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Balanced Neutrosophic Graphs

Sivasankar S [1]*, [2] Said Broumi

[1] Department of Mathematics, RV Institute of Technology and Management, Bangalore; sivshankar@gmail.com
[2] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco; broumisaid78@gmail.com

* Correspondence: sivshankar@gmail.com

Abstract: In this paper, we introduce the concept of balanced neutrosophic graphs based on density functions and investigate some of their properties. The necessary conditions for a neutrosophic graph to be a balanced neutrosophic graph are identified if graph G is a self-complementary, regular, complete, and strong neutrosophic graph. Some properties of complement neutrosophic graphs are presented here.

Keywords: Density of a neutrosophic graphs, Balanced neutrosophic graphs.

1. Introduction

Euler was the first to establish the concept of graph theory in 1736. In mathematical history, Euler's approach to the well-known Konigsberg bridge problem is considered as the first theorem of graph theory. This is now widely accepted as a branch of combinatorial mathematics. In many domains, such as geometry, combinatorics, elliptic curves, topography, decision theory, optimization, and data science, the theory of graphs provides a strong tool for determining combinatorial challenges.
The density of a graph G (D(G)) is associated with the network's connectivity patterns. Because of the rapid growth in network size, graph problems become ambiguous, which we address using the fuzzy logic method. The density $D(H) \leq D(G)$ for all subgraphs H of G in balanced graphs. Balanced graphs [10] first appeared in the work of random graphs, and the term Balanced neutrosophic graph is represented here based on the density functions given in [5]. A complete graph has the highest density, while a null graph has the lowest density. Several papers on balanced graph extension [25][32][14] have been published, and it has numerous applications in computer networks, image analysis, robotic systems, artificial intelligence , and decision making. Lotfi A Zadeh [29][30][31] developed a fuzzy set theory in 1965, and the idea of a fuzzy set is welcomed because it addresses uncertainty and vagueness that crisp set cannot, and it provides a meaningful and powerful recognition of quantification of ambiguity. Rosenfeld [24] developed the theory of fuzzy graphs in 1975 after studying fuzzy relations on fuzzy sets. Atanassov's [6][7] intuitionistic fuzzy graphs (IFGs) provide a way to incorporate uncertainty with an additional degree. A bipolar fuzzy graph is a fuzzy graph extension with a membership degree range of [-1, 1]. Akram [1][2] introduced the concept of bipolar fuzzy graphs and defined various operations on them. Talal Al Hawary [4] investigated some fuzzy graph operations and defined balanced fuzzy graphs. Balanced fuzzy graphs are increasingly

being used to represent complex systems in which the amount of data and information varies with different levels of precision.

A neutrosophic graph can comply with the uncertainty of any real-world problem's inconsistent and indeterminate information, whereas fuzzy graphs may lack sufficient satisfactory results. Florentin Smarandache et al [12][26-28] defined neutrosophic graphs and single valued neutrosophic graphs (SVNS) as a new dimension of graph theory as a generalisation of the fuzzy graph and the intuitionistic fuzzy graph. Said Broumi et al [8][9] developed the concept of SVNG and investigated its components. Motivated by the concept of a balanced graph and its extensions [3] [13] [15-20] [22][23] [27], we focused on introducing balanced and strictly balanced, in single valued neutrosophic graphs. The important properties of a balanced neutrosophic graph are discussed in this paper. Section 2 discusses the fundamental definitions and theorems required. Section 3 discusses the necessary conditions for a neutrosophic graph to be a balanced neutrosophic graph if graph G is a self-complementary, regular, complete, and strong neutrosophic graph. We also discussed some of the properties of complementary and a self-complementary balanced neutrosophic graphs. The paper is concluded in Section 4.

2. Preliminaries

Definition 2.1 [12] A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G = (A, B)$ where

1. The functions $T_A: V \to [0,1]$, $I_A: V \to [0,1]$, and $F_A: V \to [0,1]$, denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in V$, respectively, and $0 \le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3$ for all $v_i \in V$.
2. The functions $T_B: \mathrm{E} \subseteq \mathrm{V} \times \mathrm{V} \to [0,1]$, $I_B: \mathrm{E} \subseteq \mathrm{V} \times \mathrm{V} \to [0,1]$, and $F_B: \mathrm{E} \subseteq \mathrm{V} \times \mathrm{V} \to [0,1]$ are defined by $T_B(v_i, v_j) \le T_A(v_i) \wedge T_A(v_j)$, $I_B(v_i, v_j) \ge I_A(v_i) \vee I_A(v_j)$ and $F_B(v_i, v_j) \ge F_A(v_i) \vee \mathrm{F}(v_j)$ denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \le T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \le 3$ for all $(v_i, v_j) \in E$ $(\mathrm{i}, \mathrm{j} = 1, 2, \ldots, \mathrm{n})$. We call A the single valued neutrosophic vertex set of V, B the single valued neutrosophic edge set of E, respectively.

Definition 2.2 [8] A partial SVN-subgraph of SVN-graph $G = (A, B)$ is a SVN-graph $H = (V', E')$ such that $V' \subseteq \mathrm{V}$, where $T'_A(v_i) \le T_A(v_i)$, $I'_A(v_i) \ge I_A(v_i)$, and $F'_A(v_i) \ge F_A(v_i)$ for all $v_i \in V$ and $E' \subseteq \mathrm{E}$, where $T'_B(v_i, v_j) \le T_B(v_i, v_j)$, $I'_B(v_i, v_j) \ge I_B(v_i, v_j)$, $F'_B(v_i, v_j) \ge F_B(v_i, v_j)$ for all $(v_i, v_j) \in E$.

Definition 2.3 [11] Let $G = (A, B)$ be an SVNG. G is said to be a strong SVNG if

$T_{\mathrm{B}}(u, v) = T_A(u) \wedge T_A(v)$,

$I_{\mathrm{B}}(u, v) = I_A(u) \vee I_A(v)$ and

$F_{\mathrm{B}}(u, v) = F_A(u) \vee F_A(v)$ for every $(u, v) \in E$.

Definition 2.4 [11] Let $G = (A, B)$ be an SVNG. G is said to be a complete SVNG if

$T_{\mathrm{B}}(u, v) = T_A(u) \wedge T_A(v)$,

$I_{\mathrm{B}}(u, v) = I_A(u) \vee I_A(v)$ and

$F_{\mathrm{B}}(u,v) = F_A(u) \vee F_A(v)$ for every $u, v \in V$.

Definition 2.5 [11] Let $G = (A, B)$ be an SVNG. $\bar{G} = (\bar{A}, \bar{B})$ is the complement of an SVNG if $\bar{A} = A$ *and* $\bar{B}$ is computed as below.

$\overline{T_B(u,v)} = T_A(u) \wedge T_A(v) - T_{\mathrm{B}}(u,v)$,

$\overline{I_B(u,v)} = I_A(u) \vee I_A(v) - I_{\mathrm{B}}(u,v)$

and $\overline{F_B(u,v)} = F_A(u) \vee F_A(v) - F_{\mathrm{B}}(u,v)$ for every $(u, v) \in E$.

Here, $\overline{T_B(u,v)}, \overline{I_B(u,v)}$ and $\overline{F_B(u,v)}$ denote the true, intermediate, and false membership degree for edge (u, v) of $\bar{G}$.

Definition 2.6 [11] Let $G = (A, B)$ be an SVNG. G is a regular neutrosophic graph if it satisfies the following conditions.

$$\sum_{u \neq v} T_{\mathrm{B}}(u,v) = \text{constant}, \quad \sum_{u \neq v} I_{\mathrm{B}}(u,v) = \text{constant}, \quad \text{and} \quad \sum_{u \neq v} F_{\mathrm{B}}(u,v) = \text{constant}.$$

Definition 2.7 [11] Let $G = (A, B)$ be an SVNG. G is a regular strong neutrosophic graph if it satisfies the following conditions.

$T_{\mathrm{B}}(u,v) = T_A(u) \wedge T_A(v)$ and $\sum_{u \neq v} T_{\mathrm{B}}(u,v) = \text{constant}$,

$I_{\mathrm{B}}(u,v) = I_A(u) \vee I_A(v)$ and $\sum_{u \neq v} I_{\mathrm{B}}(u,v) = \text{constant}$,

$F_{\mathrm{B}}(u,v) = F_A(u) \vee F_A(v)$ and $\sum_{u \neq v} F_{\mathrm{B}}(u,v) = \text{constant}$.

Definition 2.8 [4] The density of the complete fuzzy graph $G = (V, E)$ is

$D(G) = \frac{2\sum_{u,v \epsilon V} (\mu(u,v))}{\sum_{(u,v) \epsilon V} (\sigma(u) \wedge \sigma(v))}$, for all $u, v \epsilon V$.

Definition 2.9[4] A fuzzy graph $G = (V, E)$is balanced if $D(H) \leq D(G)$, for all sub graphs H of G.

Definition 2.10 [21] A fuzzy graph $G = (V, E)$is a self-complementary if $\mu(u,v) = \frac{1}{2}(\sigma(u) \wedge \sigma(v))$ for all $u, v \epsilon V$.

Table 1: Some basic notations

Notation	Meaning
$G = (V, E)$	Fuzzy graph
$G = (A, B)$	Single Valued Neutrosophic Graph (SVNG)
V	Vertex Set
E	Edge set
$T_A(v), I_A(v), F_A(v)$	True membership value, indeterminacy membership value, falsity membership value of the vertex v of $G = (A, B)$
$T_B(u,v), I_B(u,v), F_B(u,v)$	True membership value, indeterminacy membership value, falsity membership value of the edge (u, v) of $G = (A, B)$
$\bar{G} = (\bar{A}, \bar{B})$	Complement of an SVNG
$\overline{T_B(u,v)}, \overline{I_B(u,v)}$, $\overline{F_B(u,v)}$	True membership value, indeterminacy

	membership value, falsity membership value of the edge (u, v) of $\bar{G} = (\bar{A}, \bar{B})$
$D_T(G), D_I(G), D_F(G)$	Density of true membership value, indeterminacy membership value, falsity membership value of $G = (A, B)$
$D(G) = \big(D_T(G), D_I(G), D_F(G)\big)$	Density of a SVNG $G = (A, B)$

3. Balanced Neutrosophic Graphs

Definition 3.1

The density of a single valued neutrosophic graph $G = (A, B)$ of $G^* = (V, E)$, is $D(G) = \big(D_T(G), D_I(G), D_F(G)\big)$, where

$D_T(G)$ is defined by $D_T(G) = \frac{2\sum_{u,v\epsilon V} T_B(u,v)}{\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)}$, for $u, v\epsilon V$,

$D_I(G)$ is defined by $D_I(G) = \frac{2\sum_{u,v\epsilon V} I_B(u,v)}{\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)}$, for $u, v\epsilon V$ and

$D_F(G)$ is defined by $D_I(G) = \frac{2\sum_{u,v\epsilon V} F_B(u,v)}{\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)}$, for $u, v\epsilon V$.

Definition 3.2

A single valued neutrosophic graph $G = (A, B)$ is balanced if $D(H) \leq D(G)$, that is, $D_T(H) \leq D_T(G)$, $D_I(H) \leq D_I(G)$, $D_F(H) \leq D_F(G)$ for all sub graphs H of G.

Example 1. Consider a neutrosophic graph, $G = (V, E)$ **,such that** $V = \{(v_1, v_2, v_3, v_4)\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3)\}$.

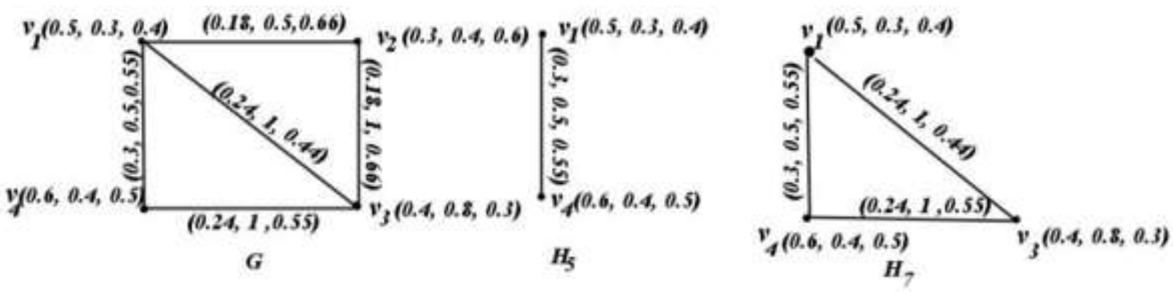

Fig.1 Balanced Neutrosophic Graph

T −density

$D_T(G) = 2\left(\frac{0.18+0.18+0.24+0.3+0.24}{0.3+0.3+0.4+0.5+0.4}\right) = 1.2$

I −density

$D_I(G) = 2\left(\frac{0.5+1+1+0.5+1}{0.4+0.8+0.8+0.4+0.8}\right) = 2.5$

F −density

$D_F(G) = 2\left(\frac{0.66+0.66+0.55+0.55+0.44}{0.6+0.6+0.5+0.5+0.4}\right) = 2.2$

$D(G) = \left(D_T(G), D_I(G), D_F(G)\right) = (1.2, 2.5, 2.2)$.

Let $H_1 = \{(v_1, v_2)\}$, $H_2 = \{(v_2, v_3)\}$, $H_3 = \{(v_3, v_4)\}$, $H_4 = \{(v_2, v_4)\}$, $H_5 = \{(v_1, v_4)\}$, $H_6 = \{(v_1, v_3)\}$, $H_7 = \{(v_1, v_3, v_4)\}$, $H_8 = \{(v_1, v_2, v_3)\}$, $H_9 = \{(v_1, v_2, v_4)\}$, $H_{10} = \{(v_2, v_3, v_4)\}$, $H_{11} = \{(v_1, v_2, v_3, v_4)\}$ be non-empty subgraphs of G. Density $\left(D_T(H), D_I(H), D_F(H)\right)$ is $D(H_1) = (1.2, 2.5, 2.2)$, $D(H_2) = (1.2, 2.5, 2.2)$, $D(H_3) = (1.2, 2.5, 2.2)$, $D(H_4) = (0, 0, 0)$, $D(H_5) = (1.2, 2.5, 2.2)$, $D(H_6) = (1.2, 2.5, 2.2)$, $D(H_7) = (1.2, 2.5, 2.2)$, $D(H_8) = (1.2, 2.5, 2.2)$, $D(H_9) = (1.2, 2.5, 2.2)$, $D(H_{10}) = (1.2, 2.5, 2.2)$, $D(H_{11}) = (1.2, 2.5, 2.2)$. So $D(H) \leq D(G)$ for all subgraphs H of G. Hence G is balanced neutrosophic graph.

Definition 3.3

A single valued neutrosophic graph $G = (A, B)$ is strictly balanced if for $u, v \in V$, $D(H) = D(G)$ for all sub graphs H of G.

Example 2. Consider a neutrosophic graph, $G = (V, E)$,such that $V = \{(v_1, v_2, v_3, v_4)\}$,
$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}$.

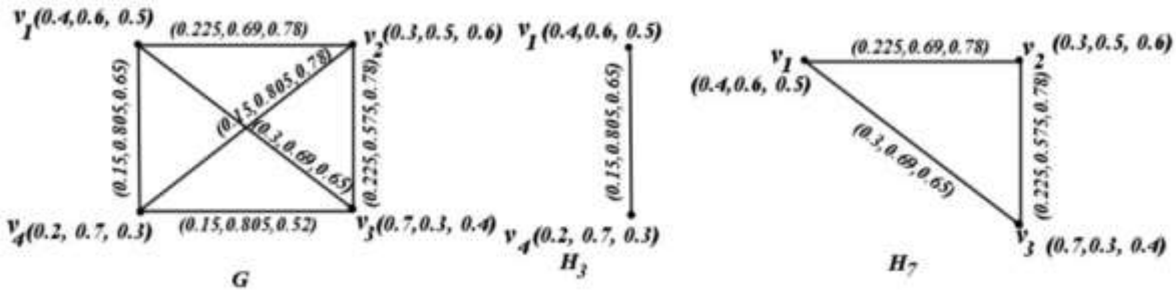

Fig. 2 Strictly Balanced Neutrosophic Graph

T −density

$D_T(G) = 2\left(\frac{0.225+0.225+0.15+0.15+0.15+0.3}{0.3+0.3+0.2+0.2+0.4+0.2}\right) = 1.5$

I −density

$D_I(G) = 2\left(\frac{0.69+0.575+0.805+0.805+0.69+0.805}{0.6+0.5+0.7+0.7+0.7+0.6}\right) = 2.3$

F −density

$D_F(G) = 2\left(\frac{0.78+0.78+0.78+0.65+0.78+0.65}{0.6+0.6+0.4+0.5+0.5+0.6}\right) = 2.6$

$D(G) = \left(D_T(G), D_I(G), D_F(G)\right) = (1.5, 2.3, 2.6)$.

Let $H_1 = \{(v_1, v_2)\}$, $H_2 = \{(v_2, v_3)\}$, $H_3 = \{(v_1, v_4)\}$, $H_4 = \{(v_2, v_4)\}$, $H_5 = \{(v_2, v_4)\}$, $H_6 = \{(v_1, v_3)\}$, $H_7 = \{(v_1, v_2, v_3)\}$,$H_8 = \{(v_1, v_3, v_4)\}$, $H_9 = \{(v_1, v_2, v_4)\}$, $H_{10} = \{(v_2, v_3, v_4)\}$, $H_{11} = \{(v_1, v_2, v_3, v_4)\}$ be non-empty subgraphs of G. Density $\left(D_T(H), D_I(H), D_F(H)\right)$ is $D(H_1) = (1.5, 2.3, 2.6)$, $D(H_2) = (1.5, 2.3, 2.6)$, $D(H_3) = (1.5, 2.3, 2.6)$, $D(H_4) = (1.5, 2.3, 2.6)$, $D(H_5) = (11.5, 2.3, 2.6)$, $D(H_6) = (1.5, 2.3, 2.6)$, $D(H_7) = (1.5, 2.3, 2.6)$, $D(H_8) = (1.5, 2.3, 2.6)$, $D(H_9) = (1.5, 2.3, 2.6)$, $D(H_{10}) =$

$(1.5, 2.3, 2.6)$, $D(H_{11}) = (1.5, 2.3, 2.6)$. So $D(H) = D(G)$ for all subgraphs H of G. Hence G is strictly balanced neutrosophic graph.

Theorem 3.4 Every complete single valued neutrosophic graph is balanced.

Proof:

Let $G = (A, B)$ be a complete single valued neutrosophic graph, then by the definition of complete neutrosophic graph, we have $T_B(u,v) = T_A(u) \wedge T_A(v)$, $I_B(u,v) = I_A(u) \vee I_A(v)$ and $F_B(u,v) = F_A(u) \vee F_A(v)$ for every $u, v \in V$.

$\therefore \sum_{u,v\epsilon V} T_B(u,v) = \sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)$

$\sum_{u,v\epsilon V} I_B(u,v) = \sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)$ and

$\sum_{u,v\epsilon V} F_B(u,v) = \sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)$.

Now $D(G) = \left(\frac{2\sum_{u,v\epsilon V} T_B(u,v)}{\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)}, \frac{2\sum_{u,v\epsilon V} I_B(u,v)}{\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)}, \frac{2\sum_{u,v\epsilon V} F_B(u,v)}{\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)}\right)$

$$D(G) = \left(\frac{2\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)}{\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)}, \frac{2\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)}{\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)}, \frac{2\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)}{\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)}\right)$$

$$D(G) = (2, 2, 2).$$

Let H be a non-empty subgraph of G then, $D(H) = (2, 2, 2)$ for every $H \subseteq G$.

Thus, G is balanced.

Note 3.5. The converse of the preceding theorem do not have to be true. Each balanced neutrosophic graph does not have to be complete.

Example 3. Consider a neutrosophic graph, $G = (V, E)$,such that $V = \{(v_1, v_2, v_3, v_4)\}$, $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$.

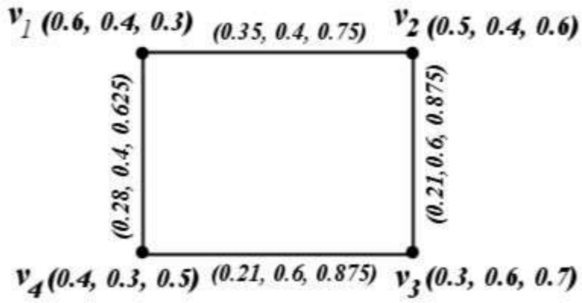

Fig. 3 Balanced but not complete neutrosophic graph

$D(G) = (D_T(G), D_I(G), D_F(G)) = (1.4, 2, 2.5)$.

Let $H_1 = \{(v_1, v_2)\}$, $H_2 = \{(v_2, v_3)\}$, $H_3 = \{(v_1, v_4)\}$, $H_4 = \{(v_2, v_4)\}$, $H_5 = \{(v_2, v_4)\}$, $H_6 = \{(v_1, v_3)\}$, $H_7 = \{(v_1, v_2, v_3)\}$, $H_8 = \{(v_1, v_3, v_4)\}$, $H_9 = \{(v_1, v_2, v_4)\}$, $H_{10} = \{(v_2, v_3, v_4)\}$, $H_{11} = \{(v_1, v_2, v_3, v_4)\}$ be non-empty subgraphs of G. Density $(D_T(H), D_I(H), D_F(H))$ is $D(H_1) = (1.4, 2, 2.5)$, $D(H_2) = (1.4, 2, 2.5)$, $D(H_3) = (1.4, 2, 2.5)$, $D(H_4) = (1.4, 2, 2.5)$, $D(H_5) = (1.4, 2, 2.5)$, $D(H_6) = (1.4, 2, 2.5)$, $D(H_7) = (1.4, 2, 2.5)$, $D(H_8) = (1.4, 2, 2.5)$, $D(H_9) = (1.4, 2, 2.5)$, $D(H_{10}) = (1.4, 2, 2.5)$, $D(H_{11}) = (1.5, 2.3, 2.6)$. So $D(H) \leq D(G)$ for all subgraphs H of G. Hence G is balanced neutrosophic graph.

From the above graph easy to see that:

$T_B(u,v) \neq T_A(u) \wedge T_A(v)$, $I_B(u,v) = I_A(u) \vee I_A(v)$ and $F_B(u,v) \neq F_A(u) \vee F_A(v)$ for every $u, v \in V$. Hence G is balanced not complete.

Corollary 3.6 Every strong single valued neutrosophic graph is balanced.

Theorem 3.7

Let G = (A, B) be a self-complementary neutrosophic graph. Then D(G) = (1,1,1).

Proof:

Let $G = (A, B)$ be a self-complementary neutrosophic graph, then

$$\sum_{u,v\epsilon V} T_{\mathrm{B}}(u,v) = \frac{1}{2}\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)$$

$$\sum_{u,v\epsilon V} I_{\mathrm{B}}(u,v) = \frac{1}{2}\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v) \text{ and}$$

$$\sum_{u,v\epsilon V} F_{\mathrm{B}}(u,v) = \frac{1}{2}\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v).$$

Now $D(G) = \left(\frac{2\sum_{u,v\epsilon V} T_{\mathrm{B}}(u,v)}{\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)}, \frac{2\sum_{u,v\epsilon V} I_{\mathrm{B}}(u,v)}{\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)}, \frac{2\sum_{u,v\epsilon V} F_{\mathrm{B}}(u,v)}{\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)}\right)$

$$D(G) = \left(\frac{2\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)}{2\sum_{(u,v)\epsilon V} T_A(u) \wedge T_A(v)}, \frac{2\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)}{2\sum_{(u,v)\epsilon V} I_A(u) \vee I_A(v)}, \frac{2\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)}{2\sum_{(u,v)\epsilon V} F_A(u) \vee F_A(v)}\right)$$

Hence $D(G) = (1, 1, 1)$.

Theorem 3.8

Let G = (A, B) be a strictly balanced neutrosophic graph and $\bar{G} = (\bar{A}, \bar{B})$ be its complement then D(G) + D($\bar{G}$) = (2, 2, 2).

Proof:

Let G = (A, B) be a strictly balanced neutrosophic graph and $\bar{G} = (\bar{A}, \bar{B})$ be its complement.

Let H be a subgraph of G which is non-empty. D(G) = D(H) for all H ⊆ G and u, v ∈ V since G is strictly balanced.

In $\bar{G}$, $\overline{T_B(u,v)} = T_A(u) \wedge T_A(v) - T_{\mathrm{B}}(u,v)$, (1)

$\overline{I_B(u,v)} = I_A(u) \vee I_A(v) - I_{\mathrm{B}}(u,v)$ (2)

and $\overline{F_B(u,v)} = F_A(u) \vee F_A(v) - F_{\mathrm{B}}(u,v)$ for every $(u,v) \in E$. (3)

Dividing (1) by $T_A(u) \wedge T_A(v)$

$$\frac{\overline{T_B(u,v)}}{T_A(u) \wedge T_A(v)} = 1 - \frac{T_{\mathrm{B}}(u,v)}{T_A(u) \wedge T_A(v)}, \quad \text{for every } u,v \in V$$

Similarly dividing (2) by $I_A(u) \vee I_A(v)$

$$\frac{\overline{I_B(u,v)}}{I_A(u) \vee I_A(v)} = 1 - \frac{I_{\mathrm{B}}(u,v)}{I_A(u) \vee I_A(v)}, \quad \text{for every } u,v \in V$$

and dividing (3) by $F_A(u) \vee F_A(v)$

$$\frac{\overline{F_B(u,v)}}{F_A(u) \vee F_A(v)} = 1 - \frac{F_{\mathrm{B}}(u,v)}{F_A(u) \vee F_A(v)}, \quad \text{for every } u,v \in V$$

then

$$\sum_{u,v\in V} \frac{\overline{T_B(u,v)}}{T_A(u) \wedge T_A(v)} = 1 - \sum_{u,v\in V} \frac{T_{\mathrm{B}}(u,v)}{T_A(u) \wedge T_A(v)}, \quad \text{for every } u,v \in V$$

$\sum_{u,v \in V} \frac{\overline{I_B(u,v)}}{I_A(u) \vee I_A(v)} = 1 - \sum_{u,v \in V} \frac{I_B(u,v)}{I_A(u) \vee I_A(v)}$, for every $u, v \in V$

$\sum_{u,v \in V} \frac{\overline{F_B(u,v)}}{F_A(u) \vee F_A(v)} = 1 - \sum_{u,v \in V} \frac{F_B(u,v)}{F_A(u) \vee F_A(v)}$, for every $u, v \in V$

Multiply the above equations by 2 on both sides

$2\sum_{u,v \in V} \frac{\overline{T_B(u,v)}}{T_A(u) \wedge T_A(v)} = 2 - 2\sum_{u,v \in V} \frac{T_B(u,v)}{T_A(u) \wedge T_A(v)}$, for every $u, v \in V$

$2\sum_{u,v \in V} \frac{\overline{I_B(u,v)}}{I_A(u) \vee I_A(v)} = 2 - 2\sum_{u,v \in V} \frac{I_B(u,v)}{I_A(u) \vee I_A(v)}$, for every $u, v \in V$

$2\sum_{u,v \in V} \frac{\overline{F_B(u,v)}}{F_A(u) \vee F_A(v)} = 2 - 2\sum_{u,v \in V} \frac{F_B(u,v)}{F_A(u) \vee F_A(v)}$, for every $u, v \in V$

$D_T(\bar{G}) = 2 - D_T(G)$, $D_I(\bar{G}) = 2 - D_I(G)$ and $D_F(\bar{G}) = 2 - D_F(G)$

Now, D(G) + D($\bar{G}$) = $\left(D_T(G), D_I(G), D_F(G)\right) + \left(D_T(\bar{G}), D_I(\bar{G}), D_F(\bar{G})\right)$

D(G) + D($\bar{G}$) =$\left(\left(D_T(G) + D_T(\bar{G})\right), \left(D_I(G) + D_I(\bar{G})\right), \left(D_F(G) + D_F(\bar{G})\right)\right)$

Hence D(G) + D($\bar{G}$) =(2, 2, 2).

Theorem 3.9

The complement of a single valued neutrosophic graph that is strictly balanced is also strictly balanced.

Proof:

Let G = (A, B) be a strictly balanced neutrosophic graph and $\bar{G} = (\bar{A}, \bar{B})$ be its complement.

Let H be a subgraph of G which is non-empty. D(G) = D(H) for all H ⊆ G and u, v ∈ V since G is strictly balanced.

As G is strictly balanced by Theorem 3.7, D(G) + D($\bar{G}$) =(2, 2, 2)

Since D(H) + D($\bar{H}$) =(2, 2, 2) for every $H \subseteq G$.

Which implies D($\bar{H}$) = D($\bar{G}$)

Hence $\bar{G}$ is strictly balanced.

Theorem 3.10

The complement of strongly regular SVNG is balanced.

Proof:

Let G = (A, B) be a strongly regular neutrosophic graph and $\bar{G} = (\bar{A}, \bar{B})$ be its complement.

Since G is strongly, we have $T_B(u,v) = T_A(u) \wedge T_A(v)$, $I_B(u,v) = I_A(u) \vee I_A(v)$ and $F_B(u,v) = F_A(u) \vee F_A(v)$ for every $(u,v) \in E$. (1)

In $\bar{G}$, $\overline{T_B(u,v)} = T_A(u) \wedge T_A(v) - T_B(u,v)$,

$\overline{I_B(u,v)} = I_A(u) \vee I_A(v) - I_B(u,v)$

and $\overline{F_B(u,v)} = F_A(u) \vee F_A(v) - F_B(u,v)$ for every $(u,v) \in E$.

Since G is strongly regular, we have $\overline{T_B(u,v)} = 0$, $\overline{I_B(u,v)} = 0$ and $\overline{F_B(u,v)} = 0$ by (1) for every $(u,v) \in E$ and

$\overline{T_B(u,v)} = T_A(u) \wedge T_A(v)$,

$\overline{I_B(u,v)} = I_A(u) \vee I_A(v)$

and $\overline{F_B(u,v)} = F_A(u) \vee F_A(v)$ for every $(u,v) \in \bar{E}$.

$\Longrightarrow \bar{G}$ is a strong neutrosophic graph. Then by Corollary 3.6, $\bar{G}$ is balanced.

Theorem 3.11

Let G = (A, B) be a SVNG and $\bar{G} = (\bar{A}, \bar{B})$ be its complement then $\bar{\bar{G}} = \mathrm{G}$.

Proof:

Let G = (A, B) be a SVNG $\bar{G} = (\bar{A}, \bar{B})$ be its complement.

In $\bar{G}$, $\overline{T_B(u,v)} = T_A(u) \wedge T_A(v) - T_{\mathrm{B}}(u,v)$, (1)

$\overline{I_B(u,v)} = I_A(u) \vee I_A(v) - I_{\mathrm{B}}(u,v)$ (2)

and $\overline{F_B(u,v)} = F_A(u) \vee F_A(v) - F_{\mathrm{B}}(u,v)$ for every $(u,v) \in E$. (3)

Taking complement for (1), we get $\overline{\overline{T_{\mathrm{B}}(u,v)}} = T_A(u) \wedge T_A(v) - \overline{T_B(u,v)}$

Substitute $T_A(u) \wedge T_A(v) = T_{\mathrm{B}}(u,v) + \overline{T_B(u,v)}$ from (1) weget, $\overline{\overline{T_{\mathrm{B}}(u,v)}} = T_{\mathrm{B}}(u,v)$

Similarly, $\overline{\overline{I_{\mathrm{B}}(u,v)}} = I_{\mathrm{B}}(u,v)$ and $\overline{\overline{F_{\mathrm{B}}(u,v)}} = F_{\mathrm{B}}(u,v)$

Hence $\bar{\bar{G}} = \mathrm{G}$.

4. Conclusion

Neutrosophic graph theory is now commonly used in numerous sciences and technology, most notably in cognitive science, genetic algorithms, optimization techniques, cluster analysis, medical diagnosis, and decision theory. Florentin Smarandache created a neutrosophic graph based on neutrosophic sets. When compared to other traditional and fuzzy models, neutrosophic models provide the system with greater precision, adaptability, and compatibility. We introduced the concept of balanced neutrosophic graphs in this paper and we plan to expand our work on the application of balancing social network connectivity using density functions in the neutrosophic environment.

Compliance with Ethical Standards

Conflict of Interest

The authors declare that they do not have any financial or associative interest indicating a conflict of interest in about submitted work.

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