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A Review Study on Some Properties of The Structure of Neutrosophic Ring

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Abstract: determinacy is a philosophical concept which introduced by Smarandache and used in structure of mathematical systems. In this article we use this concept to introduced Particular Structure of neutrosophic ring and studied some theorem and properties according to classical axiomatic ring theory.

Keywords: Neutrosophic rings; Neutrosophic rings of Integers; Neutrosophic rings of Complex; Neutrosophic rings of Integers of modulo n.

1. Introduction

Neutrosophic ring established first time by Kandasamy and Smarandache in 2006 see [19], in this paper we introduced particular neutrosophic ring depend on classical axioms of ring theory and studied some theorems and properties of neutrosophic ring theory.

2. Neutrosophic Rings and Their Examples

In this section we introduced the concept of neutrosophic ring was introduced in 2006 by Kandasamy and Smarandache see [19] with examples, but by applying the axioms of classical ring theory with concept of indeterminate. The neutrosophic element as I where I is an indeterminate and I is such that $I^2 = I$. If $I^2 = I \nRightarrow I(I-1) = 0$ or any relation is just saying $I^2 = I$

Definition 2.1. [19] Let R be any ring. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and I under the operations of R.

Theorem 2.2. [19] Let $\langle R \cup I \rangle$ be a neutrosophic ring. $\langle R \cup I \rangle$ is a ring.

Note. In sated of notation $\langle R \cup I \rangle$ and $\langle R \backslash \{0\} \cup I \rangle$, we use notation $R\,[\,I]$ and $R^*[\,I]$ respectively.

Definition 2.2. Let R be a nonempty set and the triple $(R, +, \bullet)$ be a ring, and consider the neutrosophic (NS):$R[\mathrm{I}] = \{a + bI : a, b \in R\}$, then the neutrosophic algebra structure (NAS):

$N(R) = \langle R[I], +, \bullet \rangle$ is called the neutrosophic associative ring which is a generated by I and R under operations $+$ " addition "and $\bullet$ " multiplications" respectively if satisfies the axiomatic conditions of ring:

NR1: For all x, y and $z \in N(R)$, $N(R) = \langle R[I], + \rangle$ is a neutrosophic an abelian group under addition;

NR2: For all x, y and $z \in N(R^*)$, $N(R^*) = \langle R^*[\,I], \bullet \rangle$ is a mathematical associative neutrosophic system under multiplications, that is, $N(R^*) = \langle R^*[\,I], \bullet \rangle$ is neutrosophic semi group and

NR3: $x \bullet (y + \mathrm{z}) = (x \bullet y) + (x \bullet z)$ and $(y + \mathrm{z}) \bullet x = (y \bullet x) + (z \bullet x)$ " left and right distribution laws".

Observations.

- If $N(R^*) = \langle R^*[\,I], \bullet \rangle$ has neutrosophic identity (or unit), then $N(R) = \langle R[I], +, \bullet \rangle$ is called a neutrosophic ring with a neutrosophic identity (or neutrosophic unit).
- If $N(R^*) = \langle R^*[\,I], \bullet \rangle$ has neutrosophic inverse, then $N(R) = \langle R[I], +, \bullet \rangle$ is called a neutrosophic ring with a neutrosophic inverse and the neutrosophic structure $N(R^*) = \langle R^*[\,I], \bullet \rangle$ is called neutrosophic group.
- If $N(R^*) = \langle R^*[\,I], \bullet \rangle$ is a neutrosophic abelian, that is, all $x, y \in N(R)$, we have

$x.y = y.x$, in addition,$N(R^*) = \langle R^*[I],\bullet\rangle$ is called a neutrosophic abelian group, consequently, the $N(R) = \langle R[I],+,\bullet\rangle$ is called a filed and denoted by $N(F) = \langle F[I],+,\bullet\rangle$.

Definition 2.3. Let R be a finite set and the triple $(R,+,\bullet)$ be a finite ring, then $N(R) = \langle R[I],+,\bullet\rangle$ is called a finite neutrosophic ring, otherwise, $N(R) = \langle R[I],+,\bullet\rangle$ is called is an infinite neutrosophic ring.

Definition 2.4. Let $N(R) = \langle R[I],+,\bullet\rangle$be a neutrosophic ring. Define the neutrosophic set: $N(C(R) = \{x \in N(R): xy = yx, \forall\, y \in N(R)\}$ which is called the neutrosophic center of $N(R)$. Also, $N(R)$ is abelian iff $N(R) = N(C(R)$.

Definition 2.5.[19] Let $\mathbb{Z}$ be a set of integer numbers and $\mathbb{Z}[I] = \{a + bI: a, b \in \mathbb{Z}\}$ be a neutrosophic- integer set, where $a + bI$ is a neutrosophic integer number.

Preposition 2.1. Let $(\mathbb{Z},+,\bullet)$ be a ring of integers under usual addition and multiplication, then the neutrosophic algebra structure (NAS): $N(\mathbb{Z}) = \langle \mathbb{Z}[I],+,\bullet\rangle$ is called the neutrosophic integer ring which is generated by I and $\mathbb{Z}$.

Proof. Let $(\mathbb{Z},+,\bullet)$ be a ring of integers under usual addition and multiplication and $\langle \mathbb{Z} \cup I\rangle = \{a + bI: a, b \in \mathbb{Z}\}$ be a neutrosophic- integer set, where $a + bI$ is a neutrosophic integer number. Then by proposition 2.1 in [5] , $N(\mathbb{Z}) = \langle \mathbb{Z}[I],+\rangle$ is a neutrosophic abelian group, so NR1 axioms is hold. Now Let , $N(\mathbb{Z}) = \langle \mathbb{Z}[I],\bullet\rangle$ such that all $x , y \in N(\mathbb{Z})$, then:

$$x \cdot y = \big((x_1 + x_2 I) + (y_1 + y_2 I)\big)$$

$$= \Big((x_1.y_1) + \Big(\big((x_1.y_2) + (x_2.y_1)\big) + (x_2.y_2)\Big) I\Big) \in, N(\mathbb{Z}) = \langle \mathbb{Z}[I],\cdot\rangle,$$ it's a closure,

moreover,

$$(x \cdot y) \cdot z = \big((x_1 + x_2 I) + (y_1 + y_2 I)\big) \cdot (z_1 + z_2 I)$$

$$= \Big((x_1.y_1) + \Big(\big((x_1.y_2) + (x_2.y_1)\big) + (x_2.y_2)\Big) I\Big) \cdot (z_1 + z_2 I)$$

$$= \left(\begin{array}{c}(x_1.y_1).z_1 \\ + \\ \Big((x_1.y_1).z_2 + \big((x_1.y_2) + (x_2.y_1) + (x_2.y_2)\big).z_1 + \big((x_1.y_2) + (x_2.y_1) + (x_2.y_2)\big).z_2\Big) I\end{array}\right)$$

$$= \begin{pmatrix} x_1.(y_1 . z_1) \\ + \\ \big((x_1.y_1).z_2 + \big((x_1.y_2) + (x_2.y_1) + (x_2.y_2)\big).z_1 + \big((x_1.y_2) + (x_2.y_1) + (x_2.y_2)\big).z_2\big)\, I \end{pmatrix}$$

$$= \begin{pmatrix} x_1.(y_1 . z_1) \\ + \\ \big((x_1.y_1).z_2 + \big((x_1.y_2).z_1 + (x_2.y_1).z_1 + (x_2.y_2).z_1\big) + \big((x_1.y_2).z_2 + (x_2.y_1).z_2 + (x_2.y_2).z_2\big)\big)\, I \end{pmatrix}$$

$$= \begin{pmatrix} x_1.(y_1 . z_1) \\ + \\ \big(x_1.(y_1.z_2) + \big(x_1.(y_2.z_1) + x_2.(y_1.z_1) + x_2.(y_2.z_1)\big) + \big(x_1.(y_2.z_2) + x_2.(y_1.z_2) + x_2.(y_2.z_2)\big)\big)\, I \end{pmatrix}$$

$$= (x_1 + x_2 I)\Big((y_1.z_1) + \Big(\big((y_1.z_2) + (y_2.z_1)\big) + (y_2.z_2)\Big) I\Big) = x.(y.z).$$

Hence the associative law is hold.

Finally, $x\,.(y + \mathrm{z}) = (x_1 + x_2 I).\big((y_1 + y_2 I) + (z_1 + z_2 I)\big)$

$$= (x_1 + x_2 I).\big((y_1 + z_1) + (y_2 + z_2) I\big)$$

$$= \big(x_1.(y_1 + z_1) + \big(x_1.(y_2 + z_2) + x_2.(y_1 + z_1) + x_2.(y_2 + z_2)\big) I\big)$$

$$= \Big(\big((x_1.y_1) + (x_1.+z_1)\big) + \Big(\big((x_1.y_2) + (x_1.z_2)\big) + \big((x_2.y_1) + (x_2.z_1)\big) + \big((x_2.y_2) +$$

$$(x_2.z_2)\big)\Big) I\Big)$$

$$= \Big((x_1.y_1) + \big((x_1.y_2) + (x_2.y_1) + (x_2.y_2)\big) I\Big) + \Big((x_1.+z_1) + \big((x_1.z_2) + (x_2.z_1) +$$

$$(x_2.z_2)\big) I\Big)$$

$= \big((x_1 + x_2 I).(y_1 + y_2 I)\big) + \big((y_1 + y_2 I).(z_1 + z_2 I)\big) = (x.y) + (x.z)$. By similar procedure, we can deduce that: $(y + \mathrm{z}).x = (y.x) + (z.x)$. Moreover there exists $1 \in$ $, N(\mathbb{Z}) = \langle \mathbb{Z}[I], \bullet \rangle$such that $1.x = x.1 = x$. Hence $N(\mathbb{Z}) = \langle \mathbb{Z}[I], +, \bullet \rangle$ is neutrosophic integer ring with identity■. The neutrosophic integer ring will plays an important role in the study of neutrosophic ring theory.

Example 2.1. Let $(\mathbb{Z}^+ \cup \{0\}, +, \bullet)$ be a unit ring of positive integers under neutrosophic addition and multiplication, then the neutrosophic algebra structure (NAS): $N(\mathbb{Z}^+) = \{\langle \mathbb{Z}^+ \cup \{0\} \cup I \rangle, +, \cdot\}$ is called the neutrosophic unit integer ring which is a generated by I and $\mathbb{Z}^+ \cup \{0\}$.

Defintione2.6. (Number theory) Let $\mathbb{Z}$ be the set of integers and $x \in \mathbb{Z}$, then x is called even number if there exists $k \in \mathbb{Z}$ such that $x = 2k$. If $x, y \in \mathbb{Z}$ and both are even numbers, then $x + y$ and $x.y$ are even numbers. Because, $x + y = 2k_1 + 2k_2 = 2(k_1 + k_2) = 2k_3$, where,

$k_3 = (k_1 + k_2) \in \mathbb{Z}$. Also, $x.y = (2k_1).(2k_2) = 2\big(k_1.(2k_2)\big) = 2k_3$, where, $k_3 = \big(k_1.(2k_2)\big) \in \mathbb{Z}$.

Defintione2.7. (Neutrosophic Number Theory) Let $\mathbb{Z}[I] = \{a + bI : a, b \in \mathbb{Z}\}$ be the set of neutrosophic integers and $x \in \mathbb{Z}[I]$, then $x = x_1 + x_2 I$ is called the neutrosophic even number if, x_1 and x_2 are even number. So, $0, 2I, 4I, \ldots, 2 + 2I, 2 + 4I, \ldots$ etc, are neutrosophic even integers.

Example 2.2. Let $(\mathbb{Z}_{even}, +, \bullet)$ be a ring of even integers without unit under neutrosophic addition and multiplication, then the neutrosophic algebra structure (NAS): $N(\mathbb{Z}_{even}) = \{\langle \mathbb{Z}_{even} \cup I \rangle, +, \bullet\}$ is called the neutrosophic integer ring which is a generated by I and $\mathbb{Z}_{even}$. This is a neutrosophic integer ring without neutrosophic unit elements.

Definition 2.8.[19] Let $\mathbb{R}$ be a set of real numbers and $\langle \mathbb{R} \cup I \rangle = \{a + bI : a, b \in \mathbb{R}\}$ be a neutrosophic- real set, where $a + bI$ is a neutrosophic real number.

Preposition 2.2. Let $(\mathbb{R}, +, \cdot)$ be a ring of real numbers under usual addition, then the neutrosophic algebra structure (NAS): $N(\mathbb{R}) = \langle \mathbb{R}[I], +, \cdot \rangle$ is called the neutrosophic real ring with identity which is a generated by I and $\mathbb{R}$. In addition, $N(\mathbb{R}) = \langle \mathbb{R}[I], +, \cdot \rangle$ is a neutrosophic real field**.**

Proof. By the same argument of preceding preposition 2.1. In addition, $N(\mathbb{R}^*) = \langle \mathbb{R}^*[I], \cdot \rangle$ is a commutative group. consider $a = a_1 + a_2 I \in \mathbb{R}(I)$. Suppose that $x = x_1 + x_2 I \in \mathbb{R}(I)$ is the neutrosophic inverse of a, that is,

$a.x = 1 \Leftrightarrow (a_1 + a_2 I).(x_1 + x_2 I) = 1 + 0I$

$$\Leftrightarrow \Big((a_1.x_1) + \big((a_1.x_2) + (a_2.x_1) + (a_2.x_2)\big)I\Big) = 1 + 0I.$$

$$\Rightarrow a_1.x_1 = 1 \text{ and } (a_1.x_2) + (a_2.x_1) + (a_2.x_2) = 0.$$

$\Rightarrow x_1 = \frac{1}{a_1}$ and $(a_1 + a_2)x_2 + a_2.\frac{1}{a_1} = 0 \Rightarrow x_1 = \frac{1}{a_1}$ and $x_2 = -\frac{a_2}{a_{1(a_1+a_2)}}$. To check the axiom of inverse, $a.x = (a_1 + a_2 I).\left(\frac{1}{a_1} - \frac{a_2}{a_{1(a_1+a_2)}} I\right)$

$$= \left(\left(a_1.\frac{1}{a_1}\right) + \left(-\frac{a_1.a_2}{a_{1(a_1+a_2)}}\right) + \left(a_2.\frac{1}{a_1}\right) - \left(\frac{{a_2}^2}{a_{1(a_1+a_2)}}\right) I\right)$$

$= 1 + \left(\frac{-a_1.a_2+a_2.(a_1+a_2)-{a_2}^2}{a_{1(a_1+a_2)}}\right)I.$

$= 1 + \left(\frac{-a_1.a_2+a_2.a_1+{a_2}^2-{a_2}^2}{a_{1(a_1+a_2)}}\right)I.$

$= 1 + 0I = 1$.By similar way we have $x.a = 1$. Also for all $a, b \in \mathbb{R}(I)$,we have

$ab = ba$. Hence $N(\mathbb{R}) = \langle \mathbb{R}[I], +,\cdot\rangle$ is the neutrosophic field of real ■.

Definition 2.9.[19] Let $\mathbb{C}$ be a set of complex numbers and $\mathbb{C}[I] = \{a + bI: a, b \in \mathbb{C}\}$ be a neutrosophic- complex set, where $a + bI$ is a neutrosophic complex number.

Preposition 2.3. Let $(\mathbb{C}, +.\cdot)$ be a ring of complex numbers under usual addition, then the neutrosophic algebra structure (NAS): $N(\mathbb{C}) = \langle \mathbb{C}[I], +,\cdot\rangle$ is called the neutrosophic complex ring with identity which is a generated by I and $\mathbb{C}$. Moreover, $N(\mathbb{C}) = \langle \mathbb{C}[I], +,\cdot\rangle$ is the neutrosophic field of complex numbers.

Proof. Let $N(\mathbb{C}) = \langle \mathbb{C}[I], +,\cdot\rangle$ be the neutrosophic algebra structure and let $a = a_1 + a_2I$, $b = b_1 + b_2I$ and $c = c_1 + c_2I$ be three elements in $\mathbb{C}[I]$ Then $N(\mathbb{C}) = \langle \mathbb{C}[I], +\rangle$ is a neutrosophic complex abelian group by prop2.3 in [5]. Also, $N(\mathbb{C}^*) = \langle \mathbb{C}^*[I],\cdot\rangle$ is a neutrosophic commutative complex group, 1 is the neutrosophic identity element, now if we consider

$a = a_1 + a_2I \in \mathbb{C}(I)$, $a_1, a_2 \in \mathbb{C}$, then suppose that $a^{-1} = \frac{1}{a_1} - \left(\frac{a_2}{a_{1(a_1+a_2)}}\right)I$ is the neutrosophic inverse element of a by the same argument in pervious proposition 2.2.Hence $N(\mathbb{C}^*) = \langle \mathbb{C}^*[I],\cdot\rangle$is a commutative neutrosophic complex group and consequently , $N(\mathbb{C}) = \langle \mathbb{C}[I], +,\cdot\rangle$ is neutrosophic field of complex■.

Theorem 2.2. Condiser $N(\mathbb{Z}_n) = \{\mathbb{Z}_n \cup I, \oplus_n, \otimes_n\}$ is a finite neutrosophic ring under addition and multiplication with modulo n. Moreover $N(\mathbb{Z}_n) = \{\mathbb{Z}_n \cup I, \oplus_n, \otimes_n\}$ is a finite neutrosophic ring under addition and multiplication with modulo n.In addition it is a field.

Proof. See theorems 2.4 and 2.5. in [5].

Example2.3. $N(\mathbb{Z}_3) = \langle \mathbb{Z}_3\,[I], \oplus_3, \otimes_3\rangle$ is a finite neutrosophic ring under addition and multiplication with modulo 3. Moreover, it's a finite neutrosophic field. As we know, $\mathbb{Z}_3 = \{0,1,2\}$ and,

$Z_3[I] = \{a + bI: a, b \in Z_3\} = \{0,1,2, I, 2I, 1 + I, 1 + 2I, 2 + I, 2 + 2I\}$, to construct then the neutrosophic algebra structure (NAS):$N(Z_3) = \langle Z_3[I], \oplus_3 \rangle$ by the visualizing table as shown in table.2.1.

Table.2.1, of (NAS of $N(Z_3) = \langle Z_3[I], \oplus_3 \rangle$.

$\oplus_3$	0	1	2	I	$2I$	$1+I$	$1+2I$	$2+I$	$2+2I$
0	0	1	2	I	$2I$	$1+I$	$1+2I$	$2+I$	$2+2I$
1	1	2	0	$1+I$	$1+2I$	$2+I$	$2+2I$	I	$2I$
2	2	0	1	$2+I$	$2+2I$	I	$2I$	$1+I$	$1+2I$
I	I	$1+I$	$2+I$	$2I$	0	$1+2I$	1	$2+2I$	2
$2I$	$2I$	$1+2I$	$2+2I$	0	I	1	$2+I$	2	$2+I$
$1+I$	$1+I$	$2+I$	I	$1+2I$	1	$2+2I$	2	$2I$	0
$1+2I$	$1+2I$	$2+2I$	$2I$	1	$1+I$	2	$2+I$	0	I
$2+I$	$2+I$	I	$1+I$	$2+2I$	2	$2I$	0	$1+2I$	1
$2+2I$	$2+2I$	$2I$	$1+2I$	2	$2+I$	0	I	1	$1+I$

The (NAS) is a closure under operation $\oplus_3$ modulo 3 and associative, there exists identity element is zero and for any elements in x has inverse as shown in the table 2.2.

Table 2.2, of inverse element.

x	0	1	2	I	$2I$	$1+I$	$1+2I$	$2+I$	$2+2I$
x^{-1}	0	2	1	$2I$	I	$2+2I$	$2+I$	$1+2I$	$1+I$

The (NAS) $N(Z_3) = \langle Z_3[I], \oplus_3 \rangle$ is represents a neutrosophic commutative group (NS). In addition,

$\mathbb{Z}_3{}^* = \{1,2\}$ and $\mathbb{Z}_3{}^*[\text{I}] = \{a + bI : a, b \in Z_3\} = \{1+I, 1+2I, 2+I, 2+2I\}$, to construct the neutrosophic algebra structure (NAS):$N(\mathbb{Z}_3{}^*) = \langle \mathbb{Z}_3{}^*[I], \otimes_3 \rangle$ by the visualizing table as shown in table.2.3.

Table.2.3, of (NAS of $N(\mathbb{Z}_3{}^*) = \langle \mathbb{Z}_3{}^*[I], \otimes_3 \rangle$.

$\otimes_3$	$1+I$	$1+2I$	$2+I$	$2+2I$
$1+I$	1	$1+2I$	$2+I$	2
$1+2I$	$1+2I$	$1+2I$	$2+I$	2
$2+I$	$2+I$	$2+I$	$1+2I$	$1+2I$
$2+2I$	2	$2+I$	$1+2I$	1

We see that $N(\mathbb{Z}_3{}^*) = \langle \mathbb{Z}_3{}^*[I], \otimes_3 \rangle$ is a neutrosophic semigroup, but in classical ring theory $\langle \mathbb{Z}_3{}^*, \otimes_3 \rangle$ is a group. Also, this table is a correction of table 2.1 in [5]. Moreover, NR3 is hold, for instance, $(1+I).\big((2+I)+(2+2I)\big) = (1+I).(4+3I) = 4+10I$and,

$(1+I).(2+I)+(1+I).(2+2I) = (2+4I)+(2+6I) = 4+10I.$ Hence $N(\mathbb{Z}_3) = \langle \mathbb{Z}_3\,[I], \oplus_3, \otimes_3 \rangle$ is a neutrosophic ring.

Theorem2.3. [6] Let $A, B,$ and C be three neutrosophic matrices of the same capacity, and consider x and y are two neutrosophic scalars, then:

i. $A + B = B + A$;

ii. $(A + B) + C = A + (B + C)$" " associative law";

iii. $A + 0 = A$;

iv. $x(A + B) = xA + xB$;

v. $(x+y)A = xA + yA$;

vi. $x(yA) = (xy)A$,and

vii. $1.A = A$

Theorem2.4.[6]. Let A, B, and C be three neutrosophic matrices which are defined under multiplication, with x is a neutrosophic scalars, then:

i. $(AB)C = A(BC)$ " associative law";

ii. $A(B+C) = AB + AC$ " left distributive law";

iii. $(B+C)A = BA + CA$ "right distributive law" and

iv. $x(AB) = (xA)B = A(xB)$.

v. $0A. = 0,\ B.0 = 0$. Where 0 is a neutrosophic zero matrix.

Theorem 2.5. Consider the $n - square$ neutrosophic matrix set

$M_{n\times n} = \{a_{ij} + b_{ij}I : a_{ij}, b_{ij} \in \mathbb{R}, 0I = 0 \ \&\ I^2 = I\}$, such that $M_{n\times n}$ has inverse, that is $det\left(\left[a_{ij} + b_{ij}I\right]\right) \neq 0$, then $N(M_{n\times n}) = \left\{\left[a_{ij} + b_{ij}I\right], +, \times\right\}$,where " $+$ " defined as definition 2.11 and " $\times$ " defined as definition 2.13 respectively in [4,6]. Then $N(M_{n\times n}) = \left\{\left[a_{ij} + b_{ij}I\right], +, \times\right\}$ is non- commutative neutrosophic ring with unit.

Proof.

NR1: $N(M) = \left\{\left[a_{ij} + b_{ij}I\right], +\right\}$ is a commutative group under $+$. By theorem2.2.[6]. From (i) to(iii) the neutrosophic inverse element:

$$A + (-A) = \left[a_{ij} + b_{ij}I\right] + \left[\left(-a_{ij}\right) + \left(-b_{ij}\right)I\right]$$

$$= \left[a_{ij} + \left(-a_{ij}\right) + \left(b_{ij} + \left(-b_{ij}\right)\right)I\right], \text{ for } i, j = 1,2,3, \dots, n$$

$= [0 + 0I] = 0$. By the same argument we have $-A + A = 0$. Hence,

$N(M_{n\times n}) = \left\{\left[a_{ij} + b_{ij}I\right], +\right\}$ is a neutrosophic abelian group.

NR2: $N(M) = \left\{\left[a_{ij} + b_{ij}I\right], \times\right\}$ is monoid according to theorems 2.2.and 2.3.[6].

NR3: From part (ii) and (iii) in theorem 2.4, the neutrosophic distributive law is hold. Hence

$N(M_{n\times n}) = \left\{\left[a_{ij} + b_{ij}I\right], +, \times\right\}$ is non- commutative neutrosophic ring with unit■.

Example 2.4. Consider the following two matrices: $A = \begin{bmatrix} 1+0I & 1+0I \\ 0+0I & 1+0I \end{bmatrix}$ and $B = \begin{bmatrix} 1+0I & 0+0I \\ 1+0I & 1+0I \end{bmatrix}$.

Then:$AB = \begin{bmatrix} 1+0I & 1+0I \\ 0+0I & 1+0I \end{bmatrix}\begin{bmatrix} 1+0I & 0+0I \\ 1+0I & 1+0I \end{bmatrix} = \begin{bmatrix} 2+0I & 1+0I \\ 1+0I & 1+0I \end{bmatrix}$, and,

$BA = \begin{bmatrix} 1+0I & 0+0I \\ 1+0I & 1+0I \end{bmatrix}\begin{bmatrix} 1+0I & 1+0I \\ 0+0I & 1+0I \end{bmatrix} = \begin{bmatrix} 1+0I & 1+0I \\ 1+0I & 2+0I \end{bmatrix}$, we see that $AB \neq BA$.

Definition 2.10. Let $N(R) = \langle \mathrm{R}[I],,+,\cdot \rangle$be a neutrosophic ring contains a neutrosophic unit element and $x = (x_1 + x_2 I) \neq 0 \in N(R)$ (not necessarily to be a commutative neutrosophic ring), then x is called a neutrosophic unit in $N(\mathrm{R})$ if there exists a multiplication inverse y such that

$xy = \mathrm{y}x = 1$ and y denoted by x^{-1}.

Theorem 2.6. Consider $N(R) = \langle \mathrm{R}[I],,+,\cdot \rangle$ is a neutrosophic ring contains a neutrosophic unit Let $U(N(R)) = \{x \in N(R) : \exists\, y \in N(R) \ni xy = yx = 1\}$ be the set of all units. Then: $\langle U(N(R)),\cdot \rangle$ is a neutrosophic group under multiplication.

Proof. Since $1 \in N(\mathrm{R})$, then $1 \in U(N(R))$ and $U(N(R)) \neq \emptyset$. Suppose that $x, y \in U(N(R))$,then there exists $x^{-1}, y^{-1} \in N(\mathrm{R})$ such that $xx^{-1} = x^{-1}x = 1$ and $yy^{-1} = y^{-1}y = 1$. Now,

$(y^{-1}x^{-1})(xy) = 1$ and $(xy)(y^{-1}x^{-1}) = 1$ by theorem 3.2. part.2 in [5], hence $x.y \in U(N(R))$. Also, if $x \in U(N(R))$, then $x^{-1} \in U(N(R))$, therefore for all if $x \in U(N(R))$, there is a multiplication neutrosophic inverse $x^{-1} \in U(N(R))$. Moreover, $N(R) = \langle \mathrm{R}[I],,+,\cdot \rangle$ is a neutrosophic ring , then the multiplication is associative in particular of elements of $U(N(R))$ and consequently, $\langle U(N(R)),\cdot \rangle$ is a neutrosophic group.

Definition 2.9.[19]: Let $\langle R \cup I \rangle$ be a neutrosophic ring. A proper subset P of $\langle R \cup I \rangle$ is said to be a neutrosophic subring if P itself is a neutrosophic ring under the operations of $\langle R \cup I \rangle$. It is essential that $P = \langle S \cup nI \rangle, n$ a positive integer where S is a subring of R. i.e. $\{P$ is generated by the subring S together with $n\, I. (n \in Z+)\}$. Note: Even if P is a ring and cannot be represented as $\langle S \cup nI \rangle$ where S is a subring of R then we do not call P a neutrosophic subring of $\langle R \cup I \rangle$.

Theorem 2.7. Consider $N(R) = \langle R[I], +,\bullet \rangle$ is neutrosophic ring and $N(S) \neq \emptyset \subseteq N(R)$, then $N(S)$ is called a neutrosophic subring of $N(R)$ **iff** :

1. $\forall\, a, b \in N(S) \Rightarrow a - b \in N(S)$, and,
2. $\forall\, a, b \in N(S) \Rightarrow ab \in N(S)$.

Note. If $N(S)$ is a neutrosophic subring of $N(R)$, then denoted by: $N(S) \preccurlyeq N(R)$.

Proof. Frist direction, consider $N(R) = \langle R[I], +,\bullet \rangle$ is neutrosophic ring and $N(S) \neq \emptyset \subseteq N(R)$. Assume that $a, b \in N(S) = \{a + bI : a, b \in S\}$

$\Rightarrow a - b = (a_1 + a_2 I) - (b_1 + b_2 I) = \big((a_1 - b_1) + (a_2 - b_2)I\big) \in N(S)$. Also,

$\Rightarrow a.b = (a_1 + a_2 I).(b_1 + b_2 I) = (a_1.b_1) + \big((a_1.b_2) + (a_2.b_1) + (a_2.b_2)\big)I \in N(S)$.

Conversely,

Suppose that $a + b$ and $ab \in N(S)$ for all $a, b \in N(S)$,then $N(S)$ its closure under addition, since $N(R) = \langle R[I], + \rangle$ is a commutative neutrosophic group,, then $N(S) = \langle R[I], + \rangle$ in particular elements is commutative neutrosophic group. Also, $N(R) = \langle R[I],\bullet \rangle$ is a neutrosophic semigroup, so

$N(S) = \langle S[I],\bullet \rangle$ is a neutrosophic semigroup in particular elements of $N(S)$. Finally,

$N(R) = \langle R[I], +,\bullet \rangle$ has the property of NR3, so NR3 is hold in $N(S) = \langle S[I], +,\bullet \rangle$ for particular elements, therefore $N(S) = \langle S[I], +,\bullet \rangle$ is a neutrosophic ring$\blacksquare$.

Example 2.5. Consider $N(\mathrm{Z}_6) = \langle \mathrm{Z}_6\,[I], \oplus_6, \otimes_6 \rangle$is a finite neutrosophic ring under addition and multiplication with modulo 6,where $\mathbb{Z}_6\,[I] = \{a + bI : a, b \in \mathbb{Z}_6\}$, that is,

$\mathbb{Z}_6\,[I] = \{0,1,2,3,4,5,1 + I, \ldots, 1 + 5I, 2 + I, \ldots, 2 + 5I, 3 + I, \ldots, 3 + 5I, 4 + I, \ldots, 4 + 5I, 5 + I, \ldots, 5 + 5I\}$,and

Take $S[I] = \{0, 2I, 4I\} \subseteq \mathbb{Z}_6\,[I]$. Then $S[I] \preccurlyeq \mathbb{Z}_6\,[I]$.

Table.2.4, of (NAS of $N(S[I]) = \langle S[I], \oplus_6 \rangle$.

$\oplus_6$	0	$2I$	$4I$
0	0	$2I$	$4I$
$2I$	$2I$	$4I$	0
$4I$	$4I$	0	$2I$

We see that $S[I]$is closed under addition modulo 6.

Table.2.5, of (NAS of $NS[I]) = \langle S[I], \otimes_6 \rangle$.

$\oplus_6$	0	$2I$	$4I$
0	0	0	0
$2I$	0	$4I$	2I
$4I$	0	2I	$4I$

Since $S[I]$ is closed under multiplication modulo 6.

If $S[I] = \{0,2,4,2I,4I,2+2I,2+4I,4+2I,4+4I\}$, then $S[I] \preccurlyeq \mathbb{Z}_6\,[I]$, because $S[I]$ is closed under addition and multiplication of modulo 6.

Defintion2.11. Let $N(R) = \langle R[I],+,\bullet\rangle$ is neutrosophic ring**,** then the center of neutrosophic ring is denoted by $C(N(R))$ and defined by: $C(N(R)) = \{x \in R[I]: xy = yx, \forall\, y \in R[I]\}$.

Proposition 2.5. If $N(R) = \langle R[I],+,\bullet\rangle$ is neutrosophic ring contains a neutrosophic unit element, then $C(N(R)) \preccurlyeq R[I]$.

Proof. Since $1 = 1 + 0I \in C(N(R))$, then $C(N(R)) \neq \emptyset$. Suppose that $a, b \in C(N(R))$, now, since

$a \in C\big(N(R)\big) \Longrightarrow ax = xa, \forall\, x \in R[I]$

$\Leftrightarrow (a_1 + a_2I)(x_1 + x_2I) = (x_1 + x_2I)(a_1 + a_2I), \forall\, x \in R[I]$

$\Leftrightarrow (a_1.x_1) + \big((a_1.x_2) + (a_2.x_1) + (a_2.x_2)I\big) = (x_1.a_1) + \big((x_2.a_1) + (x_1.a_2) + (x_2.a_2)I\big), \forall\, x \in R[I]$.

Also, $b \in C\big(N(R)\big) \Longrightarrow bx = xb, \forall\, x \in R[I]$

$\Leftrightarrow (b_1 + b_2I)(x_1 + x_2I) = (x_1 + x_2I)(b_1 + b_2I), \forall\, x \in R[I]$

$\Leftrightarrow (b_1.x_1) + \big((b_1.x_2) + (b_2.x_1) + (b_2.x_2)I\big) = (x_1.b_1) + \big((x_2.b_1) + (x_1.b_2) + (x_2.b_2)I\big), \forall\, x \in R[I]$.

Hence $(a - b)x = \big((a_1 + a_2I) - (b_1 + b_2I)\big).(x_1 + x_2I)$

$= \big((a_1 + a_2I).(x_1 + x_2I) - (b_1 + b_2I)\big).(x_1 + x_2I)$

$= (a_1.x_1) + \big((a_1.x_2) + (a_2.x_1) + (a_2.x_2)I\big) - (x_1.b_1) + \big((x_2.b_1) + (x_1.b_2) + (x_2.b_2)I\big)$

$= (x_1.a_1) + ((x_2.a_1) + (x_1.a_2) + (x_2.a_2)I) - (x_1.b_1) + ((x_2.b_1) + (x_1.b_2) + (x_2.b_2)I)$

$= (x_1 + x_2I)(a_1 + a_2I) - (x_1 + x_2I)(b_1 + b_2I)$

$= (x_1 + x_2I)((a_1 + a_2I) - (b_1 + b_2I))$

$= x(a - b)$. Hence $(a - b) \in C(N(R))$. Moreover,

$(ab)x = (a_1 + a_2I)((b_1 + b_2I).(x_1 + x_2I))$.

$= (a_1 + a_2I)((x_1 + x_2I).(b_1 + b_2I))$.

$= ((a_1 + a_2I).(x_1 + x_2I)).(b_1 + b_2I)$.

$= ((x_1 + x_2I).(a_1 + a_2I)).(b_1 + b_2I)$.

$= (x_1 + x_2I).((a_1 + a_2I).(b_1 + b_2I).)$

$= x(ab)$. Therefore $ab \in C(N(R))$. By theorem 2.6. $C(N(R)) \preccurlyeq R[I]$■.

Example 2.6. Consider $N(\mathrm{Z}_3) = \langle \mathrm{Z}_3[I], \oplus_3, \otimes_3 \rangle$is a finite neutrosophic ring under addition and multiplication with modulo 3, where $\mathrm{Z}_3[I] = \{a + bI : a, b \in \mathrm{Z}_3\}$, then $C(N(\mathrm{Z}_3) = \mathrm{Z}_3[I]$, because $\mathrm{Z}_3[I]$ is a commutative neutrosophic ring. Also $U(N(\mathrm{Z}_3)) = \{1 + I, 2 + 2I\}$.

3. Properties of Neutrosophic Elements in Neutrosophic Ring

Definition3.1. Let $N(\mathrm{R}) = \langle \mathrm{R}[I], +, \bullet \rangle$ be a neutrosophic commutative ring and $x \neq 0 \in N(\mathrm{R})$, then x is said to be a zero-divisor, if there exists $y \neq 0 \in N(R)$ such that $x.y = 0$.

Example 3.1. $N(\mathbb{Z}) = \langle \mathbb{Z}[I], +, \bullet \rangle, N(\mathbb{Q}) = \langle \mathbb{Q}[I], +, \bullet \rangle, N(\mathbb{R}) = \langle \mathbb{R}[I], +, \bullet \rangle$ and $N(\mathbb{C}) = \langle \mathbb{C}[I], +, \bullet \rangle$ has no zero divisor. Also $N(\mathrm{Z}_4) = \langle \mathrm{Z}_4[I], \oplus_4, \otimes_4 \rangle$ and $N(\mathrm{Z}_6) = \langle \mathrm{Z}_6[I], \oplus_6, \otimes_6 \rangle$ has no zero divisor, but

$\langle \mathrm{Z}_4, \oplus_4, \otimes_4 \rangle$ and $\langle \mathrm{Z}_6, \oplus_6, \otimes_6 \rangle$ in classical ring theory has zero divisor.

Definition3.2. Let $N(\mathrm{R}) = \langle \mathrm{R}[I], +, \bullet \rangle$ be a neutrosophic commutative ring, then $N(\mathrm{R})$ is called a neutrosophic integral domain, if $N(\mathrm{R})$ it has no zero divisor.

Example 3.2. All neutrosophic ring structure in pervious example are neutrosophic integral domain.

Theorem3.1. Consider $N(\mathrm{Z}_p) = \langle \mathrm{Z}_p\,[I], \oplus_p, \otimes_\mathrm{p} \rangle$ is a neutrosophic ring, then $N(\mathrm{Z}_p) = \langle \mathrm{Z}_p\,[I], \oplus_p, \otimes_\mathrm{p} \rangle$

Is not a neutrosophic field.

Proof. By pervious example 2.3.

Example 3.3. Consider the $N(M_{n\times n}) = \{[a_{ij} + b_{ij}I], +, \times\}$ is non- commutative neutrosophic ring with unit. Take $A = \begin{bmatrix} 0 & 2I \\ 0 & 4I \end{bmatrix} \neq 0$ and $B = \begin{bmatrix} 0 & 2+2I \\ 0 & 0 \end{bmatrix} \neq 0$, then: $AB = \begin{bmatrix} 0 & 2I \\ 0 & 4I \end{bmatrix}\begin{bmatrix} 0 & 2+2I \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$,

Hence A and B are zero dvisors.

Definition3.3.[19] Let $N(\mathrm{R}) = \langle \mathrm{R}[I], +, \bullet \rangle$ be a neutrosophic ring. A characteristic of $N(\mathrm{R})$ is the smallest positive integer n (if there is one) such that $nx = 0, \forall\, x \in \mathrm{R}[I]$. If there is no such integer, we say that neutrosophic ring $\mathrm{R}[I]$ has characteristic zero, otherwise $\mathrm{R}[I]$ has characteristic n and denoted by $N(chR[I]) = n$.

Example 3.4. $N(\mathbb{Z}) = \langle \mathbb{Z}[I], +, \bullet \rangle, N(\mathbb{Q}) = \langle \mathbb{Q}[I], +, \bullet \rangle, N(\mathbb{R}) = \langle \mathbb{R}[I], +, \bullet \rangle$ and $N(\mathbb{C}) = \langle \mathbb{C}[I], +, \bullet \rangle$ have characteristic zero.

Proposition 3.1. Let $N(\mathrm{Z}_n) = \langle \mathrm{Z}_n[I], , \oplus_n, \otimes_\mathrm{n} \rangle$ be a neutrosophic ring. Then $N(ch\,\mathrm{Z}_n[I]) = n$.

Proof. By Principle of Mathematical Induction.

First, If $n = 1$, then $\mathrm{Z}_1[I] = \{a + bI : a, b \in \mathrm{Z}_1\} = \{0 + 0I\}$ and $1.(0 + 0I) = 0$, hence $N(ch\,\mathrm{Z}_1[I]) = 1$. Hence is true statement when $n = 1$.If $n = 2$, then $\mathrm{Z}_2[I] = \{a + bI : a, b \in \mathrm{Z}_2\} = \{0,1, I + 1 + I\}$ and $2.0 = 0, 2.1 = 0(\,mod 2), 2.I = 2I = 0(mod 2), 2.(1 + I) = 2 + 2I = 0\ (\,mod\ 2)$.herefore $N(ch\,\mathrm{Z}_2[I]) = 2$. Hence is true statement when $n = 2$.

Second. Suppose that, $n = k$, then $\mathrm{Z}_k[I] = \{a + bI : a, b \in \mathrm{Z}_k\}$

$\mathrm{Z}_k[I] = \{0,1,2, \dots, k-1, I, 2I, \dots, (k-1)I, 1 + I, 1 + 2I, \dots, 1 + (k-1)I, 2 + I, 2 + 2I, \dots, 2 + (k-1)I, \dots, (k-1) + I, (k-1) + 2I, \dots, (k-1) + (k-1)I\}$. Such that $k.x = 0, \forall\, x \in \mathrm{Z}_k[I]$ is true stamen.

Third, to show that the statement $n = k + 1$ is also true, that is $(k + 1).x = 0, \forall\, x \in \mathrm{Z}_{k+1}[I]$.Now

$(k + 1).x = k.x + 1.x = 0 + x = x\ (mod\ (k + 1)) \Longrightarrow (k + 1).x = 0\ (mod\ (k + 1)$. Hence,

$N(ch\,\mathrm{Z}_{k+1}[I]) = k+1$. Is also true, we deduced that $N(ch\,\mathrm{Z}_n[I]) = n, \forall n \in \mathbb{N}$■.

Theorem3.2. Let $N(R)$ be a neutrosophic ring and x, y and $z \in N(R)$.Then:

1. $x.0 = 0.x = 0$;
2. $x.(-y) = (-x).y = -(xy)$;
3. $(-x).(-y) = xy$, and,

Proof.

1. $x.0 = (x_1 + x_2 I).(0 + 0I) = (x_1 + x_2 I).(0 + 0I)$

$$= (x_1.0 + (x_1.0 + x_2.0 + x_2.0)I)$$

$$= 0 + 0I = 0.$$ By similar way $0.x = 0$.

2. We have from (1) $0 = x.0 = (x_1 + x_2 I)\big((-y_1 - y_2 I) + (y_1 + y_2 I)\big)$

$$= (x_1 + x_2 I).(-y_1 - y_2 I) + (x_1 + x_2 I).(y_1 + y_2 I)$$

(1).

Also, $0 = -(xy) + (xy) = -\big((x_1 + x_2 I).(y_1 + y_2 I)\big) + \big((x_1 + x_2 I).(y_1 + y_2 I)\big)$

(2).

From (1) and (2), we get:

$(x_1 + x_2 I).(-y_1 - y_2 I) + (x_1 + x_2 I).(y_1 + y_2 I)$

$= -\big((x_1 + x_2 I).(y_1 + y_2 I)\big) + \big((x_1 + x_2 I).(y_1 + y_2 I)\big)$. By theorem 3.2, part 3in [5], this is

implying that, $(x_1 + x_2 I).(-y_1 - y_2 I) = -\big((x_1 + x_2 I).(y_1 + y_2 I)\big) \Leftrightarrow x.(-y) = -(xy)$.

3. By the same procedure we can deduced that $(-x).y = -(xy)$. From (2), we have

$(-x).y = -(xy) \Longrightarrow (-x).(-y) = -\big(x(-y)\big) = -\big((-x).y\big) = xy$■.

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