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Neutrosophic soft set

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Abstract. In this paper we study the concept of neutrosophic set of Smarandache. We have introduced this concept in soft sets and defined neutrosophic soft set. Some definitions and operations have been introduced on neutrosophic soft set. Some properties of this concept have been established.

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1. Introduction

The concept of fuzzy sets was introduced by Lotfi A. Zadeh in 1965 [14]. Since then the fuzzy sets and fuzzy logic have been applied in many real life problems in uncertain, ambiguous environment. The traditional fuzzy sets is characterised by the membership value or the grade of membership value. Some times it may be very difficult to assign the membership value for a fuzzy sets. Consequently the concept of interval valued fuzzy sets was proposed [13] to capture the uncertainty of grade of membership value. In some real life problems in expert system, belief system, information fusion and so on , we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Neither the fuzzy sets nor the interval valued fuzzy sets is appropriate for such a situation. Intuitionistic fuzzy sets introduced by Atanassov [3] is appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not handle the indeterminate and inconsistent information which exists in belief system. Smarandache [12] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.
Soft set theory has enriched its potentiality since its introduction by Molodtsov [11].

Based on the several operations on soft sets introduced in [2, 9, 10] some more properties and algebra may be found in [1]. Feng et al. introduced the soft semirings [5]. By means of level soft sets an adjustable approach to fuzzy soft sets based decision making can be found in [6]. We can found some new concept combined with fuzzy sets and rough sets in [7, 8]. Aygünoğlu et al. introduced the Fuzzy soft groups [4]. Works on Soft sets are progressing very rapidly. The purpose of this paper is to combine the neutrosophic set with soft sets. This combination makes a new mathematical model 'Neutrosophic Soft Set'.

2. Preliminaries

In this section we recall some relevant definitions.

Definition 2.1 ([12])**.** A neutrosophic set A on the universe of discourse X is defined as A $= \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$, where $T, I, F : X \to]^-0, 1^+[$ and

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0, 1]$.

Definition 2.2 ([12])**.** A neutrosophic set A is contained in another neutrosophic set B i.e. $A \subseteq B$ if $\forall x \in X$, $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$.

Example 2.3. Assume that the universe of discourse X $= \{x_1, x_2, x_3\}$, where x_1 characterises the capability, x_2 characterises the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a Neutrosophic Set (N S) of X, such that,
A $= \{< x_1, 0.4, 0.5, 0.3 >, < x_2, 0.7, 0.2, 0.4 >, < x_3, 0.8, 0.3, 0.4 >\}$, where the degree of goodness of capability is 0.4, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.3 etc.

Definition 2.4 ([11])**.** Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A $\subset$ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F : A $\to$ P(U).

Using the concept of neutrosophic set now we introduce the concept of neutrosophic soft set.

3. **Neutrosophic Soft Set**

Definition 3.1. Let U be an initial universe set and E be a set of parameters. Consider A $\subset$ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F : A $\to$ P(U).

For illustration we consider an example.

Example 3.2. Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E = { beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive }. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by, U = $\{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters
A = $\{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly' and the parameter e_4 stands for 'moderate'. Suppose that,
F(beautiful) = $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >,$
$< h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$,
F(wooden) = $\{< h_1, 0.6, 0.3, 0.5 >, < h_2, 0.7, 0.4, 0.3 >, < h_3, 0.8, 0.1, 0.2 >,$
$< h_4, 0.7, 0.1, 0.3 >, < h_5, 0.8, 0.3, 0.6 >\}$,
F(costly) = $\{< h_1, 0.7, 0.4, 0.3 >, < h_2, 0.6, 0.7, 0.2 >, < h_3, 0.7, 0.2, 0.5 >,$
$< h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 >\}$.
F(moderate) = $\{< h_1, 0.8, 0.6, 0.4 >, < h_2, 0.7, 0.9, 0.6 >, < h_3, 0.7, 0.6, 0.4 >,$
$< h_4, 0.7, 0.8, 0.6 >, < h_5, 0.9, 0.5, 0.7 >\}$.
The neutrosophic soft set (NSS) (F, E) is a parametrized family
$\{F(e_i), i = 1 \cdots 10\}$ of all neutrosophic sets of U and describes a collection of approximation of an object. The mapping F here is 'houses(.)', where dot(.) is to be filled up by a parameter e ∈ E. Therefore, F(e_1) means 'houses(beautiful)' whose functional-value is the neutrosophic set $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >,$
$< h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$.
Thus we can view the neutrosophic soft set (NSS) (F, A) as a collection of approximation as below:
(F, A) = { beautiful houses = $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >,$
$< h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$, wooden houses = $\{< h_1, 0.6, 0.3, 0.5 >, < h_2, 0.7, 0.4, 0.3 >, < h_3, 0.8, 0.1, 0.2 >,$
$< h_4, 0.7, 0.1, 0.3 >, < h_5, 0.8, 0.3, 0.6 >\}$, costly houses = $\{< h_1, 0.7, 0.4, 0.3 >$
$, < h_2, 0.6, 0.7, 0.2 >, < h_3, 0.7, 0.2, 0.5 >, < h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 >\}$,
moderate houses = $\{< h_1, 0.8, 0.6, 0.4 >, < h_2, 0.7, 0.9, 0.6 >, < h_3, 0.7, 0.6, 0.4 >,$
$< h_4, 0.7, 0.8, 0.6 >, < h_5, 0.9, 0.5, 0.7 >\}\}$,
where each approximation has two parts: (i) a predicate p, and (ii) an approximate value-set v (or simply to be called value-set v).
For example, for the approximation 'beautiful houses = $\{< h_1, 0.5, 0.6, 0.3 >,$
$< h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >, < h_5, 0.8, 0.2, 0.3 >\}$',
we have (i) the predicate name 'beautiful houses', and (ii) the approximate value-set is $\{< h_1, 0.5, 0.6, 0.3 >, < h_2, 0.4, 0.7, 0.6 >, < h_3, 0.6, 0.2, 0.3 >, < h_4, 0.7, 0.3, 0.2 >$
$, < h_5, 0.8, 0.2, 0.3 >\}$. Thus, a neutrosophic soft set (F, E) can be viewed as a collection of approximation like (F, E) = $\{p_1 = v_1, p_2 = v_2, \cdots p_{10} = v_{10}\}$. For the purpose of storing a neutrosophic soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the neutrosophic soft set in the above example). In this table, the entries are c_{ij} corresponding to the house h_i and the parameter e_j, where

c_{ij} = (true-membership value of h_i, indeterminacy-membership value of h_i, falsity-membership value of h_i) in F(e_j). The tabular representation of the neutrosophic soft set (F, A) is as follow:

U	beautiful	wooden	costly	moderate
h_1	(0.5, 0.6, 0.3)	(0.6, 0.3, 0.5)	(0.7, 0.4, 0.3)	(0.8, 0.6, 0.4)
h_2	(0.4, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.6, 0.7, 0.2)	(0.7, 0.9, 0.6)
h_3	(0.6, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7, 0.2, 0.5)	(0.7, 0.6, 0.4)
h_4	(0.7, 0.3, 0.2)	(0.7, 0.1, 0.3)	(0.5, 0.2, 0.6)	(0.7, 0.8, 0.6)
h_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.6)	(0.7, 0.3, 0.4)	(0.9, 0.5, 0.7)

Table 1: Tabular form of the NSS (F, A).

Definition 3.3. The class of all value sets of a neutrosophic soft set (F, E) is called value-class of the neutrosophic soft set and is denoted by $C_{(F,E)}$. For the above example, $C_{(F,E)} = \{v_1, v_2, \cdots v_{10}\}$. Clearly, $C_{(F,E)} \subset P(U)$.

Definition 3.4. Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if $A \subset B$, and $T_{F(e)}(x) \leq T_{G(e)}(x), I_{F(e)}(x) \leq I_{G(e)}(x), F_{F(e)}(x) \geq F_{G(e)}(x), \forall e \in A, x \in U$. We denote it by (F, A) $\subseteq$ (G, B).
(F, A) is said to be neutrosophic soft super set of (G, B) if (G, B)is a neutrosophic soft subset of (F, A). We denote it by (F, A) $\supseteq$ (G, B).

Example 3.5. Consider the two NSSs (F, A) and (G, B) over the common universe U = $\{o_1, o_2, o_3, o_4, o_5\}$. The NSS (F, A) describes the sizes of the objects whereas the NSS (G, B) describes its surface textures. Consider the tabular representation of the NSS (F, A) is as follows:

U	small	large	moderate
o_1	(0.4, 0.3, 0.6)	(0.6, 0.1, 0.7)	(0.5, 0.7, 0.5)
o_2	(0.3, 0.1, 0.4)	(0.6, 0.7, 0.8)	(0.6, 0.3, 0.6)
o_3	(0.6, 0.2, 0.7)	(0.3, 0.1, 0.6)	(0.5, 0.3, 0.8)
o_4	(0.7, 0.1, 0.6)	(0.1, 0.5, 0.7)	(0.7, 0.5, 0.7)
o_5	(0.3, 0.2, 0.4)	(0.6, 0.1, 0.6)	(0.3, 0.2, 0.3)

Table 2: Tabular form of the NSS (F, A).

The tabular representation of the NSS (G, B) is as follows:

U	small	large	moderate	very smooth
o_1	(0.6, 0.4, 0.3)	(0.7, 0.2, 0.5)	(0.6, 0.7, 0.4)	(0.6, 0.8, 0.7)
o_2	(0.7, 0.5, 0.2)	(0.6, 0.7, 0.6)	(0.7, 0.3, 0.5)	(0.5, 0.7, 0.6)
o_3	(0.6, 0.3, 0.5)	(0.7, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.9, 0.4)
o_4	(0.7, 0.1, 0.6)	(0.3, 0.6, 0.4)	(0.7, 0.5, 0.6)	(0.7, 0.3, 0.5)
o_5	(0.5, 0.4, 0.2)	(0.6, 0.6, 0.5)	(0.6, 0.4, 0.3)	(0.5, 0.8, 0.3)

Table 3: Tabular form of the NSS (G, B).

Here (F, A) $\subset$ (G, B).

Definition 3.6. Equality of two neutrosophic soft sets.
Two NSSs (F, A) and (G, B) over the common universe U are said to be equal

if (F, A) is neutrosophic soft subset of (G, B) and (G, B) is neutrosophic soft subset of (F, A). We denote it by (F, A) = (G, B).

Definition 3.7. NOT set of a set of parameters. Let E = $\{e_1, e_2, \cdots e_n\}$ be a set of parameters. The NOT set of E is denoted by $\rceil E$ is defined by $\rceil$E =$\{\neg e_1, \neg e_2, \cdots \neg e_n\}$, where $\neg\ e_i$ = not $e_i, \forall i$ (it may be noted that $\rceil$ and $\neg$ are different operators).

Example 3.8. Consider the example 3.2. Here $\rceil E$ = { not beautiful, not wooden, not costly, not moderate }.

Definition 3.9. Complement of a neutrosophic soft set.
The complement of a neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to$ P(U) is a mapping given by
$F^c(\alpha)$ = neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}, I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.

Example 3.10. Consider the example 3.2. Then $(F, A)^c$ describes the 'not attractiveness of the houses'. We have
F(not beautiful) = $\{< h_1, 0.3, 0.6, 0.5 >, < h_2, 0.6, 0.7, 0.4 >, < h_3, 0.3, 0.2, 0.6 >,$
$< h_4, 0.2, 0.3, 0.7 >< h_5, 0.3, 0.2, 0.8 >\}$,
F(not wooden) = $\{< h_1, 0.5, 0.3, 0.6 >, < h_2, 0.3, 0.4, 0.7 >, < h_3, 0.2, 0.1, 0.8 >,$
$< h_4, 0.3, 0.1, 0.7 >, < h_5, 0.6, 0.3, 0.8 >\}$,
F(not costly) = $\{< h_1, 0.3, 0.4, 0.7 >, < h_2, 0.2, 0.7, 0.6 >, < h_3, 0.5, 0.2, 0.7 >,$
$< h_4, 0.6, 0.2, 0.5 >, < h_5, 0.4, 0.3, 0.7 >\}$,
F(not moderate) = $\{< h_1, 0.4, 0.6, 0.8 >, < h_2, 0.6, 0.9, 0.7 >, < h_3, 0.4, 0.6, 0.7 >,$
$< h_4, 0.6, 0.8, 0.7 >< h_5, 0.7, 0.5, 0.9 >\}$.

Definition 3.11. Empty or Null neutrosophic soft set with respect to a parameter.
A neutrosophic soft set (H, A) over the universe U is termed to be empty or null neutrosophic soft set with respect to the parameter A if $T_{H(e)}(m) = 0, F_{H(e)}(m) = 0$ and $I_{H(e)}(m) = 0, \forall m \in U, \forall e \in A$.
In this case the null neutrosophic soft set (NNSS) is denoted by Φ_A.

Example 3.12. Let U = $\{h_1, h_2, h_3, h_4, h_5\}$, the set of five houses be considered as the universal set and A = { beautiful, wooden, in the green surroundings } be the set of parameters that characterizes the houses. Consider the neutrosophic soft set (H, A) which describes the cost of the houses and
H(beautiful)=$\{< h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, < h_3, 0, 0, 0 >, < h_4, 0, 0, 0 >,$
$< h_5, 0, 0, 0 >\}$,
H(wooden)=$\{< h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, < h_3, 0, 0, 0 >, < h_4, 0, 0, 0 >,$
$< h_5, 0, 0, 0 >\}$,
H(in the green surroundings)=$\{< h_1, 0, 0, 0 >, < h_2, 0, 0, 0 >, < h_3, 0, 0, 0 >,$
$< h_4, 0, 0, 0 >< h_5, 0, 0, 0 >\}$.
Here the NSS (H, A) is the null neutrosophic soft set.

Definition 3.13. Union of two neutrosophic soft sets.
Let (H, A) and (G, B) be two NSSs over the common universe U. Then the union of (H, A) and (G, B) is denoted by '(H, A) ∪ (G, B)' and is defined by (H, A) ∪ (G, B) = (K, C), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$T_{K(e)}(m) = T_{H(e)}(m)$, if $e \in A - B$,
$= T_{G(e)}(m)$, if $e \in B - A$,
$=$ max $(T_{H(e)}(m), T_{G(e)}(m))$, if $e \in A \cap B$.
$I_{K(e)}(m) = I_{H(e)}(m)$, if $e \in A - B$,
$= I_{G(e)}(m)$, if $e \in B - A$,
$= \frac{I_{H(e)}(m)+I_{G(e)}(m)}{2}$, if $e \in A \cap B$.
$F_{K(e)}(m) = F_{H(e)}(m)$, if $e \in A - B$,
$= F_{G(e)}(m)$, if $e \in B - A$,
$=$ min $(F_{H(e)}(m), F_{G(e)}(m))$, if $e \in A \cap B$.

Example 3.14. Let (H, A) and (G, B) be two NSSs over the common universe U. Consider the tabular representation of the NSS (H, A) is as follow:

U	beautiful	wooden	moderate
h_1	(0.6, 0.3, 0.7)	(0.7, 0.3, 0.5)	(0.6, 0.4, 0.5)
h_2	(0.5, 0.4, 0.5)	(0.6, 0.7, 0.3)	(0.6, 0.5, 0.4)
h_3	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.7, 0.4, 0.5)
h_4	(0.8, 0.4, 0.7)	(0.6, 0.3, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.6, 0.7, 0.2)	(0.7, 0.3, 0.4)	(0.8, 0.6, 0.5)

Table 4: Tabular form of the NSS (H, A).

The tabular representation of the NSS (G, B) is as follow:

U	costly	moderate
h_1	(0.7, 0.6, 0.6)	(0.7, 0.8, 0.6)
h_2	(0.8, 0.4, 0.5)	(0.8, 0.8, 0.3)
h_3	(0.7, 0.4, 0.6)	(0.5, 0.6, 0.7)
h_4	(0.6, 0.3, 0.5)	(0.8, 0.5, 0.6)
h_5	(0.8, 0.5, 0.4)	(0.6, 0.3, 0.5)

Table 5: Tabular form of the NSS (G, B).

Then the union of (H, A) and (G, B) is (K, C) whose tabular representation is as:

U	beautiful	wooden	moderate	costly
h_1	(0.6, 0.3, 0.7)	(0.7, 0.3, 0.5)	(0.7, 0.6, 0.5)	(0.7, 0.6, 0.6)
h_2	(0.5, 0.4, 0.5)	(0.6, 0.7, 0.3)	(0.8, 0.65, 0.3)	(0.8, 0.4, 0.5)
h_3	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.5)	(0.7, 0.5, 0.5)	(0.7, 0.4, 0.6)
h_4	(0.8, 0.4, 0.7)	(0.6, 0.3, 0.6)	(0.8, 0.5, 0.6)	(0.6, 0.3, 0.5)
h_5	(0.6, 0.7, 0.2)	(0.7, 0.3, 0.4)	(0.8, 0.45, 0.5)	(0.8, 0.5, 0.4)

Table 6: Tabular form of the NSS (K, C).

Definition 3.15. Intersection of two neutrosophic soft sets.
Let (H, A) and (G, B) be two NSSs over the same universe U. Then the intersection of (H, A) and (G, B) is denoted by '(H, A) ∩ (G, B)' and is defined by (H, A) ∩(G, B) = (K, C), where C = A ∩ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:
$T_{K(e)}(m) = min(T_{H(e)}(m), T_{G(e)}(m)), I_{K(e)}(m) = \frac{I_{H(e)}(m)+I_{G(e)}(m)}{2}$ and
$F_{K(e)}(m) = max(F_{H(e)}(m), F_{G(e)}(m)), \forall e \in C$.

Example 3.16. Consider the above example 3.14. Then that tabular representation of (H, A)∩ (G, B) is as follow:

U	moderate
h_1	(0.6, 0.6, 0.6)
h_2	(0.6, 0.65, 0.4)
h_3	(0.5, 0.5, 0.7)
h_4	(0.7, 0.5, 0.6)
h_5	(0.6, 0.45, 0.5)

Table 7: Tabular form of the NSS (K, C).

For any two NSSs (H, A) and (G, B) over the same universe U and on the basis of the operations defined above, we have the following propositions:

Proposition 3.17. (1) $(H,A) \cup (H,A) = (H,A)$.
(2) $(H,A) \cup (G,B) = (G,B) \cup (H,A)$.
(3) $(H,A) \cap (H,A) = (H,A)$.
(4) $(H,A) \cap (G,B) = (G,B) \cap (H,A)$.
(5) $(H,A) \cup \Phi = (H,A)$.
(6) $(H,A) \cap \Phi = \Phi$.
(7) $[(H,A)^c]^c = (H,A)$.

Proof. The proof of the propositions 1 to 7 are obvious. □

For any three NSSs (H, A), (G, B) and (K, C) over the same universe U, we have the following propositions:

Proposition 3.18. (1) $(H,A) \cup [(G,B) \cup (K,C)] = [(H,A) \cup (G,B)] \cup (K,C)$.
(2) $(H,A) \cap [(G,B) \cap (K,C)] = [(H,A) \cap (G,B)] \cap (K,C)$.
(3) $(H,A) \cup [(G,B) \cap (K,C)] = [(H,A) \cup (G,B)] \cap [(H,A) \cup (K,C)]$.
(4) $(H,A) \cap [(G,B) \cup (K,C)] = [(H,A) \cap (G,B)] \cup [(H,A) \cap (K,C)]$.

Proof. Proofs are simple and thus omitted. □

Definition 3.19. AND operation on two neutrosophic soft sets.
Let (H, A) and (G, B) be two NSSs over the same universe U. Then 'AND' operation on them is denoted by '(H, A) $\bigwedge$ (G, B)' and is defined by
(H, A) $\bigwedge$(G, B) = (K, A × B), where the truth-membership, indeterminacy-membership and falsity-membership of (K, A×B) are as follows:
$T_{K(\alpha,\beta)}(m) = min(T_{H(\alpha)}(m), T_{G(\beta)}(m))$, $I_{K(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m)+I_{G(\beta)}(m)}{2}$,
$F_{K(\alpha,\beta)}(m) = max(F_{H(\alpha)}(m), F_{G(\beta)}(m)), \forall\alpha \in A, \forall\beta \in B$.

Example 3.20. Consider the same example 3.14 above. Then the tabular representation of (H, A) AND (G, B) is as follow:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)
h_1	(0.6, 0.45, 0.7)	(0.6, 0.55, 0.7)	(0.7, 0.45, 0.6)
h_2	(0.5, 0.4, 0.5)	(0.5, 0.6, 0.5)	(0.6, 0.55, 0.5)
h_3	(0.7, 0.4, 0.6)	(0.5, 0.5, 0.7)	(0.7, 0.35, 0.6)
h_4	(0.6, 0.35, 0.7)	(0.8, 0.45, 0.7)	(0.6, 0.3, 0.6)
h_5	(0.6, 0.6, 0.4)	(0.6, 0.5, 0.5)	(0.7, 0.4, 0.4)

U	(wooden, moderate)	(moderate, costly)	(moderate,moderate)
h_1	(0.7, 0.55, 0.6)	(0.6, 0.5, 0.6)	(0.6, 0.6, 0.6)
h_2	(0.6, 0.75, 0.3)	(0.6, 0.45, 0.5)	(0.6, 0.65, 0.4)
h_3	(0.5, 0.45, 0.7)	(0.7, 0.4, 0.6)	(0.5, 0.5, 0.7)
h_4	(0.6, 0.4, 0.6)	(0.6, 0.4, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.6, 0.3, 0.5)	(0.8, 0.55, 0.5)	(0.6, 0.45, 0.5)

Table 8: Tabular representation of the NSS (K, $A \times B$).

Definition 3.21. If (F, A) and (G, B) be two NSSs over the common universe U then '(F, A) OR (G, B)' denoted by (F, A) $\vee$ (G, B) is defined by (F, A) $\vee$ (G, B) = (O, A $\times$ B), where, the truth-membership, indeterminacy-membership and falsity-membership of O(α,β) are given as follows:
$T_{O(\alpha,\beta)}(m) = max(T_{H(\alpha)}(m), T_{G(\beta)}(m))$,
$I_{O(\alpha,\beta)}(m) = \frac{I_{H(\alpha)}(m)+I_{G(\beta)}(m)}{2}$,
$F_{O(\alpha,\beta)}(m) = min(F_{H(\alpha)}(m), F_{G(\beta)}(m)), \forall\alpha \in A, \forall\beta \in B.$

Example 3.22. Consider the same example 3.14 above. Then the tabular representation of (H, A) OR (G, B) is as follow:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)
h_1	(0.7, 0.45, 0.6)	(0.7, 0.55, 0.6)	(0.7, 0.45, 0.5)
h_2	(0.8, 0.4, 0.5)	(0.8, 0.6, 0.3)	(0.8, 0.55, 0.3)
h_3	(0.7, 0.4, 0.3)	(0.7, 0.5, 0.3)	(0.7, 0.35, 0.5)
h_4	(0.8, 0.35, 0.5)	(0.8, 0.45, 0.6)	(0.6, 0.3, 0.5)
h_5	(0.8, 0.6, 0.2)	(0.8, 0.5, 0.2)	(0.8, 0.4, 0.4)

U	(wooden, moderate)	(moderate, costly)	(moderate,moderate)
h_1	(0.7, 0.55, 0.5)	(0.7, 0.5, 0.5)	(0.7, 0.6, 0.5)
h_2	(0.8, 0.75, 0.3)	(0.8, 0.45, 0.4)	(0.8, 0.65, 0.3)
h_3	(0.7, 0.45, 0.5)	(0.7, 0.4, 0.5)	(0.7, 0.5, 0.5)
h_4	(0.8, 0.4, 0.6)	(0.7, 0.4, 0.5)	(0.8, 0.5, 0.6)
h_5	(0.7, 0.3, 0.4)	(0.8, 0.55, 0.4)	(0.8, 0.45, 0.5)

Table 9: Tabular representation of the NSS (O, $A \times B$).

For any two NSSs (H, A) and (G, B) over the common universe U, the De Morgan's types of results are true.

Proposition 3.23. (1) $[(H, A) \vee (G, B)]^c = (H, A)^c \wedge (G, B)^c$
(2) $[(H, A) \wedge (G, B)]^c = (H, A)^c \vee (G, B)^c$

Proof 1. Let (H, A) = $\{< h, T_{H(x)}(h), I_{H(x)}(h), F_{H(x)}(h) > | h \in U\}$ and (G, B) = $\{< h, T_{G(x)}(h), I_{G(x)}(h), F_{G(x)}(h) > | h \in U\}$ be two NSSs over the common universe U. Also let $(O, A \times B) = (H, A) \vee (G, B)$, where,

$$O(\alpha, \beta) = \{< h, max(T_{H(\alpha)}(h), T_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2}, min(F_{H(\alpha)}(h), F_{G(\beta)}(h)) > | h \in U\}.$$

Therefore,

$[(H,A)\vee(G,B)]^c = (O, A\times B)^c = \{< h, min(F_{H(\alpha)}(h), F_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2},$
$max(T_{H(\alpha)}(h), T_{G(\beta)}(h)) > |h \in U\}.$

Again

$(H,A)^c \wedge (G,B)^c$
$= \{< h, min(F_{H^c(\alpha)}(h), F_{G^c(\beta)}(h)), \frac{I_{H^c(\alpha)}(h)+I_{G^c(\beta)}(h)}{2},$
$max(T_{H^c(\alpha)}(h), T_{G^c(\beta)}(h)) >| h \in U\}$
$= \{< h, max(T_{H(\alpha)}(h), T_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2},$
$min(F_{H(\alpha)}(h), F_{G(\beta)}(h)) >| h \in U\}^c$
$= \{< h, min(F_{H(\alpha)}(h), F_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2},$
$max(T_{H(\alpha)}(h), T_{G(\beta)}(h)) >| h \in U\}.$

Hence the result is proved. □

Proof 2. Let

(H, A) $= \{< h, T_{H(x)}(h), I_{H(x)}(h), F_{H(x)}(h) > |h \in U\}$

and

(G, B) $= \{< h, T_{G(x)}(h), I_{G(x)}(h), F_{G(x)}(h) > |h \in U\}$

be two NSSs over the common universe U. Also let $(K, A \times B) = (H, A) \wedge (G, B)$, where,

$K(\alpha,\beta) = \{< h, min(T_{H(\alpha)}(h), T_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2},$
$max(F_{H(\alpha)}(h), F_{G(\beta)}(h)) >| h \in U\}.$

Therefore,

$[(H,A) \wedge (G,B)]^c = (K, A \times B)^c$
$= \{< h, max(F_{H(\alpha)}(h), F_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2},$
$min(T_{H(\alpha)}(h), T_{G(\beta)}(h)) > |h \in U\}.$

Again

$(H,A)^c \vee (G,B)^c$
$= \{< h, max(F_{H^c(\alpha)}(h), F_{G^c(\beta)}(h)), \frac{I_{H^c(\alpha)}(h)+I_{G^c(\beta)}(h)}{2},$
$min(T_{H^c(\alpha)}(h), T_{G^c(\beta)}(h)) >| h \in U\}$
$= \{< h, min(T_{H(\alpha)}(h), T_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2},$
$max(F_{H(\alpha)}(h), F_{G(\beta)}(h)) >| h \in U\}^c$
$= \{< h, max(F_{H(\alpha)}(h), F_{G(\beta)}(h)), \frac{I_{H(\alpha)}(h)+I_{G(\beta)}(h)}{2},$
$min(T_{H(\alpha)}(h), T_{G(\beta)}(h)) >| h \in U\}.$

Hence the result is proved. □

4. An application of neutrosophic soft set in a decision making problem

We consider the problem to select the most suitable house which Mr. X is going to choose on the basis of his m number of parameters out of n number of houses. Let the n number of houses are $h_1, h_2, \cdots h_n$ and the m numbers of choice parameters are $e_1, e_2, \cdots e_m$. We also assume that corresponding to the parameter $e_j (j = 1, 2 \cdots m)$ the rating or performance value of the house $h_i (i = 1, 2, \cdots n)$ is a tuple $p_{ij} = (T_{H(e_j)}(h_i), I_{H(e_j)}(h_i), T_{F(e_j)}(h_i))$, such that for a fixed i that values $p_{ij} (j = 1, 2 \cdots m)$ represents a Neutrosophic Soft Set of all the n objects. Thus the performance values could be arranged in the form of a matrix called the 'criteria matrix'. If the criteria values are more, preferability of the corresponding object is also more. Our problem is to select the most suitable object i.e. the object which dominates each of the objects of the spectrum of the parameters e_j. Since the data are not crisp but neutrosophic soft the selection is not straightforward. The problem is to select the house which is the most suitable with the choice parameters of Mr. X. The house which is suitable for Mr. X need not be suitable for Mr. Y or Mr. Z, as the selection is dependent on the choice parameters of each buyer.We use the technique to calculate the score for the objects.

Definition 4.1. Comparison Matrix. It is a matrix whose rows are labelled by the object names $h_1, h_2, \cdots h_n$ and the columns are labelled by the parameters $e_1, e_2, \cdots e_m$. The entries c_{ij} are calculated by c_{ij} = a + b - c, where 'a' is the integer calculated as 'how many times $T_{h_i}(e_j)$ exceeds or equal to $T_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in$ U, 'b'is the integer calculated as 'how many times $I_{h_i}(e_j)$ exceeds or equal to $I_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in$ U and 'c' is the integer 'how many times $F_{h_i}(e_j)$ exceeds or equal to $F_{h_k}(e_j)$', for $h_i \neq h_k$, $\forall h_k \in$ U.

Definition 4.2. Score of an Object. The score of an object h_i is S_i and is calculated as $S_i = \sum_j c_{ij}$.
Now we present an algorithm for most appropriate selection of an object.

Algorithm

(1) input the Neutrosophic Soft Set (H, A)
(2) input P, the choice parameters of Mr. X which is a subset of A
(3) consider the NSS (H, P) and write it in tabular form
(4) compute the comparison matrix of the NSS (H, P)
(5) compute the score S_i of $h_i, \forall i$
(6) find $S_k = max_i S_i$
(7) if k has more than one value then any one of h_i could be the preferable choice.

Let us use the algorithm to solve the problem. Suppose
P = { beautiful, cheap, in good repairing, moderate, wooden }.
Consider the tabular representation of the NSS (H, P) is as below:

U	beautiful	cheap	in good repairing	moderate	wooden
h_1	(0.6, 0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, 0.4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)
h_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)
h_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)
h_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)
h_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7, 0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)

Table 10: Tabular form of the NSS (H, P).

The comparison-matrix of the above NSS (H, P) is as below:

U	beautiful	cheap	in good repairing	moderate	wooden
h_1	-2	0	3	2	4
h_2	0	1	2	3	0
h_3	6	6	-1	2	2
h_4	3	4	2	1	3
h_5	5	0	5	4	2

Table 11: Comparison matrix of the NSS (H, P).

Next we compute the score for each h_i as shown below:

U	score (S_i)
h_1	7
h_2	6
h_3	15
h_4	13
h_5	16

Clearly, the maximum score is 16, scored by the house h_5.
Decision Mr. X will select the house h_5. In any case if he does not want to choose h_5 due to some reasons his second choice will be h_3.

5. Conclusions

In this paper we study the notion of neutrosophic set initiated by Smarandache. We use this concept in soft sets considering the fact that the parameters (which are words or sentences) are mostly neutrosophic set. We define various operations on NSS and prove some results on them. Finally, we present an application of NSS in a decision making problem.

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