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An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces

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Abstract:

The fusion of symbolic plithogenic sets with algebraic structures generates novel algebraic neutrosophic structures that generalize the classical known structures. The objective of this paper is to define the concept of symbolic 2-plithogenic vector space over a symbolic 2-plitogenic field.

Concepts such as AH-subspace and AH-linear transformation will be presented and discussed in terms of theorems.

Keywords: 2-plithogenic symbolic set, 2-plithogenic vector space, 2-plithogenic dimension

Introduction

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17 ,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31], where the concepts such as symbolic AH-ideals, and AH-homomorphisms were presented and discussed.

In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$2-SP_R=\{a_0+a_1P_1+a_2P_2;\ a_i\in R, P_j{}^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\}$.

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2]+[b_0+b_1P_1+b_2P_2]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2$.

Multiplication:

$[a_0+a_1P_1+a_2P_2].[b_0+b_1P_1+b_2P_2]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_1b_0P_1{}^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2P_2{}^2+a_1b_1P_1P_1=a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2$.

It is clear that $(2-SP_R)$ is a ring.

Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field.

In this paper, we study the symbolic 2-plithogenic vector spaces according to many points of view, where substructures such as AH-subspaces, and AH-linear transformations will be presented in terms of theorems. In addition, many examples will be illustrated to explain the novelty of these ideas.

Main Discussion

Definition.

Let V be a vector space over the field F, let $2-SP_F$ be the corresponding symbolic 2-plithogenic field.

$2-SP_F=\{x+yP_1+zP_2;\ x,y,z\in F, P_i{}^2=P_i, P_1P_2=P_2P_1=P_2\}$.

We define the symbolic 2-plithogenic vector space as follows:

$2-SP_V=V+VP_1+VP_2=\{a+bP_1+cP_2;\ a,b,c\in V\}$.

Operations on $2-SP_V$ can be defined as follows:

Addition: $(+): 2-SP_V\to 2-SP_V$, such that:

$[x_0+x_1P_1+x_2P_2]+[y_0+y_1P_1+ys_2P_2]=(x_0+y_0)+(x_1+y_1)P_1+(x_2+y_2)P_2$

Multiplication: $(.): 2-SP_F\times 2-SP_V\to 2-SP_V$, such that:

$$[a+bP_1+cP_2].[x_0+x_1P_1+x_2P_2]$$
$$=ax_0+(ax_1+bx_0+bx_1)P_1+(ax_2+bx_2+cx_0+cx_1+cx_2)P_2$$

where $x_i,y_i\in V, a,b,c\in F$

Theorem.

Let $(2-SP_V,+,.)$ Is a module over the ring $2-SP_F$.

Proof.

Let $X=x_0+x_1P_1+x_2P_2, Y=y_0+y_1P_1+y_2P_2 \in 2-SP_V$, $A=a_0+a_1P_1+a_2P_2, B=b_0+b_1P_1+b_2P_2 \in 2-SP_F$ we have:

1. $X=X,(X+Y)+Z=X+(Y+Z),X+(-X)=-X+X=0,X+0=0+X=X$

Also

$$A(X+Y)=(a_0+a_1P_1+a_2P_2)[(x_0+y_0)+(x_1+y_1)P_1+(x_2+y_2)P_2]$$
$$=a_0(x_0+y_0)+\big(a_0(x_1+y_1)+a_1(x_0+y_0)+a_1(x_1+y_1)\big)P_1$$
$$+\big(a_0(x_2+y_2)+a_1(x_2+y_2)+a_2(x_0+y_0)+a_2(x_1+y_1)+a_2(x_2+y_2)\big)P_2$$
$$=A.X+A.Y$$

$$(A+B)X=[(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2](x_0+x_1P_1+x_2P_2)$$
$$=(a_0+b_0)x_0+\big((a_0+b_0)x_1+(a_1+b_1)x_0+(a_1+b_1)x_1\big)P_1$$
$$+\big((a_0+b_0)x_2+(a_1+b_1)x_2+(a_2+b_2)x_0+(a_2+b_2)x_1+(a_2+b_2)x_2\big)P_2$$
$$=A.X+B.X$$

$$(A.B).X=[a_0b_0+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2](x_0$$
$$+x_1P_1+x_2P_2)$$
$$=a_0b_0x_0+[a_0b_0x_1+(a_0b_1+a_1b_0+a_1b_1)x_0+(a_0b_1+a_1b_0+a_1b_1)x_1]P_1$$
$$+[a_0b_0x_2+(a_0b_2+a_2b_0+a_1b_1)x_2+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)x_0$$
$$+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)x_1$$
$$+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)x_2]P_2=A(B.X)$$

Example.

Let $V=R^3$ be the Euclidean space over the field $F=R$.

The corresponding symbolic 2-plithogenic vector space over $2-SP_F$ is:

$2-SP_{R^3}=\{(x_0,y_0,z_0)+(x_1,y_1,z_1)P_1+(x_2,y_2,z_2)P_2; x_i,y_i,z_i \in R\}$

Consider $X=(1,1,0)+(2,-1,1)P_1+(0,1,-1)P_2 \in 2-SP_{R^3}, A=2+P_1+P_2 \in 2-SP_R$. We have:

$$A.X=(2,2,0)+[(4,-2,2)+(1,1,0)+(2,-1,1)]P_1$$
$$+[(0,2,2)+(0,1,1)+(1,1,0)+(2,-1,1)+(0,1,1)]P_2$$
$$=(2,2,0)+(7,-2,3)P_1+(3,4,5)P_2$$

Definition.

Let $2-SP_V$ be a symbolic 2-plithogenic vector space over $2-SP_F$, let V_0,V_1,V_2 be the three subspaces of V, we define the AH-subspace as follows:

$$W = V_0 + V_1P_1 + V_2P_2 = \{x + yP_1 + zP_2;\ x \in V_0, y \in V_1, z \in V_2\}$$

If $V_0 = V_1 = V_2$, then W is called an AHS-subspace.

Example.

Consider $2 - SP_{R^3}$, we have $V_0 = \{(a, 0,0);\ a \in R\}, V_1 = \{(0, b, 0);\ b \in R\}, V_2 = \{(0,0, c);\ c \in R\}$ are three subspaces of $V = R^3$.

$W = V_0 + V_1P_1 + V_2P_2 = \{(a, 0,0) + (0, b, 0)P_1 + (0,0, c)P_2;\ a, b, c \in R\}$ is an AH-subspace of $2 - SP_{R^3}$.

$T = V_1 + V_1P_1 + V_1P_2 = \{(0, a, 0) + (0, b, 0)P_1 + (0, c, 0)P_2;\ a, b, c \in R\}$ is an AHS-subspace.

Theorem.

Let $2 - SP_V$ be a symbolic 2-plithogenic vector space over $2 - SP_F$, let W be an AHS-subspace of $2 - SP_V$, then W is a submodule of $2 - SP_V$.

Proof.

Suppose that W is an AHS-subspace, then there exists a subspace $V_0 \leq V$, such that

$W = V_0 + V_0P_1 + V_0P_2 = \{x + yP_1 + zP_2;\ x, y, z \in V_0\}$.

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in W$, then:

$X - Y = (x_0 - y_0) + (x_1 - y_1)P_1 + (x_2 - y_2)P_2 \in W$

$\forall A = a_0 + a_1P_1 + a_2P_2 \in 2 - SP_F$, then:

$A.X = a_0x_0 + (a_0x_1 + a_1x_0 + a_1x_1)P_1 + (a_0x_2 + a_1x_2 + a_2x_0 + a_2x_1 + a_2x_2)P_2 \in W$, that is because $a_0x_0 \in V_0, a_0x_1 + a_1x_0 + a_1x_1 \in V_0, a_0x_2 + a_1x_2 + a_2x_0 + a_2x_1 + a_2x_2 \in V_0$, this implies the proof.

Definition.

Let V, W be two vector spaces over the field F. Let $2 - SP_V$, $2 - SP_W$ be the corresponding symbolic 2-plithogenic vector spaces over $2 - SP_F$.

Let $L_0, L_1, L_2: V \to W$ be three linear transformations, we define the AH-linear transformation as follows:

$L: 2 - SP_V \to 2 - SP_W, L = L_0 + L_1P_1 + L_2P_2\ ; L(x + yP_1 + zP_2) = L_0(x) + L_1(y)P_1 + L_2(z)P_2$.

If $L_0 = L_1 = L_2$, then L is called AHS-linear transformation.

Definition.

Let $L = L_0 + L_1P_1 + L_2P_2: 2 - SP_V \to 2 - SP_W$ be an AH-linear transformation, we define:

1. $AH - ker(L) = ker(L_0) + ker(L_1)P_1 + ker(L_2)P_2 = \{x + yP_1 + zP_2\}; x \in ker(L_0), y \in ker(L_1), z \in ker(L_2)$.

2. $AH-Im(L)=Im(L_0)+Im(L_1)P_1+Im(L_2)P_2=\{a+bP_1+cP_2\}; a\in Im(L_0), b\in Im(L_1), c\in Im(L_2)$

If L is AHS-linear transformation, then we get $AHS-kernel,\ AHS-Image$.

Theorem.

Let $L=L_0+L_1P_1+L_2P_2: 2-SP_V\to 2-SP_W$ be an AH-linear transformation, then:

1. $AH-ker(L)$ is AH-subspace of $2-SP_V$.
2. $AH-Im(L)$ is AH-subspace of $2-SP_W$.

Proof.

1. Since $ker(L_0), ker(L_1), ker(L_2)$ are subspaces of V, then $AH-ker(L)$ is an AH-subspace of $2-SP_V$.
2. It is holds by the same.

Remark.

If L_0, L_1, L_2 are isomorphism, then $ker(L_0)=ker(L_1)=ker(L_2)=\{0\}, Im(L_0)=Im(L_1)=Im(L_2)=W$, thus $AH-ker(L)=\{0\}, AH-Im(L)=2-SP_W$.

Example.

Take $V=R^3, W=R^3,\ L_0, L_1, L_2: V\to W$ such that:

$L_0(x,y,z)=(x,y), L_1(x,y,z)=(2x,z), L_2(x,y,z)=(x-y,y-z)$

The corresponding AH-linear transformation is:

$L=L_0+L_1P_1+L_2P_2: 2-SP_{R^3}\to 2-SP_{R^2}$:

$$L[(x_0,y_0,z_0)+(x_1,y_1,z_1)P_1+(x_2,y_2,z_2)P_2]$$
$$=L_0(x_0,y_0,z_0)+L_1(x_1,y_1,z_1)P_1+L_2(x_2,y_2,z_2)P_2$$
$$=(x_0,y_0)+(2x_1,z_1)P_1+(x_2-y_2,y_2-z_2)P_2$$

For example, take $X=(1,2,1)+(4,3,-5)P_1+(1,1,1)P_2$, then:

$L(X)=(1,2)+(8,-5)P_1+(0,0)P_2=(1,2)+(8,-5)P_1$.

$$\begin{cases} ker(L_0)=\{(0,0,z_0);\ z_0\in R\} \\ ker(L_1)=\{(0,y_1,0);\ y_1\in R\} \\ ker(L_2)=\{(x_2,x_2,x_2);\ x_2\in R\} \\ AH-ker(L)=\{(0,0,z_0)+(0,y_1,0)P_1+(x_2,x_2,x_2)P_2; z_0,y_1,x_2\in R\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0)=R^2 \\ Im(L_1)=R^2 \\ Im(L_2)=R^2 \\ AH-Im(L)=R^2+R^2P_1+R^2P_2=2-SP_W \end{cases}$$

Example.

Take $W=V=R^2,\ L_0,L_1,L_2:V\to W$ such that:

$L_0(x,y)=(3x,-2x),L_1(x,y)=(x-y,2x),L_2(x,y,z)=(x+2y,y)$

The corresponding AH-linear transformation is $L=L_0+L_1P_1+L_2P_2:2-SP_V\to 2-SP_W$;

$$L[(x_0,y_0)+(x_1,y_1)P_1+(x_2,y_2)P_2]=L_0(x_0,y_0)+L_1(x_1,y_1)P_1+L_2(x_2,y_2)P_2$$
$$=(3x_0,-2x_0)+(x_1-y_1,2x_1)P_1+(x_2+2y_2,y_2)P_2$$

For example $X=(1,4)+(2,8)P_1+(3,-1)P_2$

$L(X)=(1,4)+(2,8)P_1+(3,-1)P_2.$

$$\begin{cases} ker(L_0)=\{(0,y_0);\ y_0\in R\} \\ ker(L_1)=\{0\} \\ ker(L_2)=\{0\} \\ AH-ker(L)=\{(0,y_0)+0P_1+0P_2;y_0\in R\} \end{cases}$$

Also,

$$\begin{cases} Im(L_0)=\{(a,0);\ a\in R\} \\ Im(L_1)=R^2 \\ Im(L_2)=R^2 \\ AH-Im(L)=\{(a,0)+(a_1,b_1)P_1+(a_2,b_2)P_2;a,a_1,a_2,b_2,b_1\in R\} \end{cases}$$

Theorem.

Let $L=f+fP_1+fP_2:2-SP_V\to 2-SP_W$ be an AHS-linear transformation, then L is a module homomorphism.

Proof.

Let $X=x_0+x_1P_1+x_2P_2,Y=y_0+y_1P_1+y_2P_2\in 2-SP_V$, then:

$$L(X+Y)=f(x_0+y_0)+f(x_1+y_1)P_1+f(x_2+y_2)P_2$$
$$=[f(x_0)+f(x_1)P_1+f(x_2)P_2]+[f(y_0)+f(y_1)P_1+f(y_2)P_2]=L(X)+L(Y)$$

Let $A=a_0+a_1P_1+a_2P_2\in 2-SP_F$, then:

$$L(A.X)=f(a_0x_0)+f(a_0x_1+a_1x_0+a_1x_1)P_1+f(a_0x_2+a_2x_0+a_2x_2+a_1x_2+a_2x_1)P_2$$
$$=a_0f(x_0)+\big(a_0f(x_1)+a_1f(x_0)+a_1f(x_1)\big)P_1$$
$$+\big(a_0f(x_2)+a_2f(x_0)+a_2f(x_2)+a_1f(x_2)+a_2f(x_1)\big)P_2$$
$$=[a_0+a_1P_1+a_2P_2].[f(x_0)+f(x_1)P_1+f(x_2)P_2]=A.L(X)$$

Thus, L is a module homomorphism.

The algebraic relations between symbolic 2-plithogenic vector spaces and neutrosophic vector spaces .

Theorem.

Let V be a vector space over the field F, consider $V(I) = V + VI = \{x + yI; x, y \in V\}$ is the corresponding neutrosophic vector space over the neutrosophic field $F(I) = \{a + bI; a, b \in F\}$.

$V(I_1, I_2) = V + VI_1 + VI_2 = \{x + yI_1 + zI_2; x, y, z \in V\}$ is the corresponding refined neutrosophic vector space over the refined neutrosophic field $F(I_1, I_2) = \{a + bI_1 + cI_2; a, b, c \in F\}$.

$2 - SP_V = V + VP_1 + VP_2 = \{x + yP_1 + zP_2; x, y, z \in V\}$ is the corresponding symbolic 2-plithogenic vector space over $2 - SP_F$, then:

1. $2 - SP_V$ is semi homomorphic to $V(I)$.
2. $2 - SP_V$ is semi isomorphic to $V(I_1, I_2)$.

Proof.

1. We define $f: 2 - SP_V \to V(I), g: 2 - SP_F \to F(I)$ such that:

$f(x + yP_1 + zP_2) = x + yI; x, y, z \in V$

$g(a + bP_1 + cP_2) = a + bI; a, b, c \in F$

We have the following:

g is a ring homomorphism, that is because:

$A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2;\ a_i, b_i \in F$, then:

If $A = B$, then $a_i = b_i$ for all i, thus $a_0 + a_1I = b_0 + b_1I, i.e.\quad g(A) = g(B)$.

$g(A + B) = g[(a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2] = a_0 + b_0 + (a_1 + b_1)I = g(A) + g(B)$.

$$g(A.B) = g[a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b + a_2b_1 + a_2b_2)P_2]$$
$$= a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)I = (a_0 + a_1I)(b_0 + b_1I) = g(A).g(B)$$

On the other hand, f is well defined, that is because:

If $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2$, then $x_i = y_i$ for all i, hence $a_0 + a_1I = b_0 + b_1I$, thus $f(X) = f(Y)$.

f preserves addition, that is because:

For $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2$, we have:

$f(X + Y) = f[(x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2] = x_0 + y_0 + (x_1 + y_1)I = f(X) + f(Y)$.

f preserves multiplication, that is because:

For $A = a_0 + a_1P_1 + a_2P_2 \in 2 - SP_V$, we have:

$f(A.X) = a_0x_0 + (a_0x_1 + a_1x_0 + a_1x_1)I = (a_0 + a_1I)(x_0 + x_1I) = g(A).f(X)$

Thus f is a semi module homomorphism.

We define $f: 2-SP_V \rightarrow V(I_1, I_2)$, $g: 2-SP_F \rightarrow F(I_1, I_2)$, where $f(x+yP_1+zP_2) = x + zI_1 + yI_2$, and $g(a+bP_1+cP_2) = a + cI_1 + bI_2; x, y, z \in V, a, b, c \in F$.

(g) is well defined, that is because:

If $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2$, then:

$a_0 = a_1, b_0 = b_1, c_0 = c_1$, hence: $a_0 + c_0I_1 + b_0I_2 = a_1 + c_1I_1 + b_1I_2$, so that $g(A) = g(B)$.

(f) is well defined by a similar discussion.

(g) is one-to-one mapping, that is because:

$ker(g) = \{a + bP_1 + cP_2; g(a + bP_1 + cP_2) = 0\} = O$

$$Im(g) = \{a + cI_1 + bI_2; g(a + bP_1 + cP_2) \in F(I_1, I_2); \exists A \in 2 - SP_F, g(A) = a + cI_1 + bI_2\}$$
$$= F(I_1, I_2)$$

(f) is one-to-one mapping, it can be proved by the same.

(g) and (f) preserve addition, that is because:

Consider $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_F$, $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_V$, then:

$$g(A + B) = g[(a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2] = a_0 + b_0 + (a_1 + b_1)I_1 + (a_2 + b_2)I_2$$
$$= g(A) + g(B)$$

$f(X + Y) = f(X) + f(Y)$ by a similar discussion.

(g) preserves multiplication, that is because:

$g(A.B) = a_0b_0 + (a_0b_2 + a_2b_0 + a_2b_2 + a_1b_2 + a_2b_1)I_1 + (a_0b_1 + a_1b_0 + a_1b_1)I_2 = g(A).g(B)$.

(f) is semi module homomorphism, that is because:

$$f(A.X) = a_0x_0 + (a_0x_2 + a_2x_0 + a_2x_2 + a_1x_2 + a_2x_1)I_1 + (a_0x_1 + a_1x_0 + a_1x_1)I_2$$
$$= (a_0 + a_1I_1 + a_2I_2)(x_0 + x_2I_1 + x_1I_2) = g(A).f(X)$$

The basis of a symbolic 2-plithogenic vector spaces:

Theorem.

Let $T = \{t_1, \dots, t_n\}$ be a basis of the vector space V over the field F, then the set:

$T_P = \{t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2; 1 \le i, j, k \le n\}$ is a basis of $2 - SP_V$.

Proof.

Let $X = x_0 + x_1P_1 + x_2P_2 \in 2 - SP_V, x_0, x_1, x_2 \in V$.

$x_0 = \sum_{i=1}^{n} \alpha_i t_i,\ x_0 + x_1 = \sum_{j=1}^{n} \beta_j t_j,\ x_0 + x_1 + x_2 = \sum_{k=1}^{n} \gamma_k t_k ; \alpha_i, \beta_j, \gamma_k \in F$.

We put $A_{i,j,k} = \alpha_i + (\beta_j - \alpha_i)P_1 + (\gamma_k - \beta_j)P_2; 1 \le i, j, k \le n$

$T_{i,j,k} = t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2; 1 \le i, j, k \le n$

$$\sum_{i,j,k=1}^{n} A_{i,j,k}\, T_{i,j,k}$$
$$= \sum_{i=1}^{n} [\alpha_i t_i + [\beta_j t_j - \beta_j t_i - \alpha_i t_j + \alpha_i t_i + \beta_j t_i - \alpha_i t_i + \alpha_i t_j - \alpha_i t_i] P_1$$
$$+ [\alpha_i t_k - \alpha_i t_j + \gamma_k t_i - \beta_j t_i - \gamma_k t_j + \gamma_k t_i - \beta_j t_j + \beta_j t_i + \gamma_k t_k - \gamma_k t_j - \beta_j t_k$$
$$+ -\beta_j t_j + \beta_j t_k - \beta_j t_j - \alpha_i t_k + \alpha_i t_j] P_2]$$
$$\sum_{i=1}^{n} \alpha_i t_i + P_1 \left[\sum_{j=1}^{n} \beta_j t_j - \sum_{i=1}^{n} \alpha_i t_i \right] + P_2 \left[\sum_{k=1}^{n} \gamma_k t_k - \sum_{j=1}^{n} \beta_j t_j \right]$$
$$= x_0 + P_1[x_0 + x_1 - x_0] + P_2[x_0 + x_1 + x_2 - (x_0 + x_1)] = x_0 + x_1 P_1 + x_2 P_2$$
$$= X$$

Thus T generates $2 - SP_V$.

On the other hand, T is linearly independent, that is because:

If $\sum_{i,j,k=1}^{n} A_{i,j,k} . X = 0$, then:

$\sum_{i=1}^{n} \alpha_i t_i = 0, \sum_{j=1}^{n} \beta_j t_j = 0, \sum_{k=1}^{n} \gamma_k t_k = 0$, hence$\alpha_i = \beta_j = \gamma_k = 0$ for all i, j, k, thus $A_{i,j,k} = 0$.

This implies that T is a basis of $2 - SP_V$.

Example.

Find a basis of $2 - SP_{R^2}$.

Solution.

First of all, we have $\{u_1 = (1,0), u_2 = (0,1)\}$ is a basis of R^2.

The corresponding basis of $2 - SP_{R^2}$ is:

$T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}$ such that:

$T_1 = (1,0), T_2 = (0,1), T_3 = u_1 + (u_2 - u_1)P_1 + (u_2 - u_2)P_2 = (1,0) + (-1,1)P_1$

$T_4 = u_1 + (u_2 - u_1)P_1 + (u_1 - u_2)P_2 = (1,0) + (-1,1)P_1 + (1,-1)P_2$

$T_5 = u_2 + (u_2 - u_1)P_1 + (u_1 - u_1)P_2 = (0,1) + (1,-1)P_1$

$T_6 = u_2 + (u_2 - u_1)P_1 + (u_2 - u_1)P_2 = (0,1) + (1,-1)P_1 + (-1,1)P_2$

$T_7 = u_1 + (u_1 - u_1)P_1 + (u_2 - u_1)P_2 = (1,0) + (-1,1)P_2$

$T_8 = u_2 + (u_2 - u_2)P_1 + (u_1 - u_2)P_2 = (0,1) + (1,-1)P_2$

Remark.

$dim\ (2 - SP_V) = (dimV)^3$.

Conclusion

In this paper we have defined the concept of symbolic 2-plithogenic vector spaces over a symbolic 2-plithogenic field, where we have presented some of their elementary properties such as basis, linear transformations, and AH-subspaces. On the other hand, we have suggested many examples to clarify the validity of our work.

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