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Separation Axioms for Intuitionistic Neutrosophic Crisp supra and Infra Topological Spaces

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Abstract

The objective of this paper is to introduce a new intuitionistic neutrosophic crisp points in intuitionistic neutrosophic crisp topological space, where the intuitionistic neutrosophic crisp limit point was defined using intuitionistic neutrosophic crisp points with some of its properties. Also, a generalized form of intuitionistic neutrosophic crisp topological space as intuitionistic neutrosophic crisp supra topological space and intuitionistic neutrosophic crisp infra topological space were defined. Moreover, the separation axioms were constructed in these new spaces and the relationship between them will be examined in details.

Keywords:

Intuitionistic neutrosophic crisp topological space, intuitionistic neutrosophic crisp supra topological space, intuitionistic neutrosophic crisp infra topological space, intuitionistic neutrosophic crisp point, intuitionistic neutrosophic crisp separation axioms.

Introduction

For the first time in the world, F. Smarandache [1,2,3] introduced the notions of neutrosophic theory as a generalization of the fuzzy and intuitionistic fuzzy theories. Also, D. Cocer [4] introduced the concept of intuitionistic sets and studied its applications in algebraic and topological structures.

As the generalization of classical sets, Salama et al. in 2014 proposed the concept of neutrosophic crisp sets [5]. Neutrosophic crisp sets is a special case of neutrosophic sets.

Recently, J .Kim et al. [6] introduced the concept of intuitionistic neutrosophic crisp sets by combined intuitionistic set and neutrosophic crisp set.

They applied it to topology by defined intuitionistic neutrosophic crisp topological space and studied some concepts related to intuitionistic neutrosophic crisp sets as intuitionistic neutrosophic crisp interior and closure.

In 2015, Adel. M. AL-Odhari [7] have discussed the concept of infra-Topological spaces as an extension of topological space.

Also, G.Jayaparthasarathy et al. presented a more general study, where he created the concept of neutrosophic supra topological spaces [8] in 2019.

A. B.AL-Nafee et al. In 2015, have been discussed the concept of neutrosophic points and separation axioms in neutrosophic crisp topological spaces [9].

In fact, the concept of neutrosophic sets represents an important idea to open the door in front of many researchers especially in pure and applied mathematics [10].

In this paper, we give some important spaces via intuitionistic neutrosophic crisp sets, where we define intuitionistic neutrosophic crisp supra topological space and intuitionistic neutrosophic crisp infra topological space, as well as new sets in these new spaces as intuitionistic neutrosophic crisp supra open (closed) sets and intuitionistic neutrosophic crisp infra open (closed) sets. On other hand we define, for the first time, the intuitionistic neutrosophic crisp points and we use these points to define separation axioms in all of this new spaces (intuitionistic neutrosophic crisp topological space , intuitionistic neutrosophic crisp supra topological space and intuitionistic neutrosophic crisp infra topological space).

1. Basic Concepts

Definition:[4]

Let $X\neq\phi$ be a set. Then A is called an intuitionistic set (IS) of X, if it is an object having the form $A = (A_{\in} , A_{\notin})$; such that $A_{\in} \cap A_{\notin}=\phi$, in this case $A_{\in}$ ($A_{\notin}$) represents the set of memberships (non-memberships) of each element in X.

- The intuitionistic empty set of X, is defined by $\overline{\phi} = (\phi , X)$.

- The intuitionistic whole set of X, is defined by $\overline{X} = (X , \phi)$

- all ISs in X as IS(X).

Definition :[6]

Let $X \neq \phi$ be a set. Then the form $<\tilde{A}_T, \tilde{A}_I, \tilde{A}_F>$;

($\tilde{A}_T = (A_{1,1}, A_{1,2}), \tilde{A}_I = (A_{2,1}, A_{2,2}), \tilde{A}_F = (A_{3,1}, A_{3,2}) \in$ IS(X)).

is called an intuitionistic neutrosophic crisp set in X (INCS), if $A_{1,1} \cap A_{3,1} = \phi$.

- $\tilde{A}_T = (A_{1,1}, A_{1,2}), \tilde{A}_I = (A_{2,1}, A_{2,2}), \tilde{A}_F = (A_{3,1}, A_{3,2})$ represent the IS of memberships, indeterminacies and non-memberships respectively of each element $x \in X$ to A.
- We will denote the set of all INCS by INCS(X).

Definition: [6]

Types of INCS $\overline{\phi}_{IN}$ & $\overline{X}_{IN}$ as follows:

1. $\overline{\phi}_{IN,i}$ may be defined in many ways as a INCS as follows: (i=1,2,3,4)
   1. $\overline{\phi}_{IN,1} = < \overline{\phi}, \overline{\phi}, \overline{X} >$
   2. $\overline{\phi}_{IN,2} = < \overline{\phi}, \overline{X}, \overline{X} >$
   3. $\overline{\phi}_{IN,3} = < \overline{\phi}, \overline{X}, \overline{\phi} >$
   4. $\overline{\phi}_{IN,4} = < \overline{\phi}, \overline{\phi}, \overline{\phi} >$.
2. $\overline{X}_{IN,i}$ may be defined in many ways as a INCS as follows: (i=1,2,3,4)
   1. $\overline{X}_{IN,1} = <\overline{X}, \overline{\phi}, \overline{\phi} >$
   2. $\overline{X}_{IN,2} = < \overline{X}, \overline{X}, \overline{\phi} >$
   3. $\overline{X}_{IN,3} = < \overline{X}, \overline{\phi}, \overline{X} >$
   4. $\overline{X}_{IN,4} = < \overline{X}, \overline{X}, \overline{X} >$.

Definition: [6]

A Intuitionistic neutrosophic crisp topology (INCT) on a non-empty set X is a family T of intuitionistic neutrosophic crisp subsets in X satisfying the following axioms:

1. $\overline{\phi}_{IN,i}$ & $\overline{X}_{IN,i} \in T$. (i=1,2,3,4)
2. $C \cap D \in T$, for any $C, D \in T$.
3. T is closed under arbitrary union.

The pair (X, T) is said to be a intuitionistic neutrosophic crisp topological space (INCTS)in X. Moreover, The elements in T are said to be intuitionistic neutrosophic crisp open sets

(INCOS), a intuitionistic neutrosophic crisp set F is intuitionistic neutrosophic crisp closed (INCCS) if and only if its complement F^c is an intuitionistic neutrosophic crisp open set.

2. Intuitionistic Neutrosophic crisp point

In this part, we will introduce the intuitionistic neutrosophic crisp point and intuitionistic neutrosophic crisp limit points with some of its properties.

Definition 2.1.

For all x, y, z belonging to a non-empty set X. Then the intuitionistic neutrosophic crisp points related to x, y, z are defined as follows:

- x_{I_1}<$(\{x\},\ X-\{x\}),\bar{\phi},\ \bar{\phi}$> is called an intuitionistic neutrosophic crisp point ($INCP_{I_1}$) in X .
- y_{I_2}<$\bar{\phi},(\{y\},X-\{y\}),\bar{\phi}$> is called an intuitionistic neutrosophic crisp point ($INCP_{I_2}$) in X.
- z_{I_3}<$\bar{\phi},\bar{\phi},(\{z\},X-\{z\})$> is called an intuitionistic neutrosophic crisp point ($INCP_{N_3}$) in X .

The set of all intuitionistic neutrosophic crisp points ($INCP_{I_1}, INCP_{I_2}, INCP_{I_3}$) is denoted by $INCP_I$

Definition 2.2.

Let X be a non-empty set and x ,y, z ∈X. Then the intuitionistic neutrosophic crisp point:

- x_{I_1} is belonging to the intuitionistic neutrosophic crisp set B◎◎($B_{1,1}$, $B_{1,2}$), ($B_{2,1}$, $B_{2,2}$), ($B_{3,1}$, $B_{3,2}$)>, denoted by $x_{I_1} \in B$, if $x \in B_{1,1}$,wherein x_{I_1} not belongs to the intuitionistic neutrosophic crisp set B denoted by $x_{I_1} \notin B$,if $x \notin B_{1,1}$.
- y_{I_2} is belonging to the intuitionistic neutrosophic crisp set B◎◎($B_{1,1}$, $B_{1,2}$), ($B_{2,1}$, $B_{2,2}$), ($B_{3,1}$, $B_{3,2}$)>, denoted by $y_{I_2} \in B$, if $y \in B_{2,1}$,wherein y_{I_2} not belongs to the intuitionistic neutrosophic crisp set B denoted by $y_{I_2} \notin B$,if $y \notin B_{2,1}$.
- z_{I_3} is belonging to the intuitionistic neutrosophic crisp set B◎◎($B_{1,1}$, $B_{1,2}$), ($B_{2,1}$, $B_{2,2}$), ($B_{3,1}$, $B_{3,2}$)>, denoted by $z_{I_3} \in B$, if $z \in B_{3,1}$,wherein z_{I_3} not belongs to the intuitionistic neutrosophic crisp set B denoted by $z_{I_3} \notin B$,if $z \notin B_{3,1}$.

Definition 2.3.

Let (X, T) be an INCTS , $P \in INCP_N$ in X , an intuitionistic neutrosophic crisp set B◎◎($B_{1,1}$, $B_{1,2}$), ($B_{2,1}$, $B_{2,2}$), ($B_{3,1}$, $B_{3,2}$)> $\in T$ is said to be intuitionistic neutrosophic crisp open nhd of P in (X, T) if $P \in B$.

Definition 2.4.

Let (X, T) be an INCTS , $P \in INCP_N$ in X , an intuitionistic neutrosophic crisp set B◎◎($B_{1,1}$, $B_{1,2}$), ($B_{2,1}$, $B_{2,2}$), ($B_{3,1}$, $B_{3,2}$)> is said to be intuitionistic neutrosophic crisp nhd of P in (X, T), if there is an intuitionistic neutrosophic crisp open set A◎◎($A_{1,1}$, $A_{1,2}$), ($A_{2,1}$, $A_{2,2}$), ($A_{3,1}$, $A_{3,2}$)> containing P such that $A \subseteq B$

Note 2.5.

Every intuitionistic neutrosophic crisp open nhd of any point $P \in INCP_N$ in X is intuitionistic neutrosophic crisp nhd of P.

3 .Separation Axioms In an intuitionistic neutrosophic Crisp Topological Space

Definition 3.1.

An intuitionistic neutrosophic. crisp topological space (X, T) is called:

- I_1-T_o-space if $\forall\ x_{I_1} \neq y_{I_1} \in X\ \exists$ an intuitionistic neutrosophic crisp open set G in X containing one of them but not the other.
- I_2-T_o-space if $\forall\ x_{I_2} \neq y_{I_2} \in X\ \exists$ an intuitionistic neutrosophic crisp open set G in X containing one of them but not the other .
- I_3-T_o-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp open set G in X containing one of them but not the other.
- I_1-T_1-space if $\forall\ x_{N_1} \neq y_{N_1} \in X\ \exists$ an intuitionistic neutrosophic crisp open sets G_1, G_2 in X such that $x_{I_1} \in G_1$, $y_{I_1} \notin G_1$ and $x_{I_1} \notin G_2$, $y_{I_1} \in G_2$.
- I_2-T_1-space if $\forall\ x_{N_2} \neq y_{N_2} \in X\ \exists$ an intuitionistic neutrosophic crisp open sets G_1, G_2 in X such that $x_{I_2} \in G_1$, $y_{I_2} \notin G_1$ and $x_{I_2} \notin G_2$, $y_{I_2} \in G_2$.
- I_3-T_1-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp open sets G_1, G_2 in X such that $x_{I_3} \in G_1$, $y_{I_3} \notin G_1$ and $x_{I_3} \notin G_2$, $y_{I_3} \in G_2$.
- I_1-T_2-space if $\forall\ x_{I_1} \neq y_{I_1} \in X\ \exists$ an intuitionistic neutrosophic crisp open sets G_1,G_2 in X such that $x_{I_1} \in G_1$, $y_{I_1} \notin G_1$ and $x_{I_1} \notin G_2$, $y_{I_1} \in G_2$ with $G_1 \cap G_2 = \overline{\phi}_{IN,i}$.

- I_2-T2-space if $\forall$ $x_{I_2} \neq y_{I_2} \in X$ $\exists$ an intuitionistic neutrosophic crisp open sets G1,G2 in X such that $x_{I_2} \in G_1$, $y_{I_2} \notin G_1$ and $x_{I_2} \notin G_2$, $y_{I_2} \in G_2$ with $G_1 \cap G_2 = \overline{\phi}_{IN,i}$.
- I_3-T2-space if $\forall$ $x_{I_3} \neq y_{I_3} \in X$ $\exists$ an intuitionistic neutrosophic crisp open sets G1,G2 in X such that $x_{I_3} \in G_1$, $y_{I_3} \notin G_1$ and $x_{I_3} \notin G_2$, $y_{I_3} \in G_2$ with $G_1 \cap G_2 = \overline{\phi}_{IN,i}$.

Example 3.2.

If $X = \{x, y\}$, $T_1 = \{\overline{\phi}_{IN}$ & $\overline{X}_{IN}, A\}$,$T_2 = \{\overline{\phi}_{IN}$ & $\overline{X}_{IN}, B\}$, $T_3 = \{X_N, \emptyset_N, G\}$, A <<({x},{y}),$\overline{\emptyset}$,$\overline{\emptyset}$>, B <$\overline{\emptyset}$,({y},{x}),$\overline{\emptyset}$> , G <$\overline{\emptyset}$,$\overline{\emptyset}$,({x},{y})>, Then (X,$T_1$) is I_1-To-space, (X,T_2) is I_2-To-space, (X,T_3) is I_3-To-space.

Remark 3.3.

For an intuitionistic neutrosophic crisp topological space (X, T)

- EveryI_i-T1-space is I_i-T0-space (i=1,2,3).
- Every I_i-T2-space is I_i-T1-space (i=1,2,3).

Proof : the proof holds directly.

Remark 3.4.

The inverse of remark (3.3) is not true as it is shown in the following example :

Example 3.5.

If $= \{x, y\}$, A <({x},{y}),$\overline{\emptyset}$,$\overline{\emptyset}$>, B <$\overline{\emptyset}$,({y},{x}),$\overline{\emptyset}$> , G <<$\overline{\emptyset}$,$\overline{\emptyset}$,({x},{y})>, Then:

- When $T = \{\overline{\phi}_{IN}$ & $\overline{X}_{IN}$, A}, then (X,T) is I_1-To-space but not I_1-T1-space.
- When $T = \{\overline{\phi}_{IN}$ & $\overline{X}_{IN}$, B}, then (X,T) is I_2-To-space but not I_2-T1-space.
- When $T = \{\overline{\phi}_{IN}$ & $\overline{X}_{IN}$, G}, then (X,T) is I_3-To-space but not I_3-T1-space.

4. Intuitionistic neutrosophic Crisp Supra Topological Space

Definition 4.1.

An intuitionistic neutrosophic crisp supra topology (INCST) on a non-empty set χ is a family T of intuitionistic neutrosophic crisp subsets in X satisfying the following axioms:

1. $\overline{\phi}_{IN,i}$, $\overline{X}_{IN,i}$ $\in$ T.
2. T is closed under arbitrary union.

The pair (X,T) is said to be a intuitionistic neutrosophic crisp supra topological space (INCSTS)in X. Moreover, The elements in T are said to be intuitionistic neutrosophic crisp supra open sets (INCSOS), a neutrosophic crisp supra set F is intuitionistic neutrosophic crisp supra closed set (INCSCS) if and only if its complement F^c is an intuitionistic neutrosophic crisp supra open set.

Remark 4.2.

Every (INCTS) is (INCSTS), But the converse not true as it is shown in the following example.

Example 4.3.

Let X={a,b,c,d,e,f,g,i} and $T = \{\bar{\phi}_{IN}, \bar{X}_{IN}, A_1, A_2, A_3\}$;

$A_1 =< (\{a,b,c\},\{d,e\}),(\{e,f\},\{g\}),(\{g,h\},\{b,i\}) >$

$A_2 =< (\{a,c,d\},\{e,i\}),(\{e,g\},\{h\}),(\{h,i\},\{a\}) >$

$A_3 =< (\{a,b,c,d\},\{e\}),(\{e,f,g\},\phi),(\{g,h,i\},\phi) >$

(X,T) is (INCSTS) , but (X ,T) is not (INCTS). because $A_1, A_2 \in T$ but $A_1 \cap A_2 =< (\{a,c\},\{d,i,e\}),(\{e\},\{g,h\}),(\{h\},\{a,b,i\}) > \notin T$.

Separation Axioms In an intuitionistic neutrosophic Crisp supra Topological Space

Definition 4.5.

An intuitionistic neutrosophic crisp supra topological space (X,T) is called:

- I_1NS-T_o-space if $\forall\ x_{I_1} \neq y_{I_1} \in X\ \exists$ an intuitionistic neutrosophic crisp supra open set G in X containing one of them but not the other.
- I_2NS-T_o-space if $\forall\ x_{I_2} \neq y_{I_2} \in X\ \exists$ an intuitionistic neutrosophic crisp supra open set G in X containing one of them but not the other.
- I_3NS-T_o-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp open set G in X containing one of them but not the other.
- I_1NS-T_1-space if $\forall\ x_{N_1} \neq y_{N_1} \in X\ \exists$ an intuitionistic neutrosophic crisp supra open sets G_1, G_2 in X such that $x_{I_1} \in G_1$, $y_{I_1} \notin G_1$ and $x_{I_1} \notin G_2$, $y_{I_1} \in G_2$.
- I_2NS-T_1-space if $\forall\ x_{N_2} \neq y_{N_2} \in X\ \exists$ an intuitionistic neutrosophic crisp supra open sets G_1, G_2 in X such that $x_{I_2} \in G_1$, $y_{I_2} \notin G_1$ and $x_{I_2} \notin G_2$, $y_{I_2} \in G_2$.

- I_3NS-T_1-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp supra open sets G_1, G_2 in X such that $x_{I_3} \in G_1$, $y_{I_3} \notin G_1$ and $x_{I_3} \notin G_2$, $y_{I_3} \in G_2$
- I_1NS-T_2-space if $\forall\ x_{I_1} \neq y_{I_1} \in X\ \exists$ an intuitionistic neutrosophic crisp supra open sets G_1,G_2 in X such that $x_{I_1} \in G_1$, $y_{I_1} \notin G_1$ and $x_{I_1} \notin G_2$, $y_{I_1} \in G_2$ with $G_1 \cap G_2 = \bar{\phi}_{IN,i}$
- I_2NS-T_2-space if $\forall\ x_{I_2} \neq y_{I_2} \in X\ \exists$ an intuitionistic neutrosophic crisp open supra sets G_1,G_2 in X such that $x_{I_2} \in G_1$, $y_{I_2} \notin G_1$ and $x_{I_2} \notin G_2$, $y_{I_2} \in G_2$ with $G_1 \cap G_2 = \bar{\phi}_{IN,i}$.
- I_3NS-T_2-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp open supra sets G_1,G_2 in X such that $x_{I_3} \in G_1$, $y_{I_3} \notin G_1$ and $x_{I_3} \notin G_2$, $y_{I_3} \in G_2$ with $G_1 \cap G_2 = \bar{\phi}_{IN,i}$.

Example 4.6.

If $X = \{x, y\}$, $T = \{\bar{\phi}_{IN}$ & $\bar{X}_{IN}, A, B, C\}$, A <({x},{y}), $\bar{\emptyset}$, $\bar{\emptyset}$ >, B M $\bar{\emptyset}$,({y},{x}), $\bar{\emptyset}$ >, C ◎◎({x},∅),({y},∅) ,$\bar{\emptyset}$>, Then (X,T) is I_1NS-T_o-space, and I_2NS-T_o-space.

Example 4.7.

If $X = \{x, y\}$, $T = \{\bar{\phi}_{IN}$ & $\bar{X}_{IN}, G, A, C\}$, A <({x},{y}), $\bar{\emptyset}$, $\bar{\emptyset}$ >, G ◎◎ $\bar{\emptyset}$, $\bar{\emptyset}$,({x},{y})>, C <({x},∅),$\bar{\emptyset}$,({y},{ x}) >, Then (X,T) is I_3NS-T_o-space.

Remark 4.8.

For an intuitionistic neutrosophic crisp supra topological space (X, T)

- Every I_iNS-T_1-space is I_iNS-T_0-space (i=1,2,3).
- Every I_iNS-T_2-space is I_iNS-T_1-space (i=1,2,3).

Proof : the proof holds directly.

Remark 4.9.

The inverse of remark (4.8) is not true as it is shown in the following example :

Example 4.10.

In example 4.6, (X,T) is I_iNS-T_o-space, but not I_iNS-T_1-space(i=1,2).

In example 4.7, (X,T) is I_iNS-T_o-space, but not I_iNS-T_1-space(i=3).

5. Intuitionistic neutrosophic Crisp Infra Topological Space

Definition 5.1.

An Intuitionistic neutrosophic crisp topology infra (INCIT) on a non-empty set X is a family T of intuitionistic neutrosophic crisp subsets in X, satisfying the following axioms:

1. $\overline{\phi}_{IN,i}, \overline{X}_{IN,i} \in T$.
2. T is closed under finite intersection.

The pair (X,T) is said to be a intuitionistic neutrosophic crisp infra topological space (INCITS)in X. Moreover, The elements in T are said to be intuitionistic neutrosophic crisp infra open sets (INCIOS), a neutrosophic crisp infra set F is neutrosophic crisp infra closed (INCICS) if and only if its complement F^c is an intuitionistic neutrosophic crisp infra open set.

Remark 5.2.

Every (INCTS) is (INCITS), But the converse not true as it is shown in the following example.

Example 5.3.

Let X={a,b,c,d,e,f,g,i} and $T = \{\overline{\phi}_{IN}, \overline{X}_{IN}, A_1, A_2, A_3\}$;

$A_1 =< (\{a,b,c\},\{d,e\}),(\{e,f\},\{g\}),(\{g,h\},\{b,i\}) >$

$A_2 =< (\{a,c,d\},\{e,i\}),(\{e,g\},\{h\}),(\{h,i\},\{a\}) >$

$A_3 =< (\{a,c\},\{d,e,i\}),(\{e\},\{g,h\}),(\{h\},\{a,b,i\}) >$

(X,T) is (INCITS), but (X ,T) is not (INCTS). because $A_1, A_2 \in$T.

But $A_1 \cup A_2 =< (\{a,b,c,d\},\{e\}),(\{e,f,g\},\phi),(\{g,h,i\},\phi) > \notin$T.

Remark 5.4.

Let (X ,T) be a (INCITS), then :

The union of two intuitionistic neutrosophic crisp infra open sets is not necessary intuitionistic neutrosophic crisp infra open set.

Proof:

In example 5.3, A_1, A_2 are intuitionistic neutrosophic crisp infra open sets but $A_1 \cup A_2 =< (\{a,b,c,d\},\{e\}),(\{e,f,g\},\phi),(\{g,h,i\},\phi) >$ is not intuitionistic neutrosophic crisp infra open set.

Remark 5.6.

(INCITS) is not necessary (INCSTS).

Example 5.7.

In example 5.3, (X ,T) is (INCITS) , but (X ,T) is not (INCSTS). because $A_1, A_2 \in$T but $A_1 \cup A_2 =< (\{a, b, c, d\}, \{e\}), (\{e, f, g\}, \phi), (\{g, h, i\}, \phi) > \notin$T.

Remark 5.8.

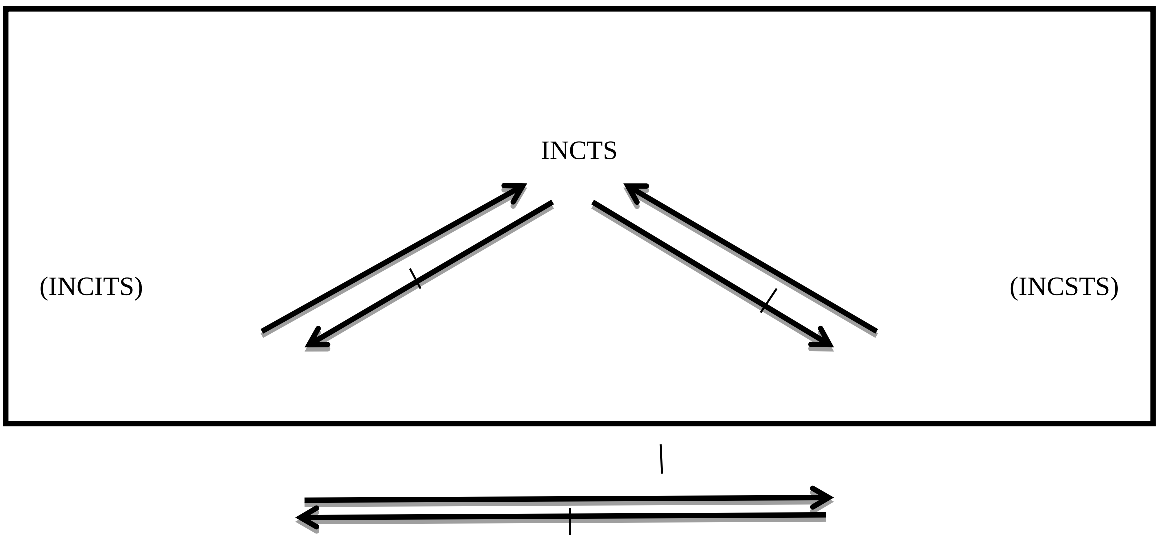
(INCSTS) is not necessary (INCITS).

Example 5.9.

In example 4.3, (X ,T) is (INCSTS), but (X ,T) is not (INCITS). Because $A_1, A_2 \in$T but $A_1 \cap A_2 =< (\{a, c\}, \{d, i, e\}), (\{e\}, \{g, h\}), (\{h\}, \{a, b, i\}) > \notin$T.

Remark 5.10.

The relations between (INCITS) , (INCSTS) and (INCTS) in the following diagram :



6 .Separation Axioms In an intuitionistic neutrosophic Crisp infra Topological Space

Definition 6.1.

An intuitionistic neutrosophic crisp infra topological space (X, T) is called:

- I_1NI-T_o-space if $\forall\ x_{I_1} \neq y_{I_1} \in X\ \exists$ an intuitionistic neutrosophic crisp infra open set G in X containing one of them but not the other.
- I_2NI-T_o-space if $\forall\ x_{I_2} \neq y_{I_2} \in X\ \exists$ an intuitionistic neutrosophic crisp infra open set G in X containing one of them but not the other .
- I_3NI-T_o-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp open set G in X containing one of them but not the other .
- I_1NI-T_1-space if $\forall\ x_{N_1} \neq y_{N_1} \in X\ \exists$ an intuitionistic neutrosophic crisp infra open sets G_1, G_2 in X such that $x_{I_1} \in G_1$, $y_{I_1} \notin G_1$ and $x_{I_1} \notin G_2$, $y_{I_1} \in G_2$
- I_2NI-T_1-space if $\forall\ x_{N_2} \neq y_{N_2} \in X\ \exists$ an intuitionistic neutrosophic crisp infra open sets G_1, G_2 in X such that $x_{I_2} \in G_1$, $y_{I_2} \notin G_1$ and $x_{I_2} \notin G_2$, $y_{I_2} \in G_2$
- I_3NI-T_1-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp infra open sets G_1, G_2 in X such that $x_{I_3} \in G_1$, $y_{I_3} \notin G_1$ and $x_{I_3} \notin G_2$, $y_{I_3} \in G_2$
- I_1NI-T_2-space if $\forall\ x_{I_1} \neq y_{I_1} \in X\ \exists$ an intuitionistic neutrosophic crisp infra open sets G_1,G_2 in X such that $x_{I_1} \in G_1$, $y_{I_1} \notin G_1$ and $x_{I_1} \notin G_2$, $y_{I_1} \in G_2$ with $G_1 \cap G_2 = \overline{\phi}_{IN,i}$.
- I_2NI-T_2-space if $\forall\ x_{I_2} \neq y_{I_2} \in X\ \exists$ an intuitionistic neutrosophic crisp open infra sets G_1,G_2 in X such that $x_{I_2} \in G_1$, $y_{I_2} \notin G_1$ and $x_{I_2} \notin G_2$, $y_{I_2} \in G_2$ with $G_1 \cap G_2 = \overline{\phi}_{IN,i}$.
- I_3NI-T_2-space if $\forall\ x_{I_3} \neq y_{I_3} \in X\ \exists$ an intuitionistic neutrosophic crisp open infra sets G_1,G_2 in X such that $x_{I_3} \in G_1$, $y_{I_3} \notin G_1$ and $x_{I_3} \notin G_2$, $y_{I_3} \in G_2$ with $G_1 \cap G_2 = \overline{\phi}_{IN.i}$.

Example 6.2.

If $X = \{x, y\}$, $T = \{\overline{\phi}_{IN}\ \&\ \overline{X}_{IN}, A, B, C\}$, A ◎◎({x},{y}), $\overline{\emptyset}$, $\overline{\emptyset}$ >, B ◎◎ $\overline{\emptyset}$,({y},{x}), $\overline{\emptyset}$ > , C ◎◎(∅,{y}),(∅,{x}) ,$\overline{\emptyset}$>, Then (X,T) is I_1NI-T_o-space, and I_2NI-T_o-space.

Example 6.3.

If $X = \{x, y\}$, $T = \{\overline{\phi}_{IN}\ \&\ \overline{X}_{IN}, G, A, C\}$, A ◎◎({x},{y}), $\overline{\emptyset}$, $\overline{\emptyset}$ >, G ◎◎ $\overline{\emptyset}$, $\overline{\emptyset}$,({x},{y})>, C ◎◎(∅,{y}),$\overline{\emptyset}$,(∅,{ y}) >, Then (X,T) is I_3NI-T_o-space.

Example 6.4.

If $X = \{x, y\}$, $T = \{\overline{\phi}_{IN}\ \&\ \overline{X}_{IN}, A, B, C\}$, A ◎◎({x},{y}),$\overline{\emptyset}$,$\overline{\emptyset}$>, B ◎◎({y},{x}),$\overline{\emptyset}$,$\overline{\emptyset}$>, C ◎◎(∅,{x,y}),$\overline{\emptyset}$,$\overline{\emptyset}$>, Then (X,T) is I_1NI-T_1-space, but(X,T) in not I_1NI-T_2-space.

Example 6.5.

If X = {x, y} , $T = \{\overline{\phi}_{IN}$ & $\overline{X}_{IN}$, G , $A, C\}$, A <({x},{y}), $\overline{\emptyset}$, $\overline{\emptyset}$ >, G ⊚⊚ $\overline{\emptyset}$, $\overline{\emptyset}$,({x},{y})>, C ⊚⊚(∅,{y}),$\overline{\emptyset}$,(∅,{ y}) >, Then (X,T) is I_3NI-T_o-space, but (X,T) in not I_3NI-T_2-space.

Remark 6.6.

For an intuitionistic neutrosophic crisp infra topological space (X, T)

- EveryI_iNI-T_1-space is I_iNI-T_0-space (i=1,2,3).
- Every I_iNI-T_2-space is I_iNI-T_1-space (i=1,2,3).

Proof the proof holds directly.

The inverse of remark (3.8) is not true as it is shown in the following example :

Remark 6.7.

- In example 6.2, (X,T) is I_1NI-T_o-space, but(X,T) is I_1NI-T_1-space, and (X,T) I_2NI-T_o-space, but(X,T) is not I_2NI-T_1-space.
- In example 6.4, (X,T) is I_1NI-T_1-space, but(X,T) is I_1NI-T_2-space.

7. Conclusion

In this paper, we have defined new topological spaces by using intuitionistic neutrosophic crisp sets. This new space is called intuitionistic neutrosophic crisp supra space and intuitionistic neutrosophic crisp infra space. Then we have introduced new intuitionistic neutrosophic crisp supra open (closed) sets and intuitionistic neutrosophic crisp infra open (closed) sets in this new spaces. Also we studied some of their basic properties and their relationship with each other. Also we defined intuitionistic neutrosophic crisp points, using these notions, various classes of separation axioms were defined. In the future, many researchers can study the intuitionistic neutrosophic crisp supra space and intuitionistic neutrosophic crisp infra space.

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