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An Exploration of Wisdom of Crowds using Neutrosophic Cognitive Maps

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Abstract. The wisdom of crowds (WOC) is a theory where it is believed that a multitude of people, unknown to each other and not experts in some subject, can reach more accurate conclusions on this subject than each of them would achieve individually; it could even have more accuracy than the result that a group of experts would obtain. This theory can be used to obtain information from the individual knowledge of an inexperienced crowd, including knowledge on complex phenomena. In this paper, the complex phenomenon is represented with the help of Neutrosophic Cognitive Maps (NCM), which allow us to capture the cause-effect relations among the concepts according to each of the individuals' judgments. In this case, a dynamic processing of the results is carried out. The NCMs are aggregated following the WOC principles using an aggregation algorithm, which is based on the Fuzzy Negative-Positive-Neutral (NPN) logic. The advantage of using NCM is that indeterminacy is included in the modeling, thus individuals can express their opinions more reliably.

Keywords: Wisdom of crowds, fuzzy cognitive map, neutrosophic cognitive map, fuzzy NPN logic.

1 Introduction

The Wisdom of Crowds is a theory introduced by James Surowiecki in his book "The Wisdom of Crowds: Why the Many are Smarter than the Few and How Collective Wisdom Shapes Business, Economies, Societies and Nations", [1]. The book proposes the thesis that the opinion's aggregation of the multitude's members that are non-experts on a subject will have a more accurate result than that of each of the members has individually; it could even be more exact than the opinion obtained by a few experts.

The explanation that is usually given to this fact is that the bias committed by one person is annulated by the bias of another, [2-4]. A much-cited example corresponds to Francis Galton's experience at a cattle fair in his county. In the example, a crowd accurately deduced the weight of an ox when their individual estimates were averaged (and the average was closer to the true weight of the ox than the estimates separately from most of the crowd and closer than any of the livestock experts' estimates).

This thesis is debated in the book through a large number of examples from the field of economics and psychology, and where it is not hidden that on some occasions group decisions can fail, as is the case with the collective analysis of market bubbles. This is because not every crowd meets the conditions necessary to get to a sufficiently approximate result. Participants need to meet certain conditions, such as: (1) Independence, which means every individual makes a judgment on his/her own, such that each individual does not discuss or even does not have knowledge on the others' opinions. (2) Diversity, which means the judgments are sufficiently different each other with respect to their perspectives, (3) Decentralization, which means that for aggregating the individual results, every member has the same weight, [3].

This theory can be applicable not only to cases such as the weighing of an ox at a fair, but also to arrive at analysis of complex phenomena. For example, [5, 6] shows the application of the WOC theory in the study of stakeholder crowds' knowledge of social-ecological systems. It is known that this type of problem involves complex interactions between human systems and natural systems, where there are different dimensions to the same problem, such as sociological, economic, ecological, cultural, among others, so that holistic knowledge of the phenomenon is difficult to capture. In [6] they experimentally demonstrated that the WOC can be used to generate collective knowledge from the knowledge of stakeholders on this issue, where preferably none of them is an expert.

The individual knowledge of each stakeholder is represented using Fuzzy Cognitive Maps (FCM), which is a tool for representing knowledge based on cause-effect relationships of two or more factors related to the phenomenon being studied. The dynamic study of FCM allows us to reach an equilibrium point using a sequence of results dependent on a discrete time variable. There can also be instability, which occurs when the sequence does not converge to any point, and in this case, the result cannot be used.

FCMs were introduced by Kosko [7]. These are directed graphs, where the nodes represent concepts and the edges represent the relationships between two concepts. In Cognitive Maps [8], each edge has a weight associated with it (-1 which means an inverse total relationship, 0 means that there is no relationship, 1 means that the two concepts are directly and totally related). For Kosko, these weights take values in the interval [-1, 1]. FCMs have been generalized to Neutrosophic Cognitive Maps (NCM), some of them are analyzed statically and others dynamically, [9-15]. The idea of the NCM is to add to the weights -1, 0, 1, the value I that represents indeterminacy.

The WOC is an approach of great interest to sociology, which has also been generalized to the field of Neutrosophy with the so-called Neutrosociology [16, 17]. This paper aims to introduce a theoretical tool based on Neutrosophy, especially NCM to represent the knowledge of each individual, and a method to apply WOC as a way to obtain collective knowledge. In FCM and NCM, the usual set of truth values [0, 1] is extended to the set [-1, 1]. This semantic is related to the bipolar fuzzy sets that have also been extended to the field of Neutrosophy with the bipolar neutrosophic sets [18-24]. This type of bipolar sets have been extended to the offsets [25], where operators such as the offuninorms have been defined, [26].

In this paper, we use NCMs as a way of representing the individual knowledge of the crowd members. The individual results are aggregated satisfying the WOC conditions and converted into a single FCM with the help of an aggregation algorithm, which is based on the Fuzzy Negative-Positive-Neutral (NPN) logic. Fuzzy NPN logic extends the range of truth values [0, 1] to logical relations represented as ordered pairs of truth values in [-1, 1], [18, 27, 28]. This logic contains operators that are used in FCM and are based on some rules specially designed to perform calculations with FCM. Fuzzy NPN logic can be considered a type of bipolar neutrosophic set, since the individual can express his/her opinion indeterminately in the form of an interval, with a minimum value that can be negative and a maximum value that can be positive.

The contribution of this paper is that for the first time is offered a tool to obtain knowledge on complex phenomena through the use of NCM and WOC. The potentialities of the Fuzzy NPN logic are also used. Even though we employ an iterative algorithm for calculating the equilibrium point, the Pool2 algorithm [29] is also recommendable to get to the result of a collective knowledge. The advantage over the proposal shown in [30] is that the NCM allow us to include indeterminacy in the knowledge representation.

This article is structured into the following sections. Section 2 recalls the main concepts of Neutrosophy, NCM, and Fuzzy NPN logic; Section 3 introduces the tools and arguments that should be used in an NCM-based WOC method. Conclusions are shown in the last section.

2 Preliminaries

This section summarizes the main concepts needed to develop the method that we propose. First, in subsection 2.1 we describe the Neutrosophic Cognitive Map theory, which is processed by a dynamic method. Subsection 2.2 exposes the definitions and main elements of Fuzzy NPN logic.

2.1 Neutrosophic Cognitive Maps

Definition 1: [31] Let X be a universe of discourse. A *Neutrosophic Set* (NS) is characterized by three membership functions, $u_A(x), r_A(x), v_A(x) : X \rightarrow \,]^-0, 1^+[$, which satisfy the condition $^-0 \leq \inf u_A(x) + \inf r_A(x) + \inf v_A(x) \leq \sup u_A(x) + \sup r_A(x) + \sup v_A(x) \leq 3^+$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$ are the membership functions of truthfulness, indeterminacy and falseness of x in A, respectively, and their images are standard or non-standard subsets of $]^-0, 1^+[$.

Definition 2: [31] Let X be a universe of discourse. A *Single-Valued Neutrosophic Set* (SVNS) A on X is a set of the form:

$$A = \{\langle x, u_A(x), r_A(x), v_A(x) \rangle : x \in X\} \quad (1)$$

Where $u_A, r_A, v_A : X \rightarrow [0,1]$, satisfy the condition $0 \leq u_A(x) + r_A(x) + v_A(x) \leq 3$ for all x∈X. $u_A(x), r_A(x)$ and $v_A(x)$ denotes the membership functions of truthfulness, indeterminacy and falseness of x in A, respectively. For convenience a *Single-Valued Neutrosophic Number* (SVNN) will be expressed as $A = (a, b, c)$, where $a, b, c \in [0,1]$ and satisfy $0 \leq a + b + c \leq 3$.

Other important definitions are related to the graphs, [14, 31].

Definition 3: [9, 11, 12, 14, 23] A *neutrosophic graph* is a graph containing at least one indeterminate edge, which is represented by dotted lines.

Definition 4: [9, 11, 12, 14, 23] A *neutrosophic directed graph* is a directed graph containing at least one indeterminate edge, which is represented by dotted lines.

Definition 5: [9, 11, 12, 14, 23] A *Neutrosophic Cognitive Map* (NCM) is a neutrosophic directed graph, whose nodes represent concepts and whose edges represent causal relationships among the edges.

If $C_1, C_2, \ldots, C_k$ are k nodes, each of the C_i $(i = 1, 2, \ldots, k)$ can be represented by a vector $(x_1, x_2, \ldots, x_k)$ where $x_i \in \{0, 1, I\}$. $x_i = 0$ means that the node C_i is in an activated state, $x_i = 1$ means that the node C_i is in a deactivated state and $x_i = I$ means that the node C_i is in an indeterminate state, in a specific time or in a specific situation.

If C_m and C_n are two nodes of the NCM, an edge directed from C_m to C_n is called a *connection* and represents the causality from C_m to C_n. Each node in the NCM is associated with a weight within the set $\{-1, 0, 1, I\}$. If α_{mn} denotes the weight of the edge C_mC_n, $\alpha_{mn} \in \{-1, 0, 1, I\}$ then we have:

$\alpha_{mn} = 0$ if C_m has no effect on C_n,

$\alpha_{mn} = 1$ if an increase (decrease) in C_m produces an increase (decrease) in C_n,

$\alpha_{mn} = -1$ if an increase (decrease) in C_m produces a decrease (increase) in C_n,

$\alpha_{mn} = I$ if the effect of C_m on C_n is indeterminate.

Definition 6: [9, 11, 12, 14, 23] A NCM having edges with weights in $\{-1, 0, 1, I\}$ is called *Simple Neutrosophic Cognitive Map*.

Definition 7: [9, 11, 12, 14, 23] If $C_1, C_2, \ldots, C_k$ are the nodes of a NCM. The *neutrosophic matrix* N(E) is defined as $N(E) = (\alpha_{mn})$, where α_{mn} denotes the weight of the directed edge C_mC_n, such that $\alpha_{mn} \in \{-1, 0, 1, I\}$. $N(E)$ is called the *neutrosophic adjacency matrix* of the NCM.

Definition 8: [9, 11, 12, 14, 23] Let $C_1, C_2, \ldots, C_k$ be the nodes of a NCM. Let $A = (a_1, a_2, \ldots, a_k)$, where $a_m \in \{-1, 0, 1, I\}$. A is called *instantaneous state neutrosophic vector* and means a position of on-off-indeterminate state of the node in a given instant.

$a_m = 0$ if C_m is deactivated (has no effect),

$a_m = 1$ if C_m is activated (has an effect),

$a_m = I$ if C_m is indeterminate (its effect cannot be determined).

Definition 9: [9, 11, 12, 14, 23] Let $C_1, C_2, \ldots, C_k$ be the nodes of a NCM. Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…, $\overrightarrow{C_mC_n}$ be the edges of the NCM, then the edges constitute a *directed cycle*.

The NCM is called *cyclic* if it has a directed cycle. It is *acyclic* if it has not a directed cycle.

Definition 10: [9, 11, 12, 14, 23] A NCM containing cycles is said to have *feedback*. When there is feedback in the NCM, it is a *dynamic system*.

Definition 11: [9, 11, 12, 14, 23] Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,…, $\overrightarrow{C_{k-1}C_k}$ be a cycle. When C_m is activated and its causality flows through the edges of the cycle and it is the cause of C_m itself, then the dynamic system circulates. This is fulfilled for each node C_m with $m = 1, 2, \ldots, k$. The equilibrium state for this dynamic system is called the *hidden pattern*.

Definition 12: [9, 11, 12, 14, 23] If the equilibrium state of a dynamic system is a single state, then it is called a *fixed point*.

An example of a fixed point is when a dynamic system starts by being activated by C_l. If it is assumed that the NCM sits on C_l and C_k, i.e. the state remains as $(1, 0, \ldots, 0, 1)$, then this vector of neutrosophic state is called *fixed point*.

Definition 13: [9, 11, 12, 14, 23] If the NCM is established with a neutrosophic state-vector that repeats itself in the form:

$A_1 \to A_2 \to \cdots \to A_m \to A_1$, then the equilibrium is called a *limit cycle* of the NCM.

Method to determine the Hidden Patterns

Let $C_1, C_2, \ldots, C_k$ be the nodes of the NCM with feedback. Assume that E is the associated adjacency matrix. A hidden pattern is found when C_l is activated and a vector input $A_1 = (1, 0, 0, \ldots, 0)$ is given. The data must be passed to the neutrosophic matrix $N(E)$, which is obtained by multiplying A_1 by the matrix $N(E)$.

Let $A_1N(E) = (\alpha_1, \alpha_2, \ldots, \alpha_k)$ with the threshold operation of replacing α_m by 1 if $\alpha_m > p$ and α_m by 0 if $\alpha_m < p$ (p is a suitable positive integer) and α_m is replaced by I if this is not an integer. The resulting concept is updated; vector C_1 is included in the updated vector by transforming the first coordinate of the resulting vector into 1.

If $A_1N(E) \rightarrow A_2$ is assumed then $A_2N(E)$ is considered and the same procedure is repeated. This procedure is repeated until a limit cycle or fixed point is reached.

Definition 14: [32-34] A *neutrosophic number* N is defined as a number in the form of:

$$N = d + I \quad (2)$$

Where d is called *determinate part* and I is called *indeterminate part*.

Given $N_1 = a_1 + b_1 I$ and $N_2 = a_2 + b_2 I$ two neutrosophic numbers, some operations between them are defined as follows:

$N_1 + N_2 = a_1 + a_1 + (b_1 + b_2)I$ (Addition);

$N_1 - N_2 = a_1 - a_1 + (b_1 - b_2)I$ (Difference),

$N_1 \times N_2 = a_1a_2 + (a_1b_2 + b_1a_2 + b_1b_2)I$ (Multiplication),

$\frac{N_1}{N_2} = \frac{a_1+b_1I}{a_2+b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1-a_1b_2}{a_2(a_2+b_2)}I$ (Division).

2.2 Fuzzy NPN logic

Fuzzy NPN logic is a generalization of the NPN logic, which consists of 6-valued semantic containing the following values, -1 (negative), 1 (positive), 0 (neutral or unrelated), (-1, 0) (negative or neutral), (0, 1) neutral or positive, (-1, 1) (negative or positive/negative, neutral, or positive).

An NPN fuzzy value pair is represented in the form of (x, y), where $x \in [-1, 0]$ and $\in [0, 1]$.

Three logical operations between one or two fuzzy NPN values are the following:

$$NEG(x, y) = (NEG(y), NEG(x)) \quad (3)$$

$$(x, y) * (u, v) = \left(\min(x * u, x * v, y * u, y * v), \max(x * u, x * v, y * u, y * v)\right) \quad (4)$$

$$(x, y)OR(u, v) = \left(\min(x, u), \max(y, v)\right) \quad (5)$$

When these operators are restricted to the crisp domain, they intuitively correspond to the rules shown in Table 1.

A)	Friend’s friend is friend
B)	Friends enemy is enemy
C)	Friend’s neutral friend is neutral
D)	Enemy’s enemy is friend
E)	Enemy’s friend is enemy
F)	Enemy’s neutral friend is neutral
G)	Neutral’s friend is neutral
H)	Neutral’s neutral friend is neutral
I)	Neutral’s enemy is neutral
J)	IF a neutral’s friend is a friend’s enemy. THEN he/she is an enemy or neutral
K)	IF a friend’s friend is another friend’s enemy. IF a friend’s friend is an enemy’s friend, or IF an enemy’s enemy is another enemy’s friend, THEN he/she might be a friend or an enemy

Table 1: Some production rules related to crisp NPN logic. Source [18, 27, 28].

Some important identities of Fuzzy NPN logic are summarized in Table 2.

Law	AND form (*)	OR (+) form
Identity law	$1(x, y) = (x, y)$	undefined
Null law	$0(x, y) = 0$	$(-1,1) + (x, y) = (-1,1)$
Idempotent law	undefined	$(x, y) + (x, y) = (x, y)$
Commutative law	$(x, y)(u, v) = (u, v)(x, y)$	$(x, y) + (u, v) = (u, v) + (x, y)$
Associative law	$\left((x, y)(u, v)\right)(w, z) = (x, y)\left((u, v)(w, z)\right)$	$\left((x, y) + (u, v)\right) + (w, z) = (x, y) + \left((u, v) + (w, z)\right)$
Distributive law	undefined	$(x, y)\left((u, v) + (w, z)\right) = (x, y)(u, v) + (x, y)(w, z)$

Table 2: Some logical identities in Fuzzy NPN logic. Source [27].

3 NCM and WOC

This section introduces the method for combining WOC and NCMs.

First, the crowd must be selected, such that it satisfies the conditions of Independence and Diversity. The larger the size of the crowd the best is the result. Because we are dealing with a complex phenomenon, the completely non-expert crew is not desirable, instead of that a "stakeholder" crowd can be selected, which are not expert peoples, with certain degree of experience with respect to the subject they are analyzing.

Other measure to take into account is to select a group of people having different point of views from the subject. For example, if the subject is the effectiveness of certain medication, the crowd can be formed by patients being treated with this drug, some physicians not experts in medicaments, and so on. On the other hand, the opinion of every one of them must be collected individually without any discussion among the members; moreover, the members of the crowd should not know each other.

A moderator informs to each crowd's member $G = \{g_1, g_2, \cdots, g_m\}$ the set of concepts $C = \{C_1, C_2, \dots, C_k\}$ he/she is dealing with. Each g_i $(i = 1, 2, \dots, m)$ assigns a value $e_{pq}^i \in \{-1, 0, 1, I\}$ $(p, q = 1, 2, \dots, k)$ to the edge which connects the two nodes p and q.

The algorithm contains two counter functions defined by equations 6 and 7:

$$f_-(e_{pq}^i) = \begin{cases} -1, \text{if } e_{pq}^i = -1 \text{ or } I \\ 0, \quad \text{otherwise} \end{cases} \quad (6)$$

$$f_+(e_{pq}^i) = \begin{cases} 1, \text{if } e_{pq}^i = 1 \text{ or } I \\ 0, \quad \text{otherwise} \end{cases} \quad (7)$$

Here we consider $I = [-1, 1]$, thus, it is included for aggregating both negative as well as positive values. The aggregated NCM is represented with the adjacency matrix $M = (M_{pq})$ such that $M_{pq} = \left(f_L(e_{pq}), f_U(e_{pq})\right)$ that is the ordered pair where:

$$f_L(e_{pq}) = \frac{\sum_{i=1}^m f_-(e_{pq}^i)}{m} \quad (8)$$

$$f_U(e_{pq}) = \frac{\sum_{i=1}^m f_+(e_{pq}^i)}{m} \quad (9)$$

This new cognitive map is a FCM with a pair of values corresponding to the Fuzzy NPN logic representation. Let us remark that every member of the crowd is equally weighted, thus Decentralization principle is complied with. Then, the HTC algorithm [29] is applied, with the effect of obtaining more negative value for $f_L(e_{pq})$ and more positive value for the $f_U(e_{pq})$ for each edge e_{pq} in the new FCM. The pseudo-code of this algorithm is the following [30]:

Given M defined like above, convert M to M' by representing each element as a pair with a lower boundary and upper boundary $M_{pj} = (a, b)$:

1) DO q = l TO k
2) DO p =1 TO k
3) IF $M_{pq} = (x, y)$ AND $(x, y) \neq (0, 0)$ THEN
4) DO j = 1 TO k
   IF $M_{qj} = (u, v)$, THEN
   $M'_{qj} = (\min(a, x * u, x * v, y * u, y * v), \max(b, x * u, x * v, y * u, y * v))$
5) END
6) END
7) END

Where the operator * is defined for $x, y \in [-1, 1]$ as follows:

$$x * y = \text{sign}(x) * \text{sign}(y) * (|x| * |y|) \quad (10)$$

The effect of this algorithm implies the satisfaction of the desirable condition of the WOC, which states that the dissidence of criteria is welcome.

For $M'_{pq} = \left(M'_{pq}(1), M'_{pq}(2)\right)$ we form a new matrix M''_{pq} applying the maximum effect, which means that if $M'_{pq}(2) \geq \left|M'_{pq}(1)\right|$ we have $M''_{pq} = M'_{pq}(2)$, else $M''_{pq} = M'_{pq}(1)$.

The equilibrium point is calculated with the formula 11.

$$A_p^t = F\left(\sum_{q=1,q\neq p}^{k} A_q^{t-1} W_{pq} + A_p^{t-1}\right) \quad (11)$$

Where, A_p^t is the value of node C_p at step t, A_p^{t-1} is the value of node C_p at step t-1, A_q^{t-1} is the value of node C_q at step t-1, W_{pq} is the weight of the interconnection between C_p and C_q, i.e., M''_{pq}. F is the threshold function; in this case, we use the sigmoid function, see Equation 12.

$$F(x) = \frac{1}{1+e^{-x}} \quad (12)$$

The previous method of convergence is simple, it can be substituted with others like Pool2 algorithm [29], and that introduced in [30].

Let us illustrate our approach with a simple hypothetical example, nevertheless in future works we use a real-life example as an experimental proof of the validity of the proposed method.

Example 1:

First, we would like to make some reflections about the method. We have to pay attention about the satisfaction of every condition of WOC. For example, the problem on migrant construction workers in West Bengala, India who are HIV carriers [11] is a complex problem with social, economical, human-rights dimensions. However, almost every citizen has an opinion on this subject, thus a crowd with a large size can be selected, where they have a variety of criteria about very controversial subjects like immigration or sexual diseases are.

In this example we select a hypothetical case with three nodes (k = 3) $C = \{C_1, C_2, C_3\}$ and the size of the crowd is of 200 members. Tables 3, 4, 5, and 6 summarize the number of members which selected -1, 0, 1, I, respectively, as the connection between every C_p and C_q, $(p, q = 1, 2, 3)$.

	C_1	C_2	C_3
C_1	89	52	5
C_2	85	129	21
C_3	3	34	40

Table 3: Number of crowd's members who evaluated as -1 the edges C_pC_q.

	C_1	C_2	C_3
C_1	14	55	23
C_2	4	14	2
C_3	77	44	65

Table 4: Number of crowd's members who evaluated as 0 the edges C_pC_q.

	C_1	C_2	C_3
C_1	67	79	84
C_2	98	19	32
C_3	20	58	56

Table 5: Number of crowd's members who evaluated as 1 the edges C_pC_q.

	C_1	C_2	C_3
C_1	30	14	88
C_2	13	38	145
C_3	101	64	39

Table 6: Number of crowd's members who evaluated as I the edges C_pC_q.

Then, calculating M with Equations 6 and 7, later applying HTC algorithm and finally calculating M'' we have the matrix summarized in Table 7.

	C_1	C_2	C_3
C_1	0.60500	0.61000	0.52030
C_2	0.60500	0.61000	0.53985
C_3	0.60500	0.61000	0.47500

Table 7: Matrix M'' of the example.

Calculating the iterative process using Equation 9, individually activating every one of the three nodes, i.e., starting with $x_0 = (1, 0, 0)$, $y_0 = (0, 1, 0)$, and $z_0 = (0, 0, 1)$, after 10 iterations we obtain $x \approx y \approx z = (0.93155, 0.93249, 0.91154)$.

Conclusion

In this paper, we propose a method where the wisdom of crowds' theory is used to obtain knowledge from complex issues represented individually in the form of neutrosophic cognitive maps. To apply the method, the principles of Independence and Diversity proposed by the WOC must be complied with. Within the method, the decentralization recommended in the WOC is guaranteed. The novelty of the method consists in representing the individual knowledge of the members of the crowd with the help of NCMs; this allows each individual to include indeterminacy in the knowledge representation. The aggregation is carried out with the help of the Fuzzy Negative-Positive-Neutral (NPN) logic, which naturally allows calculating with the symbol I, to obtain a FCM. A way to perform dynamic calculations is also developed by means of a recursive method. In future works, we will explore other methods, such as Pool2. Additionally, future studies will carry out experiments on the use of this proposal in real life problems, the results of which will be compared with other methods using experts, to test the validity of the authors' proposal.

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