1+ [. That is $T_A : X \rightarrow]0-\,,1[$ $I_A : X \rightarrow]0-\,,1+\,[$ $F_A : X \rightarrow]0-\,,1+\,[$ There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0- \leq sup\, T_A(x) + sup\, I_A(x) + sup F_A(x) \leq 3+$.

<table>
<tr>
<td>Single valued Neutrosophic</td>
<td>The set theoretic operators on an instance of Neutrosophic set is single valued Neutrosophic set (SVNS).</td>
<td>A Single Valued Neutrosophic Set (SVNS) A in X is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$.
$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$</td>
</tr>
<tr>
<td>Pythagorean fuzzy
-Yager (2014)</td>
<td>FFS is firstly proposed by Senapati and Yager (2020) as a special case of q-rung orthopair fuzzy sets (q-ROFS). The theory of q-ROFS which is developed by Yager (2017) requires the sum of the qth power of membership (e.g., support for an idea) and non-membership (e.g., support against an idea) degrees should be equal to or smaller than 1. It is obvious that when q increases the space of acceptable orthopairs will increase and this geometric area supplies more independence to users or decision-makers while declaring their preferences, ideas, and claims. By setting q = 2, Yager (2014) rename the q-ROFS as Pythagorean fuzzy sets (PFS) and developed basic operations on them. It deals with vagueness considering the membership grade, μ and nonmembership grade, ν satisfying the conditions $\mu + \nu \le 1 \; or \; \mu + \nu \ge 1$, and, it follows that $\mu^2 + \nu^2 + \pi^2 = 1$, where π is the Pythagorean fuzzy set index.</td>
<td rowspan="2">In a voting process, a judgement may give based on a candidate satisfies his expectations with a possibility of 0.80 and this candidate dissatisfies the expectations with a possibility of 0.75. But their sum is 1.55 (>1) and their square sum is 1.20 (>1). the sum of the cubes is equal to 0.93 (<1).</td>
</tr>
<tr>
<td>Fermatean Fuzzy
- Sanapati(2019)</td>
<td>Senapati and Yager (2019) set q = 3 and this novel q-ROFS is called Fermatean fuzzy sets (FFS). Under this new concept, the decision-makers have more freedom since they can specify their ideas about agreeing (membership) and/or disagreeing (non-membership) regarding the state of a subject. It deals with vagueness considering the membership grade, μ and non-membership grade, ν satisfying the conditions $\mu + \nu \le 1 \; or \; \mu + \nu \ge 1$, and, it follows that $\mu^3 + \nu^3 + \pi^3 = 1$, where π is the Pythagorean fuzzy set index.</td>
</tr>
<tr>
<td>Pythagorean Neutrosophic</td>
<td>Pythagorean fuzzy sets has limitation that their square sum is less than or equal to 1. In neutrosophic set, if truth membership and falsity membership are 100% dependent and indeterminacy is 100% independent, that is $0 \le T_A(x) + I_A(x) + F_A(x) \le 2$. Sometimes in real life, we face many problems which cannot be handled by using neutrosophic when $T_A(x) + I_A(x) + F_A(x) > 2$. In such condition, a neutrosophic set has no ability to obtain any satisfactory result. In Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] of condition is as their square sum does not exceeds 2. Here, T and F are dependent neutrosophic components and we make $T_A(x), F_A(x)$ as Pythagorean, then $\left(T_A(x)\right)^2 + \left(F_A(x)\right)^2 \le 1$ with</td>
<td>In a voting process, a judgement may give based on a candidate satisfies his expectations with a possibility of 0.80 and this candidate dissatisfies the expectations with a possibility of 0.95 and neutrally give 0.85 But their sum is 2.80 (>2) and their square sum is 2.265 (<2). the sum of the cubes is equal to 1.9835 (<2).</td>
</tr>
</table>

	$T_A(x), F_A(x)$ *in* [0,1]. If $I_A(x)$ is an Independent from them, then $0 \leq I_A(x) \leq 1$. Then $0 \leq ((T_A(x))^2 + (F_A(x))^2 + (I_A(x))^2 \leq 2$, with $T_A(x), I_A(x), F_A(x)$ in [0,1].	
Fermatean Neutrosophic sets	Fermatean neutrosophic sets, then $(T_A(x))^3 + (F_A(x))^3 \leq 1$ with $T_A(x), F_A(x)$ *in* [0,1]. If $I_A(x)$ is an Independent from them, then $0 \leq I_A(x) \leq 1$. Then $0 \leq ((T_A(x))^3 + (F_A(x))^3 + (I_A(x))^3 \leq 2, with\ T_A(x), I_A(x), F_A(x)\ in$ [0,1].	

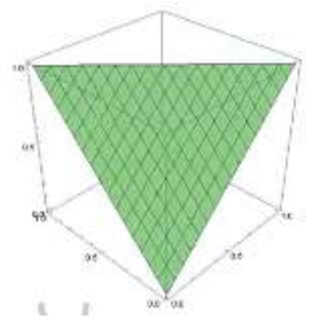
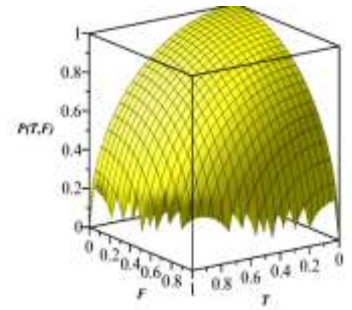
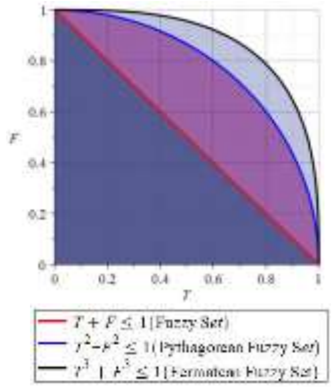
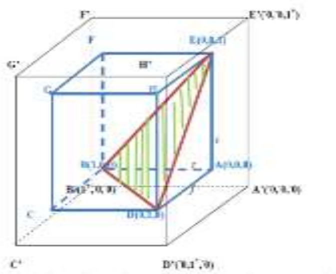
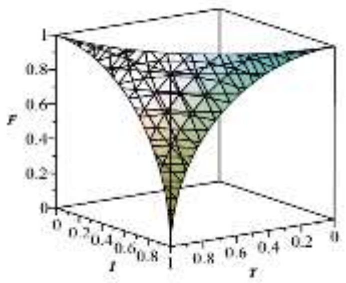
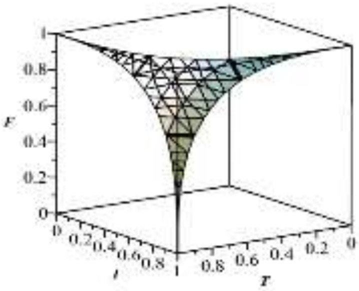
2.2 Flow chart of literature survey of uncertainty sets



2.3 Geometrical interpretation

The Graphical representation of sets which deal with uncertain may be useful to the reader to understand the flow of the T, F and I values. In this section, we give a graphical representation of membership, non-membership, and indeterminacy grades for all fuzzy sets and Neutrosophic sets.

Intuitionistic fuzzy set	**Pythagorean fuzzy set**	**Fermatean fuzzy Set (Benchmark of IFS, PFS, and FFS)**

		
Neutrosophic set	**Pythagorean Neutrosophic set**	**Fermatean Neutrosophic set**
		

3. Fermatean neutrosophic graphs

In this section, we propose the new class of graph namely, Fermatean Neutrosophic Graph which is associated with Fermatean Neutrosophic Set (FNS).

Definition 3.1: Let X be a universal set. A mapping $\mathcal{P} = (T_{\mathcal{P}}, I_{\mathcal{P}}, F_{\mathcal{P}}) : X \times X \to [0,1]$ is called a Fermatean Neutrosophic relation on X such that $T_{\mathcal{P}}(u,v)$, $I_{\mathcal{P}}(u,v)$, $F_{\mathcal{P}}(u,v) \in [0,1]$ for all $u, v \in X$.

Definition 3.2: Let $\mathcal{P} = (T_{\mathcal{P}}, I_{\mathcal{P}}, F_{\mathcal{P}})$ and $\mathcal{Q} = \left(T_{\mathcal{Q}}, I_{\mathcal{Q}}, F_{\mathcal{Q}}\right)$ be Fermatean Neutrosophic sets on X if $\mathcal{Q}$ is Fermatean Neutrosophic relation on X, then $\mathcal{Q}$ is called a Fermatean Neutrosophic relation on $\mathcal{P}$ if

$$\begin{cases} T_{\mathcal{Q}}(u,v) \leq \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\} \\ I_{\mathcal{Q}}(u,v) \geq \max\{I_{\mathcal{P}}(u), I_{\mathcal{P}}(v)\} \\ F_{\mathcal{Q}}(u,v) \geq \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\} \end{cases}$$

if $T_{\mathcal{P}}(u,v)$, $I_{\mathcal{P}}(u,v)$, $F_{\mathcal{P}}(u,v) \in [0,1]$ for all $u, v \in X$.

Definition 3.3: A **Fermatean neutrosophic graph** on a universal set X is a pair $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ where $\mathcal{P}$ is Fermatean Neutrosophic set on X and $\mathcal{Q}$ is a Fermatean Neutrosophic relation on X such that:

$$\begin{cases} T_{\mathcal{Q}}(u,v) \leq \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\} \\ I_{\mathcal{Q}}(u,v) \geq \max\{I_{\mathcal{P}}(u), I_{\mathcal{P}}(v)\} \\ F_{\mathcal{Q}}(u,v) \geq \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\} \end{cases}$$

and $0 \leq T_Q^3(u,v) + I_Q^3(u,v) + F_Q^3(u,v) \leq 2 \quad for\ all\ u,v \in X$, where , $T_Q: X \times X \to [0,1]$, $I_Q: X \times X \to [0,1]$ and $F_Q: X \times X \to [0,1]$ indicates degree of membership, degree of indeterminacy-membership and degree of non-membership of Q, correspondingly.

Here, $\mathcal{P}$ is the Fermatean Neutrosophic vertex set of $\mathbb{G}$ and Q is the Fermatean Neutrosophic edge set of $\mathbb{G}$.

An example of Fermatean Neutrosophic graph is given below.

Example 3.1 Consider a Fermatean neutrosophic graph $\mathbb{G}$=($\mathcal{P}, Q$) defined on G = (V, E), where $\mathcal{P}$ be a Fermatean Neutrosophic set on V and Q be a Fermatean Neutrosophic relation on V, defined by

$\mathcal{P}$={ $\langle v_1$,(0.6, 1,0.7)$\rangle$,$\langle$ v_2,(0.5, 0.8,0.4)$\rangle$,$\langle$ v_3,(0.7, 0.5,0.3)$\rangle$}

and

Q={ $\langle v_1v_2$, (0.4, 1,0.8)$\rangle$, $\langle v_2v_3$, (0.4, 0.9,0.6)$\rangle$, $\langle v_1v_3$, (0.5, 1,0.8)$\rangle$}

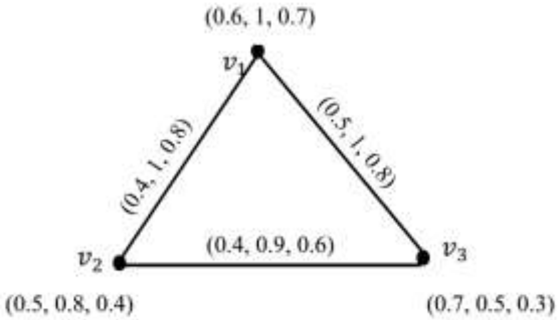


Figure 1. Fermatean Neutrosophic graph

Definition 3.4 Let $\mathbb{G}$=($\mathcal{P}, Q$) be A Fermatean neutrosophic graph $\mathbb{FNG}$ on G=(V, E). The complement of Fermatean Neutrosophic graph is $\mathbb{FNG}$ $\bar{\mathbb{G}}$=($\bar{\mathcal{P}}, \bar{Q}$) where $\bar{\mathcal{P}} = (\overline{T_{\mathcal{P}}}, \overline{I_{\mathcal{P}}}, \bar{F}_{\mathcal{P}})$ and $\bar{Q} = \left(\overline{T_Q}, \overline{I_Q}, \overline{F_Q}\right)$, defined by

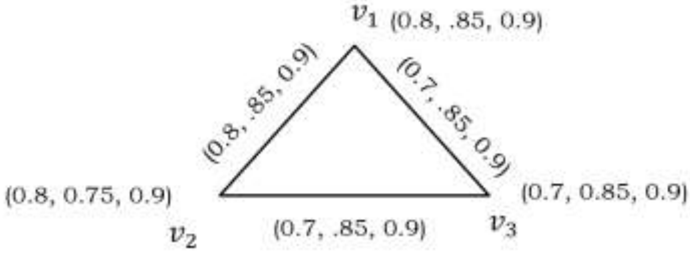
(i) $\mathcal{P} = \bar{\mathcal{P}}$

(ii) $\overline{T_{\mathcal{P}}}(u)$= $T_{\mathcal{P}}(u)$, $\overline{I_{\mathcal{P}}}(u)$= $I_{\mathcal{P}}(u)$, $\bar{F}_{\mathcal{P}}(u) = F_{\mathcal{P}}(u)\ \forall\ u \in V$

(iii)$\overline{T_Q}(uv) = \left|T_{\mathcal{P}}(u) \wedge T_{\mathcal{P}}(v) - T_Q(uv)\right|$, $\overline{T_Q}(uv) = \left|I_{\mathcal{P}}(u) \vee I_{\mathcal{P}}(v) - I_Q(uv)\right|$ and

(iv)$\overline{T_Q}(uv) = \left|F_{\mathcal{P}}(u) \vee F_{\mathcal{P}}(v) - F_Q(uv)\right|$, for all $u, v \in V$

Note: In the below example, T, I and F values are very close to 1. This situation will happen in the most of real time problems. But $0 \leq T^2 + I^2 + F^2 \nleq 2$. So , we adopt $0 \leq T^3 + I^3 + F^3 \leq 2$. Hence, we can model this situation by Fermatean Neutrosophic graphs.



4. Size and Types of degrees in Fermatean Neutrosophic graphs

The concept of regularity has been explored by many academics on fuzzy graphs and several of its generalizations. We will now propose a description on regularity of Fermatean Neutrosophic graphs ($\mathbb{FNG}$). First, we introduce few definitions in this context.

Definition 4.1 Let $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ be a Fermatean Neutrosophic graph ($\mathbb{FNG}$) defined on G = (V, E). The order of $\mathbb{G}$ is symbolized by O($\mathbb{G}$) and defined as

$$O(\mathbb{G})= (\textstyle\sum_{u\in V} T_{\mathcal{P}}(u)\,, \sum_{u\in V} I_{\mathcal{P}}(u)\,, \sum_{u\in V} F_{\mathcal{P}}(u))$$

Definition 4.2 Let $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ be a Fermatean Neutrosophic graph ($\mathbb{FNG}$) defined on G = (V, E). The size of $\mathbb{G}$ is symbolized by S($\mathbb{G}$) and defined as

$$S(\mathbb{G})= \left(\textstyle\sum_{uv\in E} T_{\mathcal{Q}}(uv)\,, \sum_{uv\in E} I_{\mathcal{Q}}(uv)\,, \sum_{uv\in E} F_{\mathcal{Q}}(uv)\right)$$

Example 4.1 Consider a Fermatean Neutrosophic graph $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ defined on G = (V, E), where $\mathcal{P}$ be a Fermatean Neutrosophic set on V and $\mathcal{Q}$ be a Fermatean Neutrosophic relation on V, defined by

$\mathcal{P}$={ $\langle v_1$,(0.6, 1,0.7)$\rangle$,$\langle v_2$,(0.5, 0.8,0.4)$\rangle$,$\langle v_3$,(0.7, 0.5,0.3)$\rangle$} and

$\mathcal{Q}$={ $\langle v_1v_2$, (0.4, 1,0.8)$\rangle$, $\langle v_2v_3$, (0.4, 0.9,0.6)$\rangle$, $\langle v_1v_3$, (0.5, 1,0.8)$\rangle$}

The order and size of Fermatean Neutrosophic graph displayed in Fig. 1 are

O($\mathbb{G}$) = (1.8, 2.3, 1.4) and S($\mathbb{G}$)= (1.3, 2.9, 2.2), respectively.

Definition 4.3 Let $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ be a Fermatean Neutrosophic graph $\mathbb{FNG}$ defined on G= (V, E). The degree of a vertex u of $\mathbb{G}$ is symbolized by $d_{\mathbb{G}}(u) = (\mathrm{d_T(u), d_I(u), d_F(u)})$ and defined as

$$d_{\mathbb{G}}(u)=(\textstyle\sum_{u\neq v} T_{\mathcal{P}}(u)\,, \sum_{u\neq v} I_{\mathcal{P}}(u)\,, \sum_{u\neq v} F_{\mathcal{P}}(u)) \text{ for } uv \in E.$$

Definition 4.4 $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ is a Fermatean Neutrosophic graph $\mathbb{FNG}$ defined on G= (V, E). The total degree of a vertex u of $\mathbb{G}$ is symbolized by $td_{\mathbb{G}}(u) = (\mathrm{td_T(u), td_I(u), td_F(u)})$ and defined as

$$td_{\mathbb{G}}(u)=\left(\textstyle\sum_{u\neq v} T_{\mathcal{Q}}(uv)+T_{\mathcal{P}}(u), \sum_{u\neq v} I_{\mathcal{Q}}(uv)+I_{\mathcal{P}}(u), \sum_{u\neq v} F_{\mathcal{Q}}(uv)+F_{\mathcal{P}}(u)\right) \text{ for } uv \in E.$$

Example 4.2. For the Fermatean Neutrosophic graph $\mathbb{G}$ in Figure 1, the degree and the total degree of the vertices are

$d_{\mathbb{G}}$ (v_1) = (1.2, 1.3, 0.7) and $td_{\mathbb{G}}$ (v_1)= (1.5, 2.8, 1.8) ;

$d_{\mathbb{G}}$ (v_2) = (1.3, 1.5, 1.0) and $td_{\mathbb{G}}$ (v_2)= (1.8, 2.5, 1.8) ;

$d_{\mathbb{G}}$ (v_3) = (1.1, 1.8, 1.1) and $td_{\mathbb{G}}$ (v_3)= (1.5, 2.8, 1.8), respectively.

The following theorem is developed to demonstrate an interesting fact regarding degree of vertices of $\mathbb{FNG}$s.

Theorem 4.1 For any Fermatean Neutrosophic graph $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ defined on V={$u_1, u_2, \dots, u_n$}, the following relation for degree of vertices of $\mathbb{G}$ must holds:

$$\sum_{j=1}^{n} \mathrm{d}_{\mathbb{G}}(\mathrm{u}_j)= 2\left(\sum_{\substack{j=1\\ i>j}}^{n-1} T_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i)\,, \sum_{\substack{j=1\\ i>j}}^{n-1} I_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i)\,, \sum_{\substack{j=1\\ i>j}}^{n-1} F_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i)\right) \text{ for all } 1 \le \mathrm{i} \le \mathrm{n}.$$

Proof : Let V={$u_1, u_2, \dots, u_n$}, and $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ be a Fermatean neutrosophic graph defined on G = (V, E)

$$\sum_{j=1}^{n} \mathrm{d}_{\mathbb{G}}(u_j)=\sum_{j=1}^{n}\left(d_T(u_j), d_I(u_j), d_F(u_j)\right)$$

$$= (d_T(u_1), d_I(u_1), d_F(u_1))+ (d_T(u_2), d_I(u_2), d_F(u_2))+\dots+(d_T(u_n), d_I(u_n), d_F(u_n))$$

$$=\left[\left(T_{\mathcal{Q}}(u_1u_2), I_{\mathcal{Q}}(u_1u_2), F_{\mathcal{Q}}(u_1u_2)\right)+\left(T_{\mathcal{Q}}(u_1u_3), I_{\mathcal{Q}}(u_1u_3), F_{\mathcal{Q}}(u_1u_3)\right)+\dots\right.$$

$$
\begin{aligned}
&+\left(T_{\mathcal{Q}}(u_1u_n), I_{\mathcal{Q}}(u_1u_n), F_{\mathcal{Q}}(u_1u_n)\right)\big] \\
&+\big[\left(T_{\mathcal{Q}}(u_2u_1), I_{\mathcal{Q}}(u_2u_1), F_{\mathcal{Q}}(u_2u_1)\right) + \left(T_{\mathcal{Q}}(u_2u_2), I_{\mathcal{Q}}(u_2u_2), F_{\mathcal{Q}}(u_2u_2)\right) +\ldots \\
&+\left(T_{\mathcal{Q}}(u_2u_n), I_{\mathcal{Q}}(u_2u_n), F_{\mathcal{Q}}(u_2u_n)\right)\big] \\
&+\big[\left(T_{\mathcal{Q}}(u_nu_1), I_{\mathcal{Q}}(u_nu_1), F_{\mathcal{Q}}(u_nu_1)\right) + \left(T_{\mathcal{Q}}(u_nu_2), I_{\mathcal{Q}}(u_nu_2), F_{\mathcal{Q}}(u_nu_2)\right) +\ldots \\
&+\left(T_{\mathcal{Q}}(u_nu_{n-1}), I_{\mathcal{Q}}(u_nu_{n-1}), F_{\mathcal{Q}}(u_nu_{n-1})\right)\big] \\
&=2\big[\left(T_{\mathcal{Q}}(u_1u_2), I_{\mathcal{Q}}(u_1u_2), F_{\mathcal{Q}}(u_1u_2)\right) + \left(T_{\mathcal{Q}}(u_1u_3), I_{\mathcal{Q}}(u_1u_3), F_{\mathcal{Q}}(u_1u_3)\right) +\ldots \\
&+\left(T_{\mathcal{Q}}(u_1u_n), I_{\mathcal{Q}}(u_1u_n), F_{\mathcal{Q}}(u_1u_n)\right)\big] \\
&+ 2\big[\left(T_{\mathcal{Q}}(u_2u_3), I_{\mathcal{Q}}(u_2u_3), F_{\mathcal{Q}}(u_2u_3)\right) +\ldots \\
&+T_{\mathcal{Q}}(u_2u_n), I_{\mathcal{Q}}(u_2u_n), F_{\mathcal{Q}}(u_2u_n)\big] +\ldots \\
&+2\left(T_{\mathcal{Q}}(u_{n-1}u_n), I_{\mathcal{Q}}(u_{n-1}u_n), F_{\mathcal{Q}}(u_{n-1}u_n)\right)\big] \\
&=2\left(\sum_{\substack{j=1\\ i>j}}^{n-1} T_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i), \sum_{\substack{j=1\\ i>j}}^{n-1} I_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i), \sum_{\substack{j=1\\ i>j}}^{n-1} F_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i)\right)
\end{aligned}
$$

Hence proved.

Theorem 4.2 For any Fermatean Neutrosophic graph $\mathbb{G}=(\mathcal{P},\mathcal{Q})$ defined on V=$\{u_1,u_2,\ldots,u_n\}$, the following relation for total degree of vertices of $\mathbb{G}$ must holds:

$$\sum_{j=1}^{n} td_{\mathbb{G}}(\mathrm{u}_j)=\left(2\sum_{\substack{j=1\\ i>j}}^{n-1} T_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i)+\sum_{j=1}^{n} T_{\mathcal{P}}(\mathrm{u}_j), 2\sum_{\substack{j=1\\ i>j}}^{n-1} I_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i)+\sum_{j=1}^{n} I_{\mathcal{P}}(\mathrm{u}_j), 2\sum_{\substack{j=1\\ i>j}}^{n-1} F_{\mathcal{Q}}(\mathrm{u}_j\mathrm{u}_i)+ + \sum_{j=1}^{n} F_{\mathcal{P}}(\mathrm{u}_j)\right), \text{ for all } 1\le i\le n.$$

Proof : The proof directly follows from Theorem 4.1 and Definition 4.4.

Definition 4.5. A Fermatean Neutrosophic graph is complete if

$$T_{\mathcal{Q}}(u,v)=\min\{T_{\mathcal{P}}(u),T_{\mathcal{P}}(v)\}$$
$$I_{\mathcal{Q}}(u,v)=\max\{I_{\mathcal{P}}(u),I_{\mathcal{P}}(v)\}$$
$$F_{\mathcal{Q}}(u,v)=\max\{F_{\mathcal{P}}(u),F_{\mathcal{P}}(v)\}$$

We illustrate it by giving an example.

Example 4.3. Let the vertex set $\mathbb{V}=\{v_1,v_2,v_3\}$ and the edge sets $\mathbb{E}$=$\{v_1v_2, v_2v_3, v_1v_3\}$ in $\mathbb{G}'$=(V,E). Take the Fermatean Neutrosophic set $\mathcal{P}=(T_{\mathcal{P}},I_{\mathcal{P}},F_{\mathcal{P}})$ in $\mathbb{V}$ and the Fermatean Neutrosophic edge sets in $\mathbb{E}\subseteq \mathbb{V}\times\mathbb{V}$ defined by

$(T_{\mathcal{P}}(v_1), I_{\mathcal{P}}(v_1), F_{\mathcal{P}}(v_1))$ = (0.6, 1, 0.7)
$(T_{\mathcal{P}}(v_2), I_{\mathcal{P}}(v_2), F_{\mathcal{P}}(v_2))$ = (0.5, 0.8, 0.4)
$(T_{\mathcal{P}}(v_3), I_{\mathcal{P}}(v_3), F_{\mathcal{P}}(v_3))$ = (0.7, 0.5, 0.3)
and
$\left(T_{\mathcal{Q}}(v_1v_2), I_{\mathcal{Q}}(v_1v_2), F_{\mathcal{Q}}(v_1v_2)\right)$ = (0.5, 1, 0.7)
$(T_{\mathcal{P}}(v_2v_3), I_{\mathcal{P}}(v_2v_3), F_{\mathcal{P}}(v_2v_3))$ = (0.5, 0.8, 0.4)
$(T_{\mathcal{P}}(v_1v_3), I_{\mathcal{P}}(v_1v_3), F_{\mathcal{P}}(v_1v_3))$ = (0.6, 1, 0.7)

Then, it is a complete $\mathbb{FNG}$.

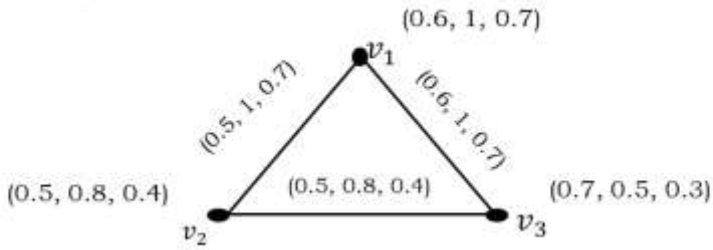


Figure 2. Complete Fermatean Neutrosophic graph

Definition 4.6: The minimum degree of Fermatean Neutrosophic graph $\mathbb{FNG}$, $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ is designated as δ($\mathbb{G}$)=($\delta_T(\mathbb{G})$, $\delta_I(\mathbb{G})$, $\delta_F(\mathbb{G})$) where,

$\delta_T(\mathbb{G})$=min$\{d_T(u)|u \in V\}$; is minimum T-degree of $\mathbb{G}$

$\delta_I(\mathbb{G})$=min$\{d_I(u)|u \in V\}$; is minimum I-degree of $\mathbb{G}$

$\delta_F(\mathbb{G})$=min$\{d_F(u)|u \in V\}$; is minimum F-degree of $\mathbb{G}$

Definition 4.7: The maximum degree of Fermatean Neutrosophic graph $\mathbb{FNG}$, $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ is designated as Δ($\mathbb{G}$)=($\Delta_T(\mathbb{G})$, $\Delta_I(\mathbb{G})$, $\Delta_F(\mathbb{G})$) where,

$\Delta_T(\mathbb{G})$=max$\{d_T(u)|u \in V\}$; is maximum T-degree of $\mathbb{G}$

$\Delta_I(\mathbb{G})$=max$\{d_I(u)|u \in V\}$; is maximum I-degree of $\mathbb{G}$

$\Delta_F(\mathbb{G})$=max$\{d_F(u)|u \in V\}$; is maximum F-degree of $\mathbb{G}$

Example 4.4. Let the vertex set $\mathbb{V} = \{v_1, v_2, v_3, v_4\}$ and the edge sets $\mathbb{E}$=$\{v_1v_2, v_2v_3, v_3v_4, v_1v_4\}$ in $\mathbb{G}'$=(V, E). Take the Fermatean Neutrosophic set $\mathcal{P} = (T_\mathcal{P}, I_\mathcal{P}, F_\mathcal{P})$ in $\mathbb{V}$ and the Fermatean Neutrosophic edge sets in $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ defined by

$(T_\mathcal{P}(v_1), I_\mathcal{P}(v_1), F_\mathcal{P}(v_1))$ = (0.3, 0.7, 0.5)

$(T_\mathcal{P}(v_2), I_\mathcal{P}(v_2), F_\mathcal{P}(v_2))$ = (0.6, 0.5, 0.7)

$(T_\mathcal{P}(v_3), I_\mathcal{P}(v_3), F_\mathcal{P}(v_3))$ = (0.8, 0.3, 0.7)

$(T_\mathcal{P}(v_4), I_\mathcal{P}(v_4), F_\mathcal{P}(v_4))$ = (0.7, 0.2, 0.4)

and

$\left(T_\mathcal{Q}(v_1v_2), I_\mathcal{Q}(v_1v_2), F_\mathcal{Q}(v_1v_2)\right)$ = (0.3, 0.7, 0.7)

$(T_\mathcal{P}(v_2v_3), I_\mathcal{P}(v_2v_3), F_\mathcal{P}(v_2v_3))$ = (0.4, 0.6, 0.7)

$(T_\mathcal{P}(v_3v_4), I_\mathcal{P}(v_3v_4), F_\mathcal{P}(v_3v_4))$ = (0.6, 0.5, 0.8)

$(T_\mathcal{P}(v_1v_4), I_\mathcal{P}(v_1v_4), F_\mathcal{P}(v_1v_4))$ = (0.3, 0.8, 0.6)

Then, it is $\mathbb{FNG}$.

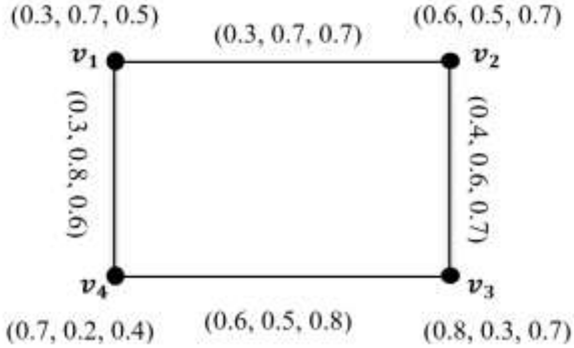


Figure 3. Minimum and maximum degree of a Fermatean Neutrosophic graph

δ(𝔾)=(0.6,1.1,1.3); Δ(𝔾)=(1,1.5,1.5)

Next, the definition of effective edge of 𝔽ℕ𝔾 are

Definition 4.9. The edge $e = (u, v)$ of 𝔾=$(\mathcal{P}, \mathcal{Q})$ be a 𝔽ℕ𝔾 is called an effective edge of 𝔾 is defined as

$$T_{\mathcal{Q}}(u, v) = \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\}$$
$$I_{\mathcal{Q}}(u, v) = \max\{I_{\mathcal{P}}(u), I_{\mathcal{P}}(v)\}$$
$$F_{\mathcal{Q}}(u, v) = \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\}$$

In Fig. 3, $v_1 v_2$ is an effective edge of 𝔽ℕ𝔾.

Definition 4.10. The effective degree of a vertex u of 𝔽ℕ𝔾, 𝔾=$(\mathcal{P}, \mathcal{Q})$ is defined by $d_{\mathcal{E}}(u)$=$(d_{\mathcal{E}_T}(u), d_{\mathcal{E}_I}(u), d_{\mathcal{E}_F}(u))$ $\forall\, u \in \mathcal{E}$; here $d_{\mathcal{E}_T}(u)$ is the sum of the T−values of the effective edges of 𝔽ℕ𝔾 incident with u, $d_{\mathcal{E}_I}(u)$is the sum of the I−values of the effective edges of 𝔽ℕ𝔾 incident with u and $d_{\mathcal{E}_F}(u)$ is the sum of the F−values of the effective edges of 𝔽ℕ𝔾 incident with u.

Definition 4.11. The minimum effective degree of 𝔾=$(\mathcal{P}, \mathcal{Q})$ in 𝔽ℕ𝔾 is designated as

$\delta_{\mathcal{E}}(𝔾)$ =($\delta_{\mathcal{E}_T}(𝔾)$, $\delta_{\mathcal{E}_I}(𝔾)$, $\delta_{\mathcal{E}_F}(𝔾)$) where,

$$\delta_{\mathcal{E}_T}(𝔾) = \wedge\{d_{\mathcal{E}_T}(u) | u \in \mathcal{P}\};$$
$$\delta_{\mathcal{E}_I}(𝔾) = \wedge\{d_{\mathcal{E}_I}(u) | u \in \mathcal{P}\};$$
$$\delta_{\mathcal{E}_F}(𝔾) = \wedge\{d_{\mathcal{E}_F}(u) | u \in \mathcal{P}\}$$

Definition 4.12. The maximum effective degree of 𝔾=$(\mathcal{P}, \mathcal{Q})$ in 𝔽ℕ𝔾 is designated as

$\Delta_{\mathcal{E}}(𝔾)$ =($\Delta_{\mathcal{E}_T}(𝔾)$, $\Delta_{\mathcal{E}_I}(𝔾)$, $\Delta_{\mathcal{E}_F}(𝔾)$) where,

$$\Delta_{\mathcal{E}_T}(𝔾) = \vee\{d_{\mathcal{E}_T}(u) | u \in \mathcal{P}\};$$
$$\Delta_{\mathcal{E}_I}(𝔾) = \vee\{d_{\mathcal{E}_I}(u) | u \in \mathcal{P}\};$$
$$\Delta_{\mathcal{E}_F}(𝔾) = \vee\{d_{\mathcal{E}_F}(u) | u \in \mathcal{P}\}$$

Example 4.6. Let the vertex set 𝕍 = $\{v_1, v_2, v_3, v_4\}$ and the edge sets 𝔼=$\{v_1 v_2, v_2 v_3, v_3 v_4, v_1 v_4\}$ in 𝔾′=(V, E).Take the Fermatean Neutrosophic set $\mathcal{P} = (T_{\mathcal{P}}, I_{\mathcal{P}}, F_{\mathcal{P}})$ in 𝕍 and the Fermatean Neutrosophic edge sets in 𝔼 ⊆ 𝕍 × 𝕍 defined by

$(T_{\mathcal{P}}(v_1), I_{\mathcal{P}}(v_1), F_{\mathcal{P}}(v_1)) = (0.3, 0.7, 0.5)$

$(T_{\mathcal{P}}(v_2), I_{\mathcal{P}}(v_2), F_{\mathcal{P}}(v_2)) = (0.6, 0.5, 0.7)$

$(T_{\mathcal{P}}(v_3), I_{\mathcal{P}}(v_3), F_{\mathcal{P}}(v_3)) = (0.8, 0.3, 0.7)$

$(T_{\mathcal{P}}(v_4), I_{\mathcal{P}}(v_4), F_{\mathcal{P}}(v_4)) = (0.7, 0.2, 0.4)$

and

$\left(T_{\mathcal{Q}}(v_1 v_2), I_{\mathcal{Q}}(v_1 v_2), F_{\mathcal{Q}}(v_1 v_2)\right) = (0.3, 0.7, 0.7)$

$(T_{\mathcal{P}}(v_2 v_3), I_{\mathcal{P}}(v_2 v_3), F_{\mathcal{P}}(v_2 v_3)) = (0.6, 0.5, 0.7)$

$(T_{\mathcal{P}}(v_3 v_4), I_{\mathcal{P}}(v_3 v_4), F_{\mathcal{P}}(v_3 v_4)) = (0.7, 0.3, 0.7)$

$(T_{\mathcal{P}}(v_1 v_4), I_{\mathcal{P}}(v_1 v_4), F_{\mathcal{P}}(v_1 v_4)) = (0.3, 0.8, 0.6)$

Then, it is 𝔽ℕ𝔾.

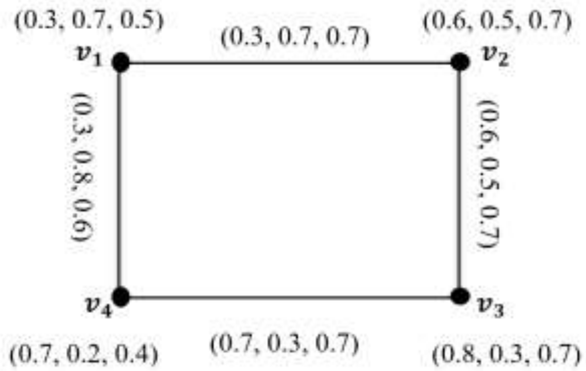


Figure 4. Fermatean Neutrosophic graph

In Fig. 4, v_1v_2, v_2v_3, v_3v_4 are the effective edges of $\mathbb{FNG}$.

$d_{\mathcal{E}}(v_1)$=(1.3,0.7,1.1)	$\delta_{\mathcal{E}}(\mathbb{G})$ =(0.8,0.3,0.7)
$d_{\mathcal{E}}(v_2)$=(1.1,1.0,1.2)	
$d_{\mathcal{E}}(v_3)$=(1.3,0.7,1.1)	$\Delta_{\mathcal{E}}(\mathbb{G})$ =(1.3,1.0,1.2)
$d_{\mathcal{E}}(v_4)$=(0.8,0.3,0.7)	

Definition 4.13. The neighborhood of any vertex u in $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$of a $\mathbb{FNG}$ is designated as $\mathcal{N}(u)$=$(\mathcal{N}_T(u), \mathcal{N}_I(u), \mathcal{N}_F(u))$ where,

$$\mathcal{N}_T(u) = \{v \in \mathcal{P}: T_{\mathcal{Q}}(u,v) = T_{\mathcal{P}}(u) \wedge T_{\mathcal{P}}(v)\};$$
$$\mathcal{N}_I(u) = \{v \in \mathcal{P}: I_{\mathcal{Q}}(u,v) = I_{\mathcal{P}}(u) \vee I_{\mathcal{P}}(v)\};$$
$$\mathcal{N}_F(u) = \{v \in \mathcal{P}: F_{\mathcal{Q}}(u,v) = F_{\mathcal{P}}(u) \vee F_{\mathcal{P}}(v)\}$$

And $\mathcal{N}[u]$= $\mathcal{N}(u) \cup u$ is called the closed neighbourhood of u.

Definition 4.14. The neighborhood degree of a vertex u in $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$of a $\mathbb{FNG}$ is designated as $\mathrm{d}_{\mathcal{N}}(u)$=$(\mathrm{d}_{\mathcal{N}_T}(u), \mathrm{d}_{\mathcal{N}_I}(u), \mathrm{d}_{\mathcal{N}_F}(u))$ where,

$$\mathrm{d}_{\mathcal{N}_T}(u) = \sum_{u\in\mathcal{N}(p)} T_{\mathcal{P}}(u),$$
$$\mathrm{d}_{\mathcal{N}_I}(u) = \sum_{u\in\mathcal{N}(p)} I_{\mathcal{P}}(u),$$
$$\mathrm{d}_{\mathcal{N}_F}(u) \sum_{u\in\mathcal{N}(p)} F_{\mathcal{P}}(u)$$

Definition 4.15. The minimum neighborhood degree of $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$ in $\mathbb{FNG}$ is designated as $\delta_{\mathcal{N}}(\mathbb{G})$=$(\delta_{\mathcal{N}_T}(\mathbb{G}), \delta_{\mathcal{N}_I}(\mathbb{G}), \delta_{\mathcal{N}_F}(\mathbb{G}))$ where,

$$\delta_{\mathcal{N}_T}(\mathbb{G}) = \wedge\{d_{\mathcal{N}_T}(u) | u \in \mathcal{P}\};$$
$$\delta_{\mathcal{N}_I}(\mathbb{G}) = \wedge\{d_{\mathcal{N}_I}(u) | u \in \mathcal{P}\};$$
$$\delta_{\mathcal{N}_F}(\mathbb{G}) = \wedge\{d_{\mathcal{N}_F}(u) | u \in \mathcal{P}\}$$

Definition 4.16. The maximum neighborhood degree of $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$ in $\mathbb{FNG}$ is designated as $\Delta_{\mathcal{N}}(\mathbb{G})$=$(\Delta_{\mathcal{N}_T}(\mathbb{G}), \Delta_{\mathcal{N}_I}(\mathbb{G}), \Delta_{\mathcal{N}_F}(\mathbb{G}))$ where,

$$\Delta_{\mathcal{N}_T}(\mathbb{G}) = \vee\{d_{\mathcal{N}_T}(u) | u \in \mathcal{P}\};$$
$$\Delta_{\mathcal{N}_I}(\mathbb{G}) = \vee\{d_{\mathcal{N}_I}(u) | u \in \mathcal{P}\};$$
$$\Delta_{\mathcal{N}_F}(\mathbb{G}) = \vee\{d_{\mathcal{N}_F}(u) | u \in \mathcal{P}\}$$

Example 4.7.

Let the vertex set $\mathbb{V} = \{v_1, v_2, v_3, v_4\}$ and the edge sets $\mathbb{E}$={$v_1v_2, v_2v_3, v_3v_4, v_1v_4$} in $\mathbb{G}'$=(V, E). Take the Fermatean Neutrosophic set $\mathcal{P} = (T_\mathcal{P}, I_\mathcal{P}, F_\mathcal{P})$ in $\mathbb{V}$ and the Fermatean Neutrosophic edge sets in $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ defined by

$(T_\mathcal{P}(v_1), I_\mathcal{P}(v_1), F_\mathcal{P}(v_1))$ = (0.3, 0.7, 0.5)

$(T_\mathcal{P}(v_2), I_\mathcal{P}(v_2), F_\mathcal{P}(v_2))$ = (0.6, 0.5, 0.7)

$(T_\mathcal{P}(v_3), I_\mathcal{P}(v_3), F_\mathcal{P}(v_3))$ = (0.8, 0.3, 0.7)

$(T_\mathcal{P}(v_4), I_\mathcal{P}(v_4), F_\mathcal{P}(v_4))$ = (0.7, 0.2, 0.4)

and

$(T_\mathcal{Q}(v_1v_2), I_\mathcal{Q}(v_1v_2), F_\mathcal{Q}(v_1v_2))$ = (0.2, 0.7, 0.8)

$(T_\mathcal{P}(v_2v_3), I_\mathcal{P}(v_2v_3), F_\mathcal{P}(v_2v_3))$ = (0.6, 0.5, 0.7)

$(T_\mathcal{P}(v_3v_4), I_\mathcal{P}(v_3v_4), F_\mathcal{P}(v_3v_4))$ = (0.7, 0.3, 0.7)

$(T_\mathcal{P}(v_1v_4), I_\mathcal{P}(v_1v_4), F_\mathcal{P}(v_1v_4))$ = (0.3, 0.8, 0.6)

Then, it is $\mathbb{FNG}$.

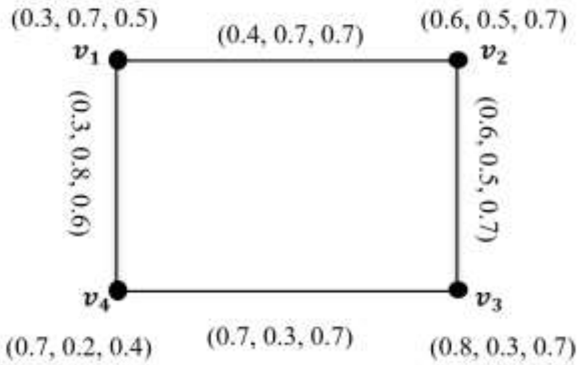


Figure 5. Fermatean Neutrosophic graph

v_1v_2, v_2v_3, v_3v_4 are the effective edges of $\mathbb{FNG}$

$\mathcal{N}(v_1)$=$(\mathcal{N}_T(v_1), \mathcal{N}_I(v_1), \mathcal{N}_F(v_1))$ $\mathcal{N}_T(v_1) = \{v_2\}$; $\mathcal{N}_I(v_1) = \{v_2\}$; $\mathcal{N}_F(v_1) = \{v_2\}$	$\mathrm{d}_\mathcal{N}(v_1) = (0.6, 0.5, 0.7)$
$\mathcal{N}(v_2)$=$(\mathcal{N}_T(v_2), \mathcal{N}_I(v_2), \mathcal{N}_F(v_2))$ $\mathcal{N}_T(v_2) = \{v_1, v_3\}$; $\mathcal{N}_I(v_2) = \{v_1, v_3\}$; $\mathcal{N}_F(v_2) = \{v_1, v_3\}$	$\mathrm{d}_\mathcal{N}(v_2) = (1.1, 1.0, 1.2)$
$\mathcal{N}(v_3)$=$(\mathcal{N}_T(v_3), \mathcal{N}_I(v_3), \mathcal{N}_F(v_3))$ $\mathcal{N}_T(v_3) = \{v_2, v_4\}$; $\mathcal{N}_I(v_3) = \{v_2, v_4\}$;$\mathcal{N}_F(v_3) = \{v_2, v_4\}$	$\mathrm{d}_\mathcal{N}(v_3) = (1.3, 0.7, 1.1)$
$\mathcal{N}(v_4)$=$(\mathcal{N}_T(v_4), \mathcal{N}_I(v_4), \mathcal{N}_F(v_4))$ $\mathcal{N}_T(v_4) = \{v_3\}$; $\mathcal{N}_I(v_4) = \{v_3\}$;$\mathcal{N}_F(v_4) = \{v_3\}$	$\mathrm{d}_\mathcal{N}(v_4) = (0.8, 0.3, 0.7)$

$$\delta_\mathcal{N}(\mathbb{G}) = = (0.6, 0.3, 0.7); \Delta_\mathcal{N}(\mathbb{G}) = (1.3, 1.0, 1.2)$$

Definition 4.17. The closed neighborhood degree of a vertex u of $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$in a $\mathbb{FNG}$ is designated as $\mathrm{d}_\mathcal{N}[u]$=$(\mathrm{d}_{\mathcal{N}_T}[u], \mathrm{d}_{\mathcal{N}_I}[u], \mathrm{d}_{\mathcal{N}_F}[u])$

where,

$$\mathrm{d}_{\mathcal{N}_T}[u] = \textstyle\sum_{v \in \mathcal{N}(p)} T_\mathcal{P}(v) + T_\mathcal{P}(u),$$

$$\mathrm{d}_{\mathcal{N}_I}[u] \textstyle\sum_{v \in \mathcal{N}(p)} I_\mathcal{P}(v) + I_\mathcal{P}(u),$$

$$d_{\mathcal{N}_F}[u]\ \sum_{v\in\mathcal{N}(p)} F_{\mathcal{P}}(v) + F_{\mathcal{P}}(u),$$

Definition 4.18. The minimum closed neighborhood degree of $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$in a $\mathbb{FNG}$ is designated as $\delta_{\mathcal{N}}[\mathbb{G}]$=($\delta_{\mathcal{N}_T}[\mathbb{G}]$, $\delta_{\mathcal{N}_I}[\mathbb{G}]$, $\delta_{\mathcal{N}_F}[\mathbb{G}]$) where,

$$\delta_{\mathcal{N}_T}[\mathbb{G}]=\wedge\{d_{\mathcal{N}_T}(u)|u\in\mathcal{P}\};$$
$$\delta_{\mathcal{N}_I}[\mathbb{G}]=\wedge\{d_{\mathcal{N}_I}(u)|u\in\mathcal{P}\};$$
$$\delta_{\mathcal{N}_F}[\mathbb{G}]=\wedge\{d_{\mathcal{N}_F}(u)|u\in\mathcal{P}\};$$

Definition 4.19. The maximum closed neighborhood degree of $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$in a $\mathbb{FNG}$ is designated as $\Delta_{\mathcal{N}}[\mathbb{G}]$=($\Delta_{\mathcal{N}_T}[\mathbb{G}]$, $\Delta_{\mathcal{N}_I}[\mathbb{G}]$, $\Delta_{\mathcal{N}_F}[\mathbb{G}]$) where,

$$\Delta_{\mathcal{N}_T}[\mathbb{G}]=\wedge\{d_{\mathcal{N}_T}(u)|u\in\mathcal{P}\};$$
$$\Delta_{\mathcal{N}_I}[\mathbb{G}]=\wedge\{d_{\mathcal{N}_I}(u)|u\in\mathcal{P}\};$$
$$\Delta_{\mathcal{N}_F}[\mathbb{G}]=\wedge\{d_{\mathcal{N}_F}(u)|u\in\mathcal{P}\};$$

Example 4.8.

From Fig. 5,

$\mathcal{N}[v_1]$=($\mathcal{N}_T[v_1]$, $\mathcal{N}_I[v_1]$, $\mathcal{N}_F[v_1]$) $\mathcal{N}_T[v_1]=\{v_1,v_2\}$; $\mathcal{N}_I[v_1]=\{v_1,v_2\}$; ; $\mathcal{N}_F[v_1]=\{v_1,v_2\}$	$d_{\mathcal{N}}[v_1]=(0.9, 1.2, 1.2)$
$\mathcal{N}[v_2]$=($\mathcal{N}_T[v_2]$, $\mathcal{N}_I[v_2]$, $\mathcal{N}_F[v_2]$) $\mathcal{N}_T[v_2]=\{v_1,v_2,v_3\}$; $\mathcal{N}_I[v_2]=\{v_1,v_2,v_3\}$; $\mathcal{N}_F[v_2]=\{v_1,v_2,v_3\}$	$d_{\mathcal{N}}(v_2)=(1.7,1.5,1.9)$
$\mathcal{N}[v_3]$=($\mathcal{N}_T[v_3]$, $\mathcal{N}_I[v_3]$, $\mathcal{N}_F[v_3]$) $\mathcal{N}_T[v_3]=\{v_2,v_3,v_4\}$; $\mathcal{N}_I[v_3]=\{v_2,v_3,v_4\}$; $\mathcal{N}_F[v_3]=\{v_2,v_3,v_4\}$	$d_{\mathcal{N}}(v_3)=(2.1,1.0,1.8)$
$\mathcal{N}[v_4]$=($\mathcal{N}_T[v_4]$, $\mathcal{N}_I[v_4]$, $\mathcal{N}_F[v_4]$) $\mathcal{N}_T[v_4]=\{v_3,v_4\}$; $\mathcal{N}_I[v_4]=\{v_3,v_4\}$; $\mathcal{N}_F[v_4]=\{v_3,v_4\}$	$d_{\mathcal{N}}(v_4)=(1.5,0.5,1.1)$

$$\delta_{\mathcal{N}}[\mathbb{G}] = = (0.9,0.5,1.2);\ \Delta_{\mathcal{N}}[\mathbb{G}] = (2.1,1.5,1.8)$$

5. Types of Fermatean neutrosophic graphs

In this section, we introduce different types of Fermatean Neutrosophic graphs based on the degree of each node in FNG such as regular, totally regular and uniform FNGs with suitable examples.

Definition 5.1 Let $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$ be A Fermatean Neutrosophic graph $\mathbb{FNG}$ defined on G = (V, E). If each vertex of $\mathbb{G}$ has same degree, that is

$$d_{\mathbb{G}}(u)=(l_1, l_2, l_3)\ \ \forall\ u\in V$$

Then $\mathbb{G}$ is called (l_1, l_2, l_3) - regular $\mathbb{FNG}$.

Example 5.2 Consider a Fermatean Neutrosophic graph $\mathbb{G}$=$(\mathcal{P},\mathcal{Q})$ defined on G = (V, E), where $\mathcal{P}$ be a Fermatean Neutrosophic set on V and $\mathcal{Q}$ be a Fermatean Neutrosophic relation on V, defined by

$\mathcal{P}$={ $\langle v_1,(0.6, 1,0.7)\rangle, \langle v_2,(0.5, 0.8,0.4)\rangle, \langle v_3,(0.7, 0.5,0.3)\rangle$ }

And $\mathcal{Q}$={ $\langle v_1v_2, (0.4, 1, 0.8)\rangle, \langle v_2v_3, (0.4, 1, 0.8)\rangle, \langle v_1v_3, (0.4, 1, 0.8)\rangle$ }

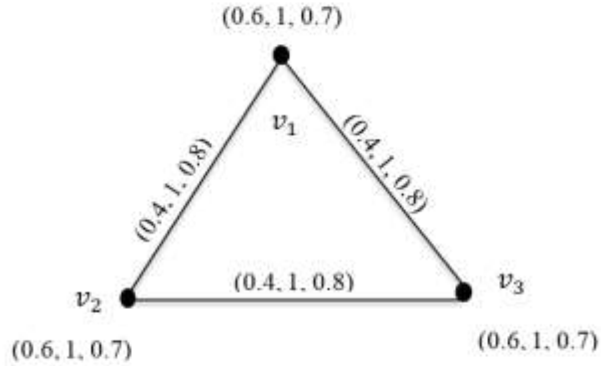


Figure 6. Regular Fermatean Neutrosophic graph

We see that the degree of each vertex in $\mathbb{G}$ is $d_{\mathbb{G}}(v_1) = d_{\mathbb{G}}(v_2) = d_{\mathbb{G}}(v_3) = (1.2, 2, 1.4)$. Hence the Fermatean Neutrosophic graph, displayed in Fig. 6, is (1.2, 2,1.4) – regular.

Definition 5.3. A Fermatean Neutrosophic graph $\mathbb{FNG}$ $\mathbb{G}$=($\mathcal{P}, \mathcal{Q}$) is called Strong Fermatean Neutrosophic graph if the following conditions are satisfied:

$$T_{\mathcal{Q}}(u, v) = \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\}$$
$$I_{\mathcal{Q}}(u, v) = \max\{I_{\mathcal{P}}(u), I_{\mathcal{P}}(v)\}$$
$$F_{\mathcal{Q}}(u, v) = \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\} \text{ for all } u, v \in E$$

That is , all the edges in a Fermatean Neutrosophic graph are effective edges.

An example of a Strong Fermatean neutrosophic graph is shown in Figure 7.

Example 5.4

Consider a graph G=(V, E) where the vertex set V=$\{v_1, v_2, v_3, v_4\}$ and the edge se E=$\{v_1v_2, v_2v_3, v_3v_4, v_1v_4\}$. Let $\mathbb{G}$=($\mathcal{P}, \mathcal{Q}$)be a Fermatean Neutrosophic graph on V as shown in Figure 7, defined by $\mathcal{P}$={ $\langle v_1, (0.3, 0.7, 0.5)\rangle, \langle v_2, (0.4, 0.6, 0.7)\rangle, \langle v_3, (0.8, 0.3, 0.7)\rangle, \langle v_4, (0.7, 0.2, 0.4)\rangle$ } and $\mathcal{Q}$={ $\langle v_1v_2, (0.3, 0.7, 0.7)\rangle, \langle v_2v_3, (0.6, 0.5, 0.7)\rangle, \langle v_3v_4, (0.7, 0.3, 0.7)\rangle, \langle v_1v_4, (0.3, 0.7, 0.5)\rangle$}.

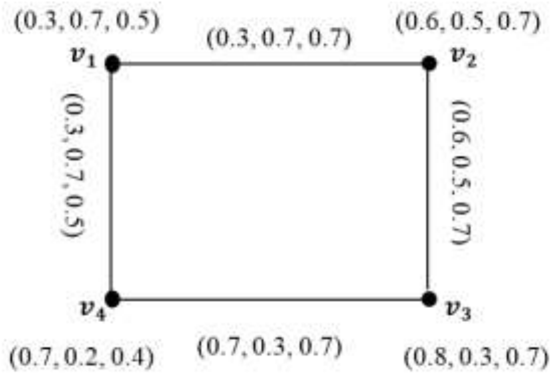


Figure 7. Strong Fermatean Neutrosophic graph

Following and extending the idea of uniform single valued neutrosophic graphs by Broumi et al. [32], we describe the concept of regularity of uniform single valued neutrosophic graphs under Fermatean neutrosophic environment.

Definition 5.5 Let $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ be a Fermatean Neutrosophic graph $\mathbb{FNG}$ defined on G= (V, E), where $\mathcal{P} = (T_{\mathcal{P}}, I_{\mathcal{P}}, F_{\mathcal{P}})$ is a Fermatean Neutrosophic sets on V and $\mathcal{Q} = \left(T_{\mathcal{Q}}, I_{\mathcal{Q}}, F_{\mathcal{Q}}\right)$ is a Fermatean Neutrosophic relation on V. $\mathbb{G}$ is called uniform Fermtean Neutrosophic of level (k_1, k_2, k_3) if $T_{\mathcal{Q}}(u, v)$= k_1, $I_{\mathcal{Q}}(u, v)$= k_2 and $F_{\mathcal{Q}}(u, v)$= k_3, $\forall$ $(u, v) \in$ V $\times$ V and $T_{\mathcal{P}}(u)$= k_1, $I_{\mathcal{P}}(u)$= k_2 and $F_{\mathcal{P}}(u)$= k_3 $\forall$ $u \in$ V, where, 0< $k_1, k_2, k_3 \leq 1$.

Example 5.5 : The following figure is an uniform Fermatean Neutrosophic graph $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$.

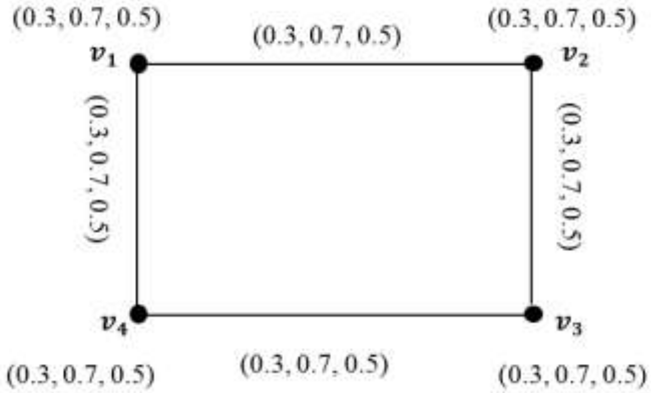


Figure 8. Uniform Fermatean Neutrosophic graph

Theorem 5.6 Every uniform Fermatean Neutrosophic graph is perfectly regular Fermatean Neutrosophic.

Proof. Let $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ be a Fermatean Neutrosophic graph $\mathbb{FNG}$ defined on G= (V, E) with V=$\{u_1, u_2, \dots, u_n\}$, then $T_{\mathcal{Q}}(u, v)$= k_1, $I_{\mathcal{Q}}(u, v)$= k_2 and $F_{\mathcal{Q}}(u, v)$= k_3, $\forall$ $(u, v) \in$ V $\times$ V and $T_{\mathcal{P}}(u)$= k_1, $I_{\mathcal{P}}(u)$= k_2 and $F_{\mathcal{P}}(u)$= k_3 $\forall$ $(u, v) \in$ V $\times$ V, where $0 < k_1, k_2, k_3 \leq 1$.

Then for each u in V,

$$d_{\mathbb{G}}(u) = (\mathrm{d_T}(u), \mathrm{d_I}(u), \mathrm{d_F}(u))$$

$$= \left(\sum_{uv\in E} T_{\mathcal{Q}}(uv), \sum_{uv\in E} I_{\mathcal{Q}}(uv), \sum_{uv\in E} F_{\mathcal{Q}}(uv)\right)$$

$$= ((n-1)k_1, (n-1)k_2, (n-1)k_3)$$

This shows that $\mathbb{G}$ is $((n-1)k_1, (n-1)k_2, (n-1)k_3)$ regular $\mathbb{FNG}$. Moreover for each vertex u in V,

$$td_{\mathbb{G}}(u) = (\mathrm{td_T}(u), \mathrm{td_I}(u), \mathrm{td_F}(u))$$

$$= \left(\sum_{uv\in E} T_{\mathcal{Q}}(uv) + T_{\mathcal{P}}(u), \sum_{uv\in E} I_{\mathcal{Q}}(uv) + I_{\mathcal{P}}(u), \sum_{uv\in E} F_{\mathcal{Q}}(uv) + F_{\mathcal{P}}(u)\right)$$

$$= \left((n-1)k_1 + k_1, (n-1)k_2 + k_2, (n-1)k_3 + k_3\right)$$

$$= (nk_1, nk_2, nk_3)$$

This shows that $\mathbb{G}$ is (nk_1, nk_2, nk_3) totally regular $\mathbb{FNG}$.

Theorem 5.7 If $\mathbb{G}$ is *a uniform* $\mathbb{FNG}$ of level (k_1, k_2, k_3) on G= (V, E), then

a) O($\mathbb{G}$)= (nk_1, nk_2, nk_3) where n=$|V|$.

b) S($\mathbb{G}$)= (mk_1, mk_2, mk_3) where m=$|E|$.

Proof. Let $\mathbb{G}$=$(\mathcal{P}, \mathcal{Q})$ be a uniform Fermatean Neutrosophic graph $\mathbb{FNG}$ defined on G= (V, E) with V=$\{u_1, u_2, \dots, u_n\}$, then $T_{\mathcal{Q}}(u,v)= k_1$, $I_{\mathcal{Q}}(u,v)= k_2$ and $F_{\mathcal{Q}}(u,v)= k_3$, $\forall$ $(u,v) \in$ V × V and $T_{\mathcal{P}}(u)= k_1$, $I_{\mathcal{P}}(u)= k_2$ and $F_{\mathcal{P}}(u)= k_3$ $\forall$ $(u,v) \in$ V × V, where $0 < k_1, k_2, k_3 \leq 1$.

a) for each vertex u in V

$$\begin{aligned} O(\mathbb{G}) &= \left(\sum_{u\in V} T_{\mathcal{P}}(u), \sum_{u\in V} I_{\mathcal{P}}(u), \sum_{u\in V} F_{\mathcal{P}}(u)\right) \\ &= \left(\sum_{u\in V} k_1, \sum_{u\in V} k_2, \sum_{u\in V} k_3\right) \\ &= (nk_1, nk_2, nk_3) \quad \text{where } n=|V|. \end{aligned}$$

b) for each edge uv in E

$$\begin{aligned} S(\mathbb{G}) &= \left(\sum_{uv\in E} T_{\mathcal{Q}}(uv), \sum_{uv\in E} I_{\mathcal{Q}}(uv), \sum_{uv\in E} F_{\mathcal{Q}}(uv)\right) \\ &= \left(\sum_{uv\in E} k_1, \sum_{uv\in E} k_2, \sum_{uv\in E} k_3\right) \\ &= (mk_1, mk_2, mk_3) \text{ where } m=|E|. \end{aligned}$$

Hence proved.

Remark 5.8 The underlying crisp graph of complement of a Fermatean Neutrosophic graph is always an empty graph.

6. Operations on Fermatean Neutrosophic Graphs

In this section, we propose some important graph-theoretic operations over Fermatean Neutrosophic graphs along with various important results and illustrative examples.

Let $\mathbb{G}_1$=$(\mathcal{P}_1, \mathcal{Q}_1)$ and $\mathbb{G}_2$=$(\mathcal{P}_2, \mathcal{Q}_2)$ be two Fermatean Neutrosophic graphs with references to the graphs G^1=(V_1, E_1) and G^2=(V_2, E_2), correspondingly, where $\mathcal{P}_1$& $\mathcal{P}_2$ are the Fermatean Neutrosophic vertex sets in V_1& V_2 corespondingly, and $\mathcal{Q}_1$& $\mathcal{Q}_2$ are the the Fermatean Neutrosophic edge sets in E_1& E_2, correspondingly.

There are many operations on two graphs G^1=(V_1, E_1) and G^2=(V_2, E_2),which result in a graph whose vertex set is the Cartesian product V_1& V_2.

In the following section, we discuss a few operations on two graphs in the structure of Fermatean Neutrosophic sets theory and investigate their properties.

6.1 Cartesian Product of Fermatean Neutrosophic Graphs

Definition 6.1.1 The Cartesian product of two Fermatean Neutrosophic graphs $\mathbb{G}_1$and $\mathbb{G}_2$, denoted by $\mathbb{G}_1$**Cartesian Product** $\mathbb{G}_2$, is defined as follows:

$\mathbb{G}_1 \times \mathbb{G}_2$=$(\mathcal{P}_1 \times \mathcal{P}_2, \mathcal{Q}_1 \times \mathcal{Q}_2)$

where

- $$T_{\mathcal{P}_1\times\mathcal{P}_2}(u_1,u_2) = \min\left(T_{\mathcal{P}_1}(u_1), T_{\mathcal{P}_2}(u_2)\right)$$
$$I_{\mathcal{P}_1\times\mathcal{P}_2}(u_1,u_2) = \max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_2}(u_2)\right)$$
$$F_{\mathcal{P}_1\times\mathcal{P}_2}(u_1,u_2) = \max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_2}(u_2)\right)\ \forall (u_1,u_2) \in V_1 \times V_2$$

The membership value of the edges in $\mathbb{G}_1 \times \mathbb{G}_2$ can be computed as

- $$T_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u,u_2),(u,v_2)\big) = \min\left(T_{\mathcal{P}_1}(u), T_{\mathcal{Q}_2}(u_2,v_2)\right)$$
$$I_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u,u_2),(u,v_2)\big) = \max\left(I_{\mathcal{P}_1}(u), I_{\mathcal{Q}_2}(u_2,v_2)\right)$$
$$F_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u,u_2),(u,v_2)\big) = \max\left(F_{\mathcal{P}_1}(u), F_{\mathcal{Q}_2}(u_2,v_2)\right)\ \forall\, u \in V_1, (u_2,v_2) \in E_2$$
- $$T_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u_1,\gamma),(v_1,\gamma)\big) = \min\left(T_{\mathcal{Q}_1}(u_1,v_1), T_{\mathcal{P}_2}(\gamma)\right),$$
$$I_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u_1,\gamma),(v_1,\gamma)\big) = \max\left(I_{\mathcal{Q}_1}(u_1,v_1), I_{\mathcal{P}_2}(\gamma)\right)$$
$$F_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u_1,\gamma),(v_1,\gamma)\big) = \max\left(F_{\mathcal{Q}_1}(u_1,v_1), F_{\mathcal{P}_2}(\gamma)\right)\ \forall\, \gamma \in V_2, (u_1,v_1) \in E_1$$

Theorem 6.1.2 The **Cartesian Product** of two Fermatean Neutrosophic graphs is a Fermatean Neutrosophic graph.

Proof suppose $u \in V_1, (u_2,v_2) \in E_2$. Then,

$$T_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u,u_2),(u,v_2)\big) = min\left(T_{\mathcal{P}_1}(u), T_{\mathcal{Q}_2}(u_2,v_2)\right),$$
$$\leq min\left(T_{\mathcal{P}_1}(u), \min\left(T_{\mathcal{P}_2}(u_2), T_{\mathcal{P}_2}(v_2)\right)\right),$$
$$= min\left(min\left(T_{\mathcal{P}_1}(u), T_{\mathcal{P}_2}(u_2)\right), \min\left(T_{\mathcal{P}_2}(u), T_{\mathcal{P}_2}(v_2)\right)\right),$$
$$= min\big(T_{\mathcal{P}_1\times\mathcal{P}_2}(u,u_2), T_{\mathcal{P}_1\times\mathcal{P}_2}(u,v_2)\big)$$
$$I_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u,u_2),(u,v_2)\big) = max\left(I_{\mathcal{P}_1}(u), I_{\mathcal{Q}_2}(u_2,v_2)\right),$$
$$\geq max\left(I_{\mathcal{P}_1}(u), \max\left(I_{\mathcal{P}_2}(u_2), I_{\mathcal{P}_2}(v_2)\right)\right),$$
$$= max\left(max\left(I_{\mathcal{P}_1}(u), I_{\mathcal{P}_2}(u_2)\right), \max\left(I_{\mathcal{P}_2}(u), I_{\mathcal{P}_2}(v_2)\right)\right),$$
$$= max\big(I_{\mathcal{P}_1\times\mathcal{P}_2}(u,u_2), I_{\mathcal{P}_1\times\mathcal{P}_2}(u,v_2)\big)$$

and

$$F_{\mathcal{Q}_1\times\mathcal{Q}_2}\big((u,u_2),(u,v_2)\big) = max\left(F_{\mathcal{P}_1}(u), F_{\mathcal{Q}_2}(u_2,v_2)\right),$$
$$\geq max\left(F_{\mathcal{P}_1}(u), \max\left(F_{\mathcal{P}_2}(u_2), F_{\mathcal{P}_2}(v_2)\right)\right),$$
$$= max\left(max\left(F_{\mathcal{P}_1}(u), F_{\mathcal{P}_2}(u_2)\right), \max\left(F_{\mathcal{P}_2}(u), F_{\mathcal{P}_2}(v_2)\right)\right),$$
$$= max\ \big(F_{\mathcal{P}_1\times\mathcal{P}_2}(u,u_2), F_{\mathcal{P}_1\times\mathcal{P}_2}(u,v_2)\big)$$

Again, let $\forall\,\gamma \in V_2, (u_1, v_1) \in E_1$, then we have

$$T_{Q_1 \times Q_2}\big((u_1,\gamma),(v_1,\gamma)\big) = min\left(T_{Q_1}(u_1,v_1), T_{\mathcal{P}_2}(\gamma)\right),$$

$$\leq\ min\left(min\left(T_{\mathcal{P}_1}(u_1), T_{\mathcal{P}_1}(v_1), T_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ min\left(min\left(T_{\mathcal{P}_1}(u_1), T_{\mathcal{P}_2}(\gamma)\right), min\left(T_{\mathcal{P}_1}(v_1), T_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$= min\big(T_{\mathcal{P}_1 \times \mathcal{P}_2}(u_1,\gamma), T_{\mathcal{P}_1 \times \mathcal{P}_2}(v_1,\gamma)\big)\ .$$

$$I_{Q_1 \times Q_2}\big((u_1,\gamma),(v_1,\gamma)\big) = max\left(I_{Q_1}(u_1,v_1), I_{\mathcal{P}_2}(\gamma)\right),$$

$$\geq\ max\left(max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_1}(v_1), I_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\left(max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_2}(\gamma)\right), max\left(I_{\mathcal{P}_1}(v_1), I_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\big(I_{\mathcal{P}_1 \times \mathcal{P}_2}(u_1,\gamma), I_{\mathcal{P}_1 \times \mathcal{P}_2}(v_1,\gamma)\big).$$

and

$$F_{Q_1 \times Q_2}\big((u_1,\gamma),(v_1,\gamma)\big) = max\left(F_{Q_1}(u_1,v_1), F_{\mathcal{P}_2}(\gamma)\right),$$

$$\geq\ max\left(max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_1}(v_1), F_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\left(max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_2}(\gamma)\right), max\left(F_{\mathcal{P}_1}(v_1), F_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\big(F_{\mathcal{P}_1 \times \mathcal{P}_2}(u_1,\gamma), F_{\mathcal{P}_1 \times \mathcal{P}_2}(v_1,\gamma)\big).$$

Thus, in view of the definition of the Fermatean Neutrosophic, the result follows. The following example illustrates the above defined graph-theoretic operation.

Example 6.3 Consider two Fermatean Neutrosophic $\mathbb{G}_1 and\ \mathbb{G}_2$ as shown in the below Figure 9.

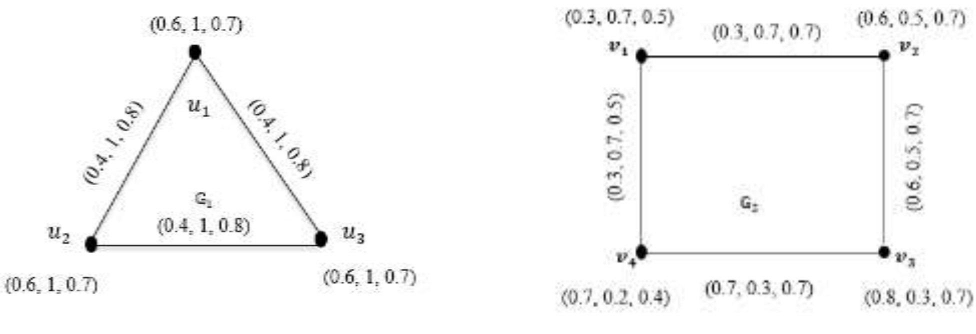


Figure 9. Fermatean Neutrosophic graphs $\mathbb{G}_1 and\ \mathbb{G}_2$

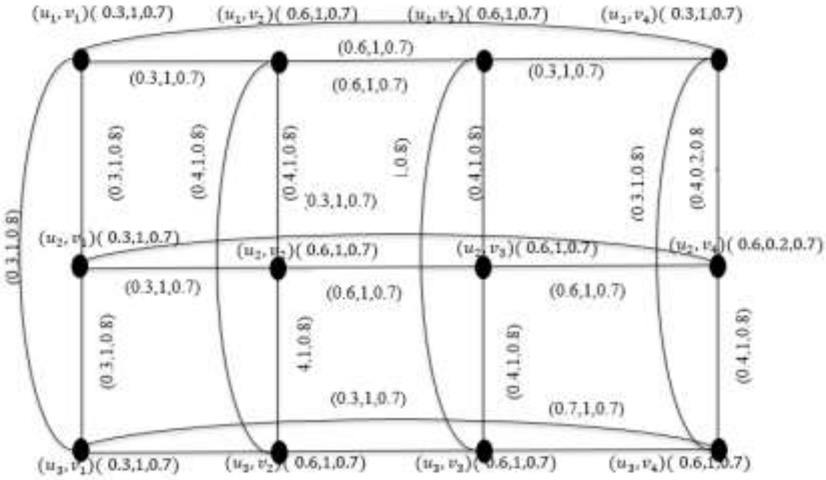


Figure 10. Composition graph $\mathbb{G}_1 \times \mathbb{G}_2$

Then, the graphs $\mathbb{G}_1$, $\mathbb{G}_2$ and their composition graph $\mathbb{G}_1 \times \mathbb{G}_2$ are being graphically presented in the above Figure 10.

6.2 Composition of Fermatean Neutrosophic Graphs

Definition 6.2.1 The composition of two Fermatean Neutrosophic graphs $\mathbb{G}_1$and $\mathbb{G}_2$, denoted by $\mathbb{G}_1 \circ \mathbb{G}_2$, is defined as follows:

$\mathbb{G}_1 \circ \mathbb{G}_2$=$(\mathcal{P}_1 \circ \mathcal{P}_2, \mathcal{Q}_1 \circ \mathcal{Q}_2)$

where

- $$T_{\mathcal{P}_1 \times \mathcal{P}_2}(u_1, u_2) = \min\left(T_{\mathcal{P}_1}(u_1), T_{\mathcal{P}_2}(u_2)\right)$$
  $$I_{\mathcal{P}_1 \times \mathcal{P}_2}(u_1, u_2) = \max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_2}(u_2)\right)$$
  $$F_{\mathcal{P}_1 \times \mathcal{P}_2}(u_1, u_2) = \max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_2}(u_2)\right)\ \forall (u_1, u_2) \in V_1 \times V_2$$

- $$T_{\mathcal{Q}_1 \circ \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = \min\left(T_{\mathcal{P}_1}(\beta), T_{\mathcal{Q}_2}(u_2, v_2)\right)$$
  $$I_{\mathcal{Q}_1 \circ \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = \max\left(I_{\mathcal{P}_1}(\beta), I_{\mathcal{Q}_2}(u_2, v_2)\right)$$
  $$F_{\mathcal{Q}_1 \circ \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = \max\left(F_{\mathcal{P}_1}(\beta), F_{\mathcal{Q}_2}(u_2, v_2)\right)\ \forall\ \beta \in V_1, (u_2, v_2) \in E_2$$

- $$T_{\mathcal{Q}_1 \circ \mathcal{Q}_2}\big((u_1, \gamma), (v_1, \gamma)\big) = \min\left(T_{\mathcal{Q}_1}(u_1, v_1), T_{\mathcal{P}_2}(\gamma)\right),$$
  $$I_{\mathcal{Q}_1 \circ \mathcal{Q}_2}\big((u_1, \gamma), (v_1, \gamma)\big) = \max\left(I_{\mathcal{Q}_1}(u_1, v_1), I_{\mathcal{P}_2}(\gamma)\right)$$
  $$F_{\mathcal{Q}_1 \circ \mathcal{Q}_2}\big((u_1, \gamma), (v_1, \gamma)\big) = \max\left(F_{\mathcal{Q}_1}(u_1, v_1), F_{\mathcal{P}_2}(\gamma)\right)\ \forall\ \gamma \in V_2, (u_1, v_1) \in E_1$$

- $$T_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,u_2),(v_1,v_2)\big)=\min\Big(T_{\mathcal{P}_2}(u_2),T_{\mathcal{P}_2}(v_2),T_{\mathcal{Q}_1}(u_1,v_1)\Big),$$
  $$I_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,u_2),(v_1,v_2)\big)=\max\Big(I_{\mathcal{P}_2}(u_2),I_{\mathcal{P}_2}(v_2),I_{\mathcal{Q}_1}(u_1,v_1)\Big)$$
  $$F_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,u_2),(v_1,v_2)\big)=\max\Big(F_{\mathcal{P}_2}(u_2),F_{\mathcal{P}_2}(v_2),F_{\mathcal{Q}_1}(u_1,v_1)\Big)$$
  $$\forall\big((u_1,u_2),(v_1,v_2)\big)\in E^{\circ}, where$$
  $$E^{\circ}=\{(u_1,u_2),(v_1,v_2)|(u_1,v_1)\in E_1\ and\ u_2\neq v_2\}$$

Theorem 6.2.2 The composition of two Fermatean Neutrosophic graphs is a Fermatean Neutrosophic graph.

Proof: Suppose $\beta\in V_1,(u_2,v_2)\in E_2$. Then,

$$\begin{aligned}T_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((\beta,u_2),(\beta,v_2)\big)&=min\Big(T_{\mathcal{P}_1}(\beta),T_{\mathcal{Q}_2}(u_2,v_2)\Big),\\&\le min\Big(T_{\mathcal{P}_1}(\beta),\min\Big(T_{\mathcal{P}_2}(u_2),T_{\mathcal{P}_2}(v_2)\Big)\Big),\\&=min\Big(min\Big(T_{\mathcal{P}_1}(\beta),T_{\mathcal{P}_2}(u_2)\Big),\min\Big(T_{\mathcal{P}_2}(\beta),T_{\mathcal{P}_2}(v_2)\Big)\Big),\\&=min\big(T_{\mathcal{P}_1\circ\mathcal{P}_2}(\beta,u_2),T_{\mathcal{P}_1\circ\mathcal{P}_2}(\beta,v_2)\big).\end{aligned}$$

$$\begin{aligned}I_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((\beta,u_2),(\beta,v_2)\big)&=max\Big(I_{\mathcal{P}_1}(\beta),I_{\mathcal{Q}_2}(u_2,v_2)\Big),\\&\ge max\Big(I_{\mathcal{P}_1}(\beta),\max\Big(I_{\mathcal{P}_2}(u_2),I_{\mathcal{P}_2}(v_2)\Big)\Big),\\&=max\Big(max\Big(I_{\mathcal{P}_1}(\beta),I_{\mathcal{P}_2}(u_2)\Big),max\Big(I_{\mathcal{P}_2}(\beta),I_{\mathcal{P}_2}(v_2)\Big)\Big),\\&=max\big(I_{\mathcal{P}_1\circ\mathcal{P}_2}(\beta,u_2),I_{\mathcal{P}_1\circ\mathcal{P}_2}(\beta,v_2)\big).\end{aligned}$$

and

$$\begin{aligned}F_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((\beta,u_2),(\beta,v_2)\big)&=max\Big(F_{\mathcal{P}_1}(\beta),F_{\mathcal{Q}_2}(u_2,v_2)\Big),\\&\ge max\Big(F_{\mathcal{P}_1}(\beta),\max\Big(F_{\mathcal{P}_2}(u_2),F_{\mathcal{P}_2}(v_2)\Big)\Big),\\&=max\Big(max\Big(F_{\mathcal{P}_1}(\beta),F_{\mathcal{P}_2}(u_2)\Big),\max\Big(F_{\mathcal{P}_2}(\beta),F_{\mathcal{P}_2}(v_2)\Big)\Big),\\&=max\big(F_{\mathcal{P}_1\circ\mathcal{P}_2}(\beta,u_2),F_{\mathcal{P}_1\circ\mathcal{P}_2}(\beta,v_2)\big).\end{aligned}$$

Again, let $\forall\gamma\in V_2,(u_1,v_1)\in E_1$, then we have

$$\begin{aligned}T_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,\gamma),(v_1,\gamma)\big)&=min\Big(T_{\mathcal{Q}_1}(u_1,v_1),T_{\mathcal{P}_2}(\gamma)\Big),\\&\le min\Big(min\Big(T_{\mathcal{P}_1}(u_1),T_{\mathcal{P}_1}(v_1),T_{\mathcal{P}_2}(\gamma)\Big)\Big),\\&=min\Big(min\Big(T_{\mathcal{P}_1}(u_1),T_{\mathcal{P}_2}(\gamma)\Big),min\Big(T_{\mathcal{P}_1}(v_1),T_{\mathcal{P}_2}(\gamma)\Big)\Big),\\&=min\big(T_{\mathcal{P}_1\circ\mathcal{P}_2}(u_1,\gamma),T_{\mathcal{P}_1\circ\mathcal{P}_2}(v_1,\gamma)\big).\end{aligned}$$

$$I_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,\gamma),(v_1,\gamma)\big) = max\left(I_{\mathcal{Q}_1}(u_1,v_1), I_{\mathcal{P}_2}(\gamma)\right),$$

$$\geq\ max\left(max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_1}(v_1), I_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\left(max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_2}(\gamma)\right), max\left(I_{\mathcal{P}_1}(v_1), I_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\big(I_{\mathcal{P}_1\circ\mathcal{P}_2}(u_1,\gamma), I_{\mathcal{P}_1\circ\mathcal{P}_2}(v_1,\gamma)\big).$$

and

$$F_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,\gamma),(v_1,\gamma)\big) = max\left(F_{\mathcal{Q}_1}(u_1,v_1), F_{\mathcal{P}_2}(\gamma)\right),$$

$$\geq\ max\left(max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_1}(v_1), F_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\left(max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_2}(\gamma)\right), max\left(F_{\mathcal{P}_1}(v_1), F_{\mathcal{P}_2}(\gamma)\right)\right),$$

$$=\ max\big(F_{\mathcal{P}_1\circ\mathcal{P}_2}(u_1,\gamma), F_{\mathcal{P}_1\circ\mathcal{P}_2}(v_1,\gamma)\big).$$

Further, if $\big((u_1,u_2),(v_1,v_2)\big) \in E^{\circ}$, $(u_1,v_1) \in E_1$ and $u_2 \neq v_2$, then we have

$$T_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,u_2),(v_1,v_2)\big) = min\left(T_{\mathcal{P}_2}(u_2), T_{\mathcal{P}_2}(v_2), T_{\mathcal{Q}_1}(u_1,v_1)\right)$$

$$\leq\ min\left(T_{\mathcal{P}_2}(u_2), T_{\mathcal{P}_2}(v_2), min\left(T_{\mathcal{P}_1}(u_1), T_{\mathcal{P}_1}(v_1)\right)\right)$$

$$=\ min\left(T_{\mathcal{P}_2}(u_2), T_{\mathcal{P}_2}(v_2), min\left(T_{\mathcal{P}_1}(u_1), T_{\mathcal{P}_1}(v_1)\right)\right)$$

$$=\ min(T_{\mathcal{P}_1\circ\mathcal{P}_2}(u_1,u_2),\ T_{\mathcal{P}_1\circ\mathcal{P}_2}(v_1,v_2)$$

$$I_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,u_2),(v_1,v_2)\big) = max\left(I_{\mathcal{P}_2}(u_2), I_{\mathcal{P}_2}(v_2), I_{\mathcal{Q}_1}(u_1,v_1)\right)$$

$$\geq\ max\left(I_{\mathcal{P}_2}(u_2), I_{\mathcal{P}_2}(v_2), max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_1}(v_1)\right)\right)$$

$$=\ max\left(I_{\mathcal{P}_2}(u_2), I_{\mathcal{P}_2}(v_2), max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_1}(v_1)\right)\right)$$

$$=\ max(I_{\mathcal{P}_1\circ\mathcal{P}_2}(u_1,u_2),\ I_{\mathcal{P}_1\circ\mathcal{P}_2}(v_1,v_2)$$

and

$$F_{\mathcal{Q}_1\circ\mathcal{Q}_2}\big((u_1,u_2),(v_1,v_2)\big) = max\left(F_{\mathcal{P}_2}(u_2), F_{\mathcal{P}_2}(v_2), F_{\mathcal{Q}_1}(u_1,v_1)\right)$$

$$\geq\ max\left(F_{\mathcal{P}_2}(u_2), F_{\mathcal{P}_2}(v_2), max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_1}(v_1)\right)\right)$$

$$=\ max\left(F_{\mathcal{P}_2}(u_2), F_{\mathcal{P}_2}(v_2), max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_1}(v_1)\right)\right)$$

$$=\ max(F_{\mathcal{P}_1\circ\mathcal{P}_2}(u_1,u_2),\ F_{\mathcal{P}_1\circ\mathcal{P}_2}(v_1,v_2)$$

Thus, in view of the definition of the Fermatean Neutrosophic, the result follows. The following example illustrates the above defined graph-theoretic operation.

Example 6.2.3

Consider two Fermatean Neutrosophic $\mathbb{G}_1 and$ $\mathbb{G}_2$ as shown in the below Figure 11.

Then, the graphs $\mathbb{G}_1$, $\mathbb{G}_2$ and their composition graph $\mathbb{G}_1 \circ \mathbb{G}_2$ are being graphically presented in the below Figure 12.

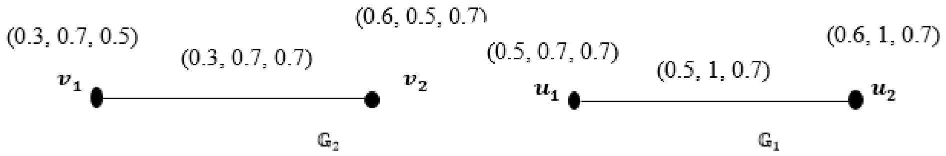


Figure 11. Fermatean Neutrosophic graphs $\mathbb{G}_1 and$ $\mathbb{G}_2$

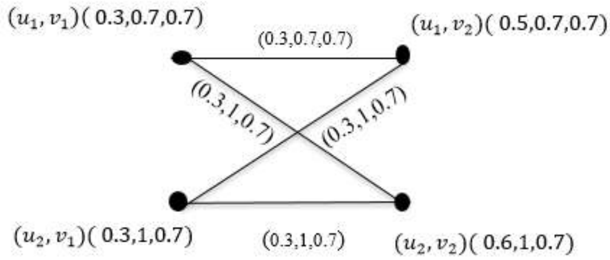


Figure 11. Composition graph $\mathbb{G}_1 \circ \mathbb{G}_2$

6.3 The lexicographic product

Definition 6.3.1 The lexicographic product of two Fermatean Neutrosophic graphs $\mathbb{G}_1$and $\mathbb{G}_2$, denoted by $\mathbb{G}_1 \cdot \mathbb{G}_2$, is defined as follows:

$\mathbb{G}_1 \cdot \mathbb{G}_2$=$(\mathcal{P}_1 \cdot \mathcal{P}_2, \mathcal{Q}_1 \cdot \mathcal{Q}_2)$

- $$T_{\mathcal{P}_1 \cdot \mathcal{P}_2}(u_1, u_2) = min\left(T_{\mathcal{P}_1}(u_1), T_{\mathcal{P}_2}(u_2)\right)$$
  $$I_{\mathcal{P}_1 \cdot \mathcal{P}_2}(u_1, u_2) = max\left(I_{\mathcal{P}_1}(u_1), I_{\mathcal{P}_2}(u_2)\right)$$
  $$F_{\mathcal{P}_1 \cdot \mathcal{P}_2}(u_1, u_2) = max\left(F_{\mathcal{P}_1}(u_1), F_{\mathcal{P}_2}(u_2)\right)\ \forall (u_1, u_2) \in \mathcal{P}_1 \cdot \mathcal{P}_2$$

- $$T_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = min\left(T_{\mathcal{P}_1}(\beta), T_{\mathcal{Q}_2}(u_2, v_2)\right)$$
  $$I_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = max\left(I_{\mathcal{P}_1}(\beta), I_{\mathcal{Q}_2}(u_2, v_2)\right)$$
  $$F_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = max\left(F_{\mathcal{P}_1}(\beta), F_{\mathcal{Q}_2}(u_2, v_2)\right)\ \forall\ \beta \in V_1, (u_2, v_2) \in E_2$$

- $$T_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((u_1, u_2), (v_1, v_2)\big) = min\left(T_{\mathcal{Q}_1}(u_1, v_1), T_{\mathcal{Q}_2}(u_2, v_2)\right),$$
  $$I_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((u_1, u_2), (v_1, v_2)\big) = max\left(I_{\mathcal{Q}_1}(u_1, v_1), I_{\mathcal{Q}_2}(u_2, v_2)\right)$$
  $$F_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((u_1, u_2), (v_1, v_2)\big) = max\left(F_{\mathcal{Q}_1}(u_1, v_1), F_{\mathcal{Q}_2}(u_2, v_2)\right)\ \forall\ (u_1, v_1) \in E_1, (u_2, v_2) \in E_2$$

Theorem 6.3.2 The lexicographic product of two Fermatean Neutrosophic graphs is also the Fermatean Neutrosophic graph.

Proof: We have two cases.

Case 1: $\forall\ \beta \in V_1, (u_2, v_2) \in E_2$. Then,

$$T_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = min\left(T_{\mathcal{P}_1}(\beta), T_{\mathcal{Q}_2}(u_2, v_2)\right),$$

$$\leq \ min\left(T_{\mathcal{P}_1}(\beta), \min\left(T_{\mathcal{P}_2}(u_2), T_{\mathcal{P}_2}(v_2)\right)\right),$$

$$= \ min\left(min\left(T_{\mathcal{P}_1}(\beta), T_{\mathcal{P}_2}(u_2)\right), \min\left(T_{\mathcal{P}_2}(\beta), T_{\mathcal{P}_2}(v_2)\right)\right),$$

$$= \ min\big(T_{\mathcal{P}_1 \circ \mathcal{P}_2}(\beta, u_2), T_{\mathcal{P}_1 \circ \mathcal{P}_2}(\beta, v_2)\big) \ .$$

$$I_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = max\left(I_{\mathcal{P}_1}(\beta), I_{\mathcal{Q}_2}(u_2, v_2)\right),$$

$$\geq \ max\left(I_{\mathcal{P}_1}(\beta), \max\left(I_{\mathcal{P}_2}(u_2), I_{\mathcal{P}_2}(v_2)\right)\right),$$

$$= \ max\left(max\left(I_{\mathcal{P}_1}(\beta), I_{\mathcal{P}_2}(u_2)\right), \max\left(I_{\mathcal{P}_2}(\beta), I_{\mathcal{P}_2}(v_2)\right)\right),$$

$$= \ max\big(I_{\mathcal{P}_1 \circ \mathcal{P}_2}(\beta, u_2), I_{\mathcal{P}_1 \circ \mathcal{P}_2}(\beta, v_2)\big).$$

and

$$F_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((\beta, u_2), (\beta, v_2)\big) = max\left(F_{\mathcal{P}_1}(\beta), F_{\mathcal{Q}_2}(u_2, v_2)\right),$$

$$\geq \ max\left(F_{\mathcal{P}_1}(\beta), \max\left(F_{\mathcal{P}_2}(u_2), F_{\mathcal{P}_2}(v_2)\right)\right),$$

$$= \ max\left(max\left(F_{\mathcal{P}_1}(\beta), F_{\mathcal{P}_2}(u_2)\right), \max\left(F_{\mathcal{P}_2}(\beta), F_{\mathcal{P}_2}(v_2)\right)\right),$$

$$= \ max\big(F_{\mathcal{P}_1 \circ \mathcal{P}_2}(\beta, u_2), F_{\mathcal{P}_1 \circ \mathcal{P}_2}(\beta, v_2)\big).$$

Case 2 : $\forall \ (u_1, v_1) \in E_1, (u_2, v_2) \in E_2$

$$T_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((u_1, u_2), (v_1, v_2)\big) = min\left(T_{\mathcal{Q}_1}(u_1, v_1), T_{\mathcal{Q}_2}(u_2, v_2)\right)$$

$$\leq \ min\left(min\left(T_{\mathcal{Q}_1}(u_1), T_{\mathcal{Q}_1}(v_1)\right), min\left(T_{\mathcal{Q}_2}(u_2), T_{\mathcal{Q}_2}(v_2)\right)\right),$$

$$= \ min\left(min\left(T_{\mathcal{Q}_1}(u_1), T_{\mathcal{Q}_2}(u_2)\right), min\left(T_{\mathcal{Q}_1}(v_1), T_{\mathcal{Q}_2}(v_2)\right)\right),$$

$$= \ min\big(T_{\mathcal{P}_1 \cdot \mathcal{P}_2}(u_1, u_2), T_{\mathcal{P}_1 \cdot \mathcal{P}_2}(v_1, v_2)\big) \ .$$

$$I_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((u_1, u_2), (v_1, v_2)\big) = max\left(I_{\mathcal{Q}_1}(u_1, v_1), I_{\mathcal{Q}_2}(u_2, v_2)\right)$$

$$\geq \ max\left(max\left(I_{\mathcal{Q}_1}(u_1), I_{\mathcal{Q}_1}(v_1)\right), max\left(I_{\mathcal{Q}_2}(u_2), I_{\mathcal{Q}_2}(v_2)\right)\right),$$

$$= \ max\left(max\left(I_{\mathcal{Q}_1}(u_1), I_{\mathcal{Q}_2}(u_2)\right), max\left(I_{\mathcal{Q}_1}(v_1), I_{\mathcal{Q}_2}(v_2)\right)\right),$$

$$= \ max\big(I_{\mathcal{P}_1 \cdot \mathcal{P}_2}(u_1, u_2), I_{\mathcal{P}_1 \cdot \mathcal{P}_2}(v_1, v_2)\big) \ .$$

and

$$F_{\mathcal{Q}_1 \cdot \mathcal{Q}_2}\big((u_1, u_2), (v_1, v_2)\big) = max\left(F_{\mathcal{Q}_1}(u_1, v_1), F_{\mathcal{Q}_2}(u_2, v_2)\right)$$

$$\geq max\left(max\left(F_{\mathcal{Q}_1}(u_1), F_{\mathcal{Q}_1}(v_1)\right), max\left(F_{\mathcal{Q}_2}(u_2), F_{\mathcal{Q}_2}(v_2)\right)\right),$$

$$= max\left(max\left(F_{\mathcal{Q}_1}(u_1), F_{\mathcal{Q}_2}(u_2)\right), max\left(F_{\mathcal{Q}_1}(v_1), F_{\mathcal{Q}_2}(v_2)\right)\right),$$

$$= max\left(F_{\mathcal{P}_1\cdot\mathcal{P}_2}(u_1, u_2), F_{\mathcal{P}_1\cdot\mathcal{P}_2}(v_1, v_2)\right).$$

Example 6.3.3

Consider two Fermatean neutrosophic $\mathbb{G}_1 and\ \mathbb{G}_2$ as shown in the below Figure 13.

Then, lexicographic product the graphs $\mathbb{G}_1$, $\mathbb{G}_2$ ($\mathbb{G}_1 \circ \mathbb{G}_2$) is graphically presented in the below Figure 14.

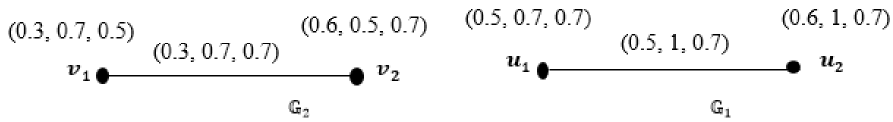


Figure 13. Fermatean neutrosophic graphs $\mathbb{G}_1 and\ \mathbb{G}_2$

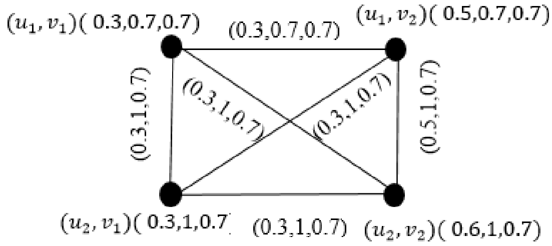


Figure 14. Lexicographic product the graphs $\mathbb{G}_1$, $\mathbb{G}_2$ ($\mathbb{G}_1 \circ \mathbb{G}_2$)

6.4 Union of Fermatean Neutrosophic Graphs

Definition 6.4.1 The union of two Fermatean Neutrosophic graphs $\mathbb{G}_1$and $\mathbb{G}_2$, denoted by $\mathbb{G}_1 \cup \mathbb{G}_2$, is defined as follows:

$\mathbb{G}_1 \cup \mathbb{G}_2$=$(\mathcal{P}_1 \cup \mathcal{P}_2, \mathcal{Q}_1 \cup \mathcal{Q}_2)$

where

- $T_{\mathcal{P}_1\cup\mathcal{P}_2}(u)$=$\begin{cases} T_{\mathcal{P}_1}(u) & if\ u \in V_1 - V_2 \\ T_{\mathcal{P}_2}(u) & if\ u \in V_2 - V_1 \\ max\left(T_{\mathcal{P}_1}(u), T_{\mathcal{P}_2}(v)\right) & if\ u \in V_1 \cup V_2 \end{cases}$

  $I_{\mathcal{P}_1\cup\mathcal{P}_2}(u)$=$\begin{cases} I_{\mathcal{P}_1}(u) & if\ u \in V_1 - V_2 \\ I_{\mathcal{P}_2}(u) & if\ u \in V_2 - V_1 \\ min\left(I_{\mathcal{P}_1}(u), I_{\mathcal{P}_2}(v)\right) & if\ u \in V_1 \cup V_2 \end{cases}$

  $F_{\mathcal{P}_1\cup\mathcal{P}_2}(u)$=$\begin{cases} F_{\mathcal{P}_1}(u) & if\ u \in V_1 - V_2 \\ F_{\mathcal{P}_2}(u) & if\ u \in V_2 - V_1 \\ min\left(F_{\mathcal{P}_1}(u), F_{\mathcal{P}_2}(v)\right) & if\ u \in V_1 \cup V_2 \end{cases}$

- $T_{\mathcal{Q}_1\cup\mathcal{Q}_2}(u,v)$=$\begin{cases} T_{\mathcal{Q}_1}(u,v) & if\ (u,v) \in E_1 - E_2 \\ T_{\mathcal{Q}_2}(u,v) & if\ (u,v) \in E_2 - E_1 \\ max\left(T_{\mathcal{Q}_1}(u,v), T_{\mathcal{Q}_2}(u,v)\right) & if\ (u,v) \in E_1 \cup E_2 \end{cases}$

  $I_{\mathcal{Q}_1\cup\mathcal{Q}_2}(u,v)$=$\begin{cases} I_{\mathcal{Q}_1}(u,v) & if\ (u,v) \in E_1 - E_2 \\ I_{\mathcal{Q}_2}(u,v) & if\ (u,v) \in E_2 - E_1 \\ min\left(I_{\mathcal{Q}_1}(u,v), I_{\mathcal{Q}_2}(u,v)\right) & if\ (u,v) \in E_1 \cup E_2 \end{cases}$

$$F_{\mathcal{Q}_1\cup\mathcal{Q}_2}(u,v)=\begin{cases} F_{\mathcal{Q}_1}(u,v) & if\ (u,v)\in E_1-E_2 \\ F_{\mathcal{Q}_2}(u,v) & if\ (u,v)\in E_2-E_1 \\ min\left(F_{\mathcal{Q}_1}(u,v),F_{\mathcal{Q}_2}(u,v)\right) & if\ (u,v)\in E_1\cup E_2 \end{cases}$$

6.5 Join of Fermatean Neutrosophic Graphs

Definition 6.5.1 The join of two Fermatean Neutrosophic graphs $\mathbb{G}_1$and $\mathbb{G}_2$, denoted by $\mathbb{G}_1+\mathbb{G}_2$, is defined as follows:

$\mathbb{G}_1+\mathbb{G}_2=(\mathcal{P}_1+\mathcal{P}_2,\mathcal{Q}_1+\mathcal{Q}_2)$

where

- $$T_{\mathcal{P}_1+\mathcal{P}_2}(u)=\begin{cases} T_{\mathcal{P}_1}(u) & if\ u\in V_1-V_2 \\ T_{\mathcal{P}_2}(u) & if\ u\in V_2-V_1 \\ T_{\mathcal{P}_1\cup\mathcal{P}_2}(u) & if\ u\in V_1\cup V_2 \end{cases}$$

  $$I_{\mathcal{P}_1+\mathcal{P}_2}(u)=\begin{cases} I_{\mathcal{P}_1}(u) & if\ u\in V_1-V_2 \\ I_{\mathcal{P}_2}(u) & if\ u\in V_2-V_1 \\ I_{\mathcal{P}_1\cup\mathcal{P}_2}(u) & if\ u\in V_1\cup V_2 \end{cases}$$

  $$F_{\mathcal{P}_1+\mathcal{P}_2}(u)=\begin{cases} F_{\mathcal{P}_1}(u) & if\ u\in V_1-V_2 \\ F_{\mathcal{P}_2}(u) & if\ u\in V_2-V_1 \\ F_{\mathcal{P}_1\cup\mathcal{P}_2}(u) & if\ u\in V_1\cup V_2 \end{cases}$$

- $$T_{\mathcal{Q}_1+\mathcal{Q}_2}(u,v)=\begin{cases} T_{\mathcal{Q}_1}(u,v) & if\ (u,v)\in E_1-E_2 \\ T_{\mathcal{Q}_2}(u,v) & if\ (u,v)\in E_2-E_1 \\ T_{\mathcal{Q}_1\cup\mathcal{Q}_2}(u,v) & if\ (u,v)\in E_1\cup E_2 \end{cases}$$

  $$I_{\mathcal{Q}_1+\mathcal{Q}_2}(u,v)=\begin{cases} I_{\mathcal{Q}_1}(u,v) & if\ (u,v)\in E_1-E_2 \\ I_{\mathcal{Q}_2}(u,v) & if\ (u,v)\in E_2-E_1 \\ I_{\mathcal{Q}_1\cup\mathcal{Q}_2}(u,v) & if\ (u,v)\in E_1\cup E_2 \end{cases}$$

  $$F_{\mathcal{Q}_1+\mathcal{Q}_2}(u,v)=\begin{cases} F_{\mathcal{Q}_1}(u,v) & if\ (u,v)\in E_1-E_2 \\ F_{\mathcal{Q}_2}(u,v) & if\ (u,v)\in E_2-E_1 \\ F_{\mathcal{Q}_1\cup\mathcal{Q}_2}(u,v) & if\ (u,v)\in E_1\cup E_2 \end{cases}$$

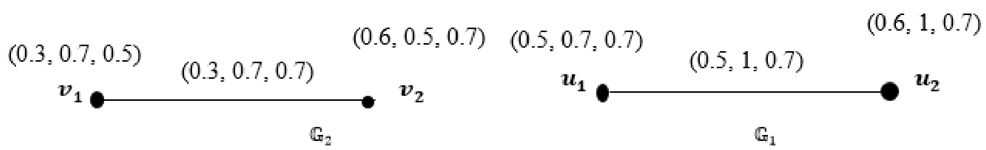
- $$T_{\mathcal{Q}_1+\mathcal{Q}_2}(u,v)=\min\left(T_{\mathcal{P}_1}(u),T_{\mathcal{P}_2}(v)\right)\text{ if }(u,v)\in E'$$
  $$I_{\mathcal{Q}_1+\mathcal{Q}_2}(u,v)=\max\left(I_{\mathcal{P}_1}(u),I_{\mathcal{P}_2}(v)\right)\text{ if }(u,v)\in E'$$
  $$F_{\mathcal{Q}_1+\mathcal{Q}_2}(u,v)=\max\left(F_{\mathcal{P}_1}(u),F_{\mathcal{P}_2}(v)\right)\text{ if }(u,v)\in E'$$

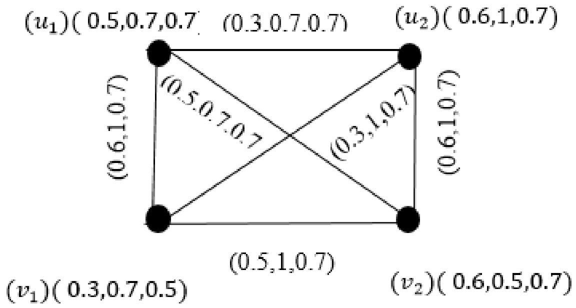
  where E'denotes the set of all the edge joining the nodes of $V_1$and V_2.

Example 6.5.2

Consider two Fermatean Neutrosophic $\mathbb{G}_1 and\ \mathbb{G}_2$ as shown in the below Figure 13.

Then, the join of two Fermatean Neutrosophic graphs $\mathbb{G}_1$and $\mathbb{G}_2$, denoted by $\mathbb{G}_1+\mathbb{G}_2$, is graphically presented in the below Figure 14.





Theorem 6.5.3 The union and join of two Fermatean Neutrosophic graphs are also Fermatean Neutrosophic graphs.

Proof The proof can be outlined similarly as the proof of Theorem 6.3.2.

7. Applications of FNG

Recent days, many researchers who have studied the decision-making problems in different sectors like production, manufacturing, social networking, etc. by using fuzzy, neutrosophic tools [49 – 66]. Sriganesh et. al. [48] investigated the selection of the best power plant among three of the major power plants like hydroelectric power plant, thermal power plant, and nuclear power plant using a graph-theoretic approach. They used digraph characteristic between the factors and cofactors in the selection of the power plant. The interdependency of the factors and their inheritances are identified and they have been represented by using numerical values in their work. Among all these decision-making problems, power plants play a prominent role in for all industry sectors that depend on exergy processes. This section reports the selection of the best power plant among six of the major power plants using Fermatean Neutrosophic graph-theoretic approach. A power plant or power generating station where electric power is generated and distributed on a mass scale. It can be classified into different types based on the fuel used for the generation of electricity. There are many power plants depend on the availability of coal, fuel, wind, and water, etc. We have considered the following six power plants in this case study.

Hydroelectric power plant (P_1): Electricity is produced in a hydroelectric power plant by the flow of water from a height that is used to drive the turbine. The fast-flowing water is converted into mechanical energy when the turbine rotates which is further converted into electric power by the generator.

Thermal power plants (P_2): It converts heat energy into electricity. The heat energy is used to convert fluid into gas which turns the turbine producing mechanical energy which is an intermediate in the process and is converted into electricity in the generators.

A nuclear power plant (P_3): It is similar to a thermal power plant but in nuclear power plants, a nuclear reactor acts as the heat source. In a nuclear reactor, controlled nuclear fission takes place which produces an enormous amount of heat. This heat is dissipated in the water, and it is converted into high-pressure steam which in turn runs the turbine.

Geothermal power plant (P_4): The geothermal power plants are related to other steam turbine thermal power plants. In this heat from the fuel source is used to heat water or any other working fluid. The working fluid is then used to rotate on the turbine of a generator, for producing electricity.
Tidal power plant (P_5): Tidal power or tidal energy is a form of hydropower that converts energy derived from tides primarily into useful forms of electricity. Although not yet generally used, tidal energy has the potential to generate future electricity.
Solar power plant (P_6): A solar power plant is based on the conversion of sunlight into electricity either directly photovoltaics or indirectly using concentrated solar power. Concentrated solar power systems use lenses, mirrors and tracking systems to focus a large area of sunlight into a small beam.

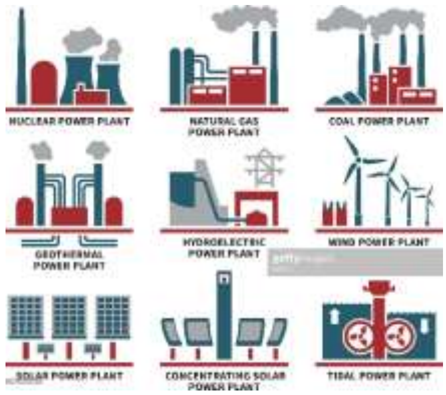


Figure 16. Different power plants

The identification of a site for a power plant selection depends on various factors like land, space, water, cost, transport, fuel, availability of cooling water, nature of the load, etc. Apart from these factors, there are a few sub-factors involving in this process (Figure. 17).

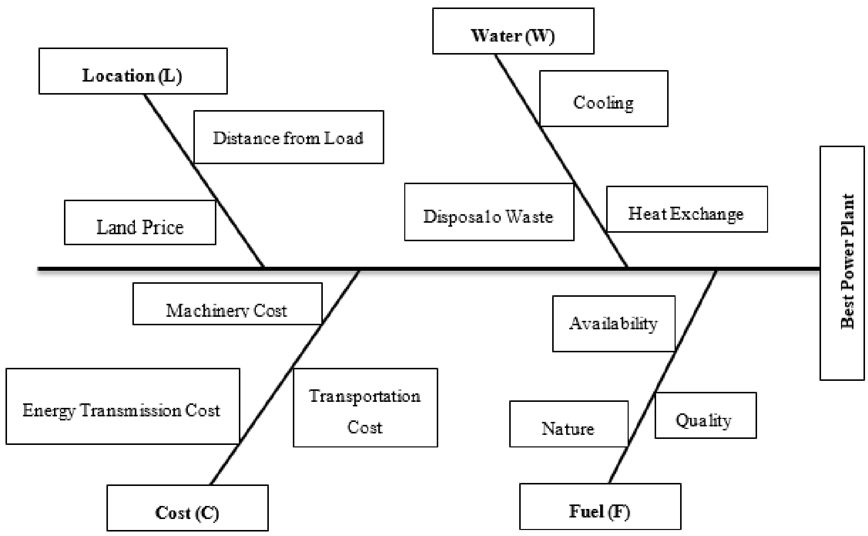


Figure 17. Fishbone diagram representing the necessities for setting up a power plant

In the process of applying FNG in finding the best power plant. FNG can be represented as a matrix whose rows and columns are the sub-factors. Let $V = \{ P_1, P_2, P_3, P_4, P_5, P_6\}$ be the six different power plants under the selection on the basis of wishing parameters or attributes set $A = \{L , W, C , F\}$. The following figures represents the Fermatean Neutrosophic graphs of location, water, cost, and fuel.

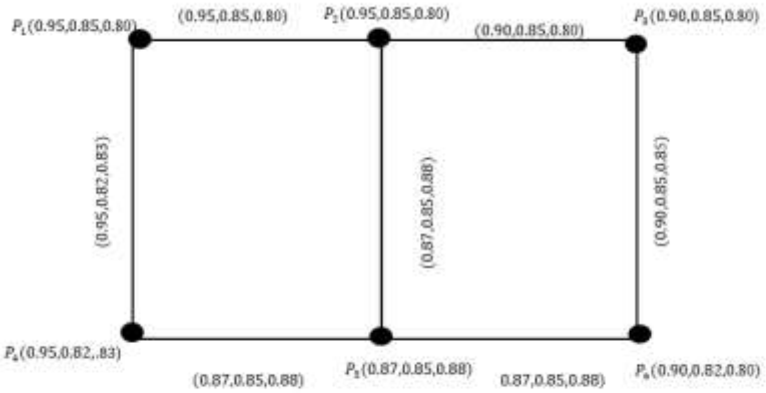


Figure 18. Location based Fermatean Neutrosophic graphs

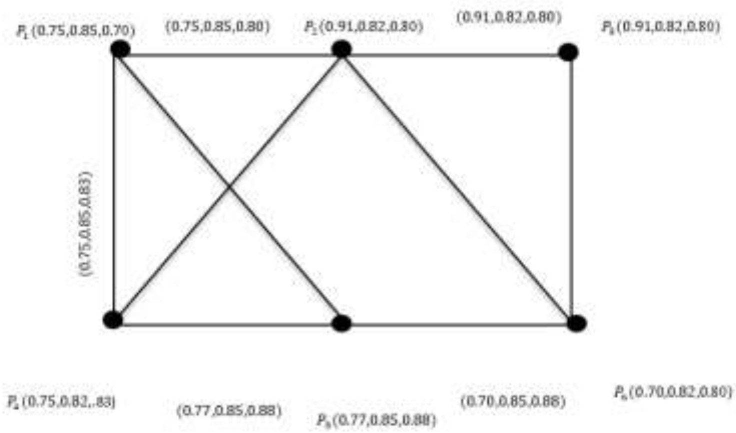


Figure 19. Water based Fermatean Neutrosophic graphs

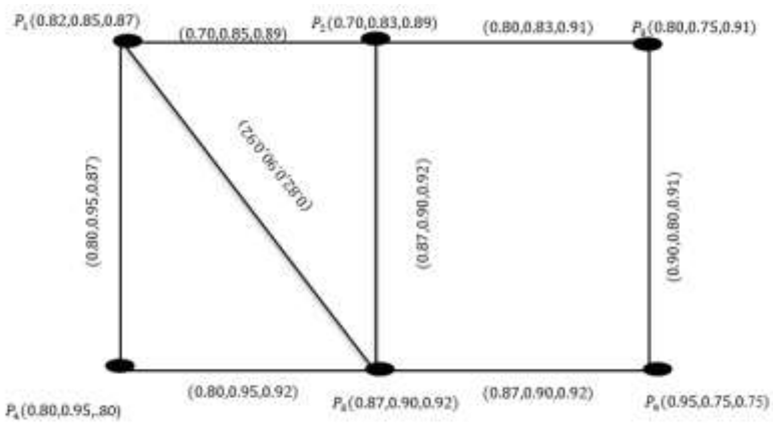


Figure 20. Cost based Fermatean Neutrosophic graphs

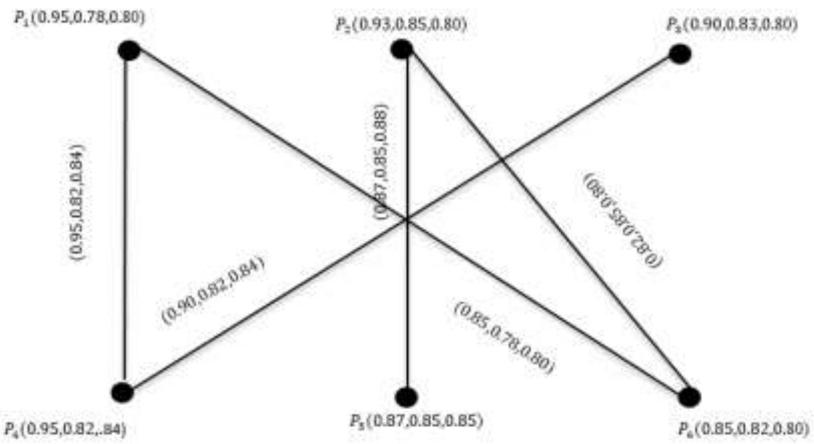


Figure 21. Fuel based Fermatean Neutrosophic graphs

We construct the incidence matrix for $P(L)$, $P(C), P(W), P(F)$ listed below:

$$\boldsymbol{P}(\boldsymbol{L}) = \begin{pmatrix} (0,0,0) & (0.95,0.85,0.80) & (0,0,0) & (0.95,0.82,0.83) & (0,0,0) & (0,0,0) \\ (0.95,0.85,0.80) & (0,0,0) & (0.90,0.85,0.80) & (0,0,0) & (0.87,0.85,0.88) & (0,0,0) \\ (0,0,0) & (0.90,0.85,0.80) & (0,0,0) & (0,0,0) & (0,0,0) & (0.90,0.85,0.85) \\ (0.95,0.82,0.83) & (0,0,0) & (0,0,0) & (0,0,0) & (0.87,0.85,0.8) & (0,0,0) \\ (0,0,0) & (0.87,0.85,0.88) & (0,0,0) & (0.87,0.85,0.8) & (0,0,0) & (0.87,0.85,0.88) \\ (0,0,0) & (0,0,0) & (0.90,0.85,0.85) & (0,0,0) & (0.87,0.85,0.88) & (0,0,0) \end{pmatrix}$$

$$\boldsymbol{P}(\boldsymbol{W}) = \begin{pmatrix} (0,0,0) & (0.75,0.85,0.80) & (0,0,0) & (0.75,0.85,0.83) & (0.75,0.85,0.88) & (0,0,0) \\ (0.91,0.82,0.80) & (0,0,0) & (0.91,0.82,0.80) & (0.75,0.82,0.83) & (0,0,0) & (0.70,0.82,0.80) \\ (0,0,0) & (0.91,0.82,0.80) & (0,0,0) & (0,0,0) & (0,0,0) & (0.70,0.82,0.80) \\ (0.75,0.85,0.83) & (0.75,0.82,0.83) & (0,0,0) & (0,0,0) & (0.95,0.85,0.80) & (0,0,0) \\ (0.75,0.85,0.88) & (0,0,0) & (0,0,0) & (0.77,0.85,0.88) & (0,0,0) & (0.70,0.85,0.88) \\ (0,0,0) & (0.70,0.82,0.80) & (0.95,0.85,0.80) & (0,0,00) & (0.70,0.85,0.88) & (0,0,0) \end{pmatrix}$$

$$\boldsymbol{P}(\boldsymbol{C}) = \begin{pmatrix} (0,0,0) & (0.70,0.85,0.89) & (0,0,0) & (0.80,0.95,0.87) & (0.95,0.85,0.80) & (0,0,0) \\ (0.70,0.85,0.80) & (0,0,0) & (0.80,0.83,0.91) & (0,0,0) & (0.87,0.90,0.92) & (0,0,0) \\ (0,0,0) & (0.80,0.83,0.91) & (0,0,0) & (0,0,0) & (0,0,0) & (0.87,0.90,0.92) \\ (0.80,0.95,0.87) & (0,0,0) & (0,0,0) & (0,0,0) & (0.80,0.95,0.92) & (0,0,0) \\ (0.82,0.90,0.92) & (0.87,0.85,0.80) & (0,0,0) & (0.80,0.95,0.92) & (0,0,0) & (0.87,0.90,0.92) \\ (0,0,0) & (0,0,0) & (0.90,0.80,0.91) & (0,0,0) & (0.87,0.90,0.92) & (0,0,0) \end{pmatrix}$$

$$\boldsymbol{P}(\boldsymbol{F}) = \begin{pmatrix} (0,0,0) & (0,0,0) & (0,0,0) & (0.95,0.85,0.80) & (0,0,0) & (0.85,0.78,0.80) \\ (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0.87,0.85,0.88) & (0.82,0.85,0.80) \\ (0.90,0.82,0.84) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0.95,0.82,0.84) & (0,0,0) & (0.90,0.82,0.84) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0.87,0.85,0.88) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0.85,0.78,0.80) & (0.82,0.85,0.80) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \end{pmatrix}$$

The incidence matrix of resultant FNG is obtained from the combination of all attributes for each power plant

	P_1	P_2	P_3	P_4	P_5	P_6	Overall
P_1	0.333333	0.32	0.333333	0.61	0.323333	0.326667	2.246667
P_2	0.35	0.333333	0.333333	0.33	0.326667	0.35	2.023333
P_3	0.326667	0.313333	0.333333	0.333333	0.343333	0.326667	1.976667
P_4	0.61	0.333333	0.326667	0.333333	0.343333	0.333333	2.28
P_5	0.326667	0.323333	0.333333	0.343333	0.333333	0.326667	1.986667
P_6	0.326667	0.313333	0.313333	0.333333	0.326667	0.333333	1.946667

$$\boldsymbol{P(with\ respect\ all\ attributes)} = \begin{pmatrix} (0,0,0) & (0,0.85,0.89) & (0,0,0) & (0.75,0.95,0.87) & (0,0.85,0.88) & (0,0.78,0.80) \\ (0,0.85,0.80) & (0,0,0) & (0,0,0) & (0,0.82,0.83) & (0,0.90,0.92) & (0,0.85,0.80) \\ (0,0.82,0.84) & (0,0.85,0.91) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0.90,0.92) \\ (0.75,0.95,0.87) & (0,0,0) & (0,0.82,0.84) & (0,0,0) & (0,0.95,0.92) & (0,0,0) \\ (0,0.90,0.92) & (0,0.85,0.88) & (0,0,0) & (0,0.95,0.92) & (0,0,0) & (0,0.90,0.92) \\ (0,0.78,0.80) & (0,0.85,0.80) & (0,0.85,0.91) & (0,0,0) & (0,0.90,0.92) & (0,0,0) \end{pmatrix}$$

Tabular representation of score values of incidence matrix of resultant FNG with average score function $S = \frac{T+I+1-F}{3}$.

Clearly, the maximum score value is 2.28, scored by the plant P_4. According the data **Geothermal power plant is the beast choice.**

8. Conclusion

Fuzzy theory plays a vital role in uncertainty situations. The extension of fuzzy sets are the popular Intuitionistic fuzzy sets and then Smarandache introduced the most general concept called the Neutrosophic sets. There are many variants of NS are available in the literature like Pythagorean Neutrosophic, Single Valued Neutrosophic, Bipolar Neutrosophic sets. In the list, we have introduced a new class of set namely, Fermatean Neutrosophic sets in this work. We have discussed various types of Fermatean Neutrosophic graphs and the properties of these graphs in this paper. We also apply this new type of graph in a decision making problem. We are extending our research on this new concept to introduce Fermatean Neutrosophic number and Fermatean triangle and trapezoidal Neutrosophic number and its applications in our future work.

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