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Application of Neutrosophic Set Theory in Generalized Assignment Problem

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Abstract. This paper presents the application of Neutrosophic Set Theory (NST) in solving Generalized Assignment Problem (GAP). GAP has been solved earlier under fuzzy environment. NST is a generalization of the concept of classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set. Elements of Neutrosophic set are characterized by a truth-membership function, falsity and also indeterminacy which is a more realistic way of expressing the parameters in real life problem. Here the elements of the cost matrix for the GAP are considered as neutrosophic elements which have not been considered earlier by any other author. The problem has been solved by evaluating score function matrix and then solving it by Extremum Difference Method (EDM) [1] to get the optimal assignment. The method has been demonstrated by a suitable numerical example.

Keywords: NST, GAP, EDM.

1. Introduction

The concept of fuzzy sets and the degree of membership/truth (T) was first introduced by Zadeh in 1965 [2]. This concept is very much useful to handle uncertainty in real life situation. After two decades, Turksen [3] introduced the concept of interval valued fuzzy set which was not enough to consider the non-membership function. In the year 1999, Atanassov [4], [5], [6] proposed the degree of nonmembership/falsehood (F) and intuitionistic fuzzy set (IFS) which is not only more practical in real life but also the generalization of fuzzy set. The paper considers both the degree of membership $\mu_A(x) \in [0, 1]$ of each element $x \in X$ to a set A and the degree of non-membership $\nu_A(x) \in [0, 1]$ s.t. $\mu_A(x) + \nu_A(x) \leq 1$. IFS deals with incomplete information both for membership and non-membership function but not with indeterminacy membership function which is also very natural and obvious part in real life situation. Wang et. Al [7] first considered this indeterminate information which is more practical and useful in real life problems. F.Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set. He coined the words “neutrosophy” and “neutrosophic”. In 2013 he refined the neutrosophic set to n components: t1, t2, ...; i1, i2, ...; f1, f2, ... So in this paper we have used the neutrosophic set theory to solve GAP which hasn’t been done till now.

2. Preliminaries

2.1 Neutrosophic Set [8]

Let U be the space of points (or objects) with generic element ‘x’. A neutrosophic set A in U is characterized by a truth membership function T_A, and indeterminacy function I_A and a falsity membership function F_A, where T_A, I_A and F_A are real standard or non-standard subsets of $]^-0, 1^+[$, i.e sup T_A: $x \rightarrow]^-0, 1^+[$

$$\sup F_A: x \rightarrow]^-0, 1^+[$$

sup I_A: x →] $^-0, 1^+$[

A neutrosophic set A upon U as an object is defined as

$$\frac{x}{T_A(x), I_A(x), F_A(x)} = \{ \frac{x}{T_A, I_A, F_A} : x \in U \},$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are subintervals or union of subintervals of [0, 1].

2.2 Algebraic Operations with Neutrosophic Set [8]

For two neutrosophic sets A and B where and

a> Complement of A
$A^{/} = \{ \frac{x}{T,I,F} \mid T = 1 - T_A, I = 1 - I_A, F = 1 - F_A \}$

b> Intersection of A and B
$A \cap B = \{ \frac{x}{T,I,F} \mid T = T_A T_B, I = I_A I_B, F = F_A F_B \}$

c> Union of A and B
$A \cup B = \{ \frac{x}{T,I,F} \mid T = T_A + T_B - T_A T_B, I = I_A + I_B - I_A I_B, F = F_A + F_B - F_A F_B \}$

d> Cartesian Product of A and B

$$A \times B = \{ (\frac{x}{T_A, I_A, F_A}, \frac{y}{T_B, I_B, F_B}) \mid \frac{x}{T_A, I_A, F_A} \in A, \frac{y}{T_B, I_B, F_B} \in B \}$$

e> A is a subset of B

$$A \subseteq B \ \forall \ \frac{x}{T_A, I_A, F_A} \in A \text{ and } \frac{y}{T_B, I_B, F_B} \in B, T_A \leq T_B, I_A \geq I_B \text{ and } F_A \geq F_B$$

f> Difference of A and B
$A \backslash B = \{ \frac{x}{T,I,F} \mid T = T_A - T_A T_B, I = I_A - I_A I_B, F = F_A - F_A F_B \}$

NST can be used in assignment problem (AP) and Generalized Assignment Problem (GAP).

3. Generalized Assignment Problem using Neutrosophic Set Theory

In this section, we have formulated the GAP using NST. GAP has been solved earlier in different ways by different mathematicians. David B. Shymos and Eva Tardos [April, 1991] considered the GAP as the problem of scheduling parallel machines and solved it by polynomial time algorithm. Dr. Zeev Nutov [June, 2005] solved GAP considering it as a Max-profit scheduling problems. Supriya Kar, Dr. Kajla Basu, Dr. Sathi Mukherjee [International Journal of Fuzzy Mathematics & Systems, Vol.4, No.2 (2014) pp.-169-180] solved GAP under fuzzy environment using Extremum Difference Method. Supriya Kar et. al solved FGAP with restriction on the cost of both job & person using EDM [International Journal of management, Vol.4, Issue5, September-October (2013) pp.-50-59]. They solved FGAP also under Hesitant Fuzzy Environment [Springer India, Opsearch, 29th October 2014, ISSN 0030-3887.

Here, we have used NST to solve GAP because in neutrosophy, every object has not only a certain degree of truth, but also a falsity degree and an indeterminacy degree that have to be considered independently.

3.1 Mathematical model for GAP under Neutrosophic Set Theory

Let us consider a GAP under neutrosophic set in which there are m jobs J= {J1, J2,…….,J_m} and n persons P= { P_1,P_2,……..,P_n}. The cost matrix of the Neutrosophic Generalized Assignment Problem (NGAP) contains neutrosophic elements denoting time for completing j-th job by i-th machine and the mathematical model for NGAP will be as follows-

Model 1.

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} \quad \text{……… [1]}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1, \ i = 1, 2, \ldots, m \quad \text{…….. [2]}$$

$$\sum_{i=1}^{m} c_{ij} x_{ij} \leq a_j, \ j = 1,2,\ldots,n \quad \text{....... [3]}$$

X_{ij} = 0 or 1, i= 1,2,……,m and j=1,2,……,n ……..[4]

Where a_j is the total cost available that worker j can be assigned.

3.2 Solution Procedure of NGAP

To solve NGAP first we have calculated the evaluation matrix for each alternative. Using the elements of Evaluation Matrix for alternatives Score function (S_{ij}) matrix has been calculated. Taking the Score function matrix (S_{ij}) as the initial input data we get the model 2.

Model 2.

Minimize Z= $\sum_{i=1}^{m} \sum_{j=1}^{n} S_{ij}X_{ij}$ …………….. [5]

s.t. the constraints [2], [3], [4].

To solve the model 2 we have used EDM and to verify it the problem has been transformed into LPP form and solved by LINGO 9.0.

3.3 Algorithm for NGAP

Step1. Construct the cost matrix of Neutrosophic generalized assignment problem D = $(C_{ij})_{m\times n}$

Step2. Determine the Evaluation Matrix of the job J_i as E(J_i) = $\left[T_{J_i}^l, T_{J_i}^u\right]$ where

$$[T_{J_i}^l, T_{J_i}^u] = \begin{bmatrix} \min\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right), \\ \max\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right) \end{bmatrix}$$

Step3. Compute the Score function S(J_{ij}) of an alternative

$$S(J_{ij}) = 2(T_{J_{ij}}^u - T_{J_{ij}}^l)$$

$$= 2\begin{bmatrix} \max\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right) - \\ \min\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right) \end{bmatrix}$$

Where $0 \le S(J_{ij}) \le 1$

Step4. Take the Score function matrix as initial input data for NGAP and solve it by EDM.

Step5. End.

4. Numerical Example

Let us consider a NGAP having four jobs and three machines where the cost matrix contains neutrosophic elements denoting time for completing jth job by ith machine. It is required to find optimal assignment of jobs to machines.

Input data table

$$\begin{array}{c} \\ J_1 \\ J_2 \\ J_3 \\ J_4 \end{array} \begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{bmatrix} [0.75,0.39,0.1] & [0.8,0.6,0.15] & [0.4,0.8,0.45] \\ [0.6,0.5,0.25] & [0.75,0.9,0.05] & [0.68,0.46,0.2] \\ [0.8,0.4,0.2] & [0.45,0.1,0.5] & [1.0,0.5,1.0] \\ [0.4,0.6,0.3] & [0.5,0.4,0.8] & [0.5,0.6,0.9] \end{bmatrix} \end{array}$$

Solution:

Evaluate E(J_i) as the evaluation function of the job J_i as

E(J_i) = $\left[T_{J_i}^l, T_{J_i}^u\right]$ where

$$[T_{J_i}^l, T_{J_i}^u] = \begin{bmatrix} \min\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right), \\ \max\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right) \end{bmatrix}$$

Therefore elements of the Evaluation matrix for alternatives

$$\left[T^l_{J_{ij}}, T^u_{J_{ji}}\right] = \begin{bmatrix} [0.57,0.645] & [0.7,0.725] & [0.6,0.675] \\ [0.55,0.625] & [0.825,0.925] & [0.57,0.63] \\ [0.6,0.6] & [0.275,0.3] & [0.25,0.75] \\ [0.5,0.65] & [0.3,0.45] & [0.35,0.55] \end{bmatrix}$$

Compute the Score function $S(J_{ij})$ of an alternative

$$S(J_{ij}) = 2(T^u_{J_{ij}} - T^l_{J_{ij}})$$

$$= 2\begin{bmatrix} \max\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right) - \\ \min\left(\left(\frac{T_{J_{ij}} + I_{J_{ij}}}{2}\right), \left(\frac{1 - F_{J_{ij}} + I_{J_{ij}}}{2}\right)\right) \end{bmatrix}.$$

Where $0 \le S(J_{ij}) \le 1$

Therefore elements of Score function matrix will be as follows-

$$S(J_{ij}) = \begin{bmatrix} 0.15 & 0.05 & 0.15 \\ 0.15 & 0.2 & 0.12 \\ 0.0 & 0.05 & 1.0 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}.$$

Solving $S(J_{ij})$ by EDM,

	M_1	M_2	M_3	Row Penalties
J_1	0.15	[0.05]	0.15	0.10
J_2	0.15	0.2	[0.12]	0.08
J_3	[0.0]	0.05	1.0	1.00
J_4	[0.3]	0.3	0.4	0.10
$a_j \rightarrow$	0.525	0.475	0.81	

Therefore optimal assignment is,

$J_1 \rightarrow M_2$, $J_2 \rightarrow M_3$, $J_3 \rightarrow M_1$, $J_4 \rightarrow M_1$.

To verify the problem, it has been transformed into LPP form and solved by LINGO 9.0 as follows-

Minimize Z=
$0.15x_{11}+0.05x_{12}+0.15x_{13}+0.15x_{21}+0.2x_{22}+0.12x_{23}+0.0x_{31}+0.05x_{32}+1.0x_{33}+0.3x_{41}+0.3x_{42}+0.4x_{43}$

s.t $\sum_{j=1}^{n} x_{ij} = 1$, i = 1, 2, ….. , m

$\sum_{i=1}^{m} c_{ij}x_{ij} \le a_j$, j = 1,2,…….,n

X_{ij} = 0 or 1, i= 1,2,……,m and j= 1,2,……,n

By LINGO 9.0, we get the solution as,

$x_{12}=1$, $x_{23}=1$, $x_{31}=1$, $x_{41}=1$

Therefore the optimal assignment is $J_1 \rightarrow M_2$, $J_2 \rightarrow M_3$, $J_3 \rightarrow M_1$, $J_4 \rightarrow M_1$.

Therefore the solution has been verified as same result has been obtained both by Model 1 and Model 2.

5. Conclusion

Neutrosophic set theory is a generalization of classical set, fuzzy set, interval-valued fuzzy set, intuitionistic fuzzy set because it not only considers the truth membership T_A and falsity membership F_A, but also an indeterminacy function I_A which is very obvious in real life situation. In this paper, we have considered the cost matrix as neutrosophic elements considering the restrictions on the available costs. By calculation Evaluation matrix and Score function matrix, the problem is solved by EDM which is very simple yet efficient method to solve GAP. Now to verify the solution the problem has been transformed to LPP form and solved by standard software LINGO 9.0.

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