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An Introduction to Neutro-Prime Topology and Decision Making Problem

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Abstract: In this industrial era the innovation of industrial machines had a significant impact on industrial evaluation which minimize manpower, time consumption for product making, material wastage. Heavy usage of machines leads to the occurrence of some faults in it. Such damaged parts of each machine have been identified to overhaul which is defined as a set called neutro-prime set under its topological structure. Some related properties of such space have been proved and some are disproved with counterexamples. Also, the idea of interior and closure dealt with this space with few basic properties. This article provides a decision-making process to identify the best fit of those damages under a neutrosophic environment and the priority is given to the heavily damaged machine. We also use step by step algorithm and formulae to compute machine values. Our objective is to demonstrate that our proposed algorithm can calculate key measurements for fault diagnostic in machines as well as to provide fair and reliable forecasted outcomes.

Keywords: Neutro-prime sets; neutro-prime topological spaces; neutro-prime interior; neutro-prime closure; neutro-prime absolute complement; decision making.

1. Introduction

Throughout history, the relation between humans and machines became most important in moral, ethical, social, economic, and the environment. Machines have confirmed to grasp the key to further developments we humans so extremely need. In the process of doing so, a machine whether or not in continuous use will get damaged and worn-out. In our daily life, we need to reduce the risk of its expensive cost, bad maintenance, and repair parts.

The principles of three autonomous membership degrees such as truth, falsity, and indeterminacy, committed to each element of a set which categorized to neutrosophic set (NS) as instigated by (1998) Smarandache [20, 21], which is an explanation of a fuzzy set (FS) defined by (1965) Zadeh [33], and intuitionistic fuzzy set (IFS) generated by (1986) Atanassov [32]. It is an active organization that hypothesizes the notion of all other sets introduced before. It goes out to be a treasured mathematical implement to observe unformed, damaged, indistinct facts. In recent years many researchers have further expanded and developed the theory and application of NSs [1, 2, 3, 5, 6, 14, 16-19]. Also, (2017) Smarandache [22] originated a new trend set called plithogenic set and others developed [4, 9, 12, 15].

Topology plays a vital role among many sets such as FS, NS, soft sets (SS), neutrosophic soft set (NSS), etc., These types of sets are extended by different researchers [7, 10, 11, 13, 23-27, 29, 30, 31] and its application in decision making (DM) problems [8]. Chinnadurai and Sindhu [28] introduced the notion of prime sets (PSs) and prime-topological spaces (PTSs) (2020), as one of the mathematical utensils for dealing with the subsets of the universe set.

The major achievements of this research are:

- Initiating a neutrosophic environment on prime sets under a topological space.
- Demonstrating the decision-making problem for analyzing the amount of damage in machines.
- An outcome of the proposed algorithm fits in a better way with the number of faults in machines by diagnosis the set values.

To overcome the disadvantages of machines, solving algorithms are presented in this study. A decision-making process delivers to identify the best fit of those damages under a neutrosophic environment and the priority is given to the heavily damaged machine with the use of step by step algorithm and formulae to compute machine values. The main tool used to find the faults in machines are complement and absolute complements of the specified set.

The structure of this study is as follows: Some significant definitions interrelated to the study are presented in part 2. Part 3 introduces the definition of neutro-prime sets, neutro-prime topological spaces, neutro-prime interior and neutro-prime closure with fundamental properties, and related examples. Part 4 explains the DM problem to repair the sample machines with some damages. The algorithm and formulae are presented to find the final result. Finally, the contributions of this study are concluded with future works in part 5.

2. Preliminaries

In this part, some essential definitions connected to this work are pointed.

Definition 2.1 Let W be a non-empty set and $w \in W$. A NS D in W is characterized by a truth-membership function T_D, an indeterminacy-membership function I_D, and a false-membership function F_D which are subsets of [0, 1] and is defined as

$$D = \{\langle w, T_D(w), I_D(w), F_D(w)\rangle : w \in W\},$$

where

$$0 \le \sup T_D(w) + \sup I_D(w) + \sup F_D(w) \le 3 .$$

Definition 2.2 Let $NS(W)$ denote the family of all NSs over W and $\tau_n \subset NS(W)$. Then τ_η is called a neutrosophic topology (NT) on W if it satisfies the following conditions

(i) $0_n, 1_n \in \tau_n$, where null NS $0_n = \{\langle w, 0, 0, 1\rangle : w \in W\}$ and an absolute NS $1_n = \{\langle w, 1, 1, 0\rangle : w \in W\}$.

(ii) the intersection of any finite number of members of τ_n belongs to τ_n.

(iii) the union of any collection of members of τ_n belongs to τ_n.

Then the pair (W, τ_n) is called a NTS.

Every member of τ_n is called τ_n-open neutrosophic set (τ_n- ONS). An NS is called τ_n-closed (τ_n-CNS) if and only if its complement is τ_n- ONS.

Definition 2.3 Let D be a NS over W. Then the complement of is denoted by D' and defined by

$$D' = \{\langle w, F_D(w), 1 - I_D(w), T_D(w)\rangle : w \in W\} .$$

Clearly, $(D')' = D$.

Definition 2.4 Let (W, τ) be a topological space (TS), where W is the universe and τ is a topology. Let K be a proper nonempty subset of W. Let D be a τ -open set, where $D \neq \phi, W$. Then the prime set (PS) over W is denoted by ξ and defined by $\xi = \{\phi, W, K : K \cap D \neq \phi\}$.

Definition 2.5 Let (W, τ) be a TS. Then τ_p is called a prime topology (PT) if it satisfies the following conditions

(i) $\phi, W \in \tau_p$.

(ii) the intersection of any finite number of members of τ_p belongs to τ_p.

(iii) the union of any collection of members of τ_p belongs to τ_p.

Then the pair (W, τ_p) is called a prime topological space (PTS).

Every member of τ_p is called τ_p-prime open set (τ_p - POS). The complement of every τ_p - POS of W is called the τ_p-prime closed set (τ_p - PCS) of W and this collection is denoted by τ_p^* .

Example 2.6 Let $W = \{w_1, w_2, w_3\}$ with topology $\tau = \{\phi, W, \{w_1\}\}$.
Clearly, (W, τ) is a TS over W.
Then

$$\tau_p = \{\phi, W, \{w_1\}, \{w_1, w_2\}, \{w_1, w_3\}\} = PS(W)$$

and its members are τ_p - POSs.
Thus (W, τ_p) is a PTS over W.
Then

$$\tau_p^* = \{\phi, W, \{w_2, w_3\}, \{w_3\}, \{w_2\}\}$$

and its members are τ_p - PCSs, whose complements are τ_p - POSs.

Definition 2.7 Let W be a set of universe and $w_i \in W$ where $i \in I$. Let D be a NS over W. Then the subset of NS (sub-NS) D is denoted as $\xi_D(W^*)$ and defined as

$$\xi_D(W^*) = \{\langle w_i, T_D(w_i), I_D(w_i), F_D(w_i)\rangle, \langle (w_i, w_j), \max(T_D(w_i), T_D(w_j)), \max(I_D(w_i), I_D(w_j)), \min(F_D(w_i), F_D(w_j))\rangle\}.$$

where $i, j \in I$ and $i \neq j$.
Clearly, $(w_i, w_j) = (w_j, w_i)$.

Example 2.8 Let $W = \{w_1, w_2, w_3\}$ be a set of features of the washing machine, where w_1 = energy efficiency, w_2 = capacity, w_3 =price. Let D be a NS over W, defined as

$$D = \{\langle w_1, .7, .5, .4\rangle, \langle w_2, .2, .7, .9\rangle, \langle w_3, .4, .1, .3\rangle\}.$$

Then the sub-NS D is

$$\xi_D(W^*) = \{\langle w_1, .7, .5, .4\rangle, \langle w_2, .2, .7, .9\rangle, \langle w_3, .4, .1, .3\rangle, \langle (w_1, w_2), .7, .7, .4\rangle, \langle (w_1, w_3), .7, .5, .3\rangle, \langle (w_2, w_3), .4, .7, .3\rangle\}.$$

Definition 2.9 Let W be a set of universe and $w_i \in W$ where $i \in I$. Let V be any proper nonempty subset of W, say $\{w_i\}$ and $\{w_i, w_j\}$. Let D be a NS over W. Then the subset of NS D with respect to w_i (sub-NS D_{w_i}) and w_i, w_j (sub-NS D_{w_i, w_j}) are denoted as $\xi_D(w_i)$ and $\xi_D(w_i, w_j)$, and defined as

$$\xi_D(w_i) = \{\langle w_i, T_D(w_i), I_D(w_i), F_D(w_i)\rangle, \langle (w_i, w_j), \max(T_D(w_i), T_D(w_j)), \max(I_D(w_i), I_D(w_j)), \min(F_D(w_i), F_D(w_j))\rangle,$$
$$\langle w_k, T_D(0_n), I_R(0_n), F_R(0_n)\rangle, \langle (w_k, w_l), T_D(0_n), I_R(0_n), F_D(0_n)\rangle\}$$

where $i \in I$, $j \in I - \{i\}$, $k, l \in I - \{i, j\}$ and $k \neq l$

and

$$\xi_D(w_i,w_j)=\{\langle w_i,T_D(w_i),I_D(w_i),F_D(w_i)\rangle,\langle w_j,T_D(w_j),I_D(w_j),F_D(w_j)\rangle,\langle w_k,T_D(0_n),I_D(0_n),F_D(0_n)\rangle,$$
$$\langle (w_i,w_j),\max(T_D(w_i),T_D(w_j)),\max(I_D(w_i),I_D(w_j)),\min(F_D(w_i),F_D(w_j))\rangle,$$
$$\langle (w_i,w_k),\max(T_D(w_i),T_D(w_k)),\max(I_D(w_i),I_D(w_k)),\min(F_D(w_i),F_D(w_k))\rangle,$$
$$\langle (w_j,w_k),\max(T_D(w_j),T_D(w_k)),\max(I_D(w_j),I_D(w_k)),\min(F_D(w_j),F_D(w_k))\rangle\}$$

where $i,j,k\in I$ and $i\neq j\neq k$, respectively.

Example 2.10 Let $W=\{w_1,w_2,w_3\}$. Let *D* and *F* be two NSs over *W* and are defined as follows

$$D=\{\langle w_1,.1,.2,.8\rangle,\langle w_2,.4,.7,.3\rangle,\langle w_3,.6,.5,.2\rangle\}$$

and

$$F=\{\langle w_1,.6,.5,.3\rangle,\langle w_2,.9,.8,.1\rangle,\langle w_3,.7,.6,.1\rangle\}.$$

Then sub-NS D_{w_2,w_3} and sub-NS F_{w_2} are defined as

$$\xi_D(w_2,w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,.4,.7,.3\rangle,\langle w_3,.6,.5,.2\rangle,\langle w_{1,2},.4,.7,.3\rangle,\langle w_{1,3},.6,.5,.2\rangle,\langle w_{2,3},.6,.7,.2\rangle\}$$ and

$$\xi_F(w_2)=\{\langle w_1,0,0,1\rangle,\langle w_2,.9,.8,.1\rangle,\langle w_3,0,0,1\rangle,\langle w_{1,2},.9,.8,.1\rangle,\langle w_{1,3},0,0,1\rangle,\langle w_{2,3},.9,.8,.1\rangle\},$$ respectively.

3. Neutro-Prime Topology

In this part, the new type of set is initiated as neutro-prime sets and defined its topological space as neutro-prime topological spaces. Some of its basic properties are examined with illustrative examples.

Definition 3.1 Let (W,τ_n) be a neutrosophic topological space (NTS), where *W* is the universe and τ_n is a neutrosophic topology (NT). Let *D* be a τ_n-open neutrosophic set, where $D\neq\phi,W$. Let *V* be any proper nonempty subset of *W*. Then

$$\eta_p D(V)=\{\xi_D(V^*):V\cap V^*\neq\phi\},$$

for all proper nonempty subset V^* of *W*.
Thus the elements belongs to $\eta_p D(V)$ are said to be neutro-prime sets (NPSs) over *W* and denoted by $\xi_D(V^*)$.

Example 3.2 Let $W=\{w_1,w_2,w_3\}$ be a set of features of the washing machine, where w_1 = energy efficiency, w_2 = capacity, w_3 =price. Let *D* be a NS over *W*, defined as

$$D=\{\langle w_1,.7,.5,.4\rangle,\langle w_2,.2,.7,.9\rangle,\langle w_3,.4,.1,.3\rangle\}.$$

Then NPS

$$\eta_p D(w_3)=\{\xi_D(w_3),\xi_D(w_1,w_3),\xi_D(w_2,w_3)\},$$

where

$$\xi_D(w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,0,0,1\rangle,\langle w_3,.4,.1,.3\rangle,\langle (w_1,w_2),0,0,1\rangle,\langle (w_1,w_3),.7,.5,.3\rangle,\langle (w_2,w_3),.4,.7,.3\rangle\},$$
$$\xi_D(w_1,w_3)=\{\langle w_1,.7,.5,.4\rangle,\langle w_2,0,0,1\rangle,\langle w_3,.4,.1,.3\rangle,\langle (w_1,w_2),.7,.7,.4\rangle,\langle (w_1,w_3),.7,.5,.3\rangle,\langle (w_2,w_3),.4,.7,.3\rangle\}$$

and

$$\xi_D(w_2,w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,.2,.7,.9\rangle,\langle w_3,.4,.1,.3\rangle,\langle (w_1,w_2),.7,.7,.4\rangle,\langle (w_1,w_3),.7,.5,.3\rangle,\langle (w_2,w_3),.4,.7,.3\rangle\}.$$

Definition 3.3 Let *W* be a set of the universe and *V* be any proper nonempty subset of the *W*. Then the null NPS is denoted as 0_{np} and defined as

$$0_{np}=\{\langle V,T_R(V)=0,I_R(V)=0,F_R(V)=1\rangle:\forall V\}.$$

Definition 3.4 Let *W* be a set of the universe and *V* be any proper nonempty subset of the *W*. Then the absolute NPS is denoted as 1_{np} and defined as

$$1_{np} = \{\langle V, T_R(V) = 1, I_R(V) = 1, F_R(V) = 0\rangle : \forall V\}.$$

Definition 3.5 Let $\xi_D(V_1^*)$ and $\xi_D(V_2^*)$ be two NPSs over *W*. Then their union is denoted as $\xi_D(V_1^*) \cup \xi_D(V_2^*) = \xi_D(V_{1\vee 2}^*)$ and is defined as

$$\xi_D(V_{1\vee 2}^*) = \left\{\left\langle V_{1\vee 2}^*, \max\left(T_R(V_1^*), T_R(V_2^*)\right), \max\left(I_R(V_1^*), I_R(V_2^*)\right), \min\left(F_R(V_1^*), F_R(V_2^*)\right)\right\rangle\right\}.$$

Definition 3.6 Let $\xi_D(V_1^*)$ and $\xi_D(V_2^*)$ be two NPSs over *W*. Then their intersection is denoted as $\xi_D(V_1^*) \cap \xi_D(V_2^*) = \xi_D(V_{1\wedge 2}^*)$ and is defined as

$$\xi_D(V_{1\wedge 2}^*) = \left\{\left\langle V_{1\wedge 2}^*, \min\left(T_R(V_1^*), T_R(V_2^*)\right), \min\left(I_R(V_1^*), I_R(V_2^*)\right), \max\left(F_R(V_1^*), F_R(V_2^*)\right)\right\rangle\right\}.$$

Definition 3.7 Let $\xi_D(V^*)$ be a NPS over *W*. Then its complement is denoted as $\xi_D(V^*)'$ and is defined as

$$\xi_D(V^*)' = \left\{\left\langle V^*, T_D(V^*), 1 - I_D(V^*), F_D(V^*)\right\rangle\right\}.$$

Clearly, the complement of $\xi_D(V^*)'$ equals $\xi_D(V^*)$. i.e., $\left(\xi_D(V^*)'\right)' = \xi_D(V^*)$.

Definition 3.8 Let $\xi_D(V_1^*)$ and $\xi_D(V_2^*)$ be two NPSs over *W*. Then $\xi_D(V_1^*)$ is said to be a neutro-prime subset of $\xi_D(V_2^*)$ if

$$T_R(V_1^*) \le T_R(V_2^*),\ T_R(I_1^*) \le T_R(I_2^*),\ F_R(V_1^*) \ge F_R(V_2^*).$$

It is denoted by $\xi_D(V_1^*) \subseteq \xi_D(V_2^*)$.

Also $\xi_D(V_1^*)$ is said to be neutro-prime equal to $\xi_D(V_2^*)$ if $\xi_D(V_1^*)$ is a neutro-prime subset of $\xi_D(V_2^*)$ and $\xi_D(V_2^*)$ is a neutro-prime subset of $\xi_D(V_1^*)$. It is denoted by $\xi_D(V_1^*) = \xi_D(V_2^*)$.

Proposition 3.9 Let $\xi_D(V_1^*)$, $\xi_D(V_2^*)$ and $\xi_D(V_3^*)$ be NPSs over *W*. Then,

(i) $\xi_D(V_1^*) \cup 0_{np} = \xi_D(V_1^*)$.

(ii) $\xi_D(V_1^*) \cup 1_{np} = 1_{np}$.

(iii) $\xi_D(V_1^*) \cup \left[\xi_D(V_2^*) \cup \xi_D(V_3^*)\right] = \left[\xi_D(V_1^*) \cup \xi_D(V_2^*)\right] \cup \xi_D(V_3^*)$.

(iv) $\xi_D(V_1^*) \cup \left[\xi_D(V_2^*) \cap \xi_D(V_3^*)\right] = \left[\xi_D(V_1^*) \cup \xi_D(V_2^*)\right] \cap \left[\xi_D(V_1^*) \cup \xi_D(V_3^*)\right]$.

Proof. Straightforward.

Proposition 3.10 Let $\xi_D(V_1^*)$, $\xi_D(V_2^*)$ and $\xi_D(V_3^*)$ be NPSs over *W*. Then,

(i) $\xi_D(V_1^*) \cap 0_{np} = 0_{np}$.

(ii) $\xi_D(V_1^*) \cap 1_{np} = \xi_D(V_1^*)$.

(iii) $\xi_D(V_1^*) \cap \left[\xi_D(V_2^*) \cap \xi_D(V_3^*)\right] = \left[\xi_D(V_1^*) \cap \xi_D(V_2^*)\right] \cap \xi_D(V_3^*)$.

(iv) $\xi_D(V_1^*) \cap \left[\xi_D(V_2^*) \cup \xi_D(V_3^*)\right] = \left[\xi_D(V_1^*) \cap \xi_D(V_2^*)\right] \cup \left[\xi_D(V_1^*) \cap \xi_D(V_3^*)\right]$.

Proof. Straightforward.

Proposition 3.11 Let $\xi_D(V_1^*)$ and $\xi_D(V_2^*)$ be two NPSs over *W*. Then,

(i) $\left[\xi_D(V_1^*) \cup \xi_D(V_2^*)\right]' = \xi_D(V_1^*)' \cap \xi_D(V_2^*)'$.

(ii) $\left[\xi_D(V_1^*) \cap \xi_D(V_2^*)\right]' = \xi_D(V_1^*)' \cup \xi_D(V_2^*)'$.

Proof. Straightforward.

Proposition 3.12 Let $\xi_D(V_1^*)$, $\xi_D(V_2^*)$ and $\xi_F(V_1^*)$ be NPSs over *W* for NSs *D* and *F*. Then,

(i) $D \subseteq F \Rightarrow \xi_D(V_1^*) \subseteq \xi_F(V_1^*)$.

(ii) $\xi_D(V_1^*) \cup \xi_D(V_2^*) = \xi_D(V_1^* \cup V_2^*)$.

(iii) $\xi_D(V_1^*) \cap \xi_D(V_2^*) \subseteq \xi_D(V_1^*)$ and $\xi_D(V_1^*) \cap \xi_D(V_2^*) \subseteq \xi_D(V_2^*)$.

(iv) $\xi_D(V_1^*) \cup \xi_D(V_2^*) \supseteq \xi_D(V_1^*)$ and $\xi_D(V_1^*) \cup \xi_D(V_2^*) \supseteq \xi_D(V_2^*)$.

(v) $\xi_D(V_1^*) \subseteq \xi_D(V_2^*) \Rightarrow \xi_D(V_1^*)' \subseteq \xi_D(V_2^*)'$.

Proof. Straightforward.

Definition 3.13 Let (W, τ_n) be a NTS. Let *NPS*(*W*) be the collection of NPSs $\xi_D(V^*)$ over *W* and *D* be a τ_n -open neutrosophic set (ONS), where $D \neq \phi, W$. Then $\tau_{np} \subset NPS(W)$ is called a neutro-prime topology (NPT) if it satisfies the following conditions

(i) $0_{np}, 1_{np} \in \tau_{np}$.

(ii) the union of any collection of members of τ_{np} belongs to τ_{np}.

(iii) the intersection of any finite number of members of τ_{np} belongs to τ_{np}.

Then the pair (W, τ_{np}) is said to be a neutro-prime topological space (NPTS).

Every member of τ_{np} is said to be a τ_{np}-neutro-prime open set (τ_{np}- NPOS). The complement of every τ_{np}- NPOS of *W* is said to be a τ_{np}-neutro-prime closed set (τ_{np}- NPCS) of *W* and this collection is denoted by τ_{np}^*.

Example 3.14 Let $W = \{w_1, w_2, w_3\}$ and $\tau_n = \{0_n, 1_n, D, F\}$ where *D* and *F* are NSs over *W* and are defined as follows

$$D = \{\langle w_1, .1, .2, .8\rangle, \langle w_2, .4, .7, .3\rangle, \langle w_3, .6, .5, .2\rangle\}$$

and

$$F = \{\langle w_1, .6, .5, .3\rangle, \langle w_2, .9, .8, .1\rangle, \langle w_3, .7, .6, .1\rangle\}.$$

Thus (W, τ_n) is a NTS over *W*.

Here NPSs are

$$\eta_p D(w_1, w_3) = \{\xi_D(w_1), \xi_D(w_3), \xi_D(w_1, w_2), \xi_D(w_1, w_3), \xi_D(w_2, w_3)\},$$

where

$$\xi_D(w_1) = \{\langle w_1, .1, .2, .8\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .7, .3\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), 0, 0, 1\rangle\},$$

$$\xi_D(w_3) = \{\langle w_1, 0, 0, 1\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .6, .5, .2\rangle, \langle (w_1, w_2), 0, 0, 1\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), .6, .7, .2\rangle\},$$

$$\xi_D(w_1, w_2) = \{\langle w_1, .1, .2, .8\rangle, \langle w_2, .4, .7, .3\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .7, .3\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), .6, .7, .2\rangle\},$$

$$\xi_D(w_1, w_3) = \{\langle w_1, .1, .2, .8\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .6, .5, .2\rangle, \langle (w_1, w_2), .4, .7, .3\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), .6, .7, .2\rangle\},$$

$$\xi_D(w_2, w_3) = \{\langle w_1, 0, 0, 1\rangle, \langle w_2, .4, .7, .3\rangle, \langle w_3, .6, .5, .2\rangle, \langle (w_1, w_2), .4, .7, .3\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), .6, .7, .2\rangle\},$$

and

$$\eta_p F(w_2) = \{\xi_F(w_2), \xi_F(w_1, w_2), \xi_F(w_2, w_3)\},$$

where

$$\xi_F(w_2) = \{\langle w_1, 0, 0, 1\rangle, \langle w_2, .9, .8, .1\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .9, .8, .1\rangle, \langle (w_1, w_3), 0, 0, 1\rangle, \langle (w_2, w_3), .9, .8, .1\rangle\},$$

$\xi_F(w_1,w_2)=\{\langle w_1,.6,.5,.3\rangle,\langle w_2,.9,.8,.1\rangle,\langle w_3,0,0,1\rangle,\langle (w_1,w_2),.9,.8,.1\rangle,\langle (w_1,w_3),.7,.6,.1\rangle,\langle (w_2,w_3),.9,.8,.1\rangle\}$,
$\xi_F(w_2,w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,.9,.8,.1\rangle,\langle w_3,.7,.6,.1\rangle,\langle (w_1,w_2),.9,.8,.1\rangle,\langle (w_1,w_3),.7,.6,.1\rangle,\langle (w_2,w_3),.9,.8,.1\rangle\}$.

Then

$\tau_{np}=\{0_{np},1_{np},\xi_D(w_1,w_2),\xi_F(w_1,w_2)\}$ is a NPT.

Thus (W, τ_{np}) is a NPTS over W.

Also, the complement of the NPT τ_{np} is

$\tau_{np}{}^*=\{0_{np},1_{np},\xi_D(w_1,w_2)',\xi_F(w_1,w_2)'\}$,

where

$\xi_D(w_1,w_2)'=\{\langle w_1,.8,.8,.1\rangle,\langle w_2,.3,.3,.4\rangle,\langle w_3,1,1,0\rangle,\langle (w_1,w_2),.3,.3,.4\rangle,\langle (w_1,w_3),.2,.5,.6\rangle,\langle (w_2,w_3),.2,.3,.6\rangle\}$

and

$\xi_F(w_1,w_2)'=\{\langle w_1,.3,.5,.6\rangle,\langle w_2,.1,.2,.9\rangle,\langle w_3,1,1,0\rangle,\langle (w_1,w_2),.1,.2,.9\rangle,\langle (w_1,w_3),.1,.4,.7\rangle,\langle (w_2,w_3),.1,.2,.9\rangle\}$.

Remark 3.15 The collection of NPS $\eta_p D(V)$ can generate one or more NPT, which is illustrated in the following example.

Example 3.16 Consider Example 3.14.

Here

$1\tau_{np}=\{0_{np},1_{np},\xi_D(w_1,w_2),\xi_F(w_1,w_2)\}$ and

$2\tau_{np}=\{0_{np},1_{np},\xi_D(w_2,w_3),\xi_F(w_2,w_3)\}$ are NPTs

Thus (W, $1\tau_{np}$) and (W, $2\tau_{np}$) are NPTSs over W.

Also, the complement of the NPTs $1\tau_{np}$ and $2\tau_{np}$ are

$1\tau_{np}{}^*=\{0_{np},1_{np},\xi_D(w_1,w_2)',\xi_F(w_1,w_2)'\}$ and

$2\tau_{np}{}^*=\{0_{np},1_{np},\xi_D(w_2,w_3)',\xi_F(w_2,w_3)'\}$, respectively,

where

$\xi_D(w_1,w_2)'=\{\langle w_1,.8,.8,.1\rangle,\langle w_2,.3,.3,.4\rangle,\langle w_3,1,1,0\rangle,\langle (w_1,w_2),.3,.3,.4\rangle,\langle (w_1,w_3),.2,.5,.6\rangle,\langle (w_2,w_3),.2,.3,.6\rangle\}$
$\xi_F(w_1,w_2)'=\{\langle w_1,.3,.5,.6\rangle,\langle w_2,.1,.2,.9\rangle,\langle w_3,1,1,0\rangle,\langle (w_1,w_2),.1,.2,.9\rangle,\langle (w_1,w_3),.1,.4,.7\rangle,\langle (w_2,w_3),.1,.2,.9\rangle\}$

and

$\xi_D(w_2,w_3)'=\{\langle w_1,1,1,0\rangle,\langle w_2,.3,.3,.4\rangle,\langle w_3,.2,.5,.6\rangle,\langle (w_1,w_2),.3,.3,.4\rangle,\langle (w_1,w_3),.2,.5,.6\rangle,\langle (w_2,w_3),.2,.3,.6\rangle\}$
$\xi_F(w_2,w_3)'=\{\langle w_1,1,1,0\rangle,\langle w_2,.1,.2,.9\rangle,\langle w_3,.1,.4,.7\rangle,\langle (w_1,w_2),.1,.2,.9\rangle,\langle (w_1,w_3),.1,.4,.7\rangle,\langle (w_2,w_3),.1,.2,.9\rangle\}$.

Definition 3.17 A NPT τ_{np} is said to be a neutro-prime discrete topology if $\tau_{np}=NPS(W)$ for all the subsets of W.

Definition 3.18 A NPT τ_{np} is said to be a neutro-prime indiscrete topology if τ_{np} contains only 0_{np} and 1_{np}.

Proposition 3.19 Let (W, $1\tau_{np}$) and (W, $2\tau_{np}$) be two NPTSs over W and let $1\tau_{np}\cap 2\tau_{np}=\{D\in NPS(W): D\in 1\tau_{np}\cap 2\tau_{np}\}$. Then $1\tau_{np}\cap 2\tau_{np}$ is also a NPT over W.

Proof. Let (W, $1\tau_{np}$) and (W, $2\tau_{np}$) be two NPTSs over W.

(i) $0_{np},1_{np}\in 1\tau_{np}\cap 2\tau_{np}$.

(ii) Let $D_1,D_2\in 1\tau_{np}\cap 2\tau_{np}$.

Then $D_1,D_2\in 1\tau_{np}$ and $D_1,D_2\in 2\tau_{np}$.

$\Rightarrow D_1 \cap D_2 \in 1\tau_{np}$ and $D_1 \cap D_2 \in 2\tau_{np}$.

$\Rightarrow D_1 \cap D_2 \in 1\tau_{np} \cap 2\tau_{np}$.

(iii) Let $D_i \in 1\tau_{np} \cap 2\tau_{np}$, $i \in \mathrm{I}$.

Then $D_i \in 1\tau_{np}$ and $D_i \in 2\tau_{np}$, $i \in \mathrm{I}$.

$\Rightarrow \bigcup_{i \in \mathrm{I}} D_i \in 1\tau_{np}$ and $\Rightarrow \bigcup_{i \in \mathrm{I}} D_i \in 2\tau_{np}$, $i \in \mathrm{I}$.

$\Rightarrow \bigcup_{i \in \mathrm{I}} D_i \in 1\tau_{np} \cup 2\tau_{np}$.

Thus $1\tau_{np} \cap 2\tau_{np}$ is also a NPT over *W*.

Remark 3.20 The union of two NPTs need not be a NPT. The following example illustrates this remark.

Example 3.21 Consider Example 3.14.
Here NPTs are

$$1\tau_{np} = \{0_{np}, 1_{np}, \xi_D(w_1, w_2), \xi_F(w_1, w_2)\},$$

where

$$\xi_D(w_1, w_2) = \{\langle w_1, .1, .2, .8\rangle, \langle w_2, .4, .7, .3\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .7, .3\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), .6, .7, .2\rangle\},$$

$$\xi_F(w_1, w_2) = \{\langle w_1, .6, .5, .3\rangle, \langle w_2, .9, .8, .1\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .9, .8, .1\rangle, \langle (w_1, w_3), .7, .6, .1\rangle, \langle (w_2, w_3), .9, .8, .1\rangle\}$$

and

$$2\tau_{np} = \{0_{np}, 1_{np}, \xi_D(w_2, w_3), \xi_F(w_2, w_3)\},$$

where

$$\xi_D(w_2, w_3) = \{\langle w_1, 0, 0, 1\rangle, \langle w_2, .4, .7, .3\rangle, \langle w_3, .6, .5, .2\rangle, \langle (w_1, w_2), .4, .7, .3\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), .6, .7, .2\rangle\},$$

$$\xi_F(w_2, w_3) = \{\langle w_1, 0, 0, 1\rangle, \langle w_2, .9, .8, .1\rangle, \langle w_3, .7, .6, .1\rangle, \langle (w_1, w_2), .9, .8, .1\rangle, \langle (w_1, w_3), .7, .6, .1\rangle, \langle (w_2, w_3), .9, .8, .1\rangle\}.$$

Thus $(W, 1\tau_{np})$ and $(W, 2\tau_{np})$ are NPTSs over *W*.

Clearly,

$$1\tau_{np} \cup 2\tau_{np} = \{0_{np}, 1_{np}, \xi_D(w_1, w_2), \xi_F(w_1, w_2), \xi_D(w_2, w_3), \xi_F(w_2, w_3)\}.$$

Then

$$\xi_D(w_1, w_2) \cap \xi_D(w_2, w_3)$$
$$= \{\langle w_1, 0, 0, 1\rangle, \langle w_2, .4, .7, .3\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .7, .3\rangle, \langle (w_1, w_3), .6, .5, .2\rangle, \langle (w_2, w_3), .6, .7, .2\rangle\}$$

Thus $1\tau_{np} \cup 2\tau_{np}$ is not a NPTS, since $\xi_D(w_1, w_2) \cap \xi_D(w_2, w_3) \notin 1\tau_{np} \cup 2\tau_{np}$.

Hence the union of two NPTs need not be a NPT.

Proposition 3.22 Let $\xi_D(V_1^*)$ and $\xi_D(V_2^*)$ be two τ_{np}-NPOSs over *W*. Then

(i) $\left(\xi_D(V_1^*) \cup \xi_D(V_2^*)\right)' = \xi_D(V_1^*)' \cap \xi_D(V_2^*)'$.

(ii) $\left(\xi_D(V_1^*) \cap \xi_D(V_2^*)\right)' = \xi_D(V_1^*)' \cup \xi_D(V_2^*)'$.

Proof. Straightforward.

Definition 3.23 Let (W, τ_{np}) be a NPTS over *W*. Let $\xi_D(V^*)$ be any NPSs over *W*. Then the neutro-prime interior of $\xi_D(V^*)$ is denoted by $\operatorname{int}_{np}\left(\xi_D(V^*)\right)$ and defined by

$$\operatorname{int}_{np}\left(\xi_D(V^*)\right) = \cup\{\xi_D(U^*) : \xi_D(U^*) \in \tau_{np} \quad and \quad \xi_D(U^*) \subseteq \xi_D(V^*)\}$$

Clearly, it is the union of all τ_{np}-NPOSs contained in $\xi_D(V^*)$.

Definition 3.24 Let (W, τ_{np}) be a NPTS over *W*. Let $\xi_D(V^*)$ be any NPSs over *W*. Then the neutro-prime closure of $\xi_D(V^*)$ is denoted by $cl_{np}\left(\xi_D(V^*)\right)$ and defined by

$$cl_{np}\left(\xi_D(V^*)\right) = \bigcap\left\{\xi_D(U^*) : \xi_D(U^*) \in \tau_{np}^* \quad and \quad \xi_D(U^*) \supseteq \xi_D(V^*)\right\}$$

Clearly, it is the intersection of all τ_{np}^*-NPCSs containing $\xi_D(V^*)$.

Example 3.25 Let $W = \{w_1, w_2, w_3\}$ and $\tau_n = \{0_n, 1_n, A, B, C, D\}$ where *A, B, C,* and *D* are NSs over *W* and are defined as follows

$$A = \left\{\langle w_1, .1, .2, .3\rangle, \langle w_2, .4, .5, .6\rangle, \langle w_3, .2, .4, .6\rangle\right\},$$
$$B = \left\{\langle w_1, .4, .5, .6\rangle, \langle w_2, .7, .8, .2\rangle, \langle w_3, .1, .2, .3\rangle\right\},$$
$$C = \left\{\langle w_1, .1, .2, .6\rangle, \langle w_2, .4, .5, .6\rangle, \langle w_3, .1, .2, .6\rangle\right\} \text{ and}$$
$$D = \left\{\langle w_1, .4, .5, .3\rangle, \langle w_2, .7, .8, .2\rangle, \langle w_3, .2, .4, .3\rangle\right\}.$$

Here $A \bigcup B = D$, $A \bigcup D = D$, $A \bigcup C = A$, $B \bigcup D = D$, $B \bigcup C = B$, $D \bigcup C = D$ and $A \bigcap B = A$, $A \bigcap D = A$, $A \bigcap C = C$, $B \bigcap D = B$, $B \bigcap C = C$, $D \bigcap C = C$.

Then *A, B, C,* and *D* are τ_n- ONSs over *W*.

Thus (W, τ_n) is a NTS over *W*.

Here NPSs are

$$\eta_p A(w_2) = \{\xi_A(w_2), \xi_A(w_1, w_2), \xi_A(w_2, w_3)\},$$

where

$$\xi_A(w_2) = \left\{\langle w_1, 0, 0, 1\rangle, \langle w_2, .4, .5, .6\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .5, .3\rangle, \langle (w_1, w_3), 0, 0, 1\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\right\},$$
$$\xi_A(w_1, w_2) = \left\{\langle w_1, .1, .2, .3\rangle, \langle w_2, .4, .5, .6\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .5, .3\rangle, \langle (w_1, w_3), .2, .4, .3\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\right\},$$
$$\xi_A(w_2, w_3) = \left\{\langle w_1, 0, 0, 1\rangle, \langle w_2, .4, .5, .6\rangle, \langle w_3, .2, .4, .6\rangle, \langle (w_1, w_2), .4, .5, .3\rangle, \langle (w_1, w_3), .2, .4, .3\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\right\};$$

$$\eta_p B(w_1) = \{\xi_B(w_1), \xi_B(w_1, w_2), \xi_B(w_1, w_3)\},$$

where

$$\xi_B(w_1) = \left\{\langle w_1, .4, .5, .6\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .7, .8, .2\rangle, \langle (w_1, w_3), .4, .5, .3\rangle, \langle (w_2, w_3), 0, 0, 1\rangle\right\},$$
$$\xi_B(w_1, w_2) = \left\{\langle w_1, .4, .5, .6\rangle, \langle w_2, .7, .8, .2\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .7, .8, .2\rangle, \langle (w_1, w_3), .4, .5, .3\rangle, \langle (w_2, w_3), .7, .8, .2\rangle\right\}$$
$$\xi_B(w_1, w_3) = \left\{\langle w_1, .4, .5, .6\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .1, .2, .3\rangle, \langle (w_1, w_2), .7, .8, .2\rangle, \langle (w_1, w_3), .4, .5, .3\rangle, \langle (w_2, w_3), .7, .8, .2\rangle\right\};$$

$$\eta_p C(w_1, w_3) = \{\xi_C(w_1), \xi_C(w_3), \xi_C(w_1, w_2), \xi_C(w_1, w_3), \xi_C(w_2, w_3)\},$$

where

$$\xi_C(w_1) = \left\{\langle w_1, .1, .2, .6\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .5, .6\rangle, \langle (w_1, w_3), .1, .2, .6\rangle, \langle (w_2, w_3), 0, 0, 1\rangle\right\},$$
$$\xi_C(w_3) = \left\{\langle w_1, 0, 0, 1\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .1, .2, .6\rangle, \langle (w_1, w_2), 0, 0, 1\rangle, \langle (w_1, w_3), .1, .2, .6\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\right\},$$
$$\xi_C(w_1, w_2) = \left\{\langle w_1, .1, .2, .6\rangle, \langle w_2, .4, .5, .6\rangle, \langle w_3, 0, 0, 1\rangle, \langle (w_1, w_2), .4, .5, .6\rangle, \langle (w_1, w_3), .1, .2, .3\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\right\},$$
$$\xi_C(w_1, w_3) = \left\{\langle w_1, .1, .2, .6\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .1, .2, .6\rangle, \langle (w_1, w_2), .4, .5, .6\rangle, \langle (w_1, w_3), .1, .2, .3\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\right\},$$
$$\xi_C(w_2, w_3) = \left\{\langle w_1, .0, 0, 1\rangle, \langle w_2, .4, .5, .6\rangle, \langle w_3, 1, .2, .6\rangle, \langle (w_1, w_2), .4, .5, .6\rangle, \langle (w_1, w_3), .1, .2, .3\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\right\};$$

$$\eta_p D(w_3) = \{\xi_D(w_3), \xi_D(w_1, w_3), \xi_D(w_2, w_3)\},$$

where

$$\xi_D(w_3) = \left\{\langle w_1, 0, 0, 1\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .2, .4, .3\rangle, \langle (w_1, w_2), 0, 0, 1\rangle, \langle (w_1, w_3), .4, .5, .3\rangle, \langle (w_2, w_3), .7, .8, .2\rangle\right\},$$
$$\xi_D(w_1, w_3) = \left\{\langle w_1, .4, .5, .3\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .2, .4, .3\rangle, \langle (w_1, w_2), .7, .8, .2\rangle, \langle (w_1, w_3), .4, .5, .3\rangle, \langle (w_2, w_3), .7, .8, .2\rangle\right\},$$
$$\xi_D(w_2, w_3) = \left\{\langle w_1, 0, 0, 1\rangle, \langle w_2, .7, .8, .2\rangle, \langle w_3, .2, .4, .3\rangle, \langle (w_1, w_2), .7, .8, .2\rangle, \langle (w_1, w_3), .4, .5, .3\rangle, \langle (w_2, w_3), .7, .8, .2\rangle\right\}.$$

Then

$$\tau_{np} = \{0_{np}, 1_{np}, \xi_B(w_1), \xi_C(w_1)\} \text{ is a NPT.}$$

Thus (W, τ_{np}) is a NPTS over W.

Also, the complement of the NPT τ_{np} is

$$\tau_{np}^{*} = \{0_{np}, 1_{np}, \xi_B(w_1)', \xi_C(w_1)'\},$$

where

$$\xi_B(w_1)' = \{\langle w_1, .6, .5, .4\rangle, \langle w_2, 1, 1, 0\rangle, \langle w_3, 1, 1, 0\rangle, \langle (w_1, w_2), .2, .2, .7\rangle, \langle (w_1, w_3), .3, .5, .4\rangle, \langle (w_2, w_3), 1, 1, 0\rangle\}$$

and

$$\xi_C(w_1)' = \{\langle w_1, .6, .8, .1\rangle, \langle w_2, 1, 1, 0\rangle, \langle w_3, 1, 1, 0\rangle, \langle (w_1, w_2), .6, .5, .4\rangle, \langle (w_1, w_3), .6, .8, .1\rangle, \langle (w_2, w_3), 1, 1, 0\rangle\}.$$

Consider a NPS for NS B,

$$\xi_B(w_1, w_3) = \{\langle w_1, .4, .5, .6\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .1, .2, .3\rangle, \langle (w_1, w_2), .7, .8, .2\rangle, \langle (w_1, w_3), .4, .5, .3\rangle, \langle (w_2, w_3), .7, .8, .2\rangle\}.$$

Clearly,

$$\xi_B(w_1, w_3) \supseteq 0_{np}, \xi_B(w_1).$$

Thus

$$\operatorname{int}_{np}(\xi_B(w_1, w_3)) = 0_{np} \cup \xi_B(w_1) = \xi_B(w_1).$$

Also,

$$\xi_B(w_1, w_3) \subseteq 1_{np}.$$

Thus

$$cl_{np}(\xi_B(w_1, w_3)) = 1_{np}.$$

Consider a NPS for NS C,

$$\xi_C(w_3) = \{\langle w_1, 0, 0, 1\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .1, .2, .6\rangle, \langle (w_1, w_2), 0, 0, 1\rangle, \langle (w_1, w_3), .1, .2, .6\rangle, \langle (w_2, w_3), .4, .5, .6\rangle\}.$$

Clearly,

$$\xi_C(w_3) \supseteq 0_{np}.$$

Thus

$$\operatorname{int}_{np}(\xi_C(w_3)) = 0_{np}.$$

Also,

$$\xi_C(w_3) \subseteq 1_{np}, \xi_C(w_1)'.$$

Thus

$$cl_{np}(\xi_C(w_3)) = 1_{np} \cap \xi_C(w_1)' = \xi_C(w_1)'.$$

Theorem 3.26 Let (W, τ_{np}) be a NPTS over W. Let $\xi_D(V_1^*)$ and $\xi_F(V_1^*)$ be NPSs over W for NSs D and F. Then

(i) $\operatorname{int}_{np}(\xi_D(V_1^*)) \subseteq \xi_D(V_1^*)$ and $\operatorname{int}_{np}(\xi_D(V_1^*))$ is the largest τ_{np}-NPOS.

(ii) $D \subseteq F \Rightarrow \operatorname{int}_{np}(\xi_D(V_1^*)) \subseteq \operatorname{int}_{np}(\xi_F(V_1^*))$.

(iii) $\operatorname{int}_{np}(\xi_D(V_1^*))$ is an τ_{np}-NPOS.

(iv) $\xi_D(V_1^*)$ is a τ_{np}-NPOS iff $\operatorname{int}_{np}(\xi_D(V_1^*)) = \xi_D(V_1^*)$.

(v) $\operatorname{int}_{np}(\operatorname{int}_{np}(\xi_D(V_1^*))) = \operatorname{int}_{np}(\xi_D(V_1^*))$.

(vi) $\operatorname{int}_{np}(0_{np}) = 0_{np}$ and $\operatorname{int}_{np}(1_{np}) = 1_{np}$.

(vii) $\operatorname{int}_{np}(\xi_D(V_1^*) \cap \xi_F(V_1^*)) = \operatorname{int}_{np}(\xi_D(V_1^*)) \cap \operatorname{int}_{np}(\xi_F(V_1^*))$.

(viii) $\operatorname{int}_{np}(\xi_D(V_1^*) \cup \xi_F(V_1^*)) \subseteq \operatorname{int}_{np}(\xi_D(V_1^*)) \cup \operatorname{int}_{np}(\xi_F(V_1^*))$.

Proof. Follows from Definition 3.23.

Theorem 3.27 Let (W, τ_{np}) be a NPTS over W. Let $\xi_D(V_1^*)$ and $\xi_F(V_1^*)$ be NPSs over W for NSs D and F. Then

(i) $\xi_D(V_1^*) \subseteq cl_{np}(\xi_D(V_1^*))$ and $cl_{np}(\xi_D(V_1^*))$ is the smallest τ_{np}-NPCS.

(ii) $D \subseteq F \Rightarrow cl_{np}(\xi_D(V_1^*)) \subseteq cl_{np}(\xi_F(V_1^*))$.

(iii) $cl_{np}(\xi_D(V_1^*))$ is an τ_{np}-NPCS.

(iv) $\xi_D(V_1^*)$ is a τ_{np}-NPCS iff $cl_{np}(\xi_D(V_1^*)) = \xi_D(V_1^*)$.

(v) $cl_{np}(cl_{np}(\xi_D(V_1^*))) = cl_{np}(\xi_D(V_1^*))$.

(vi) $cl_{np}(0_{np}) = 0_{np}$ and $cl_{np}(1_{np}) = 1_{np}$.

(vii) $cl_{np}(\xi_D(V_1^*) \cup \xi_F(V_1^*)) = cl_{np}(\xi_D(V_1^*)) \cup cl_{np}(\xi_F(V_1^*))$.

(viii) $cl_{np}(\xi_D(V_1^*) \cap \xi_F(V_1^*)) \subseteq cl_{np}(\xi_D(V_1^*)) \cap cl_{np}(\xi_F(V_1^*))$.

Proof. Follows from Definition 3.24.

Theorem 3.28 Let (W, τ_{np}) be a NPTS over W. Let $\xi_D(V^*)$ be a NPS over W for a NS D. Then

(i) $\left(\operatorname{int}_{np}(\xi_D(V_1^*))\right)' = cl_{np}(\xi_D(V^*)')$.

(ii) $\left(cl_{np}(\xi_D(V_1^*))\right)' = \operatorname{int}_{np}(\xi_D(V^*)')$.

Proof. Follows from Definitions 3.23 and 3.24.

4. Decision Making in NPTS

In this section, the real-life application dealt to repair the sample machines with some damages. To repair it, priority is given to the high damaged machine. The solving techniques are given in the algorithm and formulae for evaluation are given. Some examples are considered to decide on these DM problems.

Definition 4.1 Let $\xi_D(V^*)$ be a τ_{np}-NPOS over W of a NPTS (W, τ_{np}). Then the neutro-prime absolute complement of $\xi_D(V^*)$ is denoted as $\tilde{\xi}_D((V^*)')$ and defined as $\tilde{\xi}_D((V^*)') = \tilde{\xi}_D((W - V^*))$.

Thus the collection of $\tilde{\xi}_D((V^*)')$ is denoted as $\tilde{\tau}_{np}$ and defined as $\tilde{\tau}_{np} = \{0_{np}, 1_{np}, \tilde{\xi}_D((V^*)')\}$. The elements belong to $\tilde{\xi}_D((V^*)')$ are said to be neutro-prime absolute open sets (NPAOSs) over (W, τ_{np}) and the complement of NPOSs are said to be neutro-prime absolute closed sets (NPACSs) over (W, τ_{np}) and denote the collection by $\tilde{\tau}_{np}^*$.

Example 4.2 Let $W = \{w_1, w_2, w_3\}$ and $\tau_n = \{0_n, 1_n, D\}$ where D is a NS over W and are defined as follows

$$D = \{\langle w_1, .9, .4, .6\rangle, \langle w_2, .6, .5, .1\rangle, \langle w_3, .7, .8, .1\rangle\}.$$

Thus (W, τ_n) is a NTS over W.

Then NPS

$$\eta_p D(w_3) = \{\xi_D(w_3), \xi_D(w_1, w_3), \xi_D(w_2, w_3)\},$$

where

$\xi_D(w_3) = \{\langle w_1, 0, 0, 1\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .7, .8, .1\rangle, \langle (w_1, w_2), 0, 0, 1\rangle, \langle (w_1, w_3), .9, .8, .1\rangle, \langle (w_2, w_3), .7, .8, .1\rangle\}$,

$\xi_D(w_1, w_3) = \{\langle w_1, .9, .4, .6\rangle, \langle w_2, 0, 0, 1\rangle, \langle w_3, .7, .8, .1\rangle, \langle (w_1, w_2), .9, .5, .1\rangle, \langle (w_1, w_3), .9, .8, .1\rangle, \langle (w_2, w_3), .7, .8, .1\rangle\}$

and

$\xi_D(w_2,w_3)=\{\langle w_1,0,0,1\rangle,\langle w_2,.6,.5,.1\rangle,\langle w_3,.7,.8,.1\rangle,\langle (w_1,w_2),.9,.5,.1\rangle,\langle (w_1,w_3),.9,.8,.1\rangle,\langle (w_2,w_3),.7,.8,.1\rangle\}$.

Then

$\tau_{np}=\{0_{np},1_{np},\xi_D(w_1,w_3)\}$ is a NPT.

Thus (W, τ_{np}) is a NPTS over W.

Also, the complement of the NPT τ_{np} is

$\tau_{np}{}^{*}=\{0_{np},1_{np},\xi_D(w_1,w_3)'\}$,

where

$\xi_D(w_1,w_3)'=\{\langle w_1,.6,.6,.9\rangle,\langle w_2,1,1,0\rangle,\langle w_3,.1,.2,.7\rangle,\langle (w_1,w_2),.1,.5,.9\rangle,\langle (w_1,w_3),.1,.2,.9\rangle,\langle (w_2,w_3),.1,.2,.7\rangle\}$.

Then NPAOSs over (W, τ_{np}) is

$\tilde{\tau}_{np}=\{0_{np},1_{np},\tilde{\xi}_D((w_1,w_3)')\}=\{0_{np},1_{np},\tilde{\xi}_D(w_2)\}$,

where

$\tilde{\xi}_D((w_1,w_3)')=\tilde{\xi}_D(w_2)=\{\langle w_1,0,0,1\rangle,\langle w_2,.6,.5,.1\rangle,\langle w_3,0,0,1\rangle,\langle (w_1,w_2),.9,.5,.1\rangle,\langle (w_1,w_3),0,0,1\rangle,\langle (w_2,w_3),.7,.8,.1\rangle\}$.

Also, NPACSs over (W, τ_{np}) is

$\tilde{\tau}_{np}{}^{*}=\{0_{np},1_{np},\tilde{\xi}_D((w_1,w_3)')'\}$

where

$\tilde{\xi}_D((w_1,w_3)')'=\tilde{\xi}_D(w_2)'=\{\langle w_1,1,1,0\rangle,\langle w_2,.1,.5,.6\rangle,\langle w_3,1,1,0\rangle,\langle (w_1,w_2),.1,.5,.9\rangle,\langle (w_1,w_3),1,1,0\rangle,\langle (w_2,w_3),.1,.2,.7\rangle\}$.

Definition 4.3 Let W be a set of universe and $w\in W$. Let D be a NS over W and U be any proper non-empty subset of W. Let $\xi_D(U)$ be a τ_{np}-NPOS over W of a NFTS (W, τ_{np}).

Then the value of D with respect to U is denoted by $Val[D(U)]$ and is calculated by the formula

$$Val[D(U)]=\left|\frac{\left[\sum_i\left(\tilde{T}_D(U')\right)_i-\sum_i\left(F_D(U)'\right)_i\right]+\left[\sum_i\left(T_D(U)'\right)_i-\sum_i\left(\tilde{F}_D(U')\right)_i\right]}{2}\times\left[1-\frac{\sum_i\left(\tilde{I}_D(U')\right)_i-\sum_i\left(I_D(U)'\right)_i}{2}\right]\right|, \quad (4.3.1)$$

where

$\sum_i\left(T_D(U)'\right)_i$, $\sum_i\left(I_D(U)'\right)_i$ and $\sum_i\left(F_D(U)'\right)_i$ are the sum of all truth, indeterminacy and falsity values of $\xi_D(U)'$ respectively, and

$\sum_i\left(\tilde{T}_R(U')\right)_i$, $\sum_i\left(\tilde{I}_R(U')\right)_i$ and $\sum_i\left(\tilde{F}_D(U')\right)_i$ are the sum of all truth, indeterminacy, and falsity values of $\tilde{\xi}_D((U'))$ respectively.

Then the grand value of D is denoted by $GV[D]$ and is calculated by the formula

$$GV[D]=\sum_i Val[D(U_i)], \text{ for all } i. \quad (4.3.2)$$

Algorithm

Step 1: List the set of machines for the sample.

Step 2: List some of its damaged parts as the universe W, where $w\in W$.

Step 3: Go through the damages of the machines.
Step 4: Define each machines as NSs, say *M*.
Step 5: Collect these NSs which defines a NT τ_n and so (W, τ_n) is a NTS.
Step 6: Define NPSs for each NS with respect to their damaged parts, say $\xi_M(U)$, where *U* is a proper non-empty subset of *W*.

Step 7: Define all possible NPTs τ_{np} and $\xi_M(U) \in \tau_{np}$, where *U* is a proper non-empty singleton subset of *W*.

Step 8: Define NPTSs (W, τ_{np}) for all possible NPTs τ_{np}.

Step 9: Find the complement and neutro-prime absolute complement of each NPTs.
Step 10: Calculate *Val*[*M*(*U*)] for all *M* with respect to some *U*, by using the formula 4.3.1.
Step 11: Tabulate all the estimated values of *Val*[*M*(*U*)].
Step 12: Calculate *GV*[*M*] for all *M*, by using the formula 4.3.2.
Step 13: Tabulate all the estimated values of *GV*[*M*].
Step 14: Select the highest value among all the *GV*[*M*].
Step 15: If two or more *GV*[*M*] are similar for a particular *U*, replace that *U* with some other damaged parts and repeat the process.
Step 16: End the process, till getting a unique *GV*[*M*].

Example 4.4 Consider the problem that a technician came to repair damaged machines. Let *M*I, *M*II, *M*III, and *M*IV be sample machines whose damages to be repaired. Let $W = \{p_1, p_2, p_3\}$ be some parts of each damaged machine, where p_1 –part 1, p_2 –part 1 and p_3 –part 3. Here the technician gives priority to the high damaged machine and to repair it initially.
1. Let *M*I, *M*II, *M*III, and *M*IV be sample machines whose damages to be repaired.
2. Let $W = \{p_1, p_2, p_3\}$ be the universe, where p_1 –part 1, p_2 –part 2, and p_3 –part 3.
3. The technician goes through the damages on each machine.
4. Define *M*I, *M*II, *M*III, and *M*IV as NSs.

$$MI = \{\langle p_1,.7,.6,.3\rangle, \langle p_2,.4,.5,.4\rangle, \langle p_3,.5,.3,.4\rangle\},$$

$$MII = \{\langle p_1,.6,.3,.1\rangle, \langle p_2,.6,.8,.4\rangle, \langle p_3,.2,.2,.7\rangle\},$$

$$MIII = \{\langle p_1,.7,.6,.1\rangle, \langle p_2,.6,.8,.4\rangle, \langle p_3,.5,.3,.4\rangle\} \quad \text{and}$$

$$MIV = \{\langle p_1,.6,.3,.3\rangle, \langle p_2,.4,.5,.4\rangle, \langle p_3,.2,.2,.7\rangle\}.$$

5. Thus $\tau_n = \{0_n, 1_n, MI, MII, MIII, MIV\}$ is a NT and so (W, τ_n) is a NTS.
6. Define NPSs for each NS with respect to their damaged parts as follows:

$$\eta_p MI(p_1, p_3) = \{\xi_{MI}(p_1), \xi_{MI}(p_3), \xi_{MI}(p_1, p_2), \xi_{MI}(p_1, p_3), \xi_{MI}(p_2, p_3)\},$$

where

$$\xi_{MI}(p_1) = \{\langle p_1,.7,.6,.3\rangle, \langle p_2,0,0,1\rangle, \langle p_3,0,0,1\rangle, \langle (p_1, p_2),.7,.6,.3\rangle, \langle (p_1, p_3),.7,.6,.3\rangle, \langle (p_2, p_3),0,0,1\rangle\},$$

$\xi_{MI}(p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),0,0,1\rangle,\langle (p_1,p_3),.7,.6,.3\rangle,\langle (p_2,p_3),.5,.5,.4\rangle\}$,
$\xi_{MI}(p_1,p_2)=\{\langle p_1,.7,.6,.3\rangle,\langle p_2,.4,.5,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.6,.3\rangle,\langle (p_1,p_3),.7,.6,.3\rangle,\langle (p_2,p_3),.5,.5,.4\rangle\}$,
$\xi_{MI}(p_1,p_3)=\{\langle p_1,.7,.6,.3\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.6,.3\rangle,\langle (p_1,p_3),.7,.6,.3\rangle,\langle (p_2,p_3),.5,.5,.4\rangle\}$,
$\xi_{MI}(p_2,p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,.4,.5,.4\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.6,.3\rangle,\langle (p_1,p_3),.7,.6,.3\rangle,\langle (p_2,p_3),.5,.5,.4\rangle\}$;

$$\eta_p MII(p_2)=\{\xi_{MII}(p_2),\xi_{MII}(p_1,p_2),\xi_{MII}(p_2,p_3)\},$$

where
$\xi_{MII}(p_2)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),0,0,1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$,
$\xi_{MII}(p_1,p_2)=\{\langle p_1,.6,.3,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),.6,.3,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$,
$\xi_{MII}(p_2,p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.2,.2,.7\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),.6,.3,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$;

$$\eta_p MIII(p_1,p_2)=\{\xi_{MIII}(p_1),\xi_{MIII}(p_2),\xi_{MIII}(p_1,p_2),\xi_{MIII}(p_1,p_3),\xi_{MIII}(p_2,p_3)\},$$

where
$\xi_{MIII}(p_1)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),0,0,1\rangle\}$,
$\xi_{MIII}(p_2)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),0,0,1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$,
$\xi_{MIII}(p_1,p_2)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$,
$\xi_{MIII}(p_1,p_3)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$,
$\xi_{MIII}(p_2,p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$
and

$$\eta_p MIV(p_1)=\{\xi_{MIV}(p_1),\xi_{MIV}(p_1,p_2),\xi_{MIV}(p_1,p_3)\},$$

where
$\xi_{MIV}(p_1)=\{\langle p_1,.6,.3,.3\rangle,\langle p_2,0,0,1\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.6,.5,.3\rangle,\langle (p_1,p_3),.6,.3,.3\rangle,\langle (p_2,p_3),0,0,1\rangle\}$,
$\xi_{MIV}(p_1,p_2)=\{\langle p_1,.6,.3,.3\rangle,\langle p_2,.4,.5,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.6,.5,.3\rangle,\langle (p_1,p_3),.6,.3,.3\rangle,\langle (p_2,p_3),.4,.5,.4\rangle\}$,
$\xi_{MIV}(p_1,p_3)=\{\langle p_1,.6,.3,.3\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.2,.2,.7\rangle,\langle (p_1,p_2),.6,.5,.3\rangle,\langle (p_1,p_3),.6,.3,.3\rangle,\langle (p_2,p_3),.4,.5,.4\rangle\}$.

7. The possible NPTs for all proper non-empty singleton subset of *W* are defined as follows:
$1\tau_{np}=\{0_{np},1_{np},\xi_{MI}(p_1),\xi_{MIII}(p_1),\xi_{MIV}(p_1)\}$,
$2\tau_{np}=\{0_{np},1_{np},\xi_{MII}(p_2),\xi_{MIII}(p_2)\}$ and
$3\tau_{np}=\{0_{np},1_{np},\xi_{MI}(p_3)\}$.

8. Thus $(W,1\tau_{np})$, $(W,2\tau_{np})$ and $(W,3\tau_{np})$ are NPTSs over *W*.

9. The complement and neutro-prime absolute complement of NPT $1\tau_{np}$ are as follows,

$$1\tau_{np}^{*}=\{0_{np},1_{np},\xi_{MI}(p_1)',\xi_{MIII}(p_1)',\xi_{MIV}(p_1)'\},$$

where
$\xi_{MI}(p_1)'=\{\langle p_1,.3,.4,.7\rangle,\langle p_2,1,1,0\rangle,\langle p_3,1,1,0\rangle,\langle (p_1,p_2),.3,.4,.7\rangle,\langle (p_1,p_3),.3,.4,.7\rangle,\langle (p_2,p_3),1,1,0\rangle\}$
$\xi_{MIII}(p_1)'=\{\langle p_1,.1,.4,.7\rangle,\langle p_2,1,1,0\rangle,\langle p_3,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.7\rangle,\langle (p_1,p_3),.1,.4,.7\rangle,\langle (p_2,p_3),1,1,0\rangle\}$
$\xi_{MIV}(p_1)'=\{\langle p_1,.3,.7,.6\rangle,\langle p_2,1,1,0\rangle,\langle p_3,1,1,0\rangle,\langle (p_1,p_2),.3,.5,.6\rangle,\langle (p_1,p_3),.3,.7,.6\rangle,\langle (p_2,p_3),1,1,0\rangle\}$
and

$$1\tilde{\tau}_{np}=\{0_{np},1_{np},\tilde{\xi}_{MI}(p_2,p_3),\tilde{\xi}_{MIII}(p_2,p_3),\tilde{\xi}_{MIV}(p_2,p_3)\},$$

where
$\tilde{\xi}_{MI}(p_2,p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,.4,.5,.4\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.6,.3\rangle,\langle (p_1,p_3),.7,.6,.3\rangle,\langle (p_2,p_3),.5,.5,.4\rangle\}$
$\tilde{\xi}_{MIII}(p_2,p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$

$\tilde{\xi}_{MIV}(p_2,p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,.4,.5,.4\rangle,\langle p_3,.2,.2,.7\rangle,\langle (p_1,p_2),.6,.5,.3\rangle,\langle (p_1,p_3),.6,.3,.3\rangle,\langle (p_2,p_3),.4,.5,.4\rangle\}.$

The complement and neutro-prime absolute complement of NPT $2\tau_{np}$ are as follows,

$2\tau_{np}{}^{*}=\{0_{np},1_{np},\xi_{MII}(p_2)',\xi_{MIII}(p_2)'\},$

where

$\xi_{MII}(p_2)'=\{\langle p_1,1,1,0\rangle,\langle p_2,.4,.2,.6\rangle,\langle p_3,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.6\rangle,\langle (p_1,p_3),1,1,0\rangle,\langle (p_2,p_3),.4,.2,.6\rangle\}$

$\xi_{MIII}(p_2)'=\{\langle p_1,1,1,0\rangle,\langle p_2,.4,.2,.6\rangle,\langle p_3,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.7\rangle,\langle (p_1,p_3),1,1,0\rangle,\langle (p_2,p_3),.4,.2,.6\rangle\}$

and

$2\tilde{\tau}_{np}=\{0_{np},1_{np},\tilde{\xi}_{MII}(p_1,p_3),\tilde{\xi}_{MIII}(p_1,p_3)\},$

where

$\tilde{\xi}_{MII}(p_1,p_3)=\{\langle p_1,.6,.3,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.2,.2,.7\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),.6,.3,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$

$\tilde{\xi}_{MIII}(p_1,p_3)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}.$

The complement and neutro-prime absolute complement of NPT $3\tau_{np}$ are as follows,

$3\tau_{np}{}^{*}=\{0_{np},1_{np},\xi_{MI}(p_3)'\},$

where

$\xi_{MI}(p_3)'=\{\langle p_1,1,1,0\rangle,\langle p_2,1,1,0\rangle,\langle p_3,.4,.7,.5\rangle,\langle (p_1,p_2),1,1,0\rangle,\langle (p_1,p_3),.3,.4,.7\rangle,\langle (p_2,p_3),.4,.5,.5\rangle\}$

and

$3\tilde{\tau}_{np}=\{0_{np},1_{np},\tilde{\xi}_{MI}(p_1,p_2)\},$

where

$\tilde{\xi}_{MI}(p_1,p_2)=\{\langle p_1,.7,.6,.3\rangle,\langle p_2,.4,.5,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.6,.3\rangle,\langle (p_1,p_3),.7,.6,.3\rangle,\langle (p_2,p_3),.5,.5,.4\rangle\}.$

10. By using the formula 4.3.1, evaluated the values of all machines with respect to each proper non-empty singleton subset of *W*.
    i.e. *Val*[*M*I(p_i)], *Val*[*M*II(p_i)], *Val*[*M*III(p_i)] and *Val*[*M*IV(p_i)], for i = 1, 2, 3.
11. These estimated values are tabulated in the following table.

Table 4.4.1. Value Table

	p_1	p_2	p_3
***M*I**	2.025	0	3.645
***M*II**	0	2.3	0
***M*III**	1.59	3.6425	0
***M*IV**	1.35	0	1.59

12. By using the formula 4.3.2, evaluated the grand values of all machines.
    i.e. *GV*[*M*I], *GV*[*M*II], *GV*[*M*III], and *GV*[*M*IV].
13. These estimated values are tabulated in the following table.

Table 4.4.2. Grand Value Table

	p_1	p_2	p_3	GV
***M*I**	2.025	0	3.645	**5.67**
***M*II**	0	2.3	0	2.3
***M*III**	1.59	3.6425	0	5.2325
***M*IV**	1.35	0	1.59	1.35

14. Thus $GV[MI]$ is the highest value.

Hence the technician gives priority to repairing the damaged machine MI.

Example 4.5 Consider the problem explained in Example 4.4.

1. Let MI, MII, and MIII be sample machines whose damages to be repaired.
2. Let $W=\{p_1,p_2,p_3\}$ be the universe, where p_1 –part 1, p_2 –part 2, and p_3 –part 3.
3. The technician goes through the damages on each machine.
4. Define MI, MII, and MIII as NSs.

$$MI=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.7,.6,.1\rangle\},$$

$$MII=\{\langle p_1,.6,.3,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.2,.2,.7\rangle\} \text{ and}$$

$$MIII=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.7,.6,.1\rangle\}\ .$$

5. Thus $\tau_n=\{0_n,1_n,MI,MII,MIII\}$ is a NT and so (W,τ_n) is a NTS.
6. Define NPSs for each NS with respect to their damaged parts as follows:

$$\eta_p MI(p_3)=\{\xi_{MI}(p_3),\xi_{MI}(p_1,p_3),\xi_{MI}(p_2,p_3)\},$$

where

$$\xi_{MI}(p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.7,.6,.1\rangle,\langle (p_1,p_2),0,0,1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.7,.8,.1\rangle\},$$
$$\xi_{MI}(p_1,p_3)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.7,.6,.1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.7,.8,.1\rangle\},$$
$$\xi_{MI}(p_2,p_3)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.7,.8,.1\rangle\};$$

$$\eta_p MII(p_2)=\{\xi_{MII}(p_2),\xi_{MII}(p_1,p_2),\xi_{MII}(p_2,p_3)\},$$

where

$$\xi_{MII}(p_2)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),0,0,1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\},$$
$$\xi_{MII}(p_1,p_2)=\{\langle p_1,.6,.3,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),.6,.3,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\},$$
$$\xi_{MII}(p_2,p_3)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.2,.2,.7\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),.6,.3,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}$$

and

$$\eta_p MIII(p_1)=\{\xi_{MIII}(p_1),\xi_{MIII}(p_1,p_2),\xi_{MIII}(p_1,p_3)\},$$

where

$$\xi_{MIII}(p_1)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),0,0,1\rangle\},$$
$$\xi_{MIII}(p_1,p_2)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.7,.8,.1\rangle\},$$
$$\xi_{MIII}(p_1,p_3)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.7,.6,.1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.7,.8,.1\rangle\},$$

7. The possible NPTs for all proper non-empty singleton subset of W are defined as follows:

$$1\tau_{np}=\{0_{np},1_{np},\xi_{MIII}(p_1)\}\ ,$$
$$2\tau_{np}=\{0_{np},1_{np},\xi_{MII}(p_2)\} \text{ and}$$
$$3\tau_{np}=\{0_{np},1_{np},\xi_{MI}(p_3)\}.$$

8. Thus $(W,1\tau_{np})$, $(W,2\tau_{np})$ and $(W,3\tau_{np})$ are NPTSs over W.
9. The complement and neutro-prime absolute complement of NPT $1\tau_{np}$ are as follows,

$$1\tau_{np}{}^{*}=\{0_{np},1_{np},\ \xi_{MIII}(p_1)'\},$$

where

$\xi_{MIII}(p_1)' = \{\langle p_1,.1,.4,.7\rangle,\langle p_2,1,1,0\rangle,\langle p_3,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.7\rangle,\langle (p_1,p_3),.1,.4,.7\rangle,\langle (p_2,p_3),1,1,0\rangle\}$

and

$$1\tilde{\tau}_{np} = \{0_{np}, 1_{np}, \tilde{\xi}_{MIII}(p_2,p_3)'\},$$

where

$\tilde{\xi}_{MIII}(p_2,p_3) = \{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}.$

The complement and neutro-prime absolute complement of NPT $2\tau_{np}$ are as follows,

$$2\tau_{np}^{*} = \{0_{np}, 1_{np}, \xi_{MII}(p_2)'\},$$

where

$\xi_{MII}(p_2)' = \{\langle p_1,1,1,0\rangle,\langle p_2,.4,.2,.6\rangle,\langle p_3,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.6\rangle,\langle (p_1,p_3),1,1,0\rangle,\langle (p_2,p_3),.4,.2,.6\rangle\}$

and

$$2\tilde{\tau}_{np} = \{0_{np}, 1_{np}, \tilde{\xi}_{MII}(p_1,p_3)\},$$

where

$\tilde{\xi}_{MII}(p_1,p_3) = \{\langle p_1,.6,.3,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_3,.2,.2,.7\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_3),.6,.3,.1\rangle,\langle (p_2,p_3),.6,.8,.4\rangle\}.$

The complement and neutro-prime absolute complement of NPT $3\tau_{np}$ are as follows,

$$3\tau_{np}^{*} = \{0_{np}, 1_{np}, \xi_{MI}(p_3)'\},$$

where

$\xi_{MI}(p_3)' = \{\langle p_1,1,1,0\rangle,\langle p_2,1,1,0\rangle,\langle p_3,.1,.4,.7\rangle,\langle (p_1,p_2),1,1,0\rangle,\langle (p_1,p_3),.1,.4,.7\rangle,\langle (p_2,p_3),.1,.2,.7\rangle\}$

and

$$3\tilde{\tau}_{np} = \{0_{np}, 1_{np}, \tilde{\xi}_{MI}(p_1,p_2)\},$$

where

$\tilde{\xi}_{MI}(p_1,p_2) = \{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_3,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_3),.7,.6,.1\rangle,\langle (p_2,p_3),.7,.8,.1\rangle\}.$

10. By using the formula 4.3.1, evaluated the values of all machines with respect to each proper non-empty singleton subset of *W*.

    i.e. *Val*[*M*I(p_i)], *Val*[*M*II(p_i)] and *Val*[*M*III(p_i)], for i = 1, 2, 3.

11. These estimated values are tabulated in the following table.

Table 4.5.1. Value Table

	p1	**p2**	**p3**
***M*I**	0	0	3.92
***M*II**	0	2.3	0
***M*III**	3.92	0	0

12. By using the formula 4.3.2, evaluated the grand values of all machines.

    i.e. *GV*[*M*I], *GV*[*M*II] and *GV*[*M*III].

13. These estimated values are tabulated in the following table.

Table 4.5.2. Grand Value Table

	p1	**p2**	**p3**	**GV**
***M*I**	0	0	3.92	**3.92**
***M*II**	0	2.3	0	2.3
***M*III**	3.92	0	0	**3.92**

14. Thus both $GV[MI]$ and $GV[MIII]$ are the highest value.
15. In this situation, replace part 3 (p_3) with some other damaged part, say p_4, and repeat the process.
1. Let MI, MII, and $MIII$ be sample machines whose damages to be repaired.
2. Let $W=\{p_1,p_2,p_4\}$ be the universe, where p_1 –part 1, p_2 –part 2, and p_4 –part 4.
3. The technician goes through the damages on each machine.
4. Define MI, MII, and $MIII$ as NSs.

$$MI=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,.5,.3,.4\rangle\},$$

$$MII=\{\langle p_1,.6,.3,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,.4,.3,.6\rangle\} \text{ and}$$

$$MIII=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,.5,.3,.4\rangle\}\ .$$

5. Thus $\tau_n=\{0_n,1_n,MI,MII,MIII\}$ is a NT and so (W,τ_n) is a NTS.
6. Define NPSs for each NS with respect to their damaged parts as follows:

$$\eta_p MI(p_4)=\{\xi_{MI}(p_4),\xi_{MI}(p_1,p_4),\xi_{MI}(p_2,p_4)\},$$

where

$$\xi_{MI}(p_4)=\{\langle p_1,0,0,1\rangle,\langle p_2,0,0,1\rangle,\langle p_4,.5,.3,.4\rangle,\langle (p_1,p_2),0,0,1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\},$$
$$\xi_{MI}(p_1,p_4)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_4,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\},$$
$$\xi_{MI}(p_2,p_4)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\};$$

$$\eta_p MII(p_2)=\{\xi_{MII}(p_2),\xi_{MII}(p_1,p_2),\xi_{MII}(p_2,p_4)\},$$

where

$$\xi_{MII}(p_2)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,0,0,1\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_4),0,0,1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\},$$
$$\xi_{MII}(p_1,p_2)=\{\langle p_1,.6,.3,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,0,0,1\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_4),.6,.3,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\},$$
$$\xi_{MII}(p_2,p_4)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,.4,.3,.6\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_4),.6,.3,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\}$$

and

$$\eta_p MIII(p_1,p_2)=\{\xi_{MIII}(p_1),\xi_{MIII}(p_2),\xi_{MIII}(p_1,p_2),\xi_{MIII}(p_1,p_4),\xi_{MIII}(p_2,p_4)\},$$

where

$$\xi_{MIII}(p_1)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_4,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),0,0,1\rangle\},$$
$$\xi_{MIII}(p_2)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),0,0,1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\},$$
$$\xi_{MIII}(p_1,p_2)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\},$$
$$\xi_{MIII}(p_1,p_4)=\{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_4,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\},$$
$$\xi_{MIII}(p_2,p_4)=\{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\}$$

7. The possible NPTs for all proper non-empty singleton subset of W are defined as follows:

$1\tau_{np}=\{0_{np},1_{np},\xi_{MIII}(p_1)\}$,

$2\tau_{np}=\{0_{np},1_{np},\xi_{MII}(p_2),\xi_{MIII}(p_2)\}$ and

$3\tau_{np}=\{0_{np},1_{np},\xi_{MI}(p_4)\}$.

8. Thus $(W,1\tau_{np})$, $(W,2\tau_{np})$ and $(W,3\tau_{np})$ are NPTSs over W.

9. The complement and neutro-prime absolute complement of NPT $1\tau_{np}$ are as follows,

$$1\tau_{np}{}^{*} = \{0_{np}, 1_{np}, \xi_{MIII}(p_1)'\},$$

where

$$\xi_{MIII}(p_1)' = \{\langle p_1,.1,.4,.7\rangle,\langle p_2,1,1,0\rangle,\langle p_4,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.7\rangle,\langle (p_1,p_4),.1,.4,.7\rangle,\langle (p_2,p_4),1,1,0\rangle\}$$

and

$$1\tilde{\tau}_{np} = \{0_{np}, 1_{np}, \tilde{\xi}_{MIII}(p_2,p_4)'\},$$

where

$$\tilde{\xi}_{MIII}(p_2,p_4) = \{\langle p_1,0,0,1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\}.$$

The complement and neutro-prime absolute complement of NPT $2\tau_{np}$ are as follows,

$$2\tau_{np}{}^{*} = \{0_{np}, 1_{np}, \xi_{MII}(p_2)', \xi_{MIII}(p_2)'\},$$

where

$$\xi_{MII}(p_2)' = \{\langle p_1,1,1,0\rangle,\langle p_2,.4,.2,.6\rangle,\langle p_4,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.6\rangle,\langle (p_1,p_4),1,1,0\rangle,\langle (p_2,p_4),.4,.2,.6\rangle\}$$

$$\xi_{MIII}(p_2)' = \{\langle p_1,1,1,0\rangle,\langle p_2,.4,.2,.6\rangle,\langle p_4,1,1,0\rangle,\langle (p_1,p_2),.1,.2,.7\rangle,\langle (p_1,p_4),1,1,0\rangle,\langle (p_2,p_4),.4,.2,.6\rangle\}$$

and

$$2\tilde{\tau}_{np} = \{0_{np}, 1_{np}, \tilde{\xi}_{MII}(p_1,p_4)\tilde{\xi}_{MIII}(p_1,p_4)\},$$

where

$$\tilde{\xi}_{MII}(p_1,p_4) = \{\langle p_1,.6,.3,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_4,.2,.2,.7\rangle,\langle (p_1,p_2),.6,.8,.1\rangle,\langle (p_1,p_4),.6,.3,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\}$$

$$\tilde{\xi}_{MIII}(p_1,p_4) = \{\langle p_1,.7,.6,.1\rangle,\langle p_2,0,0,1\rangle,\langle p_4,.5,.3,.4\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\}.$$

The complement and neutro-prime absolute complement of NPT $3\tau_{np}$ are as follows,

$$3\tau_{np}{}^{*} = \{0_{np}, 1_{np}, \xi_{MI}(p_4)'\},$$

where

$$\xi_{MI}(p_4)' = \{\langle p_1,1,1,0\rangle,\langle p_2,1,1,0\rangle,\langle p_4,.4,.7,.5\rangle,\langle (p_1,p_2),1,1,0\rangle,\langle (p_1,p_4),.1,.4,.7\rangle,\langle (p_2,p_4),.4,.2,.6\rangle\}$$

and

$$3\tilde{\tau}_{np} = \{0_{np}, 1_{np}, \tilde{\xi}_{MI}(p_1,p_2)\},$$

where

$$\tilde{\xi}_{MI}(p_1,p_2) = \{\langle p_1,.7,.6,.1\rangle,\langle p_2,.6,.8,.4\rangle,\langle p_4,0,0,1\rangle,\langle (p_1,p_2),.7,.8,.1\rangle,\langle (p_1,p_4),.7,.6,.1\rangle,\langle (p_2,p_4),.6,.8,.4\rangle\}.$$

10. By using the formula 4.3.1, evaluated the values of all machines with respect to each proper non-empty singleton subset of *W*.

i.e. *Val*[MI(p_i)], *Val*[MII(p_i)] and *Val*[MIII(p_i)], for i = 1, 2, 3.

11. These estimated values are tabulated in the following table.

Table 4.5.3. Value Table

	p_1	p_2	p_4
***M*I**	0	0	4.8675
***M*II**	0	2.665	0
***M*III**	1.59	3.6425	0

12. By using the formula 4.3.2, evaluated the grand values of all machines.

i.e. *GV*[*M*I], *GV*[*M*II] and *GV*[*M*III].

13. These estimated values are tabulated in the following table.

Table 4.5.4. Grand Value Table

	p_1	p_2	p_4	GV
***M*I**	0	0	4.8675	4.8675
***M*II**	0	2.665	0	2.665
***M*III**	1.59	3.6425	0	**5.2325**

14. Thus *GV*[*M*III] is the highest value.
Hence the technician gives priority to repairing the damaged machine *M*III.

5. Conclusions

The major contribution of this work is initiating a neutrosophic environment on prime sets under a topological space. Some related properties of NPTSs have been proved and some are disproved with counterexamples. Also, the idea of interior and closure dealt with such space with few basic properties. The novelty of this study is to merge two different poles. The decision-making problem is demonstrated with an example to analyze the number of faults in industrial machines. Sample machines are represented as NSs and their damages are represented as NPSs under its topological space. The values of fault machines are detected by finding the complement and absolute complement of each NPS. The various values of faults are taken as different subsets, for analysis. The proposed algorithm analyzes through the NPSs and finds the best suitable set values which indicate the heavy damage in machines. The lower fault machines are neglected by decision-making problems.

The primary results of this study are:

- Prime set is studied under the environment of neutrosophic.
- Related properties are stated with proof and also disproved in counter examples.
- The intersection of two NPTs is a NPT but not for its union.
- Demonstrating the decision-making problem of analyzing the number of damages in machines.
- The complement and absolute complement of NPSs are evaluated to find the best fit of fault by diagnosis the machines.

In the future, this set may develop more genetic algorithms to predict multi-criteria DM problems. Many more sets like soft sets, rough sets, crisp sets, etc., can be developed on NPTSs. More ideas may be claimed and investigated to achieve a deeper understanding of the economic and social consequences of robotization.

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