



An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings

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Abstract:

The symbolic n-plithogenic sets and algebraic structures are a new branch of pure algebra released as new generalizations of classical algebraic structures.

The main goal of this paper is to define for the first time the concept of symbolic 2-plithogenic module over a symbolic 2-plithogenic ring. Algebraic substructures of symbolic 2-plithogenic modules such as sub-modules, AH-homomorphisms, and algebraic basis.

Keywords: 2-plithogenic symbolic set, 2-plithogenic module, 2-plithogenic ring

Introduction

The concept of symbolic plithogenic sets was defined by Smarandache in [13-17,30], and he suggested an algebraic approach of these sets. Laterally, the concept of symbolic 2-plithogenic rings [31], where the concepts such as symbolic AH-ideals, and AH-homomorphisms were presented and discussed.

In general, we can say that symbolic plithogenic structures are very close to neutrosophic algebraic structures with many differences in the definition of multiplication operation [1-10].

Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2 - SP_R = \{a_0 + a_1P_1 + a_2P_2; a_i \in R, P_j^2 = P_j, P_1 \times P_2 = P_{\max(1,2)} = P_2\}.$$

Smarandache has defined algebraic operations on $2 - SP_R$ as follows:

Addition:

$$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2.$$

Multiplication:

$$[a_0 + a_1P_1 + a_2P_2] \cdot [b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_1P_1P_1 = a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2.$$

In this paper, we study the symbolic 2-plithogenic modules according to many points of view, where substructures such as AH-submodules, and AH-homomorphisms will be presented in terms of theorems. In addition, many examples will be illustrated to explain the novelty of these ideas.

Main Discussion

Definition.

Let M be a module over the ring R , let $2 - SP_R$ be the corresponding symbolic 2-plithogenic ring.

$$2 - SP_R = \{x + yP_1 + zP_2; x, y, z \in R, P_i^2 = P_i, P_1P_2 = P_2P_1 = P_2\}.$$

We define the symbolic 2-plithogenic module as follows:

$$2 - SP_M = M + MP_1 + MP_2 = \{a + bP_1 + cP_2; a, b, c \in M\}.$$

Operations on $2 - SP_M$ can be defined as follows:

Addition: (+): $2 - SP_M \rightarrow 2 - SP_M$, such that:

$$[x_0 + x_1P_1 + x_2P_2] + [y_0 + y_1P_1 + y_2P_2] = (x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2$$

Multiplication: (.): $2 - SP_R \times 2 - SP_M \rightarrow 2 - SP_M$, such that:

$$[a + bP_1 + cP_2] \cdot [x_0 + x_1P_1 + x_2P_2] = ax_0 + (ax_1 + bx_0 + bx_1)P_1 + (ax_2 + bx_2 + cx_0 + cx_1 + cx_2)P_2.$$

where $x_i, y_i \in M, a, b, c \in R$

Theorem.

Let $(2 - SP_M, +, \cdot)$ is a module over the ring $2 - SP_R$.

Proof.

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_M$, $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_R$ we have:

$$1. X = X, (X + Y) + Z = X + (Y + Z), X + (-X) = -X + X = 0, X + 0 = 0 + X = X$$

Also

$$\begin{aligned} A(X + Y) &= (a_0 + a_1P_1 + a_2P_2)[(x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2] \\ &= a_0(x_0 + y_0) + (a_0(x_1 + y_1) + a_1(x_0 + y_0) + a_1(x_1 + y_1))P_1 \\ &\quad + (a_0(x_2 + y_2) + a_1(x_2 + y_2) + a_2(x_0 + y_0) + a_2(x_1 + y_1) + a_2(x_2 + y_2))P_2 \\ &= A.X + A.Y \end{aligned}$$

$$\begin{aligned} (A + B)X &= [(a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2](x_0 + x_1P_1 + x_2P_2) \\ &= (a_0 + b_0)x_0 + ((a_0 + b_0)x_1 + (a_1 + b_1)x_0 + (a_1 + b_1)x_1)P_1 \\ &\quad + ((a_0 + b_0)x_2 + (a_1 + b_1)x_2 + (a_2 + b_2)x_0 + (a_2 + b_2)x_1 + (a_2 + b_2)x_2)P_2 \\ &= A.X + B.X \end{aligned}$$

$$\begin{aligned} (A.B).X &= [a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2](x_0 + \\ &\quad x_1P_1 + x_2P_2) = a_0b_0x_0 + [a_0b_0x_1 + (a_0b_1 + a_1b_0 + a_1b_1)x_0 + (a_0b_1 + a_1b_0 + a_1b_1)x_1]P_1 + \\ &\quad [a_0b_0x_2 + (a_0b_2 + a_2b_0 + a_1b_1)x_2 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)x_0 + (a_0b_2 + a_1b_2 + \\ &\quad a_2b_0 + a_2b_1 + a_2b_2)x_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)x_2]P_2 = A(B.X). \end{aligned}$$

Example.

Let $M = Z^3$ be the module over the ring $R =$.

The corresponding symbolic 2-plithogenic vector space over $2 - SP_Z$ is:

$$2 - SP_{Z^3} = \{(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2; x_i, y_i, z_i \in Z\}$$

Consider $X = (1,1,0) + (2, -1,1)P_1 + (0,1, -1)P_2 \in 2 - SP_{Z^3}, A = 2 + P_1 + P_2 \in 2 - SP_Z$. We have:

$$\begin{aligned} A.X &= (2,2,0) + [(4, -2,2) + (1,1,0) + (2, -1,1)]P_1 + [(0,2,2) + (0,1,1) + (1,1,0) + \\ &\quad (2, -1,1) + (0,1,1)]P_2 = (2,2,0) + (7, -2,3)P_1 + (3,4,5)P_2. \end{aligned}$$

Definition.

Let $2 - SP_M$ be a symbolic 2-plithogenic module over $2 - SP_R$, let M_0, M_1, M_2 be the three sub-modules of V , we define the AH-submodule as follows:

$$W = M_0 + M_1P_1 + M_2P_2 = \{x + yP_1 + zP_2; x \in M_0, y \in M_1, z \in M_2\}.$$

If $M_0 = M_1 = M_2$, then W is called an AHS-sub-module.

Example.

Consider $2 - SP_{Z^3}$, we have $M_0 = \{(a, 0,0); a \in R\}, M_1 = \{(0, b, 0); b \in R\}, M_2 = \{(0,0, c); c \in Z\}$ are three sub-modules of $M = Z^3$.

$W = M + M_1P_1 + M_2P_2 = \{(a, 0,0) + (0, b, 0)P_1 + (0,0, c)P_2; a, b, c \in Z\}$ is an AH-submodule of $2 - SP_{Z^3}$.

$T = M_1 + MP_1 + M_1P_2 = \{(0, a, 0) + (0, b, 0)P_1 + (0, c, 0)P_2; a, b, c \in Z\}$ is an AHS-submodule.

Theorem.

Let $2 - SP_M$ be a symbolic 2-plithogenic module over $2 - SP_R$, let W be an AHS-submodule of $2 - SP_M$, then W is a submodule of $2 - SP_M$.

Proof.

Suppose that W is an AHS-submodule, then there exists a submodule $M_0 \leq M$, such that $W = M_0 + M_0P_1 + M_0P_2 = \{x + yP_1 + zP_2; x, y, z \in M_0\}$.

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in W$, then:

$$X - Y = (x_0 - y_0) + (x_1 - y_1)P_1 + (x_2 - y_2)P_2 \in W$$

$\forall A = a_0 + a_1P_1 + a_2P_2 \in 2 - SP_R$, then:

$A.X = a_0x_0 + (a_0x_1 + a_1x_0 + a_1x_1)P_1 + (a_0x_2 + a_1x_2 + a_2x_0 + a_2x_1 + a_2x_2)P_2 \in W$, that is because $a_0x_0 \in M_0, a_0x_1 + a_1x_0 + a_1x_1 \in M_0, a_0x_2 + a_1x_2 + a_2x_0 + a_2x_1 + a_2x_2 \in M_0$, this implies the proof.

Definition.

Let V, W be two modules over the ring R . Let $2 - SP_V, 2 - SP_W$ be the corresponding symbolic 2-plithogenic modules over $2 - SP_R$.

Let $L_0, L_1, L_2: V \rightarrow W$ be three homomorphisms, we define the AH-homomorphism as follows:

$$L: 2 - SP_V \rightarrow 2 - SP_W, L = L_0 + L_1P_1 + L_2P_2; L(x + yP_1 + zP_2) = L_0(x) + L_1(y)P_1 + L_2(z)P_2.$$

If $L_0 = L_1 = L_2$, then L is called AHS-homomorphism.

Definition.

Let $L = L_0 + L_1P_1 + L_2P_2: 2 - SP_V \rightarrow 2 - SP_W$ be an AH-homomorphism, we define:

1. $AH - \ker(L) = \ker(L_0) + \ker(L_1)P_1 + \ker(L_2)P_2 = \{x + yP_1 + zP_2; x \in \ker(L_0), y \in \ker(L_1), z \in \ker(L_2)\}$.
2. $AH - \text{Im}(L) = \text{Im}(L_0) + \text{Im}(L_1)P_1 + \text{Im}(L_2)P_2 = \{a + bP_1 + cP_2; a \in \text{Im}(L_0), b \in \text{Im}(L_1), c \in \text{Im}(L_2)\}$

If L is AHS-linear homomorphism, then we get *AHS - kernel*, *AHS - Image*.

Theorem.

Let $L = L_0 + L_1P_1 + L_2P_2: 2 - SP_V \rightarrow 2 - SP_W$ be an AH-homomorphism, then:

1. $AH - \ker(L)$ is AH-submodule of $2 - SP_V$.

2. $AH - Im(L)$ is AH-submodule of $2 - SP_W$.

Proof.

1. Since $ker(L_0), ker(L_1), ker(L_2)$ are submodules of V , then $AH - ker(L)$ is an AH-submodule of $2 - SP_V$.
2. It is holds by the same.

Remark.

If L_0, L_1, L_2 are isomorphisms, then $ker(L_0) = ker(L_1) = ker(L_2) = \{0\}, Im(L_0) = Im(L_1) = Im(L_2) = W$, thus $AH - ker(L) = \{0\}, AH - Im(L) = 2 - SP_W$.

Example.

Take $V = Z^3, W = Z, L_0, L_1, L_2: V \rightarrow W$ such that:

$$L_0(x, y, z) = (x), L_1(x, y, z) = (y), L_2(x, y, z) = (z)$$

The corresponding AH-homomorphism is:

$$L = L_0 + L_1P_1 + L_2P_2: 2 - SP_{Z^3} \rightarrow 2 - SP_Z:$$

$$L[(x_0, y_0, z_0) + (x_1, y_1, z_1)P_1 + (x_2, y_2, z_2)P_2] = L_0(x_0, y_0, z_0) + L_1(x_1, y_1, z_1)P_1 + L_2(x_2, y_2, z_2)P_2 = (x_0) + (y_1)P_1 + (z_2)P_2.$$

For example, take $X = (1, 9, 8) + (9, 10, -9)P_1 + (3, 2, 1)P_2$, then:

$$L(X) = 1 + (10)P_1 + P_2.$$

$$\left\{ \begin{array}{l} ker(L_0) = \{(0, y_0, z_0); y_0, z_0 \in Z\} \\ ker(L_1) = \{(x_1, 0, z_1); x_1, z_1 \in Z\} \\ ker(L_2) = \{(x_2, y_2, 0); x_2, y_2 \in Z\} \\ AH - ker(L) = \{(0, y_0, z_0) + (x_1, 0, z_1)P_1 + (x_2, y_2, 0)P_2; y_0, z_0, x_1, z_1, x_2, y_2 \in Z\} \end{array} \right.$$

Also,

$$\left\{ \begin{array}{l} Im(L_0) = Z \\ Im(L_1) = Z \\ Im(L_2) = Z \\ AH - Im(L) = Z + ZP_1 + ZP_2 = 2 - SP_W \end{array} \right.$$

Theorem.

Let $L = f + fP_1 + fP_2: 2 - SP_V \rightarrow 2 - SP_W$ be an AHS-homomorphism, then L is a module homomorphism.

Proof.

Let $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_V$, then:

$$\begin{aligned} L(X + Y) &= f(x_0 + y_0) + f(x_1 + y_1)P_1 + f(x_2 + y_2)P_2 \\ &= [f(x_0) + f(x_1)P_1 + f(x_2)P_2] + [f(y_0) + f(y_1)P_1 + f(y_2)P_2] = L(X) + L(Y) \end{aligned}$$

Let $A = a_0 + a_1P_1 + a_2P_2 \in 2 - SP_F$, then:

$$\begin{aligned} L(A.X) &= f(a_0x_0) + f(a_0x_1 + a_1x_0 + a_1x_1)P_1 + f(a_0x_2 + a_2x_0 + a_2x_2 + a_1x_2 + a_2x_1)P_2 \\ &= a_0f(x_0) + (a_0f(x_1) + a_1f(x_0) + a_1f(x_1))P_1 \\ &\quad + (a_0f(x_2) + a_2f(x_0) + a_2f(x_2) + a_1f(x_2) + a_2f(x_1))P_2 \\ &= [a_0 + a_1P_1 + a_2P_2]. [f(x_0) + f(x_1)P_1 + f(x_2)P_2] = A.L(X) \end{aligned}$$

Thus, L is a module homomorphism.

The algebraic relations between symbolic 2-plithogenic modules and neutrosophic modules .

Theorem.

Let M be a module over the ring R , consider $M(I) = M + MI = \{x + yI; x, y \in M\}$ is the corresponding neutrosophic module over the neutrosophic ring $R(I) = \{a + bI; a, b \in R\}$.

$M(I_1, I_2) = M + MI_1 + MI_2 = \{x + yI_1 + zI_2; x, y, z \in M\}$ is the corresponding refined neutrosophic module over the refined neutrosophic ring $R(I_1, I_2) = \{a + bI_1 + cI_2; a, b, c \in R\}$.

$2 - SP_M = M + MP_1 + MP_2 = \{x + yP_1 + zP_2; x, y, z \in M\}$ is the corresponding symbolic 2-plithogenic module over $2 - SP_R$, then:

1. $2 - SP_M$ is semi homomorphic to $M(I)$.
2. $2 - SP_M$ is semi isomorphic to $M(I_1, I_2)$.

Proof.

1. We define $f: 2 - SP_M \rightarrow M(I), g: 2 - SP_R \rightarrow R(I)$ such that:

$$f(x + yP_1 + zP_2) = x + yI; x, y, z \in M$$

$$g(a + bP_1 + cP_2) = a + bI; a, b, c \in R$$

We have the following:

g is a ring homomorphism, that is because:

$$A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2; a_i, b_i \in R, \text{ then:}$$

$$\text{If } A = B, \text{ then } a_i = b_i \text{ for all } i, \text{ thus } a_0 + a_1I = b_0 + b_1I, \text{ i.e. } g(A) = g(B).$$

$$g(A + B) = g[(a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2] = a_0 + b_0 + (a_1 + b_1)I = g(A) + g(B).$$

$$\begin{aligned} g(A.B) &= g[a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b + a_2b_1 + a_2b_2)P_2] = \\ &= a_0b_0 + (a_0b_1 + a_1b_0 + a_1b_1)I = (a_0 + a_1I)(b_0 + b_1I) = g(A).g(B). \end{aligned}$$

On the other hand, f is well defined, that is because:

If $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2$, then $x_i = y_i$ for all i , hence $a_0 + a_1I = b_0 + b_1I$, thus $f(X) = f(Y)$.

f preserves addition, that is because:

For $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2$, we have:

$$f(X + Y) = f[(x_0 + y_0) + (x_1 + y_1)P_1 + (x_2 + y_2)P_2] = x_0 + y_0 + (x_1 + y_1)I = f(X) + f(Y).$$

f preserves multiplication, that is because:

For $A = a_0 + a_1P_1 + a_2P_2 \in 2 - SP_M$, we have:

$$f(A.X) = a_0x_0 + (a_0x_1 + a_1x_0 + a_1x_1)I = (a_0 + a_1I)(x_0 + x_1I) = g(A).f(X)$$

Thus f is a semi module homomorphism.

We define $f: 2 - SP_M \rightarrow M(I_1, I_2)$, $g: 2 - SP_R \rightarrow M(I_1, I_2)$, where $f(x + yP_1 + zP_2) = x + zI_1 + yI_2$, and $g(a + bP_1 + cP_2) = a + cI_1 + bI_2; x, y, z \in M, a, b, c \in R$.

(g) is well defined, that is because:

If $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2$, then:

$$a_0 = a_1, b_0 = b_1, c_0 = c_1, \text{ hence: } a_0 + c_0I_1 + b_0I_2 = a_1 + c_1I_1 + b_1I_2, \text{ so that } g(A) = g(B).$$

(f) is well defined by a similar discussion.

(g) is one-to-one mapping, that is because:

$$\ker(g) = \{a + bP_1 + cP_2; g(a + bP_1 + cP_2) = 0\} = 0$$

$$\text{Im}(g) = \{a + cI_1 + bI_2; g(a + bP_1 + cP_2) \in R(I_1, I_2); \exists A \in 2 - SP_R, g(A) = a + cI_1 + bI_2\} = R(I_1, I_2).$$

(f) is one-to-one mapping, it can be proved by the same.

(g) and (f) preserve addition, that is because:

Consider $A = a_0 + a_1P_1 + a_2P_2, B = b_0 + b_1P_1 + b_2P_2 \in 2 - SP_R$, $X = x_0 + x_1P_1 + x_2P_2, Y = y_0 + y_1P_1 + y_2P_2 \in 2 - SP_M$, then:

$$\begin{aligned} g(A + B) &= g[(a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2] = a_0 + b_0 + (a_1 + b_1)I_1 + (a_2 + b_2)I_2 \\ &= g(A) + g(B) \end{aligned}$$

$f(X + Y) = f(X) + f(Y)$ by a similar discussion.

(g) preserves multiplication, that is because:

$$\begin{aligned} g(A.B) &= a_0b_0 + (a_0b_2 + a_2b_0 + a_2b_2 + a_1b_2 + a_2b_1)I_1 + (a_0b_1 + a_1b_0 + a_1b_1)I_2 = \\ &= g(A).g(B). \end{aligned}$$

(f) is semi module homomorphism, that is because:

$$\begin{aligned} f(A.X) &= a_0x_0 + (a_0x_2 + a_2x_0 + a_2x_2 + a_1x_2 + a_2x_1)I_1 + (a_0x_1 + a_1x_0 + a_1x_1)I_2 \\ &= (a_0 + a_1I_1 + a_2I_2)(x_0 + x_2I_1 + x_1I_2) = g(A).f(X) \end{aligned}$$

The basis of a symbolic 2-plithogenic module:

Theorem.

Let $T = \{t_1, \dots, t_n\}$ be a basis of the module V over the ring R , then the set:

$T_P = \{t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2; 1 \leq i, j, k \leq n\}$ is a basis of $2 - SP_V$.

Proof.

Let $X = x_0 + x_1P_1 + x_2P_2 \in 2 - SP_M, x_0, x_1, x_2 \in M$.

$$x_0 = \sum_{i=1}^n \alpha_i t_i, \quad x_0 + x_1 = \sum_{j=1}^n \beta_j t_j, \quad x_0 + x_1 + x_2 = \sum_{k=1}^n \gamma_k t_k; \quad \alpha_i, \beta_j, \gamma_k \in R.$$

We put $A_{i,j,k} = \alpha_i + (\beta_j - \alpha_i)P_1 + (\gamma_k - \beta_j)P_2; 1 \leq i, j, k \leq n$

$$T_{i,j,k} = t_i + (t_j - t_i)P_1 + (t_k - t_j)P_2; 1 \leq i, j, k \leq n$$

$$\begin{aligned} &\sum_{i,j,k=1}^n A_{i,j,k} T_{i,j,k} \\ &= \sum_{i=1}^n [\alpha_i t_i + [\beta_j t_j - \beta_j t_i - \alpha_i t_j + \alpha_i t_i + \beta_j t_i - \alpha_i t_i + \alpha_i t_j - \alpha_i t_i]P_1 \\ &\quad + [\alpha_i t_k - \alpha_i t_j + \gamma_k t_i - \beta_j t_i - \gamma_k t_j + \gamma_k t_i - \beta_j t_j + \beta_j t_i + \gamma_k t_k - \gamma_k t_j - \beta_j t_k \\ &\quad + -\beta_j t_j + \beta_j t_k - \beta_j t_j - \alpha_i t_k + \alpha_i t_j]P_2] \\ &= \sum_{i=1}^n \alpha_i t_i + P_1 \left[\sum_{j=1}^n \beta_j t_j - \sum_{i=1}^n \alpha_i t_i \right] + P_2 \left[\sum_{k=1}^n \gamma_k t_k - \sum_{j=1}^n \beta_j t_j \right] \\ &= x_0 + P_1 [x_0 + x_1 - x_0] + P_2 [x_0 + x_1 + x_2 - (x_0 + x_1)] = x_0 + x_1 P_1 + x_2 P_2 \\ &= X \end{aligned}$$

Thus T generates $2 - SP_M$.

On the other hand, T is linearly independent, that is because:

If $\sum_{i,j,k=1}^n A_{i,j,k} \cdot X = 0$, then:

$$\sum_{i=1}^n \alpha_i t_i = 0, \sum_{j=1}^n \beta_j t_j = 0, \sum_{k=1}^n \gamma_k t_k = 0, \text{ hence } \alpha_i = \beta_j = \gamma_k = 0 \text{ for all } i, j, k, \text{ thus } A_{i,j,k} = 0.$$

This implies that T is a basis of $2 - SP_M$.

Example.

Find a basis of $2 - SP_{Z^2}$.

Solution.

First of all, we have $\{u_1 = (1,0), u_2 = (0,1)\}$ is a basis of Z^2 .

The corresponding basis of $2 - SP_{Z^2}$ is:

$T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}$ such that:

$$T_1 = (1,0), T_2 = (0,1), T_3 = u_1 + (u_2 - u_1)P_1 + (u_2 - u_2)P_2 = (1,0) + (-1,1)P_1$$

$$T_4 = u_1 + (u_2 - u_1)P_1 + (u_1 - u_2)P_2 = (1,0) + (-1,1)P_1 + (1, -1)P_2$$

$$T_5 = u_2 + (u_2 - u_1)P_1 + (u_1 - u_1)P_2 = (0,1) + (1, -1)P_1$$

$$T_6 = u_2 + (u_2 - u_1)P_1 + (u_2 - u_1)P_2 = (0,1) + (1, -1)P_1 + (-1,1)P_2$$

$$T_7 = u_1 + (u_1 - u_1)P_1 + (u_2 - u_1)P_2 = (1,0) + (-1,1)P_2$$

$$T_8 = u_2 + (u_2 - u_2)P_1 + (u_1 - u_2)P_2 = (0,1) + (1, -1)P_2$$

Remark.

$$\dim(2 - SP_M) = (\dim M)^3$$

Conclusion

In this paper we have defined the concept of symbolic 2-plithogenic modules over a symbolic 2-plithogenic ring, where we have presented some of their elementary properties such as basis, homomorphisms, and AH-submodules. On the other hand, we have suggested many examples to clarify the validity of our work.

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