

# Linear weighted averaging method on SVN-sets and its sensitivity analysis based on multi-attribute decision making problems

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## Abstract

In this paper, we develop a method of multi-attribute decision-making with both weights and attribute ratings expressed by single valued neutrosophic sets(SVN-sets). The method is called linear weighted averaging method of SVN-sets. Then, we present a sensitivity analysis of attribute weights which give changing intervals of attribute weights in which the ranking order of the alternatives is required to remain unchanging. Finally, validity and applicability of the proposed method are illustrated with a real application.

**Keyword 0.1** *Neutrosophic set, single valued neutrosophic numbers, linear weighted averaging method, sensitivity analysis, multi-attribute decision making.*

## 1 Introduction

Since Atanassov [1] proposed intuitionistic fuzzy set theory, some generalized applications have been proposed and studied to handle vagueness in [5, 6, 9, 17, 20, 22, 16, 32, 42]. The intuitionistic fuzzy set considers only the degree of membership and non-membership which is one sum of degree of membership and non-membership. To handle such situation, Smarandache [27, 28] introduced the notions of neutrosophic sets which allow to incorporate simultaneously the truth-membership degree, indeterminacy-membership degree and falsity-membership degree of each element, as a generalization of the notion of an intuitionistic fuzzy set. Each element of neutrosophic set is characterized by a the truth-membership degree, indeterminacy-membership degree and falsity-membership degree such that  $-0 \leq \text{truth-membership degree} + \text{indeterminacy-membership degree} + \text{falsity-membership degree} \leq 3^+$  where  $-0 \leq \text{truth-membership degree} \leq 1^+$ ,  $-0 \leq \text{indeterminacy-membership degree} \leq 1^+$  and  $-0 \leq \text{falsity-membership degree} \leq 1^+$ . After Smarandache, Wang et al. [31] introduced the definition of single valued neutrosophic set to apply the idea of neutrosophic

set to real life applications in scientific and engineering problems. The neutrosophic sets has been applied to many different fields, such as; supplier selection [30], medical diagnoses [35], image thresholding [7, 15], image segmentation [14], matrices [8], decision making [2, 3, 4, 10, 11, 12], topological spaces [26], clustering [39, 40], and so on.

In multiple-attribute decision-making problems, decision maker usually need to compare a set of alternatives by using attributes with different weight. Thereafter, the decision maker need to rank the given alternatives. But the ranking of alternatives with neutrosophic sets is a significant issue. Methodically to rank alternatives with neutrosophic values, one neutrosophic element needs to be compared with the others, but it is arduous to determine clearly which of them is larger or smaller. Therefore, many methods have been raised in literature to rank alternatives with neutrosophic values (e.g. [24, 25, 33, 34, 36, 38, 43]).

Feng [13] said that "Decision may change with the changes of time, conditions or environments. The problem how changes of ratings of alternatives on attributes or attribute weights affect final decision results is of useness in theoretical and practical research. In other words, we highly concern with what conditions should be satisfied if the final decision results are required to remain unchanging. These problems are called sensitivity analysis". In this study we extend the linear weighted averaging method and sensitivity analysis as well as applications of intuitionistic fuzzy sets by given [13, 18, 19, 21] to neutrosophic sets. Therefore, we organize the rest of the paper as follows: in the following section, we present preliminary definitions of intuitionistic fuzzy set, neutrosophic sets and single valued neutrosophic sets. In Section 3, we proposed a method of multi-attribute decision-making with both weights and attribute ratings expressed by single valued neutrosophic sets(SVN-sets). In Section 4, we introduced a sensitivity analysis of attribute weights by using the linear weighted averaging method. In Section 5, validity and applicability of the proposed method are illustrated with a real application. In last section, conclusion are presented.

## 2 Preliminaries

**Definition 2.1** [41] *Let  $E$  be a universe. Then a fuzzy set  $X$  over  $E$  is defined by*

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where  $\mu_X$  is called membership function of  $X$  and defined by  $\mu_X : E \rightarrow [0,1]$ . For each  $x \in E$ , the value  $\mu_X(x)$  represents the degree of  $x$  belonging to the fuzzy set  $X$ .

**Definition 2.2** [1] *Let  $E$  be a universe. An intuitionistic fuzzy set  $K$  over  $E$  is defined by*

$$K = \{< x, \mu_K(x), \gamma_K(x) > : x \in E\}$$

where  $\mu_K : E \rightarrow [0, 1]$  and  $\gamma_K : E \rightarrow [0, 1]$  such that  $0 \leq \mu_K(x) + \gamma_K(x) \leq 1$  for any  $x \in E$ . For each  $x \in E$ , the values  $\mu_K(x)$  and  $\gamma_K(x)$  are the degree of membership and degree of non-membership of  $x$ , respectively.

**Definition 2.3** [27] Let  $E$  be a universe. A neutrosophic set  $A$  over  $E$  is defined by

$$A = \{ \langle x, (T_A(x), I_A(x), F_A(x)) \rangle : x \in E \}.$$

where  $T_A(x)$ ,  $I_A(x)$  and  $F_A(x)$  are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively. They are respectively defined by

$$T_A : E \rightarrow ]^{-}0, 1^{+}[ , \quad I_A : E \rightarrow ]^{-}0, 1^{+}[ , \quad F_A : E \rightarrow ]^{-}0, 1^{+}[$$

such that  $0^{-} \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$ .

**Definition 2.4** [31] Let  $E$  be a universe. An SVN-set over  $E$  is a neutrosophic set over  $E$ , but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by

$$T_A : E \rightarrow [0, 1], \quad I_A : E \rightarrow [0, 1], \quad F_A : E \rightarrow [0, 1]$$

such that  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ .

**Definition 2.5** [24] Let  $A$  and  $B$  be two SVN-sets. Then the operations of SVN-sets can be defined as follows:

1.  $A + B = \{ \langle x, (T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x)) \rangle : x \in E \}$
2.  $A.B = \{ \langle x, (T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x)I_B(x), F_A(x) + F_B(x) - F_A(x)F_B(x)) \rangle : x \in E \}$
3.  $kA = \{ \langle x, (1 - (1 - T_A(x))^k, I_A(x)^k, F_A(x)^k) \rangle : x \in E \}$
4.  $A^k = \{ \langle x, (T_A(x)^k, 1 - (1 - I_A(x))^k, 1 - (1 - F_A(x))^k) \rangle : x \in E \}$

where  $k \in \mathbb{R}$ .

**Definition 2.6** [24] Let  $X = \{x_1, x_2, \dots, x_n\}$  be a set of alternatives,  $U = \{o_1, o_2, \dots, o_m\}$  be the set of attributes. The ratings (or evaluations) of alternatives  $x_j \in X (j = 1, 2, \dots, n)$  on attributes  $o_i \in O(o_1, o_2, \dots, o_m)$  are expressed with SVN-sets  $A_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ , where  $T_A : E \rightarrow [0, 1]$ ,  $I_A : E \rightarrow [0, 1]$ ,  $F_A : E \rightarrow [0, 1]$

such that  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ . Then

$$[A_{ij}]_{m \times n} = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ \begin{matrix} o_1 \\ o_2 \\ \vdots \\ o_m \end{matrix} & \left( \begin{array}{cccc} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{array} \right) \end{matrix}$$

is called an SVN-multi-criteria decision making matrix.

In [24], the authors assumed that the weights  $\omega_i$  of the attributes  $o_i \in O (i = 1, 2, \dots, m)$  are a fuzzy concept, which is difficult to be precisely determined in real applications. Therefore, by taking inspiration [30], we assume that weight of each attribute  $o_i \in O (i = 1, 2, \dots, m)$  is expressed with the SVN-set  $\omega_i = \langle \alpha_i, \beta_i, \gamma_i \rangle$ , where  $\alpha_i \in [0, 1]$ ,  $\beta_i \in [0, 1]$ ,  $\gamma_i \in [0, 1]$  such that  $0 \leq \alpha_i + \beta_i + \gamma_i \leq 3$ , which is truth-degree, indeterminacy-degree and falsity-degree of the attribute  $o_i \in O$ , respectively. The weights of all attributes are also written shortly as follows:

$$\omega = (\omega_1, \omega_2, \dots, \omega_m) = (\langle \alpha_1, \beta_1, \gamma_1 \rangle, \langle \alpha_2, \beta_2, \gamma_2 \rangle, \dots, \langle \alpha_m, \beta_m, \gamma_m \rangle)$$

which is called the SVN-weight vector.

**Definition 2.7** Let  $A_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle (i = 1, 2, \dots, m; j = 1, 2, \dots, n)$  be a SVN-multi-criteria decision making matrix and  $\omega_i = \langle \alpha_i, \beta_i, \gamma_i \rangle (i = 1, 2, \dots, m)$  be a SVN-weight vector. Then, the products of the SVN-sets  $A_{ij}$  and  $\omega_i$ , denoted by  $\bar{A}_{ij} = \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle$ , is given as

$$[\bar{A}_{ij}]_{m \times n} = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ \begin{matrix} o_1 \\ o_2 \\ \vdots \\ o_m \end{matrix} & \left( \begin{array}{cccc} \langle \bar{T}_{11}, \bar{I}_{11}, \bar{F}_{11} \rangle & \langle \bar{T}_{12}, \bar{I}_{12}, \bar{F}_{12} \rangle & \cdots & \langle \bar{T}_{1n}, \bar{I}_{1n}, \bar{F}_{1n} \rangle \\ \langle \bar{T}_{21}, \bar{I}_{21}, \bar{F}_{21} \rangle & \langle \bar{T}_{22}, \bar{I}_{22}, \bar{F}_{22} \rangle & \cdots & \langle \bar{T}_{2n}, \bar{I}_{2n}, \bar{F}_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \bar{T}_{m1}, \bar{I}_{m1}, \bar{F}_{m1} \rangle & \langle \bar{T}_{m2}, \bar{I}_{m2}, \bar{F}_{m2} \rangle & \cdots & \langle \bar{T}_{mn}, \bar{I}_{mn}, \bar{F}_{mn} \rangle \end{array} \right) \end{matrix}$$

where

$$\langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle = \omega_i A_{ij} = \langle \alpha_i, \beta_i, \gamma_i \rangle \langle T_{ij}, I_{ij}, F_{ij} \rangle = \langle \alpha_i T_{ij}, \beta_i + I_{ij} - \beta_i I_{ij}, \gamma_i + F_{ij} - \gamma_i F_{ij} \rangle$$

which is called weighted SVN-multi-criteria decision making matrix.

**Definition 2.8** Let  $\bar{A}_{ij} = \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle (i = 1, 2, \dots, m; j = 1, 2, \dots, n)$  be a weighted SVN-multi-criteria decision making matrix. Then, comprehensive evaluation of each alternative  $x_i \in X (j = 1, 2, \dots, n)$ , denoted

$V_j$ , is given by;

$$V_j = \sum_{i=1}^m \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle = \langle T_j, I_j, F_j \rangle$$

**Definition 2.9** [23] Let  $\bar{A}_{ij} = \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ) be a weighted SVN-multi-criteria decision making matrix and  $V_j = \sum_{i=1}^m \langle \bar{T}_{ij}, \bar{I}_{ij}, \bar{F}_{ij} \rangle = \langle T_j, I_j, F_j \rangle$  be a comprehensive evaluation. Then,

1. score function of  $V_j$  ( $j=1,2,\dots,n$ ), denoted  $s(V_j)$ , defined as;

$$s(V_j) = 2 + \bar{T}_j - \bar{F}_j - \bar{I}_j$$

2. accuracy function of  $V_j$  ( $j=1,2,\dots,n$ ), denoted  $a(V_j)$ , defined as;

$$a(V_j) = \bar{T}_j - \bar{F}_j$$

and then,

(a) If  $s(V_j) < s(V_i)$ , then  $V_i$  is smaller than  $V_j$ , denoted by  $V_i < V_j$

(b) If  $s(V_j) = s(V_i)$ ;

i. If  $a(V_j) < a(V_i)$ , then  $V_i$  is smaller than  $V_j$ , denoted by  $V_i < V_j$

ii. If  $s(V_j) = s(V_i)$ , then  $V_i$  and  $V_j$  are the same, denoted by  $V_i = V_j$

According to the scoring function ranking method of SVN-sets, the ranking order of the set of the alternatives can be generated and the best alternative can be determined.

### 3 Sensitivity analysis of the linear weighted averaging method for multiattribute decision-making with SVN-sets

In this section, we present sensitivity analysis of attribute weights in the linear weighted averaging method of multiattribute decision-making with SVN-sets.

”Decision may change with the changes of time, conditions or environments. The problem how changes of ratings of alternatives on attributes or attribute weights affect final decision results is of usefulness in theoretical and practical research. In other words, we highly concern with what conditions should be satisfied if the final decision results are required to remain unchanging. These problems are called sensitivity analysis” [13].

**Definition 3.1 (Sensitivity analysis)** Let  $A_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ) be a SVN-multi-criteria decision making matrix,  $\omega_i = \langle \alpha_i, \beta_i, \gamma_i \rangle$  ( $i = 1, 2, \dots, m$ ) be a SVN-weight vector and  $\omega' =$

$(\omega_1, \omega_2, \dots, \omega_m) = (\langle \alpha_1, \beta_1, \gamma_1 \rangle, \langle \alpha_2, \beta_2, \gamma_2 \rangle, \dots, \langle \alpha_k + \Delta\alpha_k, \beta_k + \Delta\beta_k, \gamma_k + \Delta\gamma_k \rangle, \dots, \langle \alpha_m, \beta_m, \gamma_m \rangle)^T$  be a changed SVN-weight vector where  $\Delta\alpha_k$ ,  $\Delta\beta_k$  and  $\Delta\gamma_k$  are increments of  $\alpha_k$ ,  $\beta_k$  and  $\gamma_k$ , respectively. Then, comprehensive evaluation  $V_j$  of the alternative  $x_j$  can be rewritten as follows:

$$\begin{aligned} V_j &= \sum_{i=1, i \neq k}^m \omega_i A_{ij} + \omega_k A_{kj} \\ &= \langle x_j, y_j, z_j \rangle + \langle \alpha_k T_{kj}, \beta_k + I_{kj} - \beta_k I_{kj}, \gamma_k + F_{kj} - \gamma_k F_{kj} \rangle \\ &= \langle x_j + \alpha_k T_{kj} - x_j \alpha_k T_{kj}, y_j (\beta_k + I_{kj} - \beta_k I_{kj}), z_j (\gamma_k + F_{kj} - \gamma_k F_{kj}) \rangle \end{aligned}$$

where

$$\langle x_j, y_j, z_j \rangle = \sum_{i=1, i \neq k}^m \omega_i A_{ij}$$

and

$$\omega_k A_{kj} = \langle \alpha_k, \beta_k, \gamma_k \rangle \langle T_{kj}, I_{kj}, F_{kj} \rangle = \langle \alpha_k T_{kj}, \beta_k + I_{kj} - \beta_k I_{kj}, \gamma_k + F_{kj} - \gamma_k F_{kj} \rangle$$

Therefore, we have:

$$T_j = x_j + \alpha_k T_{kj} - x_j \alpha_k T_{kj},$$

$$I_j = y_j (\beta_k + I_{kj} - \beta_k I_{kj})$$

and

$$F_j = z_j (\gamma_k + F_{kj} - \gamma_k F_{kj}).$$

Likewise, the changed comprehensive evaluation  $V'_j$  of the alternative  $x_j$  with the weight change of the attribute  $o_k$  can be calculated as follows;

$$\begin{aligned} V'_j &= \langle x_j, y_j, z_j \rangle + \langle (\alpha_k + \Delta\alpha_k) T_{kj}, \beta_k + \Delta\beta_k + I_{kj} - (\beta_k + \Delta\beta_k) I_{kj}, \gamma_k + \Delta\gamma_k + F_{kj} - (\gamma_k + \Delta\gamma_k) F_{kj} \rangle \\ &= \langle x_j + \alpha_k T_{kj} + \Delta\alpha_k T_{kj} - x_j \alpha_k T_{kj} - x_j \Delta\alpha_k T_{kj}, y_j (\beta_k + I_{kj} - \beta_k I_{kj}) + y_j (\Delta\beta_k - \Delta\beta_k I_{kj}), \\ &\quad z_j (\gamma_k + F_{kj} - \gamma_k F_{kj}) + z_j (\Delta\gamma_k - \Delta\gamma_k F_{kj}) \rangle \\ &= \langle T_j + \Delta\alpha_k T_{kj} (1 - x_j), I_j + \Delta\beta_k y_j (1 - I_{kj}), F_j + \Delta\gamma_k z_j (1 - F_{kj}) \rangle \end{aligned}$$

where

$$\begin{aligned} \omega_k A_{kj} &= \langle \alpha_k + \Delta\alpha_k, \beta_k + \Delta\beta_k, \gamma_k + \Delta\gamma_k \rangle \langle T_{kj}, I_{kj}, F_{kj} \rangle \\ &= \langle (\alpha_k + \Delta\alpha_k) T_{kj}, \beta_k + \Delta\beta_k + I_{kj} - (\beta_k + \Delta\beta_k) I_{kj}, \gamma_k + \Delta\gamma_k + F_{kj} - (\gamma_k + \Delta\gamma_k) F_{kj} \rangle \end{aligned}$$

In a similar way, the changed comprehensive evaluations  $V'_s$  and  $V'_t$  of the alternatives  $x_s$  and  $x_t$  with the weight change of the attribute  $o_k$  can be calculated as follows:

$$\begin{aligned} V'_s &= \langle x_s, y_s, z_s \rangle + \langle (\alpha_k + \Delta\alpha_k) T_{ks}, \beta_k + \Delta\beta_k + I_{ks} - (\beta_k + \Delta\beta_k) I_{ks}, \gamma_k + \Delta\gamma_k + F_{ks} - (\gamma_k + \Delta\gamma_k) F_{ks} \rangle \\ &= \langle T_s + \Delta\alpha_k T_{ks} (1 - x_s), I_s + \Delta\beta_k y_s (1 - I_{ks}), F_s + \Delta\gamma_k z_s (1 - F_{ks}) \rangle \end{aligned}$$

and

$$\begin{aligned} V'_t &= \langle x_t, y_t, z_t \rangle + \langle (\alpha_k + \Delta\alpha_k)T_{kt}, \beta_k + \Delta\beta_k + I_{kt} - (\beta_k + \Delta\beta_k)I_{kt}, \gamma_k + \Delta\gamma_k + F_{kt} - (\gamma_k + \Delta\gamma_k)F_{kt} \rangle \\ &= \langle T_t + \Delta\alpha_k T_{kt}(1 - x_t), I_t + \Delta\beta_k y_t(1 - I_{kt}), F_t + \Delta\gamma_k z_t(1 - F_{kt}) \rangle \end{aligned}$$

respectively, where

$$\begin{aligned} T_s &= x_s + \alpha_k T_{ks} - x_s \alpha_k T_{ks}, \\ I_s &= y_s(\beta_k + I_{ks} - \beta_k I_{ks}) \\ F_s &= z_s(\gamma_k + F_{ks} - \gamma_k F_{ks}), \\ T_t &= x_t + \alpha_k T_{kt} - x_t \alpha_k T_{kt}, \\ I_t &= y_t(\beta_k + I_{kt} - \beta_k I_{kt}) \\ \text{and} \\ F_t &= z_t(\gamma_k + F_{kt} - \gamma_k F_{kt}). \end{aligned}$$

Then, we can calculate the scores of  $V'_j$ ,  $V'_s$ , and  $V'_t$  as follows:

$$\begin{aligned} s(V'_j) &= 2 + T_j - I_j - F_j + \Delta\alpha_k T_{kj}(1 - x_j) - \Delta\beta_k y_j(1 - I_{kj}) - \Delta\gamma_k z_j(1 - F_{kj}) \\ s(V'_s) &= 2 + T_s - I_s - F_s + \Delta\alpha_k T_{ks}(1 - x_s) - \Delta\beta_k y_s(1 - I_{ks}) - \Delta\gamma_k z_s(1 - F_{ks}) \\ s(V'_t) &= 2 + T_t - I_t - F_t + \Delta\alpha_k T_{kt}(1 - x_t) - \Delta\beta_k y_t(1 - I_{kt}) - \Delta\gamma_k z_t(1 - F_{kt}) \end{aligned}$$

Also, we can obtain the accuracies of  $V'_j$ ,  $V'_s$ , and  $V'_t$  as follows:

$$\begin{aligned} a(V'_j) &= T_j - F_j + \Delta\alpha_k T_{kj}(1 - x_j) - \Delta\gamma_k z_j(1 - F_{kj}) \\ a(V'_s) &= T_s - F_s + \Delta\alpha_k T_{ks}(1 - x_s) - \Delta\gamma_k z_s(1 - F_{ks}) \\ a(V'_t) &= T_t - F_t + \Delta\alpha_k T_{kt}(1 - x_t) - \Delta\gamma_k z_t(1 - F_{kt}) \end{aligned}$$

Without loss of generality, assume that the first ranking order of the three alternatives  $x_j, x_s$  and  $x_t$  is  $x_j > x_s > x_t$ . When the weight  $\omega_t$  of the attribute  $o_k$  is changed to  $\omega'_t$ , if the ranking order of the alternatives  $x_j, x_s$  and  $x_t$  are required to remain unchanging, then according to the scoring function ranking method of SVN-sets, the scores and accuracies of  $V'_j, V'_s$  and  $V'_t$  should satisfy either

1.  $s(V'_j) > s(V'_s)$  and  $s(V'_s) > s(V'_t)$

or

2.  $s(V'_j) = s(V'_s)$ ,  $s(V'_s) = s(V'_t)$ ,  $a(V'_j) > a(V'_s)$ , and  $a(V'_s) > a(V'_t)$ .

Finally, we have the systems of inequalities as follows:

1.

$$\begin{aligned}
s(V'_j) &> s(V'_s) \\
s(V'_s) &> s(V'_t) \\
0 &\leq \alpha_k + \Delta\alpha_k + \beta_k + \Delta\beta_k + \gamma_k + \Delta\gamma_k \leq 3, \\
0 &\leq \alpha_k + \Delta\alpha_k \leq 1 \\
0 &\leq \beta_k + \Delta\beta_k \leq 1 \\
0 &\leq \gamma_k + \Delta\gamma_k \leq 1
\end{aligned}$$

2.

$$\begin{aligned}
s(V'_j) &= s(V'_s) \\
s(V'_s) &= s(V'_t) \\
a(V'_j) &> a(V'_s) \\
a(V'_s) &> a(V'_t) \\
0 &\leq \alpha_k + \Delta\alpha_k + \beta_k + \Delta\beta_k + \gamma_k + \Delta\gamma_k \leq 3, \\
0 &\leq \alpha_k + \Delta\alpha_k \leq 1 \\
0 &\leq \beta_k + \Delta\beta_k \leq 1 \\
0 &\leq \gamma_k + \Delta\gamma_k \leq 1
\end{aligned}$$

and

Solving either 1. or 2., we can obtain the changing ranges  $\Delta\alpha_k$ ,  $\Delta\beta_k$  and  $\Delta\gamma_k$  of the weight  $\omega_k$  of the attribute  $o_k$ . Namely, if the weight  $\omega_k$  takes any value between  $\langle \alpha_k, \beta_k, \gamma_k \rangle$ ; and  $\langle \alpha_k + \Delta\alpha_k, \beta_k + \Delta\beta_k, \gamma_k + \Delta\gamma_k \rangle$ , then, the ranking order of the alternatives still remains unchanging.

## 4 SVN-Linear Weighted Averaging Method

In this section, we give a method, which is called linear weighted averaging method, for sensitivity analysis of SVN-weights of the attributes;

Let  $X = \{x_1, x_2, \dots, x_n\}$  be a set of alternatives,  $O = \{o_1, o_2, \dots, o_m\}$  be the set of attributes.

**Algorithm:**

**Step 1.** Construct the SVN-multi-attribute decision making matrix  $[A_{ij}]_{m \times n}$ ;

**Step 2.** Determine the SVN-weight vector  $\omega = (\langle \alpha_i, \beta_i, \gamma_i \rangle)_{m \times 1}$

**Step 3.** Compute the weighted the SVN-multi-attribute decision making matrix  $[\bar{A}_{ij}]_{m \times n}$ ;

**Step 4.** Compute the comprehensive evaluations  $V_j$  of the alternatives  $x_j \in X (j = 1, 2, \dots, n)$ ;

**Step 5.** Rank the comprehensive evaluations  $V_j (j = 1, 2, \dots, n)$ ;



**Step 6.** Find the sensitivity analysis of SVN-weights of the attributes in the linear weighted averaging method;

## 5 Application

In this section, we cite the commonly used example [24] and extend it to the SVN-sets.

An investment company wants to invest a sum of money in the best option considering three criteria which are denoted by  $O = (o_1 = \text{risk analysis}, o_2 = \text{growth analysis}, o_3 = \text{environmental impact analysis})$ . The company has set up a panel which has to choose between three possible alternatives for investing the money which are denoted by  $X = (x_1 = \text{car company}, x_2 = \text{food company}, x_3 = \text{computer company})$

**Step 1.** Construct the SVN-multi-attribute decision making matrix  $[A_{ij}]_{3 \times 3}$  as;

$$[A_{ij}]_{3 \times 3} = \begin{pmatrix} \langle 0.7, 0.1, 0.8 \rangle & \langle 0.7, 0.6, 0.8 \rangle & \langle 0.1, 0.4, 0.7 \rangle \\ \langle 0.5, 0.2, 0.8 \rangle & \langle 0.4, 0.2, 0.3 \rangle & \langle 0.2, 0, 1, 0.9 \rangle \\ \langle 0.1, 0.1, 0.6 \rangle & \langle 0.8, 0.5, 0.4 \rangle & \langle 0, 6, 0.3, 0.7 \rangle \end{pmatrix}$$

**Step 2.** Determine the SVN-weight vector as

$$\omega = (\langle 0.2, 0.9, 0.8 \rangle, \langle 0.8, 0.4, 0.9 \rangle, \langle 0.7, 0.6, 0.3 \rangle)$$

**Step 3.** Compute the weighted the SVN-multi-attribute decision making matrix  $[\bar{A}_{ij}]_{m \times n}$  as;

$$[\bar{A}_{ij}]_{3 \times 3} = \begin{pmatrix} \langle 0.14, 0.91, 0.96 \rangle & \langle 0.14, 0.96, 0.96 \rangle & \langle 0.02, 0.94, 0.94 \rangle \\ \langle 0.40, 0.52, 0.98 \rangle & \langle 0.32, 0.52, 0.93, \rangle & \langle 0.16, 0, 46, 0.99 \rangle \\ \langle 0.07, 0.64, 0.72 \rangle & \langle 0.56, 0.80, 0.58 \rangle & \langle 0, 42, 0.72, 0.79 \rangle \end{pmatrix}$$

**Step 4.** Compute the comprehensive evaluations  $V_j$  of the alternatives  $x_j \in X (j = 1, 2, \dots, n)$  as;

$$\begin{aligned} V_1 &= \langle 1 - (1 - 0.14)(1 - 0.40)(1 - 0.07), 0.91 \times 0.52 \times 0.64, 0.96 \times 0.98 \times 0.72 \rangle \\ &= \langle 0.52012, 0.30285, 0.67738 \rangle \end{aligned}$$

$$\begin{aligned} V_2 &= \langle 1 - (1 - 0.14)(1 - 0.32)(1 - 0.56), 0.96 \times 0.52 \times 0.80, 0.96 \times 0.93 \times 0.58 \rangle \\ &= \langle 0.74269, 0.39936, 0.51782 \rangle \end{aligned}$$

and

$$\begin{aligned} V_3 &= \langle 1 - (1 - 0.02)(1 - 0.16)(1 - 0.42), 0.94 \times 0.46 \times 0.72, 0.94 \times 0.99 \times 0.79 \rangle \\ &= \langle 0.52254, 0.31133, 0.73517 \rangle \end{aligned}$$

respectively.

**Step 5.** Rank the comprehensive evaluations  $V_j(j = 1, 2, \dots, n)$  as;

$$s(V_1) = 1.53990$$

$$s(V_2) = 1.82550$$

and

$$s(V_3) = 1.47604$$

respectively. Then, it is obvious that the best alternative is  $x_2$  and the ranking order of the three alternatives is  $x_2 > x_1 > x_3$ .

**Step 6.** Find the sensitivity analysis of SVN-weights of the attributes in the linear weighted averaging method as;

Firstly, we assume that only weight  $\omega_2 = \langle \alpha_2, \beta_2, \gamma_2 \rangle$  of the attribute  $o_2$  is changed to the weight  $\bar{\omega}_2 = \langle \alpha_2 + \Delta\alpha_2, \beta_2 + \Delta\beta_2, \gamma_2 + \Delta\gamma_2 \rangle$  and the weights of other attributes  $o_i(i = 1, 3)$  remain the same as the original weights  $\omega_1$  and  $\omega_3$ . Then, we have the system of inequalities with respect to  $\Delta\alpha_2$ ,  $\Delta\beta_2$  and  $\Delta\gamma_2$  is obtained as follows: Finally, we have the systems of inequalities as follows:

$$s(V'_2) > s(V'_1)$$

$$s(V'_1) > s(V'_3)$$

$$0 \leq \alpha_2 + \Delta\alpha_2 + \beta_2 + \Delta\beta_2 + \gamma_2 + \Delta\gamma_2 \leq 3,$$

$$0 \leq 0.8 + \Delta\alpha_2 \leq 1$$

$$0 \leq 0.4 + \Delta\beta_2 \leq 1$$

$$0 \leq 0.9 + \Delta\gamma_2 \leq 1$$

where

$$\begin{aligned} s(V'_1) &= 2 + T_1 - I_1 - F_1 + \Delta\alpha_2 T_{21}(1 - x_1) - \Delta\beta_2 y_1(1 - I_{21}) - \Delta\gamma_2 z_1(1 - F_{21}) \\ &= 1.35176 + 0.31992\Delta\alpha_2 - 0.27955\Delta\beta_2 - 0.01382\Delta\gamma_2 \end{aligned}$$

$$\begin{aligned} s(V'_2) &= 2 + T_2 - I_2 - F_2 + \Delta\alpha_2 T_{22}(1 - x_2) - \Delta\beta_2 y_2(1 - I_{22}) - \Delta\gamma_2 z_2(1 - F_{22}) \\ &= 1.38793 + 0.12109\Delta\alpha_2 - 0.36864\Delta\beta_2 - 0.03898\Delta\gamma_2 \end{aligned}$$

$$\begin{aligned} s(V'_3) &= 2 + T_3 - I_3 - F_3 + \Delta\alpha_2 T_{23}(1 - x_3) - \Delta\beta_2 y_3(1 - I_{23}) - \Delta\gamma_2 z_3(1 - F_{23}) \\ &= 1.30498 + 0.09094\Delta\alpha_2 - 0.36547\Delta\beta_2 - 0.00743\Delta\gamma_2 \end{aligned}$$

and where

$$\begin{aligned}
T_1 &= 0.45614 \\
I_1 &= 0.41467 \\
F_1 &= 0.68982 \\
T_2 &= 0.71847 \\
I_2 &= 0.46694 \\
F_2 &= 0.86360 \\
T_3 &= 0.50436 \\
I_3 &= 0.45752 \\
F_3 &= 0.74186
\end{aligned}$$

which can be simplified into the system of inequalities as follows:

$$\begin{aligned}
0.03628 - 0.19883\Delta\alpha_2 - 0.08909\Delta\beta_2 - 0.02515\Delta\gamma_2 &> 0 \\
0,04667 + 0,22898\Delta\alpha_2 + 0,08592\Delta\beta_2 - 0,00640\Delta\gamma_2 &> 0 \\
-0.8 \leq \Delta\alpha_2 \leq 0.2 \\
-0.4 \leq \Delta\beta_2 \leq 0.6 \\
-0.9 \leq \Delta\gamma_2 \leq 0.1
\end{aligned}$$

Some solutions of the system is given by Fig. 1.

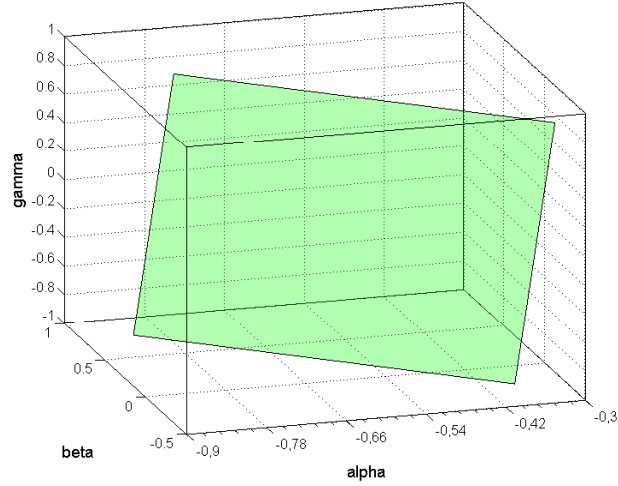


Figure 1: Some solutions of the system

Likewise, we assume that only weight  $\omega_1 = \langle \alpha_1, \beta_1, \gamma_1 \rangle$  of the attribute  $o_1$  is changed to the weight  $\bar{\omega}_1 = \langle \alpha_1 + \Delta\alpha_1, \beta_1 + \Delta\beta_1, \gamma_1 + \Delta\gamma_1 \rangle$  and the weights of other attributes  $o_i (i = 2, 3)$  remain the same as the original weights or that only weight  $\omega_3 = \langle \alpha_3, \beta_3, \gamma_3 \rangle$  of the attribute  $o_3$  is changed to the weight  $\bar{\omega}_3 = \langle \alpha_3 + \Delta\alpha_3, \beta_3 + \Delta\beta_3, \gamma_3 + \Delta\gamma_3 \rangle$  and the weights of other attributes  $o_i (i = 1, 2)$  remain the same as the

original weights, then the solutions can easily be made in a similar way for  $o_1$  and  $o_3$ .

## 6 Conclusion

This paper presents a method of multi-attribute decision-making with both weights and attribute ratings expressed by single valued neutrosophic sets(SVN-sets). Then, we developed a sensitivity analysis of attribute weights by using the linear weighted averaging method. This analysis give changing intervals of attribute weights in which the ranking order of the alternatives is required to remain unchanging. It is easily seen that the proposed the method can be extended to attribute ratings in a straightforward manner. In the future, we may extend the method by n-Valued Refined Neutrosophic Logic by given in [29]. Also, more effective methods of SVN-sets will be investigated in the near future and applied this concepts to game theory, algebraic structure, optimization, and so on.

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