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# A novel method for single-valued neutrosophic multi-criteria decision making with incomplete weight information

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**Abstract**.
A single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) are two instances of a neutrosophic set, which can efficiently deal with uncertain, imprecise, incomplete, and inconsistent information. In this paper, we develop a novel method for solving single-valued neutrosophic multi-criteria decision making with incomplete weight information, in which the criterion values are given in the form of single-valued neutrosophic sets (SVNSs), and the information about criterion weights is incompletely known or completely unknown. The developed method consists of two stages. The first stage is to use the maximizing deviation method to establish an optimization model, which derives the optimal weights of criteria under single-valued neutrosophic environments. After obtaining the weights of criteria through the above stage, the second stage is to develop a single-valued neutrosophic TOPSIS (SVNTOPSIS) method to determine a solution with the shortest distance to the single-valued neutrosophic positive ideal solution (SVNPIS) and the greatest distance from the single-valued neutrosophic negative ideal solution (SVNNIS). Moreover, a best global supplier selection problem is used to demonstrate the validity and applicability of the developed method. Finally, the extended results in interval neutrosophic situations are pointed out and a comparison analysis with the other methods is given to illustrate the advantages of the developed methods.

**Keywords:** neutrosophic set, single-valued neutrosophic set (SVNS), interval neutrosophic set (INS), multi-criteria decision making (MCDM), maximizing deviation method; TOPSIS.

## 1. Introduction

Neutrosophy, originally introduced by Smarandache [12], is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra [12]. As a powerful general formal framework, neutrosophic set [12] generalizes the concept of the classic set, fuzzy set [24], interval-valued fuzzy set [14,25], vague set [4], intuitionistic fuzzy set [1], interval-valued intuitionistic fuzzy set [2], paraconsistent set [12], dialetheist set [12], paradoxist set [12], and tautological set [12]. In the neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, and falsity-membership are independent, which is a very important assumption in many applications such as information fusion in which the data are combined from different sensors [12]. Recently, neutrosophic sets have been successfully applied to image processing [3,5,6].

The neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications [16,17]. Therefore, Wang et al. [17] defined a single valued neutrosophic set (SVNS), and then provided the set theoretic operators and various properties of single valued neutrosophic sets (SVNSs). Furthermore, Wang et al. [16] proposed the set-theoretic operators on an instance of neutrosophic set called interval neutrosophic set (INS). A single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) are two instances of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. Single valued neutrosophic sets and interval neutrosophic sets are different from intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. Intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets can only handle incomplete information, but cannot handle the indeterminate information and inconsistent information which exist commonly in real situations. The connectors in the intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set are defined with respect to membership and non-membership only (hence the indeterminacy is what is left from 1), while in the single valued neutrosophic set and interval neutrosophic set, they can be defined with respect to any of them (no restriction). For example [17], when we ask the opinion of an expert about certain statement, he or she may say that the possibility in which the statement is true is 0.6 and the statement is false is 0.5 and the degree in which he or she is not sure is 0.2. This situation can be expressed as a single valued neutrosophic set $\langle 0.6, 0.2, 0.5\rangle$, which is beyond the scope of the intuitionistic fuzzy set.

For another example [16], suppose that an expert may say that the possibility that the statement is true is between 0.5 and 0.6, and the statement is false is between 0.7 and 0.9, and the degree that he or she is not sure is between 0.1 and 0.3. This situation can be expressed as an interval neutrosophic set $\langle [0.5,0.6],[0.1,0.3],[0.7,0.9] \rangle$, which is beyond the scope of the interval-valued intuitionistic fuzzy set.

Due to their abilities to easily reflect the ambiguous nature of subjective judgments, single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs) are suitable for capturing imprecise, uncertain, and inconsistent information in the multi-criteria decision analysis [20,21,22,23]. Most recently, some methods [20,21,22,23] have been developed for solving the multi-criteria decision making (MCDM) problems with single-valued neutrosophic or interval neutrosophic information. For example, Ye [20] developed a multi-criteria decision making method using the correlation coefficient under single-valued neutrosophic environments. Ye [21] defined the single valued neutrosophic cross entropy, based on which, a multi-criteria decision making method is established in which criteria values for alternatives are single valued neutrosophic sets (SVNSs). Ye [23] proposed a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator, and then utilized two aggregation operators to develop a method for multi-criteria decision making problems under simplified neutrosophic environments. Ye [22] defined the similarity measures between interval neutrosophic sets (INSs), and then utilized the similarity measures between each alternative and the ideal alternative to rank the alternatives and to determine the best one. However, it is noted that the aforementioned methods need the information about criterion weights to be exactly known. When using these methods, the associated weighting vector is more or less determined subjectively and the decision information itself is not taken into consideration sufficiently. In fact, in the process of multi-criteria decision making (MCDM), we often encounter the situations in which the criterion values take the form of single valued neutrosophic sets (SVNSs) or interval neutrosophic sets (INSs), and the information about attribute weights is incompletely known or completely unknown because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain [18]. Considering that the existing methods are inappropriate for dealing with such situations, in this paper, we develop a novel method for single valued neutrosophic or interval neutrosophic MCDM with incomplete weight information, in which the criterion values take the form of single valued neutrosophic sets (SVNSs) or interval neutrosophic sets (INSs), and the information about criterion weights is incompletely known or completely unknown. The developed method is composed of two parts. First, we establish an optimization model based on the maximizing deviation method to objectively determine the optimal criterion weights. Then, we develop an extended TOPSIS method, which we call the single valued neutrosophic or interval neutrosophic TOPSIS, to calculate the relative closeness coefficient of each alternative to the single valued neutrosophic or interval neutrosophic positive ideal solution and to select the optimal one with the maximum relative closeness coefficient. Two illustrative examples and comparison analysis with the existing methods show that the developed methods can not only relieve the influence of subjectivity of the decision maker but also remain the original decision information sufficiently.

To do so, the remainder of this paper is set out as follows. Section 2 briefly recalls some basic concepts of neutrosophic sets, single-valued neutrosophic sets (SVNSs), and interval neutrosophic sets (INSs). Section 3 develops a novel method based on the maximizing deviation method and the single-valued neutrosophic TOPSIS (SVNTOPSIS) for solving the single-valued neutrosophic multi-criteria decision making with incomplete weight information. Section 4 develops a novel method based on the maximizing deviation method and the interval neutrosophic TOPSIS (INTOPSIS) for solving the interval neutrosophic multi-criteria decision making with incomplete weight information. Section 5 provides two practical examples to illustrate the effectiveness and practicality of the developed methods. Section 6 ends the paper with some concluding remarks.

## 2 Neutrosophic sets and and SVNSs

In this section, we will give a brief overview of neutrosophic sets [12], single-valued neutrosophic set (SVNSs) [17], and interval neutrosophic sets (INSs) [16].

### 2.1 Neutrosophic sets

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [12], and is a powerful general formal framework, which generalizes the above mentioned sets from philosophical point of view.

Smarandache [12] defined a neutrosophic set as follows:

**Definition 2.1** [12]. Let $X$ be a space of points (objects), with a generic element in $X$ denoted by $x$. A neutrosophic set $A$ in $X$ is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-,1^+[$. That is $T_A(x): X \to ]0^-,1^+[$, $I_A(x): X \to ]0^-,1^+[$, and $F_A(x): X \to ]0^-,1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

**Definition 2.2** [12]. The complement of a neutrosophic set $A$ is denoted by $A^c$ and is defined as $T_{A^c}(x) = \{1^+\} \ominus T_A(x)$, $I_{A^c}(x) = \{1^+\} \ominus I_A(x)$, and $F_{A^c}(x) = \{1^+\} \ominus F_A(x)$ for every $x$ in $X$.

**Definition 2.3** [12]. A neutrosophic set $A$ is contained in the other neutrosophic set $B$, $A \subseteq B$ if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$, and $\sup F_A(x) \ge \sup F_B(x)$ for every $x$ in $X$.

## 2.2 Single-valued neutrosophic sets (SVNSs)

A single-valued neutrosophic set (SVNS) is an instance of a neutrosophic set, which has a wide range of applications in real scientific and engineering fields. In the following, we review the definition of a SVNS proposed by Wang et al. [17].

**Definition 2.4** [17]. Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $x$. A single-valued neutrosophic set (SVNS) $A$ in $X$ is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$ for each point $x$ in $X$.

A SVNS $A$ can be written as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\} \quad (1)$$

Let $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle | x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle | x_i \in X\}$ be two single-valued neutrosophic sets (SVNSs) in $X = \{x_1, x_2, \cdots, x_n\}$. Then we define the following distances for $A$ and $B$.

(i) The Hamming distance

$$d(A,B) = \frac{1}{3}\sum_{i=1}^{n}\left(\begin{array}{l}|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + \\ |F_A(x_i) - F_B(x_i)|\end{array}\right) \quad (2)$$

(ii) The normalized Hamming distance

$$d(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{l}|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + \\ |F_A(x_i) - F_B(x_i)|\end{array}\right) \quad (3)$$

(iii) The Euclidean distance

$$d(A,B) = \sqrt{\frac{1}{3}\sum_{i=1}^{n}\left(\begin{array}{l}|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 \\ + |F_A(x_i) - F_B(x_i)|^2\end{array}\right)} \quad (4)$$

(iv) The normalized Euclidean distance

$$d(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}\left(\begin{array}{l}|T_A(x_i) - T_B(x_i)|^2 + |I_A(x_i) - I_B(x_i)|^2 \\ + |F_A(x_i) - F_B(x_i)|^2\end{array}\right)} \quad (5)$$

## 2.3 Interval neutrosophic sets (INSs)

**Definition 2.5** [16]. Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $x$. An interval neutrosophic set (INS) $\tilde{A}$ in $X$ is characterized by a truth-membership function $\tilde{T}_{\tilde{A}}(x)$, an indeterminacy-membership function $\tilde{I}_{\tilde{A}}(x)$, and a falsity-membership function $\tilde{F}_{\tilde{A}}(x)$. For each point $x$ in $X$, we have that $\tilde{T}_{\tilde{A}}(x) = [\inf \tilde{T}_{\tilde{A}}(x), \sup \tilde{T}_{\tilde{A}}(x)] \subseteq [0,1]$,
$\tilde{I}_{\tilde{A}}(x) = [\inf \tilde{I}_{\tilde{A}}(x), \sup \tilde{I}_{\tilde{A}}(x)] \subseteq [0,1]$,
$\tilde{F}_{\tilde{A}}(x) = [\inf \tilde{F}_{\tilde{A}}(x), \sup \tilde{F}_{\tilde{A}}(x)] \subseteq [0,1]$,
and $0 \le \sup \tilde{T}_{\tilde{A}}(x) + \sup \tilde{I}_{\tilde{A}}(x) + \sup \tilde{F}_{\tilde{A}}(x) \le 3$.

Let $\tilde{A} = \{\langle x_i, \tilde{T}_{\tilde{A}}(x_i), \tilde{I}_{\tilde{A}}(x_i), \tilde{F}_{\tilde{A}}(x_i)\rangle | x_i \in X\}$ and $\tilde{B} = \{\langle x_i, \tilde{T}_{\tilde{B}}(x_i), \tilde{I}_{\tilde{B}}(x_i), \tilde{F}_{\tilde{B}}(x_i)\rangle | x_i \in X\}$ be two interval neutrosophic sets (INSs) in $X = \{x_1, x_2, \cdots, x_n\}$,
where $\tilde{T}_{\tilde{A}}(x_i) = [\inf \tilde{T}_{\tilde{A}}(x_i), \sup \tilde{T}_{\tilde{A}}(x_i)]$,
$\tilde{I}_{\tilde{A}}(x_i) = [\inf \tilde{I}_{\tilde{A}}(x_i), \sup \tilde{I}_{\tilde{A}}(x_i)]$,
$\tilde{F}_{\tilde{A}}(x_i) = [\inf \tilde{F}_{\tilde{A}}(x_i), \sup \tilde{F}_{\tilde{A}}(x_i)]$,
$\tilde{T}_{\tilde{B}}(x_i) = [\inf \tilde{T}_{\tilde{B}}(x_i), \sup \tilde{T}_{\tilde{B}}(x_i)]$,
$\tilde{I}_{\tilde{B}}(x_i) = [\inf \tilde{I}_{\tilde{B}}(x_i), \sup \tilde{I}_{\tilde{B}}(x_i)]$,
and $\tilde{F}_{\tilde{B}}(x_i) = [\inf \tilde{F}_{\tilde{B}}(x_i), \sup \tilde{F}_{\tilde{B}}(x_i)]$. Then Ye [22] defined the following distances for $A$ and $B$.

(i) The Hamming distance

$$d(A,B) = \frac{1}{6}\sum_{i=1}^{n}\left(\begin{array}{l}|\inf \tilde{T}_{\tilde{A}}(x_i) - \inf \tilde{T}_{\tilde{B}}(x_i)| + |\sup \tilde{T}_{\tilde{A}}(x_i) - \sup \tilde{T}_{\tilde{B}}(x_i)| + \\ |\inf \tilde{I}_{\tilde{A}}(x_i) - \inf \tilde{I}_{\tilde{B}}(x_i)| + |\sup \tilde{I}_{\tilde{A}}(x_i) - \sup \tilde{I}_{\tilde{B}}(x_i)| + \\ |\inf \tilde{F}_{\tilde{A}}(x_i) - \inf \tilde{F}_{\tilde{B}}(x_i)| + |\sup \tilde{F}_{\tilde{A}}(x_i) - \sup \tilde{F}_{\tilde{B}}(x_i)|\end{array}\right) \quad (6)$$

(ii) The normalized Hamming distance

$$d(A,B)=\frac{1}{6n}\sum_{i=1}^{n}\begin{pmatrix}\left|\inf\tilde{T}_{\tilde{A}}(x_i)-\inf\tilde{T}_{\tilde{B}}(x_i)\right|+\left|\sup\tilde{T}_{\tilde{A}}(x_i)-\sup\tilde{T}_{\tilde{B}}(x_i)\right|+\\ \left|\inf\tilde{I}_{\tilde{A}}(x_i)-\inf\tilde{I}_{\tilde{B}}(x_i)\right|+\left|\sup\tilde{I}_{\tilde{A}}(x_i)-\sup\tilde{I}_{\tilde{B}}(x_i)\right|+\\ \left|\inf\tilde{F}_{\tilde{A}}(x_i)-\inf\tilde{F}_{\tilde{B}}(x_i)\right|+\left|\sup\tilde{F}_{\tilde{A}}(x_i)-\sup\tilde{F}_{\tilde{B}}(x_i)\right|\end{pmatrix} \quad (7)$$

(iii) The Euclidean distance

$$d(A,B)=\sqrt{\frac{1}{6}\sum_{i=1}^{n}\begin{pmatrix}\left|\inf\tilde{T}_{\tilde{A}}(x_i)-\inf\tilde{T}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{T}_{\tilde{A}}(x_i)-\sup\tilde{T}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{I}_{\tilde{A}}(x_i)-\inf\tilde{I}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{I}_{\tilde{A}}(x_i)-\sup\tilde{I}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{F}_{\tilde{A}}(x_i)-\inf\tilde{F}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{F}_{\tilde{A}}(x_i)-\sup\tilde{F}_{\tilde{B}}(x_i)\right|^2\end{pmatrix}} \quad (8)$$

(iv) The normalized Euclidean distance

$$d(A,B)=\sqrt{\frac{1}{6n}\sum_{i=1}^{n}\begin{pmatrix}\left|\inf\tilde{T}_{\tilde{A}}(x_i)-\inf\tilde{T}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{T}_{\tilde{A}}(x_i)-\sup\tilde{T}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{I}_{\tilde{A}}(x_i)-\inf\tilde{I}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{I}_{\tilde{A}}(x_i)-\sup\tilde{I}_{\tilde{B}}(x_i)\right|^2+\\ \left|\inf\tilde{F}_{\tilde{A}}(x_i)-\inf\tilde{F}_{\tilde{B}}(x_i)\right|^2+\left|\sup\tilde{F}_{\tilde{A}}(x_i)-\sup\tilde{F}_{\tilde{B}}(x_i)\right|^2\end{pmatrix}} \quad (9)$$

## 3 A novel method for single-valued neutrosophic multi-criteria decision making with incomplete weight information

### 3.1 Problem description

The aim of multi-criteria decision making (MCDM) problems is to find the most desirable alternative(s) from a set of feasible alternatives according to a number of criteria or attributes. In general, the multi-criteria decision making problem includes uncertain, imprecise, incomplete, and inconsistent information, which can be represented by SVNSs. In this section, we will present a method for handling the MCDM problem under single-valued neutrosophic environments. First, a MCDM problem with single-valued neutrosophic information can be outlined as: let $A=\{A_1,A_2,\cdots,A_m\}$ be a set of $m$ alternatives and $C=\{c_1,c_2,\ldots,c_n\}$ be a collection of $n$ criteria, whose weight vector is $w=(w_1,w_2,\cdots,w_n)^T$, with $w_j\in[0,1]$, $j=1,2,\cdots,n$, and $\sum_{j=1}^{n}w_j=1$. In this case, the characteristic of the alternative $A_i$ $(i=1,2,\cdots,m)$ with respect to all the criteria is represented by the following SVNS:

$$A_i=\left\{\left\langle c_j,T_{A_i}(c_j),I_{A_i}(c_j),F_{A_i}(c_j)\right\rangle \middle| c_j\in C\right\}$$

where $T_{A_i}(c_j),I_{A_i}(c_j),F_{A_i}(c_j)\in[0,1]$, and $0\le T_{A_i}(c_j)+I_{A_i}(c_j)+F_{A_i}(c_j)\le 3$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$).

Here, $T_{A_i}(c_j)$ indicates the degree to which the alternative $A_i$ satisfies the criterion $c_j$, $I_{A_i}(c_j)$ indicates the indeterminacy degree to which the alternative $A_i$ satisfies or does not satisfy the criterion $c_j$, and $F_{A_i}(c_j)$ indicates the degree to which the alternative $A_i$ does not satisfy the criterion $c_j$. For the sake of simplicity, a criterion value $\left\langle c_j,T_{A_i}(c_j),I_{A_i}(c_j),F_{A_i}(c_j)\right\rangle$ in $A_i$ is denoted by a single-valued neutrosophic value (SVNV) $a_{ij}=\left\langle T_{ij},I_{ij},F_{ij}\right\rangle$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$), which is usually derived from the evaluation of an alternative $A_i$ with respect to a criterion $C_j$ by means of a score law and data processing in practice [19,22]. All $a_{ij}$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$) constitute a single valued neutrosophic decision matrix $A=(a_{ij})_{m\times n}=\left(\left\langle T_{ij},I_{ij},F_{ij}\right\rangle\right)_{m\times n}$ (see Table 1):

**Table 1:** Single valued neutrosophic decision matrix $A$.

| **1** | $c_1$ | $\cdots$ | $c_j$ | $\cdots$ | $c_n$ |
|---|---|---|---|---|---|
| $A_1$ | $\langle T_{11},I_{11},F_{11}\rangle$ | $\cdots$ | $\langle T_{1j},I_{1j},F_{1j}\rangle$ | $\cdots$ | $\langle T_{1n},I_{1n},F_{1n}\rangle$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_i$ | $\langle T_{i1},I_{i1},F_{i1}\rangle$ | $\cdots$ | $\langle T_{ij},I_{ij},F_{ij}\rangle$ | $\cdots$ | $\langle T_{in},I_{in},F_{in}\rangle$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_m$ | $\langle T_{m1},I_{m1},F_{m1}\rangle$ | $\cdots$ | $\langle T_{mj},I_{mj},F_{mj}\rangle$ | $\cdots$ | $\langle T_{mn},I_{mn},F_{mn}\rangle$ |

### 3.2 Obtaining the optimal weights of criteria by the maximizing deviation method

Due to the fact that many practical MCDM problems are complex and uncertain and human thinking is inherently subjective, the information about criterion weights is usually incomplete. For convenience, let $\Delta$ be a set of the known weight information [8,9,10,11], where $\Delta$ can be constructed by the following forms, for $i\ne j$:

**Form 1.** A weak ranking: $\{w_i\ge w_j\}$;

**Form 2.** A strict ranking: $\{w_i - w_j \geq \alpha_i\}$ ($\alpha_i > 0$);

**Form 3.** A ranking of differences: $\{w_i - w_j \geq w_k - w_l\}$, for $j \neq k \neq l$;

**Form 4.** A ranking with multiples: $\{w_i \geq \alpha_i w_j\}$ ($0 \leq \alpha_i \leq 1$);

**Form 5.** An interval form: $\{\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i\}$ ($0 \leq \alpha_i \leq \alpha_i + \varepsilon_i \leq 1$).

Wang [15] proposed the maximizing deviation method for estimating the criterion weights in MCDM problems with numerical information. According to Wang [15], if the performance values of all the alternatives have small differences under a criterion, it shows that such a criterion plays a less important role in choosing the best alternative and should be assigned a smaller weight. On the contrary, if a criterion makes the performance values of all the alternatives have obvious differences, then such a criterion plays a much important role in choosing the best alternative and should be assigned a larger weight. Especially, if all available alternatives score about equally with respect to a given criterion, then such a criterion will be judged unimportant by most decision makers and should be assigned a very small weight. Wang [15] suggests that zero weight should be assigned to the criterion of this kind.

Here, based on the maximizing deviation method, we construct an optimization model to determine the optimal relative weights of criteria under single valued neutrosophic environments. For the criterion $c_j \in C$, the deviation of the alternative $A_i$ to all the other alternatives can be defined as below:

$$D_{ij} = \sum_{q=1}^{m} d\left(a_{ij}, a_{qj}\right) = \sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$$
$$i = 1, 2, \cdots, m, \ j = 1, 2, \cdots, n \tag{10}$$

where $d\left(a_{ij}, a_{qj}\right) = \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$ denotes the single valued neutrosophic Euclidean distance between two single-valued neutrosophic values (SVNVs) $a_{ij}$ and $a_{qj}$ defined as in Eq. (4).

Let

$$D_j = \sum_{i=1}^{m} D_{ij} = \sum_{i=1}^{m}\sum_{q=1}^{m} d\left(a_{ij}, a_{qj}\right) = \sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$$
$$j = 1, 2, \cdots, n \tag{11}$$

then $D_j$ represents the deviation value of all alternatives to other alternatives for the criterion $c_j \in C$.

Further, let

$$D(w) = \sum_{j=1}^{n} w_j D_j = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}} \tag{12}$$

then $D(w)$ represents the deviation value of all alternatives to other alternatives for all the criteria.

Based on the above analysis, we can construct a nonlinear programming model to select the weight vector $w$ by maximizing $D(w)$, as follows:

$$\max D(w) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}$$
$$\text{s.t.} \quad w_j \geq 0, \ j = 1, 2, \cdots, n, \quad \sum_{j=1}^{n} w_j^2 = 1 \tag{M-1}$$

To solve this model, we construct the Lagrange function as follows:

$$L(w, \lambda) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}} + \frac{\lambda}{2}\left(\sum_{j=1}^{n} w_j^2 - 1\right) \tag{13}$$

where $\lambda$ is the Lagrange multiplier.

Differentiating Eq. (13) with respect to $w_j$ ($j = 1, 2, \cdots, n$) and $\lambda$, and setting these partial derivatives equal to zero, then the following set of equations is obtained:

$$\frac{\partial L}{\partial w_j} = \sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}} + \lambda w_j = 0 \tag{14}$$

$$\frac{\partial L}{\partial \lambda} = \frac{1}{2}\left(\sum_{j=1}^{n} w_j^2 - 1\right) = 0 \tag{15}$$

It follows from Eq. (14) that

$$w_j = \frac{-\sum_{i=1}^{m}\sum_{q=1}^{m} \sqrt{\frac{\left|T_{ij} - T_{qj}\right|^2 + \left|I_{ij} - I_{qj}\right|^2 + \left|F_{ij} - F_{qj}\right|^2}{3}}}{\lambda} \tag{16}$$

Putting Eq. (16) into Eq. (15), we get

$$\lambda = -\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}\right)^2} \quad (17)$$

Then, by combining Eqs. (16) and (17), we have

$$w_j = \frac{\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}\right)^2}} \quad (18)$$

By normalizing $w_j$ ( $j=1,2,\cdots,n$ ), we make their sum into a unit, and get

$$w_j^* = \frac{w_j}{\sum_{j=1}^{n} w_j} = \frac{\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}} \quad (19)$$

which can be considered as the optimal weight vector of criteria.

However, it is noted that there are practical situations in which the information about the weight vector is not completely unknown but partially known. For such cases, we establish the following constrained optimization model:

$$\max D(w) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m} w_j \sqrt{\frac{\left|T_{ij}-T_{qj}\right|^2+\left|I_{ij}-I_{qj}\right|^2+\left|F_{ij}-F_{qj}\right|^2}{3}}$$
$$\text{s.t.} \quad w \in \Delta, \quad w_j \ge 0, \; j=1,2,\cdots,n, \quad \sum_{j=1}^{n} w_j = 1 \quad \text{(M-2)}$$

It is noted that the model (M-2) is a linear programming model that can be solved using the MATLAB mathematics software package. Suppose that the optimal solution to the model (M-2) is $w=\left(w_1,w_2,\cdots,w_n\right)^T$, which can be considered as the weight vector of criteria.

### 3.3. Extended TOPIS method for the MCDM with single valued neutrosophic information

TOPSIS method, initially introduced by Hwang and Yoon [7], is a widely used method for dealing with MCDM problems, which focuses on choosing the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). After obtaining the criterion weight values on basis of the maximizing deviation method, in the following, we will extend the TOPSIS method to take single-valued neutrosophic information into account and utilize the distance measures of SVNVs to obtain the final ranking of the alternatives.

In general, the criteria can be classified into two types: benefit criteria and cost criteria. The benefit criterion means that a higher value is better while for the cost criterion is valid the opposite. Let $C_1$ be a collection of benefit criteria and $C_2$ be a collection of cost criteria, where $C_1 \cup C_2 = C$ and $C_1 \cap C_2 = \varnothing$. Under single-valued neutrosophic environments, the single-valued neutrosophic PIS (SVNPIS), denoted by $A^+$, can be identified by using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives as follows:

$$A^+ = \left\{a_1^+, a_2^+, \cdots, a_n^+\right\} \quad (20)$$

where

$$a_j^+ = \begin{cases} \left\langle \max_i\left\{T_{ij}\right\}, \min_i\left\{I_{ij}\right\}, \min_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j \in C_1, \\ \left\langle \min_i\left\{T_{ij}\right\}, \max_i\left\{I_{ij}\right\}, \max_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j \in C_2. \end{cases} \quad (21)$$

The single-valued neutrosophic NIS (SVNNIS), denoted by $A^-$, can be identified by using a minimum operator for the benefit criteria and a maximum operator for the cost criteria to determine the worst value of each criterion among all alternatives as follows:

$$A^- = \left\{a_1^-, a_2^-, \cdots, a_n^-\right\} \quad (22)$$

where

$$a_j^- = \begin{cases} \left\langle \min_i\left\{T_{ij}\right\}, \max_i\left\{I_{ij}\right\}, \max_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j \in C_1, \\ \left\langle \max_i\left\{T_{ij}\right\}, \min_i\left\{I_{ij}\right\}, \min_i\left\{F_{ij}\right\}\right\rangle, & \text{if } j \in C_2. \end{cases} \quad (23)$$

The separation measures, $d_i^+$ and $d_i^-$, of each alternative from the SVNPIS $A^+$ and the SVNNIS $A^-$, respectively, are derived from

$$d_i^+ = \sum_{j=1}^{n} w_j d\left(a_{ij}, a_j^+\right)$$
$$= \sum_{j \in C_1} w_j \sqrt{\frac{\left(\left|T_{ij} - \max_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \min_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \min_i\left\{F_{ij}\right\}\right|^2\right)}{3}} +$$
$$\sum_{j \in C_2} w_j \sqrt{\frac{\left(\left|T_{ij} - \min_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \max_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \max_i\left\{F_{ij}\right\}\right|^2\right)}{3}} \quad (24)$$

$$d_i^- = \sum_{j=1}^{n} w_j d\left(a_{ij}, a_j^-\right)$$
$$= \sum_{j \in C_1} w_j \sqrt{\frac{\left(\left|T_{ij} - \min_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \max_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \max_i\left\{F_{ij}\right\}\right|^2\right)}{3}} +$$
$$\sum_{j \in C_2} w_j \sqrt{\frac{\left(\left|T_{ij} - \max_i\left\{T_{ij}\right\}\right|^2 + \left|I_{ij} - \min_i\left\{I_{ij}\right\}\right|^2 + \left|F_{ij} - \min_i\left\{F_{ij}\right\}\right|^2\right)}{3}} \quad (25)$$

The relative closeness coefficient of an alternative $A_i$ with respect to the single-valued neutrosophic PIS $A^+$ is defined as the following formula:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (26)$$

where $0 \le C_i \le 1$, $i = 1, 2, \cdots, m$. Obviously, an alternative $A_i$ is closer to the single-valued neutrosophic PIS $A^+$ and farther from the single-valued neutrosophic NIS $A^-$ as $C_i$ approaches 1. The larger the value of $C_i$, the more different between $A_i$ and the single-valued neutrosophic NIS $A^-$, while the more similar between $A_i$ and the single-valued neutrosophic PIS $A^+$. Therefore, the alternative(s) with the maximum relative closeness coefficient should be chosen as the optimal one(s).

Based on the above analysis, we will develop a practical approach for dealing with MCDM problems, in which the information about criterion weights is incompletely known or completely unknown, and the criterion values take the form of single-valued neutrosophic information.

The flowchart of the proposed approach for MCDM is provided in Fig. 1. The proposed approach is composed of the following steps:

**Step 1.** For a MCDM problem, the decision maker (DM) constructs the single-valued neutrosophic decision matrix $A = \left(a_{ij}\right)_{m \times n} = \left(\left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle\right)_{m \times n}$, where $a_{ij} = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle$ is a single-valued neutrosophic value (SVNV), given by the DM, for the alternative $A_i$ with respect to the attribute $c_j$.

**Step 2.** If the information about the criterion weights is completely unknown, then we use Eq. (19) to obtain the criterion weights; if the information about the criterion weights is partly known, then we solve the model (M-2) to obtain the criterion weights.

**Step 3.** Utilize Eqs. (20), (21), (22), and (23) to determine the single-valued neutrosophic positive ideal solution (SVNPIS) $A^+$ and the single-valued neutrosophic negative ideal solution (SVNNIS) $A^-$.

**Step 4.** Utilize Eqs. (24) and (25) to calculate the separation measures $d_i^+$ and $d_i^-$ of each alternative $A_i$ from the single-valued neutrosophic positive ideal solution (SVNPIS) $A^+$ and the single-valued neutrosophic negative ideal solution (SVNNIS) $A^-$, respectively.

**Step 5.** Utilize Eq. (26) to calculate the relative closeness coefficient $C_i$ of each alternative $A_i$ to the single-valued neutrosophic positive ideal solution (SVNPIS) $A^+$.

**Step 6.** Rank the alternatives $A_i$ ($i = 1, 2, \cdots, m$) according to the relative closeness coefficients $C_i$ ($i = 1, 2, \cdots, m$) to the single-valued neutrosophic positive ideal solution (SVNPIS) $A^+$ and then select the most desirable one(s).

## 4 A novel method for interval neutrosophic multi-criteria decision making with incomplete weight information

In this section, we will extend the results obtained in Section 3 to interval neutrosophic environments.

### 4.1. Problem description

Similar to Subsection 3.1, a MCDM problem under interval neutrosophic environments can be summarized as follows: let $A = \left\{A_1, A_2, \cdots, A_m\right\}$ be a set of $m$ alternatives and $C = \left\{c_1, c_2, \ldots, c_n\right\}$ be a collection of $n$ criteria, whose weight vector is $w = \left(w_1, w_2, \cdots, w_n\right)^T$, with $w_j \in [0,1]$, $j = 1, 2, \cdots, n$, and $\sum_{j=1}^{n} w_j = 1$. In this case, the characteristic of the alternative $A_i$ ($i = 1, 2, \cdots, m$) with respect to all the criteria is represented by the following INS:

$$\tilde{A}_i = \left\{\left\langle c_j, \tilde{T}_{\tilde{A}_i}\left(c_j\right), \tilde{I}_{\tilde{A}_i}\left(c_j\right), \tilde{F}_{\tilde{A}_i}\left(c_j\right)\right\rangle \middle| c_j \in C\right\}$$
$$= \left\{\left\langle \begin{matrix} c_j, \left[\inf \tilde{T}_{\tilde{A}}\left(c_j\right), \sup \tilde{T}_{\tilde{A}}\left(c_j\right)\right], \left[\inf \tilde{I}_{\tilde{A}}\left(c_j\right), \sup \tilde{I}_{\tilde{A}}\left(c_j\right)\right], \\ \left[\inf \tilde{F}_{\tilde{A}}\left(c_j\right), \sup \tilde{F}_{\tilde{A}}\left(c_j\right)\right] \end{matrix} \right\rangle \middle| c_j \in C\right\}$$

where $\tilde{T}_{\tilde{A}_i}\left(c_j\right) = \left[\inf \tilde{T}_{\tilde{A}}\left(c_j\right), \sup \tilde{T}_{\tilde{A}}\left(c_j\right)\right] \subseteq [0,1]$, $\tilde{I}_{\tilde{A}_i}\left(c_j\right) = \left[\inf \tilde{I}_{\tilde{A}}\left(c_j\right), \sup \tilde{I}_{\tilde{A}}\left(c_j\right)\right] \subseteq [0,1]$,

$\tilde{F}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{F}_{\tilde{A}}(c_j),\sup \tilde{F}_{\tilde{A}}(c_j)\right]\subseteq[0,1]$, and $\sup\tilde{T}_{\tilde{A}}(c_j)+\sup\tilde{I}_{\tilde{A}}(c_j)+\sup\tilde{F}_{\tilde{A}}(c_j)\le 3$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$).

Here, $\tilde{T}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{T}_{\tilde{A}}(c_j),\sup \tilde{T}_{\tilde{A}}(c_j)\right]$ indicates the interval degree to which the alternative $A_i$ satisfies the criterion $c_j$, $\tilde{I}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{I}_{\tilde{A}}(c_j),\sup \tilde{I}_{\tilde{A}}(c_j)\right]$ indicates the indeterminacy interval degree to which the alternative $A_i$ satisfies or does not satisfy the criterion $c_j$, and $\tilde{F}_{\tilde{A}_i}(c_j)=\left[\inf \tilde{F}_{\tilde{A}}(c_j),\sup \tilde{F}_{\tilde{A}}(c_j)\right]$ indicates the interval degree to which the alternative $A_i$ does not satisfy the criterion $c_j$. For the sake of simplicity, a criterion value $\left\langle c_j,\tilde{T}_{\tilde{A}_i}(c_j),\tilde{I}_{\tilde{A}_i}(c_j),\tilde{F}_{\tilde{A}_i}(c_j)\right\rangle$ in $\tilde{A}_i$ is denoted by an interval neutrosophic value (INV) $\tilde{a}_{ij}=\left\langle \tilde{T}_{ij},\tilde{I}_{ij},\tilde{F}_{ij}\right\rangle=\left\langle\left[T_{ij}^L,T_{ij}^U\right],\left[I_{ij}^L,I_{ij}^U\right],\left[F_{ij}^L,F_{ij}^U\right]\right\rangle$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$), which is usually derived from the evaluation of an alternative $A_i$ with respect to a criterion $c_j$ by means of a score law and data processing in practice [19,22]. All $\tilde{a}_{ij}$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$) constitute an interval neutrosophic decision matrix $\tilde{A}=(\tilde{a}_{ij})_{m\times n}=\left(\left\langle \tilde{T}_{ij},\tilde{I}_{ij},\tilde{F}_{ij}\right\rangle\right)_{m\times n}=\left(\left\langle\left[T_{ij}^L,T_{ij}^U\right],\left[I_{ij}^L,I_{ij}^U\right],\left[F_{ij}^L,F_{ij}^U\right]\right\rangle\right)_{m\times n}$ (see Table 2):

**Table 2:** Interval neutrosophic decision matrix $\tilde{A}$.

| 2 | $c_1$ | … | $c_j$ | … | $c_n$ |
|---|---|---|---|---|---|
| $A_1$ | $\langle\tilde{T}_{11},\tilde{I}_{11},\tilde{F}_{11}\rangle$ | … | $\langle\tilde{T}_{1j},\tilde{I}_{1j},\tilde{F}_{1j}\rangle$ | … | $\langle\tilde{T}_{1n},\tilde{I}_{1n},\tilde{F}_{1n}\rangle$ |
| … | … | … | … | … | … |
| $A_i$ | $\langle\tilde{T}_{i1},\tilde{I}_{i1},\tilde{F}_{i1}\rangle$ | … | $\langle\tilde{T}_{ij},\tilde{I}_{ij},\tilde{F}_{ij}\rangle$ | … | $\langle\tilde{T}_{in},\tilde{I}_{in},\tilde{F}_{in}\rangle$ |
| … | … | … | … | … | … |
| $A_m$ | $\langle\tilde{T}_{m1},\tilde{I}_{m1},\tilde{F}_{m1}\rangle$ | … | $\langle\tilde{T}_{mj},\tilde{I}_{mj},\tilde{F}_{mj}\rangle$ | … | $\langle\tilde{T}_{mn},\tilde{I}_{mn},\tilde{F}_{mn}\rangle$ |

### 4.2. Obtaining the optimal weights of criteria under interval neutrosophic environments by the maximizing deviation method

In what follows, similar to Subsection 3.2, based on the maximizing deviation method, we construct an optimization model to determine the optimal relative weights of criteria under interval neutrosophic environments. For the attribute $c_j\in C$, the deviation of the alternative $A_i$ to all the other alternatives can be defined as below:

$$D_{ij}=\sum_{q=1}^{m} d(\tilde{a}_{ij},\tilde{a}_{qj})=\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}$$

$$i=1,2,\cdots,m,\ j=1,2,\cdots,n \tag{27}$$

where $d(\tilde{a}_{ij},\tilde{a}_{qj})=\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}$ denotes the interval neutrosophic Euclidean distance between two interval neutrosophic values (INVs) $\tilde{a}_{ij}$ and $\tilde{a}_{qj}$ defined as in Eq. (8).

Let

$$D_j=\sum_{i=1}^{m}D_{ij}=\sum_{i=1}^{m}\sum_{q=1}^{m}d(\tilde{a}_{ij},\tilde{a}_{qj})=\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}$$

$$j=1,2,\cdots,n \tag{28}$$

then $D_j$ represents the deviation value of all alternatives to other alternatives for the criterion $c_j\in C$.

Further, let

$$D(w)=\sum_{j=1}^{n}w_jD_j=\sum_{j=1}^{n}w_j\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^L-T_{qj}^L\right|^2+\left|T_{ij}^U-T_{qj}^U\right|^2+\left|I_{ij}^L-I_{qj}^L\right|^2+\left|I_{ij}^U-I_{qj}^U\right|^2+\left|F_{ij}^L-F_{qj}^L\right|^2+\left|F_{ij}^U-F_{qj}^U\right|^2}{6}}\right) \tag{29}$$

then $D(w)$ represents the deviation value of all alternatives to other alternatives for all the criteria.

From the above analysis, we can construct a non-linear programming model to select the weight vector $w$ by maximizing $D(w)$, as follows:

$$\max D(w)=\sum_{j=1}^{n} w_j\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}\right)$$

$$\text{s.t.}\quad w_j\geq 0,\ \ j=1,2,\cdots,n,\quad \sum_{j=1}^{n} w_j^2=1 \tag{M-3}$$

To solve this model, we construct the Lagrange function:

$$L(w,\lambda)=\sum_{j=1}^{n} w_j\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}\right)+\frac{\lambda}{2}\left(\sum_{j=1}^{n} w_j^2-1\right) \tag{30}$$

where $\lambda$ is the Lagrange multiplier.

Differentiating Eq. (30) with respect to $w_j$ ($j=1,2,\cdots,n$) and $\lambda$, and setting these partial derivatives equal to zero, then the following set of equations is obtained:

$$\frac{\partial L}{\partial w_j}=\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}+\lambda w_j=0 \tag{31}$$

$$\frac{\partial L}{\partial \lambda}=\frac{1}{2}\left(\sum_{j=1}^{n} w_j^2-1\right)=0 \tag{32}$$

It follows from Eq. (32) that

$$w_j=\frac{-\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}}{\lambda} \tag{33}$$

Putting Eq. (33) into Eq. (32), we get

$$\lambda=-\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}\right)^2} \tag{34}$$

Then, by combining Eqs. (33) and (34), we have

$$w_j=\frac{\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}\right)^2}} \tag{35}$$

By normalizing $w_j$ ($j=1,2,\cdots,n$), we make their sum into a unit, and get

$$w_j^*=\frac{w_j}{\sum_{j=1}^{n} w_j}=\frac{\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}} \tag{36}$$

which can be considered as the optimal weight vector of criteria.

However, it is noted that there are practical situations in which the information about the weight vector is not completely unknown but partially known. For such cases, we establish the following constrained optimization model:

$$\max D(w)=\sum_{j=1}^{n} w_j\left(\sum_{i=1}^{m}\sum_{q=1}^{m}\sqrt{\frac{\left|T_{ij}^{L}-T_{qj}^{L}\right|^2+\left|T_{ij}^{U}-T_{qj}^{U}\right|^2+\left|I_{ij}^{L}-I_{qj}^{L}\right|^2+\left|I_{ij}^{U}-I_{qj}^{U}\right|^2+\left|F_{ij}^{L}-F_{qj}^{L}\right|^2+\left|F_{ij}^{U}-F_{qj}^{U}\right|^2}{6}}\right)$$

$$\text{s.t.}\quad w\in\Delta,\quad w_j\geq 0,\ \ j=1,2,\cdots,n,\quad \sum_{j=1}^{n} w_j=1 \tag{M-4}$$

It is noted that the model (M-4) is a linear programming model that can be solved using the MATLAB mathematics software package. Suppose that the optimal solution to the model (M-4) is $w=\left(w_1,w_2,\cdots,w_n\right)^T$, which can be considered as the weight vector of criteria.

### 4.3. Extended TOPIS method for the MCDM with interval neutrosophic information

After obtaining the weights of criteria on basis of the maximizing deviation method, similar to Subsection 3.3, we next extend the TOPSIS method to interval neutrosophic environments and develop an extended TOPSIS method to obtain the final ranking of the alternatives.

Under interval neutrosophic environments, the interval neutrosophic PIS (INPIS), denoted by $\tilde{A}^+$, and the interval neutrosophic NIS (INNIS), denoted by $\tilde{A}^-$, can be defined as follows:

$$\tilde{A}^+=\left\{\tilde{a}_1^+,\tilde{a}_2^+,\cdots,\tilde{a}_n^+\right\} \tag{37}$$

$$\tilde{A}^-=\left\{\tilde{a}_1^-,\tilde{a}_2^-,\cdots,\tilde{a}_n^-\right\} \tag{38}$$

where

$$\tilde{a}_j^+=\begin{cases}\left\langle \max\limits_i\left\{\tilde{T}_{ij}\right\},\min\limits_i\left\{\tilde{I}_{ij}\right\},\min\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle = \\ \left\langle \begin{matrix}\left[\max\limits_i T_{ij}^L,\max\limits_i T_{ij}^U\right],\left[\min\limits_i I_{ij}^L,\min\limits_i I_{ij}^U\right],\\ \left[\min\limits_i F_{ij}^L,\min\limits_i F_{ij}^U\right]\end{matrix}\right\rangle, & \text{if } j\in C_1,\\ \left\langle \min\limits_i\left\{\tilde{T}_{ij}\right\},\max\limits_i\left\{\tilde{I}_{ij}\right\},\max\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle \\ =\left\langle \begin{matrix}\left[\min\limits_i T_{ij}^L,\min\limits_i T_{ij}^U\right],\left[\max\limits_i I_{ij}^L,\max\limits_i I_{ij}^U\right],\\ \left[\max\limits_i F_{ij}^L,\max\limits_i F_{ij}^U\right]\end{matrix}\right\rangle, & \text{if } j\in C_2.\end{cases} \tag{39}$$

$$\tilde{a}_j^-=\begin{cases}\left\langle \min\limits_i\left\{\tilde{T}_{ij}\right\},\max\limits_i\left\{\tilde{I}_{ij}\right\},\max\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle = \\ \left\langle \begin{matrix}\left[\min\limits_i T_{ij}^L,\min\limits_i T_{ij}^U\right],\left[\max\limits_i I_{ij}^L,\max\limits_i I_{ij}^U\right],\\ \left[\max\limits_i F_{ij}^L,\max\limits_i F_{ij}^U\right]\end{matrix}\right\rangle, & \text{if } j\in C_1,\\ \left\langle \max\limits_i\left\{\tilde{T}_{ij}\right\},\min\limits_i\left\{\tilde{I}_{ij}\right\},\min\limits_i\left\{\tilde{F}_{ij}\right\}\right\rangle = \\ \left\langle \begin{matrix}\left[\max\limits_i T_{ij}^L,\max\limits_i T_{ij}^U\right],\left[\min\limits_i I_{ij}^L,\min\limits_i I_{ij}^U\right],\\ \left[\min\limits_i F_{ij}^L,\min\limits_i F_{ij}^U\right]\end{matrix}\right\rangle, & \text{if } j\in C_2.\end{cases} \tag{40}$$

The separation measures, $\tilde{d}_i^+$ and $\tilde{d}_i^-$, of each alternative $A_i$ from the INPIS $\tilde{A}^+$ and the INNIS $\tilde{A}^-$, respectively, are derived from

$$\begin{aligned}\tilde{d}_i^+ &= \sum_{j=1}^n w_j d\left(\tilde{a}_{ij},\tilde{a}_j^+\right)\\ &= \sum_{j\in C_1} w_j\sqrt{\frac{\left(\begin{matrix}\left|T_{ij}^L-\max\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\max\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\min\limits_i\left\{I_{ij}^L\right\}\right|^2+\\ \left|I_{ij}^U-\min\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\min\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\min\limits_i\left\{F_{ij}^U\right\}\right|^2\end{matrix}\right)}{6}}+\\ &\quad \sum_{j\in C_2} w_j\sqrt{\frac{\left(\begin{matrix}\left|T_{ij}^L-\min\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\min\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\max\limits_i\left\{I_{ij}^L\right\}\right|^2+\\ \left|I_{ij}^U-\max\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\max\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\max\limits_i\left\{F_{ij}^U\right\}\right|^2\end{matrix}\right)}{6}}\end{aligned} \tag{41}$$

$$\begin{aligned}\tilde{d}_i^- &= \sum_{j=1}^n w_j d\left(\tilde{a}_{ij},\tilde{a}_j^-\right)\\ &= \sum_{j\in C_1} w_j\sqrt{\frac{\left(\begin{matrix}\left|T_{ij}^L-\min\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\min\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\max\limits_i\left\{I_{ij}^L\right\}\right|^2+\\ \left|I_{ij}^U-\max\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\max\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\max\limits_i\left\{F_{ij}^U\right\}\right|^2\end{matrix}\right)}{6}}+\\ &\quad \sum_{j\in C_2} w_j\sqrt{\frac{\left(\begin{matrix}\left|T_{ij}^L-\max\limits_i\left\{T_{ij}^L\right\}\right|^2+\left|T_{ij}^U-\max\limits_i\left\{T_{ij}^U\right\}\right|^2+\left|I_{ij}^L-\min\limits_i\left\{I_{ij}^L\right\}\right|^2+\\ \left|I_{ij}^U-\min\limits_i\left\{I_{ij}^U\right\}\right|^2+\left|F_{ij}^L-\min\limits_i\left\{F_{ij}^L\right\}\right|^2+\left|F_{ij}^U-\min\limits_i\left\{F_{ij}^U\right\}\right|^2\end{matrix}\right)}{6}}\end{aligned} \tag{42}$$

The relative closeness coefficient of an alternative $A_i$ with respect to the interval neutrosophic PIS $\tilde{A}^+$ is defined as the following formula:

$$\tilde{C}_i=\frac{\tilde{d}_i^-}{\tilde{d}_i^+ + \tilde{d}_i^-} \tag{43}$$

where $0\le\tilde{C}_i\le 1$, $i=1,2,\cdots,m$. Obviously, an alternative $A_i$ is closer to the interval neutrosophic PIS $\tilde{A}^+$ and farther from the interval neutrosophic NIS $\tilde{A}^-$ as $\tilde{C}_i$ approaches 1. The larger the value of $\tilde{C}_i$, the more different between $A_i$ and the interval neutrosophic NIS $\tilde{A}^-$, while the more similar between $A_i$ and the interval neutrosophic PIS $\tilde{A}^+$. Therefore, the alternative(s) with the maximum relative closeness coefficient should be chosen as the optimal one(s).

Based on the above analysis, analogous to Subsection 3.3, we will develop a practical approach for dealing with MCDM problems, in which the information about criterion weights is incompletely known or completely unknown, and the criterion values take the form of interval neutrosophic information.

The flowchart of the proposed approach for MCDM is provided in Fig. 1. The proposed approach is composed of the following steps:

**Step 1.** For a MCDM problem, the decision maker constructs the interval neutrosophic decision matrix $\tilde{A}=\left(\tilde{a}_{ij}\right)_{m\times n}=\left(\left\langle \tilde{T}_{ij},\tilde{I}_{ij},\tilde{F}_{ij}\right\rangle\right)_{m\times n}$, where $\tilde{a}_{ij}=\left\langle \tilde{T}_{ij},\tilde{I}_{ij},\tilde{F}_{ij}\right\rangle$ is an interval neutrosophic value (INV), given by the DM, for the alternative $A_i$ with respect to the criterion $c_j$.

**Step 2.** If the information about the criterion weights is completely unknown, then we use Eq. (36) to obtain the criterion weights; if the information about the criterion weights is partly known, then we solve the model (M-4) to obtain the criterion weights.

**Step 3.** Utilize Eqs. (37), (38), (39), and (40) to determine the interval neutrosophic positive ideal solution (INPIS) $\tilde{A}^{+}$ and the interval neutrosophic negative ideal solution (INNIS) $\tilde{A}^{-}$.

**Step 4.** Utilize Eqs. (41) and (42) to calculate the separation measures $\tilde{d}_i^{+}$ and $\tilde{d}_i^{-}$ of each alternative $A_i$ from the interval neutrosophic positive ideal solution (INPIS) $\tilde{A}^{+}$ and the interval neutrosophic negative ideal solution (INNIS) $\tilde{A}^{-}$, respectively.

**Step 5.** Utilize Eq. (43) to calculate the relative closeness coefficient $\tilde{C}_i$ of each alternative $A_i$ to the interval neutrosophic positive ideal solution (INPIS) $\tilde{A}^{+}$.

**Step 6.** Rank the alternatives $A_i$ ($i=1,2,\cdots,m$) according to the relative closeness coefficients $\tilde{C}_i$ ($i=1,2,\cdots,m$) to the interval neutrosophic positive ideal solution (INPIS) $A^{+}$ and then select the most desirable one(s).

## 5 Illustrative examples

### 5.1. A practical example under single-valued neutrosophic environments

**Example 5.1** [13]. In order to demonstrate the application of the proposed approach, a multi-criteria decision making problem adapted from Tan and Chen [13] is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of suppliers. Now suppose that there are a set of four suppliers $A=\{A_1,A_2,A_3,A_4,A_5\}$ whose core competencies are evaluated by means of the following four criteria:

(1) the level of technology innovation ($c_1$),

(2) the control ability of flow ($c_2$),

(3) the ability of management ($c_3$),

(4) the level of service ($c_4$).

It is noted that all the criteria $c_j$ ($j=1,2,3,4$) are the benefit type attributes. The selection of the best global supplier can be modeled as a hierarchical structure, as shown in Fig. 2. According to [21], we can obtain the evaluation of an alternative $A_i$ ($i=1,2,3,4,5$) with respect to a criterion $c_j$ ($j=1,2,3,4$) from the questionnaire of a domain expert. Take $a_{11}$ as an example. When we ask the opinion of an expert about an alternative $A_1$ with respect to a criterion $c_1$, he or she may say that the possibility in which the statement is good is 0.5 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.1. In this case, the evaluation of the alternative $A_1$ with respect to the criterion $c_1$ is expressed as a single-valued neutrosophic value $a_{11}=\langle 0.5,0.1,0.3\rangle$. Through the similar method from the expert, we can obtain all the evaluations of all the alternatives $A_i$ ($i=1,2,3,4,5$) with respect to all the criteria $c_j$ ($j=1,2,3,4$), which are listed in the following single valued neutrosophic decision matrix $A=\left(a_{ij}\right)_{m\times n}=\left(\left\langle T_{ij},I_{ij},F_{ij}\right\rangle\right)_{m\times n}$ (see Table 3).

**Table 3:** Single valued neutrosophic decision matrix $A$.

| 3 | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $A_1$ | $\langle 0.5,0.1,0.3\rangle$ | $\langle 0.5,0.1,0.4\rangle$ | $\langle 0.3,0.2,0.3\rangle$ | $\langle 0.7,0.2,0.1\rangle$ |
| $A_2$ | $\langle 0.6,0.1,0.2\rangle$ | $\langle 0.5,0.2,0.2\rangle$ | $\langle 0.5,0.4,0.1\rangle$ | $\langle 0.4,0.2,0.3\rangle$ |
| $A_3$ | $\langle 0.9,0.0,0.1\rangle$ | $\langle 0.3,0.2,0.3\rangle$ | $\langle 0.2,0.2,0.5\rangle$ | $\langle 0.4,0.3,0.2\rangle$ |
| $A_4$ | $\langle 0.8,0.1,0.1\rangle$ | $\langle 0.5,0.0,0.4\rangle$ | $\langle 0.6,0.2,0.1\rangle$ | $\langle 0.2,0.3,0.4\rangle$ |
| $A_5$ | $\langle 0.7,0.2,0.1\rangle$ | $\langle 0.4,0.3,0.2\rangle$ | $\langle 0.6,0.1,0.3\rangle$ | $\langle 0.5,0.4,0.1\rangle$ |

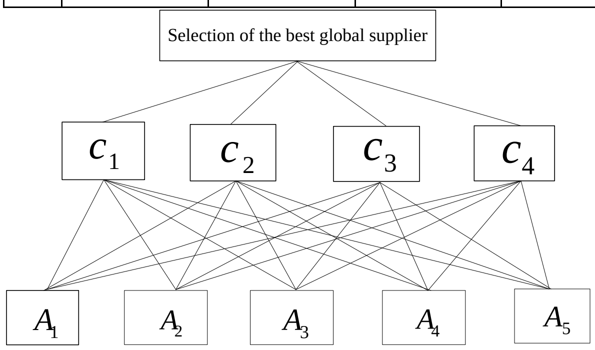


**Fig. 2:** Hierarchical structure.

In what follows, we utilize the developed method to find the best alternative(s). We now discuss two different cases.

**Case 1:** Assume that the information about the criterion weights is completely unknown; in this case, we use the following steps to get the most desirable alternative(s).

**Step 1.** Considering that the information about the criterion weights is completely unknown, we utilize Eq. (19) to get the optimal weight vector of attributes:

$$w = \left(0.2184, 0.2021, 0.3105, 0.2689\right)^T$$

**Step 2.** Utilize Eqs. (20), (21), (22), and (23) to determine the single valued neutrosophic PIS $A^+$ and the single valued neutrosophic NIS $A^-$, respectively:

$$A^+ = \{\langle 0.9,0.0,0.1\rangle, \langle 0.5,0.0,0.2\rangle, \langle 0.6,0.1,0.1\rangle, \langle 0.7,0.2,0.1\rangle\}$$

$$A^- = \{\langle 0.5,0.2,0.3\rangle, \langle 0.3,0.3,0.4\rangle, \langle 0.2,0.4,0.5\rangle, \langle 0.2,0.4,0.4\rangle\}$$

**Step 3:** Utilize Eqs. (24) and (25) to calculate the separation measures $d_i^+$ and $d_i^-$ of each alternative $A_i$ from the single valued neutrosophic PIS $A^+$ and the single valued neutrosophic NIS $A^-$, respectively:

$d_1^+ = 0.1510$, $d_1^- = 0.1951$, $d_2^+ = 0.1778$, $d_2^- = 0.1931$, $d_3^+ = 0.1895$, $d_3^- = 0.1607$, $d_4^+ = 0.1510$, $d_4^- = 0.2123$, $d_5^+ = 0.1523$, $d_5^- = 0.2242$.

**Step 4:** Utilize Eq. (26) to calculate the relative closeness coefficient $C_i$ of each alternative $A_i$ to the single valued neutrosophic PIS $A^+$:

$C_1 = 0.5638$, $C_2 = 0.5205$, $C_3 = 0.4589$, $C_4 = 0.5845$, $C_5 = 0.5954$.

**Step 5:** Rank the alternatives $A_i$ ($i = 1,2,3,4,5$) according to the relative closeness coefficient $C_i$ ($i = 1,2,3,4,5$). Clearly, $A_5 \succ A_4 \succ A_1 \succ A_2 \succ A_3$, and thus the best alternative is $A_5$.

**Case 2:** The information about the criterion weights is partly known and the known weight information is given as follows:

$$\Delta = \left\{ \begin{array}{l} 0.15 \le w_1 \le 0.25,\ 0.25 \le w_2 \le 0.3,\ 0.3 \le w_3 \le 0.4, \\ 0.35 \le w_4 \le 0.5,\ w_j \ge 0,\ j = 1,2,3,4,\ \sum_{j=1}^{4} w_j = 1 \end{array} \right\}$$

**Step 1:** Utilize the model (M-2) to construct the single-objective model as follows:

$$\begin{cases} \max D(w) = 2.9496w_1 + 2.7295w_2 + 4.1923w_3 + 3.6315w_4 \\ \text{s.t.} \quad w \in \Delta \end{cases}$$

By solving this model, we get the optimal weight vector of criteria $w = \left(0.15, 0.25, 0.3, 0.35\right)^T$.

**Step 2.** Utilize Eqs. (20), (21), (22), and (23) to determine the single valued neutrosophic PIS $A^+$ and the single valued neutrosophic NIS $A^-$, respectively:

$$A^+ = \{\langle 0.9,0.0,0.1\rangle, \langle 0.5,0.0,0.2\rangle, \langle 0.6,0.1,0.1\rangle, \langle 0.7,0.2,0.1\rangle\}$$

$$A^- = \{\langle 0.5,0.2,0.3\rangle, \langle 0.3,0.3,0.4\rangle, \langle 0.2,0.4,0.5\rangle, \langle 0.2,0.4,0.4\rangle\}$$

**Step 3:** Utilize Eqs. (24) and (25) to calculate the separation measures $d_i^+$ and $d_i^-$ of each alternative $A_i$ from the single valued neutrosophic PIS $A^+$ and the single valued neutrosophic NIS $A^-$, respectively:

$d_1^+ = 0.1368$, $d_1^- = 0.2260$, $d_2^+ = 0.1852$, $d_2^- = 0.2055$, $d_3^+ = 0.2098$, $d_3^- = 0.1581$, $d_4^+ = 0.1780$, $d_4^- = 0.2086$, $d_5^+ = 0.1619$, $d_5^- = 0.2358$.

**Step 4:** Utilize Eq. (26) to calculate the relative closeness coefficient $C_i$ of each alternative $A_i$ to the single valued neutrosophic PIS $A^+$:

$C_1 = 0.6230$, $C_2 = 0.5260$, $C_3 = 0.4297$, $C_4 = 0.5396$, $C_5 = 0.5928$.

**Step 5:** Rank the alternatives $A_i$ ($i = 1,2,3,4,5$) according to the relative closeness coefficient $C_i$ ($i = 1,2,3,4,5$). Clearly, $A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3$, and thus the best alternative is $A_1$.

### 5.2. The analysis process under interval neutrosophic environments

**Example 5.2.** Let's revisit Example 5.1. Suppose that the five possible alternatives are to be evaluated under the above four criteria by the form of INVs, as shown in the following interval neutrosophic decision matrix $\tilde{A}$ (see Table 4).

**Table 4:** Interval neutrosophic decision matrix $\tilde{A}$.

| 4 | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $A_1$ | <[0.7, 0.9], [0.1, 0.2], [0.5, 0.6]> | <[0.3, 0.4], [0.2, 0.3], [0.4, 0.5]> | <[0.3, 0.5], [0.2, 0.3], [0.6, 0.7]> | <[0.7, 0.9], [0.3, 0.4], [0.5, 0.6]> |
| $A_2$ | <[0.5, 0.6], [0.2,0.3], [0.2, 0.4]> | <[0.2, 0.3], [0.1, 0.3], [0.7, 0.8]> | < [0.5, 0.7], [0.2, 0.3], [0.7, 0.8]> | <[0.8, 0.9], [0.1, 0.2], [0.5, 0.7]> |
| $A_3$ | < [0.4, 0.5], [0.2, 0.3], [0.4, 0.6]> | <[0.3, 0.4], [0.1, 0.2], [0.7, 0.9]> | <[0.3, 0.5], [0.2, 0.3], [0.6, 0.7]> | <[0.7, 0.9], [0.2, 0.3], [0.5, 0.6]> |
| $A_4$ | < [0.2,0.3], [0.1, 0.2], [0.4, 0.5]> | <[0.4, 0.5], [0.3, 0.5], [0.2, 0.3]> | <[0.8, 0.9], [0.1, 0.3], [0.3, 0.4]> | < [0.2,0.3], [0.3, 0.5], [0.6, 0.8]> |
| $A_5$ | <[0.7, 0.8], [0.3,0.4], [0.6,0.7] > | < [0.6, 0.7], [0.1, 0.2], [0.7, 0.9]> | <[0.2, 0.3], [0.1, 0.2], [0.7, 0.8]> | <[0.6, 0.7], [0.3, 0.4], [0.4, 0.5]> |

In what follows, we proceed to utilize the developed method to find the most optimal alternative(s), which consists of the following two cases:

**Case 1:** Assume that the information about the criterion weights is completely unknown; in this case, we use the following steps to get the most desirable alternative(s).

**Step 1.** Considering that the information about the criterion weights is completely unknown, we utilize Eq. (36) to get the optimal weight vector of attributes:

$$w = (0.2490, 0.2774, 0.2380, 0.2356)^T$$

**Step 2.** Utilize Eqs. (37), (38), (39), and (40) to determine the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$$\tilde{A}^+ = \begin{Bmatrix} \langle 0.7, 0.9, 0.1, 0.2, 0.2, 0.4 \rangle, \langle 0.6, 0.7, 0.1, 0.2, 0.2, 0.3 \rangle, \\ \langle 0.8, 0.9, 0.1, 0.2, 0.3, 0.4 \rangle, \langle 0.8, 0.9, 0.1, 0.2, 0.4, 0.5 \rangle \end{Bmatrix}$$

$$\tilde{A}^- = \begin{Bmatrix} \{0.2, 0.3, 0.3, 0.4, 0.6, 0.7\}, \{0.2, 0.3, 0.3, 0.5, 0.7, 0.9\}, \\ \{0.2, 0.3, 0.2, 0.3, 0.7, 0.8\}, \{0.2, 0.3, 0.3, 0.5, 0.6, 0.8\} \end{Bmatrix}$$

**Step 3:** Utilize Eqs. (41) and (42) to calculate the separation measures $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative $A_i$ from the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$\tilde{d}_1^+ = 0.2044$, $\tilde{d}_1^- = 0.2541$, $\tilde{d}_2^+ = 0.2307$,
$\tilde{d}_2^- = 0.2405$, $\tilde{d}_3^+ = 0.2582$, $\tilde{d}_3^- = 0.1900$,
$\tilde{d}_4^+ = 0.2394$, $\tilde{d}_4^- = 0.2343$, $\tilde{d}_5^+ = 0.2853$,
$\tilde{d}_5^- = 0.2268$.

**Step 4:** Utilize Eq. (43) to calculate the relative closeness coefficient $\tilde{C}_i$ of each alternative $A_i$ to the interval neutrosophic PIS $\tilde{A}^+$:

$\tilde{C}_1 = 0.5543$, $\tilde{C}_2 = 0.5104$, $\tilde{C}_3 = 0.4239$,
$\tilde{C}_4 = 0.4946$, $\tilde{C}_5 = 0.4429$.

**Step 5:** Rank the alternatives $A_i$ ($i = 1,2,3,4,5$) according to the relative closeness coefficient $\tilde{C}_i$ ($i = 1,2,3,4,5$). Clearly, $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$, and thus the best alternative is $A_1$.

**Case 2:** The information about the attribute weights is partly known and the known weight information is given as follows:

$$\Delta = \begin{Bmatrix} 0.25 \le w_1 \le 0.3,\ 0.25 \le w_2 \le 0.35,\ 0.35 \le w_3 \le 0.4, \\ 0.4 \le w_4 \le 0.45,\ w_j \ge 0,\ j = 1,2,3,4,\ \sum_{j=1}^{4} w_j = 1 \end{Bmatrix}$$

**Step 1:** Utilize the model (M-4) to construct the single-objective model as follows:

$$\begin{cases} \max D(w) = 4.2748w_1 + 4.7627w_2 + 4.0859w_3 + 4.0438w_4 \\ \text{s.t.} \quad w \in \Delta \end{cases}$$

By solving this model, we get the optimal weight vector of criteria $w = (0.25, 0.25, 0.35, 0.4)^T$.

**Step 2.** Utilize Eqs. (37), (38), (39), and (40) to determine the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$$\tilde{A}^+ = \begin{Bmatrix} \langle 0.7, 0.9, 0.1, 0.2, 0.2, 0.4 \rangle, \langle 0.6, 0.7, 0.1, 0.2, 0.2, 0.3 \rangle, \\ \langle 0.8, 0.9, 0.1, 0.2, 0.3, 0.4 \rangle, \langle 0.8, 0.9, 0.1, 0.2, 0.4, 0.5 \rangle \end{Bmatrix}$$

$$\tilde{A}^- = \begin{Bmatrix} \{0.2, 0.3, 0.3, 0.4, 0.6, 0.7\}, \{0.2, 0.3, 0.3, 0.5, 0.7, 0.9\}, \\ \{0.2, 0.3, 0.2, 0.3, 0.7, 0.8\}, \{0.2, 0.3, 0.3, 0.5, 0.6, 0.8\} \end{Bmatrix}$$

**Step 3:** Utilize Eqs. (41) and (42) to calculate the separation measures $\tilde{d}_i^+$ and $\tilde{d}_i^-$ of each alternative $A_i$ from the interval neutrosophic PIS $\tilde{A}^+$ and the interval neutrosophic NIS $\tilde{A}^-$, respectively:

$\tilde{d}_1^+ = 0.2566$, $\tilde{d}_1^- = 0.3152$, $\tilde{d}_2^+ = 0.2670$,
$\tilde{d}_2^- = 0.3228$, $\tilde{d}_3^+ = 0.2992$, $\tilde{d}_3^- = 0.2545$,
$\tilde{d}_4^+ = 0.3056$, $\tilde{d}_4^- = 0.2720$, $\tilde{d}_5^+ = 0.3503$,
$\tilde{d}_5^- = 0.2716$.

**Step 4:** Utilize Eq. (43) to calculate the relative closeness coefficient $\tilde{C}_i$ of each alternative $A_i$ to the interval neutrosophic PIS $\tilde{A}^+$:

$\tilde{C}_1 = 0.5513$, $\tilde{C}_2 = 0.5474$, $\tilde{C}_3 = 0.4596$,
$\tilde{C}_4 = 0.4709$, $\tilde{C}_5 = 0.4368$.

**Step 5:** Rank the alternatives $A_i$ ($i = 1,2,3,4,5$) according to the relative closeness coefficient $\tilde{C}_i$ ($i = 1,2,3,4,5$). Clearly, $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$, and thus the best alternative is $A_1$.

### 5.3. Comparison analysis with the existing single-valued neutrosophic or interval neutrosophic multi-criteria decision making methods

Recently, some methods [20,21,22,23] have been developed for solving the MCDM problems with single-valued neutrosophic or interval neutrosophic information. In this section, we will perform a comparison analysis between our new methods and these existing methods, and then highlight the advantages of the new methods over these existing methods.

It is noted that these existing methods have some inherent drawbacks, which are shown as follows:

(1) The existing methods [20,21,22,23] need the decision maker to provide the weights of criteria in advance, which is subjective and sometime cannot yield the persuasive results. In contrast, our methods utilize the maximizing deviation method to determine the weight values of criteria, which is more objective and reasonable than the other existing methods [20,21,22,23].

(2) In Ref. [23], Ye proposed a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator, and then utilized two aggregation operators to develop a method for multi-criteria decision making problems under simplified neutrosophic environments. However, it is noted that these

operators and method need to perform an aggregation on the input simplified neutrosophic arguments, which may increase the computational complexity and therefore lead to the loss of information. In contrast, our methods do not need to perform such an aggregation but directly deal with the input simplified neutrosophic arguments, thereby can retain the original decision information as much as possible.

(3) In Ref. [22], Ye defined the Hamming and Euclidean distances between interval neutrosophic sets (INSs) and proposed the similarity measures between INSs on the basis of the relationship between similarity measures and distances. Moreover, Ye [22] utilized the similarity measures between each alternative and the ideal alternative to rank the alternatives and to determine the best one. In order to clearly demonstrate the comparison results, we use the method proposed in [22] to revisit Example 5.2, which is shown as follows:

First, we identify an ideal alternative by using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives as:

$$\tilde{A}^{+}=\left\{\begin{array}{l}\langle 0.7,0.9,0.1,0.2,0.2,0.4\rangle,\langle 0.6,0.7,0.1,0.2,0.2,0.3\rangle,\\ \langle 0.8,0.9,0.1,0.2,0.3,0.4\rangle,\langle 0.8,0.9,0.1,0.2,0.4,0.5\rangle\end{array}\right\}$$

In order to be consistent with Example 5.2, the same distance measure and the same weights for criteria are adopted here. Then, we apply Eq. (8) to calculate the similarity measure between an alternative $A_i$ ($i=1,2,3,4,5$) and the ideal alternative $\tilde{A}^{+}$ as follows:

$$s\left(\tilde{A}^{+},A_1\right)=1-d\left(\tilde{A}^{+},A_1\right)=1-0.2044=0.7956$$

$$s\left(\tilde{A}^{+},A_2\right)=1-d\left(\tilde{A}^{+},A_2\right)=1-0.2307=0.7693$$

$$s\left(\tilde{A}^{+},A_3\right)=1-d\left(\tilde{A}^{+},A_3\right)=1-0.2582=0.7418$$

$$s\left(\tilde{A}^{+},A_4\right)=1-d\left(\tilde{A}^{+},A_4\right)=1-0.2394=0.7606$$

$$s\left(\tilde{A}^{+},A_5\right)=1-d\left(\tilde{A}^{+},A_5\right)=1-0.2853=0.7147$$

Finally, through the similarity measure $s\left(\tilde{A}^{+},A_i\right)$ ($i=1,2,3,4,5$) between each alternative and the ideal alternative, the ranking order of all alternatives can be determined as: $A_1 \succ A_2 \succ A_4 \succ A_3 \succ A_5$. Thus, the optimal alternative is $A_1$.

It is easy to see that the optimal alternative obtained by the Ye' method [22] is the same as our method, which shows the effectiveness, preciseness, and reasonableness of our method. However, it is noticed that the ranking order of the alternatives obtained by our method is $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$, which is different from the ranking order obtained by the Ye' method [22]. Concretely, the ranking order between $A_3$ and $A_5$ obtained by two methods are just converse, i.e., $A_5 \succ A_3$ for our method while $A_3 \succ A_5$ for the Ye' method [22]. The main reason is that the Ye' method determines a solution which is the closest to the positive ideal solution (PIS), while our method determines a solution with the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). Therefore, the Ye' method is suitable for those situations in which the decision maker wants to have maximum profit and the risk of the decisions is less important for him, while our method is suitable for cautious (risk avoider) decision maker, because the decision maker might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible.

## Conclusions

Considering that some multi-criteria decision making problems contain uncertain, imprecise, incomplete, and inconsistent information, and the information about criterion weights is usually incomplete, this paper has developed a novel method for single-valued neutrosophic or interval neutrosophic multi-criteria decision making with incomplete weight information. First, motivated by the idea that a larger weight should be assigned to the criterion with a larger deviation value among alternatives, a maximizing deviation method has been presented to determine the optimal criterion weights under single-valued neutrosophic or interval neutrosophic environments, which can eliminate the influence of subjectivity of criterion weights provided by the decision maker in advance. Then, a single-valued neutrosophic or interval neutrosophic TOPSIS is proposed to calculate the relative closeness coefficient of each alternative to the single-valued neutrosophic or interval neutrosophic positive ideal solution, based on which the considered alternatives are ranked and then the most desirable one is selected. The prominent advantages of the developed methods are that they can not only relieve the influence of subjectivity of the decision maker but also remain the original decision information sufficiently. Finally, the effectiveness and practicality of the developed methods have been illustrated with a best global supplier selection example, and the advantages of the developed methods have been demonstrated with a comparison with the other existing methods.

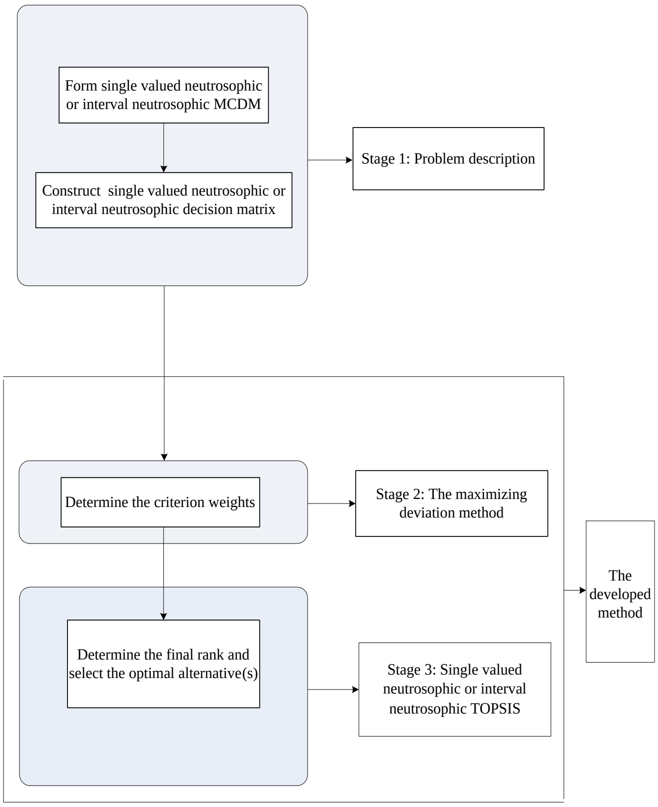


Fig. 1: The flowchart of the developed methods

**Acknowledgment**

The authors would like to thank the anonymous reviewers for their valuable suggestions as to how to improve this paper. This work was supported by the National Natural Science Foundation of China (Grant Nos. 61073121, 71271070 and 61375075) and the Natural Science Foundation of Hebei Province of China (Grant Nos. F2012201020 and A2012201033).

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Received: June 21, 2014. Accepted: July 1, 2014.