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# Neutrosophic Games Applied to Political Situations

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**Abstract**. Game theory is the branch of applied mathematics dedicated to modeling and resolve conflict situations. This has great application in other sciences such as economics, military sciences, biology, sociology, cybernetics, and political sciences. Conflict situations in politics are common and may reach high degrees of complexity. Opponents tend to change strategies during the course of time; they can cooperate with each other at a certain moment and suddenly take totally opposite positions. In addition, the actions they take at each step can be confusing and ambiguous for the adversary. That is why Neutrosophy can be an adequate theory to model this type of situation. In this paper, we propose a neutrosophic model for non-cooperative games in matrix form that generalizes a previous solution where triangular intuitionistic fuzzy payoffs were used. This generalization allows us to define the indeterminacy membership function, which is not restricted to any condition of dependency between the membership and non-membership functions. Specifically, the elements of the matrix are payoffs of single-valued triangular neutrosophic numbers. The advantage of the neutrosophic solution is that the ambiguity that is typical in political conflicts can be expressed more precisely. The use of the proposed solution is illustrated with two examples.

**Keywords:** Single-valued triangular neutrosophic number, matrix games, neutrosophic games, political conflicts.

## 1 Introduction

Game Theory is the branch of applied mathematics that proposes a solution to conflict situations between two or more agents, [1, 2]. This is a part of mathematics much appreciated for its applicability to solving real-life problems. It is common for two or more agents to reach an agreement when negotiating with each other. Each agent wants to obtain the highest possible profit, which produces contradictions between the opposing parties. However, it is essential that they reach an agreement, which should ideally be the fairest one.

The game consists of the pair formed by the set of agents called players, and the set of strategies that are the possible actions or moves that the players can perform. The solution to the game consists of finding the combination of strategies, one for each player, such that there is equilibrium in the gains.

In the so-called pure strategies, a payoff function is defined for each combination of strategies for each player, which determines the players' profits that would emerge from combining the strategies of each of the players. The obtained solution is the optimal value corresponding to one pure strategy per player.

There is not always a solution to game theory problems using pure strategies. On other occasions, it is not convenient to have this type of strategies because they are predictable. That is why mixed strategies are introduced, where each player chooses a strategy according to a probability.

A classic decision method in non-cooperative game theory, or games where the parties do not form coalitions to cooperate, is the minimax method. This method consists in each party determines its optimal strategy considering the highest payoff within the set of strategies with the lowest payoff with respect to the contrary's strategies. Another classical solution is Nash equilibrium, [3].

Basically in game theory it is considered that the players try to carry out actions to obtain the greatest number of their own possible gains, which are modeled with non-cooperative models. There is also, the cooperative game theory, where it is established that the parties form coalitions can negotiate, so as to obtain greater individual gains from jointly negotiating with other parties, [1, 4, 5].

Wide group of sciences use game theory to solve their own problems, some of them are: economics, biology, sociology, psychology, computer science, military science and political science.

In this paper, we emphasize game theory to solve political situations. Politics is usually characterized by high conflict between parties that have different ideologies, often opposite, with certain degrees of power that cannot be overridden by the opposing party, [6-8]. In many political negotiations, there are multiple conditions on each of the parties that can determine high degrees of complexity in the decisions that are made at any time.

In certain moments, the parties may tend towards cooperation, in others towards confrontation. The actions of the parties can be ambiguous, indeterminate and imprecise [9-11]. That is why Neutrosophy is an appropriate theory to deal with this type of situation. Neutrosophy is the branch of philosophy that studies all related to neutralities, where lack of information, contradictions, paradoxes and ambiguity are modeled, [12].

It is in political sciences where neutrosophic games have mainly developed, [6, 7, 13]. In [14] an example may be found of the application of neutrosophic sets, especially offsets, to solve problems of cooperative games using what was called off-uninorms. Also, in [15] it is developed a matrix game approach in a neutrosophic framework, although it is not based on single-valued triangular neutrosophic numbers [16].

There are solutions to cooperative and non-cooperative games that use fuzzy theories, such as fuzzy sets or intuitionistic fuzzy sets, [4, 17]. However, they may be limited in their application to political sciences, because the indeterminacy membership function is not defined independently, and this constitutes a fundamental function to be able to define some situations having degrees of cooperation and non-cooperation, and ambiguity in actions and speeches in political negotiations. Politics may have contradictions between what is said and what is shown. Additionally, we can find an interval-valued solution of matrix games, which includes imprecision and indeterminacy, [18].

This paper aims to extend a solution to non-cooperative games that can be found in [17], to the framework of neutrosophic sets. In the aforementioned solution, a score function is used to de-fuzzify triangular intuitionistic fuzzy payoffs in the so-called matrix games, or games where the payoffs for each pair of strategies, one for each player, are represented with the help of a matrix. The indeterminacy in the intuitionistic fuzzy sets is expressed through the degree of hesitation that depends on the degrees of membership and non-membership of the intuitionistic fuzzy set, which are restricted by the condition that their sum is less than or equal to unity [19]. The advantage of the proposed method is that a membership function of indeterminacy is explicitly defined, in addition to a membership function and a non-membership function, and the three of them are independent of each other. This model better captures the essence of political conflicts.

This paper is structured as follows: Section 1 describes the fundamental elements of game theory and Neutrosophy. Section 2 proposes a neutrosophic method for solving matrix games, including two examples from politics. The paper ends with the conclusions.

## 2 Preliminaries

In this section, we describe the main concepts needed to understand the proposed method. The first subsection contains the basic concepts of matrix games. The second subsection shows the concepts of Neutrosophy.

### 2.1 Matrix games

A game consists of a nonempty set of players, denoted by $\mathrm{N} = \{1, 2, \cdots, \mathrm{n}\}$, a set of moves (or pure strategies) available to those players, denoted by $\mathrm{A} = \{\mathrm{A}_1, \mathrm{A}_2, \cdots, \mathrm{A}_\mathrm{p}\}$, and a specification of rewards for each combination of strategies, [1]. In the case where two players are considered, the rewards of the players are represented using a payoff matrix, one player selects the row and the other one the column. The element of the i-th row and the j-th column contains the utility obtained by player I (by rows) when applying the i-th strategy ($i \in \{1, 2, \cdots, p\}$, $p \geq 1$) when player II (by columns) applies the j-th strategy ($j \in \{1, 2, \cdots, q\}$, $q \geq 1$). Let us call $\mathrm{u}_{\mathrm{ij}} = \mathrm{U}(\mathrm{A}_\mathrm{i}, \mathrm{B}_\mathrm{j})$ the payoff, where $\mathrm{U}: \mathrm{A} \times \mathrm{B} \to \mathbb{R}$, A is the set of strategies of player I and B is the set of strategies of player II.

The "maximin" and "minimax" criteria establish that each player should minimize his/her maximum loss:

"Maximin" criterion: player I chooses that his/her minimum possible payoff is the highest.

"Minimax" criterion: player II chooses that the maximum payoff to player I is the lowest possible.

Let us remark that these definitions correspond to a two-person zero sum non-cooperative games, i.e., a matrix game, where the sum of profits of the two players for every pair of strategies is null.

**Definition 1** [1]: A *Saddle Point* is the (k, r)-th position of the payoff matrix, where the following condition is satisfied:

$$max_i min_j \mathrm{u}_{\mathrm{ij}} = min_j max_i \mathrm{u}_{\mathrm{ij}}$$

The mixed strategies are defined as pure strategies; each of them is associated with one probability.

**Definition 2** [1]:The *mixed strategies* in the game of two players I and II, with strategies $\mathrm{A} = \{\mathrm{A}_1, \mathrm{A}_2, \cdots, \mathrm{A}_\mathrm{p}\}$ for player I and $\mathrm{B} = \{\mathrm{B}_1, \mathrm{B}_2, \cdots, \mathrm{B}_\mathrm{q}\}$ for player II, are defined as the vectors $x = (x_1, x_2, \cdots, x_p) \in [0, 1]^p$ and $y = (y_1, y_2, \cdots, y_q) \in [0, 1]^q$, such that $\sum_{i=1}^{p} x_i = \sum_{j=1}^{q} y_i = 1$. The payoff function of player I by player II is defined as:

$$\mathrm{E}(\mathrm{x}, \mathrm{y}) = \sum_{\mathrm{i}=1}^{\mathrm{p}} \sum_{\mathrm{j}=1}^{\mathrm{q}} \mathrm{x}_\mathrm{i} \mathrm{u}_{\mathrm{ij}} \mathrm{y}_\mathrm{j} = x^T u y, \text{ where } u = (u_{ij})_{1 \leq i \leq p, 1 \leq j \leq q}.$$

**Definition 3** [1, 17]: A *Saddle Point* with mixed strategies is the pair of vectors $(x^*, y^*)$ which satisfies the following condition:

$$min_y max_x \mathrm{E}(x, y) = max_x min_y \mathrm{E}(x, y) = \mathrm{E}(x^*, y^*) \qquad (1)$$

## 2.2 Basic concepts on Neutrosophy

**Definition 4**: [20-26] The *Neutrosophic set* N is characterized by three membership functions, which are the truth-membership function $T_A$, indeterminacy-membership function $I_A$, and falsehood-membership function $F_A$, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq ]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that according to Definition 4, $T_A(x), I_A(x), F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x), F_A(x)$ can be subintervals of $[0, 1]$.

**Definition 5**: [20-26] The *Single-Valued Neutrosophic Set* (SVNS) N over U is $A = \{< x; T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0, 1]$, $I_A: U \to [0, 1]$, and $F_A: U \to [0, 1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The *Single-Valued Neutrosophic Number* (SVNN) is represented by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

**Definition 6:** [20-26] The *single-valued triangular neutrosophic number* $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy and falsehood membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \qquad (2)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \beta_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \frac{\left(x - a_2 + \beta_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \qquad (3)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \gamma_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \frac{\left(x - a_2 + \gamma_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \leq a_3 \\ 1, & \text{otherwise} \end{cases} \qquad (4)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \leq a_2 \leq a_3$.

**Definition 7**: [20-26] Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued triangular neutrosophic numbers and $\lambda$ any non-null number in the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
3. Inversion: $\tilde{a}^{-1} = \langle (a_3^{-1}, a_2^{-1}, a_1^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$$

5. Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle\left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 > 0 \; and \; b_3 > 0 \\ \langle\left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 < 0 \; and \; b_3 > 0 \\ \langle\left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, a_3 < 0 \; and \; b_3 < 0 \end{cases}$$

6. Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle(a_1b_1, a_2b_2, a_3b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 > 0 \; and \; b_3 > 0 \\ \langle(a_1b_3, a_2b_2, a_3b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 < 0 \; and \; b_3 > 0 \\ \langle(a_3b_3, a_2b_2, a_1b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}}\rangle, & a_3 < 0 \; and \; b_3 < 0 \end{cases}$$

Where, $\wedge$ is a t-norm and $\vee$ is a t-conorm.

Let $\tilde{a} = \langle(a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$ be a single-valued triangular neutrosophic number, then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (5)$$

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (6)$$

They are called the score and accuracy degrees of $\tilde{a}$, respectively.

**Definition 8**: Let $\tilde{a}$ and $\tilde{b}$ be two SVTNNs. Let us define the order relation denoted by $\preccurlyeq$, such that $\tilde{a} \preccurlyeq \tilde{b}$ if and only if $\mathbf{A}(\tilde{a}) \leq \mathbf{A}(\tilde{b})$.

Let $\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\}$ be a set of n SVTNNs, where $\tilde{A}_j = \langle(a_j, b_j, c_j); \alpha_{\tilde{a}_j}, \beta_{\tilde{a}_j}, \gamma_{\tilde{a}_j}\rangle$ (j = 1, 2, …, n), then the *weighted mean of the SVTNNs* is calculated with the following equation:

$$\tilde{A} = \sum_{j=1}^{n} \lambda_j \tilde{A}_j \quad (7)$$

Where $\lambda_j$ is the weight of $A_j$, $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

## 3 Matrix games with single-valued triangular neutrosophic number payoffs

This section introduces the matrix games proposed by the authors, which extends the model in [17] from the intuitionistic fuzzy framework to the neutrosophic framework .

**Definition 9**: A *SVTNN Matrix Game* is a game whose elements of the payoff matrix are single-valued triangular neutrosophic numbers, as shown below:

$$\begin{array}{c c} & \begin{array}{cccc} B_1 & B_2 & \cdots & B_q \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_p \end{array} & \begin{pmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1q} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2q} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{t}_{p1} & \tilde{t}_{p2} & \cdots & \tilde{t}_{pq} \end{pmatrix} \end{array} \quad (8)$$

Where $\tilde{t}_{ij}$ (i = 1,2,…,p; j = 1, 2,…, q) are SVTNNs, which means the payoffs for player I to carry out the strategy $A_i$ when player II carries out the strategy $B_j$.

This game incorporates the indeterminacy mentioned in [17], but here it is explicitly defined and it is independent.

**Definition 10**: Let G be a SVTNN Matrix Game whose payoff function is: $\tilde{u}(A_i, B_j) = \langle(s_{1ij}, s_{2ij}, s_{3ij}); \alpha_{s_{ij}}, \beta_{s_{ij}}, \gamma_{s_{ij}}\rangle$. Then, considering pure strategies we have the saddle point is defined as the (k,r)-th position $\tilde{u}(A_k, B_r) = \langle(s_{1kr}, s_{2kr}, s_{3kr}); \alpha_{s_{kr}}, \beta_{s_{kr}}, \gamma_{s_{kr}}\rangle = \vee_i \wedge_j \tilde{u}(A_i, B_j) = \wedge_j \vee_i \tilde{u}(A_i, B_j)$.

Let us note that $\vee := max$ and $\wedge := min$, according to the order relation $\preccurlyeq$ defined in definition 8.

**Definition 11**: Let G be a SVTNN Matrix Game whose payoff function is: $\tilde{u}(A_i, B_j) = \langle(s_{1ij}, s_{2ij}, s_{3ij}); \alpha_{s_{ij}}, \beta_{s_{ij}}, \gamma_{s_{ij}}\rangle$. Then, considering mixed strategies $x = (x_1, x_2, \cdots, x_p)$ and $y = (y_1, y_2, \cdots, y_q)$, we have the expected payoff of player I by player II is defined by Equation 9:

$$\tilde{E}(x, y) = \sum_{i=1}^{p} \sum_{j=1}^{q} x_i \tilde{u}_{ij} y_j = x^T \tilde{u} y \quad (9)$$

Where $\tilde{u} = (\tilde{u}_{ij})_{1 \leq i \leq p, 1 \leq j \leq q}$.

In such a situation, player I chooses x so as to maximize his/her expectation and player II chooses y so as to minimize player II's maximum expectation, thus it is calculated by formula 10:

$$min_y max_x \widetilde{\mathrm{E}}(x,y) = max_x min_y \widetilde{\mathrm{E}}(x,y) = \widetilde{\mathrm{E}}(x^*,y^*) \quad (10)$$

Where, $(x^*,y^*)$ is the *saddle point* of the game and $\widetilde{\mathrm{E}}(x^*,y^*)$ is the solution of the game.

For simplicity, we consider another approach defining the expected payoff as a crisp value based on the accuracy degree, which is a score function, as follows:

$$\tilde{E}_{\mathbf{A}}(x,y) = \sum_{i=1}^{p}\sum_{j=1}^{q} \mathrm{x_i}\mathbf{A}(\tilde{u}_{ij})\mathrm{y_j} = x^T\mathbf{A}(\tilde{u})y \quad (11)$$

The saddle point of the matrix game with $\tilde{E}_{\mathbf{A}}(x,y)$ is $(x^*,y^*)$ satisfying conditions in Equation 12.

$$min_y max_x \tilde{E}_{\mathbf{A}}(x,y) = max_x min_y \tilde{E}_{\mathbf{A}}(x,y) = \tilde{E}_{\mathbf{A}}(x^*,y^*) \quad (12)$$

Where, $\tilde{E}_{\mathbf{A}}(x^*,y^*)$ is the solution of the game and $\mathrm{A}(\cdot)$ is the accuracy degree shown in Equation 6.

Next, we use two examples to illustrate the application of this neutrosophic game solution in political situations.

**Example 1 (A voter problem)**: [17]

Two major political parties, denoted by A and B are involved in an election, where the total number of voters is constant, thus, when the number of voters increases for A, then the number of voters for B decreases and vice versa. Let us suppose A has two strategies, which are the following:

$A_1$: Giving importance in door-to-door campaigning and carrying their ideology and issues to people.

$A_2$: Cooperating with other small political parties to reduce secured votes of the opposition.

Whereas, party B has these two strategies:

$B_1$: Campaigning by celebrities and big rallies.

$B_2$: Making lot of promises to the people.

Chief voting agents have to forecast the results of these strategies in the future, thus, there is uncertainty and indeterminacy in the possible results, which are approximates. Payoff matrix contains the following SVTNNs:

$$\begin{array}{cc} & \begin{array}{cc} \mathrm{B}_1 & \mathrm{B}_2 \end{array} \\ \begin{array}{c} \mathrm{A}_1 \\ \mathrm{A}_2 \end{array} & \begin{pmatrix} \langle(4,6,9);0.5,0.1,0.3\rangle & \langle(5,7,8);0.6,0.1,0.2\rangle \\ \langle(4,7,8);0.4,0.4,0.3\rangle & \langle(3,5,6);0.5,0.3,0.2\rangle \end{pmatrix} \end{array}$$

This matrix game means that, for example, when party A performs strategy $\mathrm{A}_1$ and party B performs strategy $\mathrm{B}_1$ the results is that A gains approximately $6\times10^5$ votes whose values may change in the interval $(4\times10^5, 9\times10^5)$, with 0.5 degree of truthiness, 0.3 degree of falsehood and 0.1 degree of indeterminacy.

An equivalent crisp matrix of payoffs is obtained by calculating the accuracy degree with Equation 6:

$$\begin{array}{cc} & \begin{array}{cc} \mathrm{B}_1 & \mathrm{B}_2 \end{array} \\ \begin{array}{c} \mathrm{A}_1 \\ \mathrm{A}_2 \end{array} & \begin{pmatrix} 6.4125 & 6.75 \\ 5.4625 & 4.20 \end{pmatrix} \end{array}$$

Considering the maximin and minimax criteria we have one equilibrium point, (2,1) with solution 5.4625 or from the original payoff matrix $\langle(4,7,8);0.4,0.4,0.3\rangle$. Thus, the optimal solution for party A is to cooperate with other small political parties and they will get a number of voters in the interval $(4\times10^5, 8\times10^5)$.

**Example 2** : [7]

Two Asian countries, India and Pakistan have a territorial dispute over Jammu and Kashmir since 1947. Both countries should achieve an accordance to finish this historical conflict. There are three strategies $\mathrm{S}_1$, $\mathrm{S}_2$, or $\mathrm{S}_3$ to resolve this difference. The neutrosophic payoff matrix is the following:

$$\begin{array}{ccc} & & \text{Pakistan} \\ & & \begin{array}{ccc} \mathrm{S}_1 & \mathrm{S}_2 & \mathrm{S}_3 \end{array} \\ \text{India} & \begin{array}{c} \mathrm{S}_1 \\ \mathrm{S}_2 \\ \mathrm{S}_3 \end{array} & \begin{pmatrix} \langle(0,0,0);0.50,0.50,0.50\rangle & \langle(-3,-2,-1);0.40,0.65,0.60\rangle & \langle(1,2,3);0.40,0.65,0.60\rangle \\ \langle(1,2,3);0.40,0.65,0.60\rangle & \langle(0,0,0);0.50,0.50,0.50\rangle & \langle(-3,-2,-1);0.40,0.65,0.60\rangle \\ \langle(-3,-2,-1);0.40,0.65,0.60\rangle & \langle(1,2,3);0.40,0.65,0.60\rangle & \langle(0,0,0);0.50,0.50,0.50\rangle \end{pmatrix} \end{array}$$

The crisp matrix game after applying accurate degree is the following:

$$\begin{array}{ccc} & & \text{Pakistan} \\ & & \begin{array}{ccc} \mathrm{S}_1 & \mathrm{S}_2 & \mathrm{S}_3 \end{array} \\ \text{India} & \begin{array}{c} \mathrm{S}_1 \\ \mathrm{S}_2 \\ \mathrm{S}_3 \end{array} & \begin{pmatrix} 0 & -1.7625 & 1.7625 \\ 1.7625 & 0 & -1.7625 \\ -1.7625 & 1.7625 & 0 \end{pmatrix} \end{array}$$

This game has not a pure strategy solution, thus, the optimal mixed strategy is $x^* = y^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.
Therefore, $\widetilde{\mathrm{E}}(x^*, y^*) = \langle\left(-\frac{2}{3}, 0, \frac{2}{3}\right); 0.40, 0.65, 0.60\rangle$.

## Conclusion

This paper presents a new neutrosophic solution to matrix games, which can be applied in any real life problem. Specifically, it is recommendable to use this solution in political situations, where ambiguity of the parties is usual, as well as their changes of strategies during the time span. Here, the payoff matrix is defined using single-valued triangular neutrosophic numbers, which are de-neutrosophied with a score function. The solution extends another one defined for intuitionistic fuzzy payoffs, however, in this approach, indeterminacy is independent, and so it is a more accurate solution for matrix game theory. Two examples of political situations illustrate the applicability of the solution in politics. Future works will study to extend this neutrosophic approach to bimatrix games [27] and Nash equilibrium points will be also considered, [1, 3].

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