Inverse Problem in DSmT and Its Applications in Trust Management*

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Abstract

DSmT (Dezert-Smarandache Theory) is actually a natural extension of DST (Dempster-Shafer Evidence Theory) and Bayes Probability Theory, which can handle both uncertainty and contradiction efficiently. We present the inverse problem in DSmT and research on the existence of its unique exact solution. In particular, we adopt PSO (Particle Swarm Optimization) arithmetic to find an approximate solution if the inverse problem dose not have a unique exact solution. Trust management has become a key technology and hot topic in open computational systems in recent years. We apply DSmT to the trust management field as a new way of trust representation. Moreover, the simulation suggests that the difference between the second-hand evidential trust evaluation and the first-hand evidential trust evaluation can be greatly reduced by the modification based on the inverse problem framework in DSmT.

Keywords: Trust Management; Dezert-Smarandache Theory; Particle Swarm Optimization.

1 Introduction

In recent years trust management has attracted a lot of attention in many fields, such as electronic commerce, grids, multi-agents systems and p2p systems. Researchers try to establish a trust-management mechanism which can help these open communities work effectively and efficiently. And a crucial step to design this mechanism is how to decide the representation of trust. Generally, existing trust models represent trust in one of the three ways: (1) they use certainty factors which are scalar values to rating others, e.g. ebay.com and [1, 2, 3]; (2) they adopt Bayesian probability theory, e.g. [4, 5, 6]; (3) they apply Dempster-Shafer theory [7], e.g. [8]. Jean Dezert and Florentin Smarandache have recently proposed a new information fusion theory (DSmT) [9, 10] which is an extension of the Dempster-Shafer theory. Compared with Dempster-Shafer theory, DSmT can also express contradiction. In order to get trust represented more explicitly, we apply DSmT to trust representation. Moreover, an inverse problem in DSmT is proposed. Based on it, we propose the Disturbing Model to explain the difference between the second-hand evidential trust evaluation and the first-hand evidential trust evaluation. And we attempt to solve the inverse problem. In particular, if the inverse problem does not have a unique exact solution, we adopt PSO [11] arithmetic to find an approximate solution. We developed a simulation experiment to estimate the Disturbing Model and show its effectiveness.

Our work makes the following contributes:

- Theoretically, we present the inverse problem in DSmT and study on the existence of its unique exact solution.
- We apply DSmT to the trust management as a method of trust representation.
- As there is some difference between the second-hand evidential trust evaluation and the first-hand evidential trust evaluation, we propose the Disturbing Model based on the inverse problem in DSmT to explain and reduce this difference.

The paper is organized as follows. In section 2, we give a brief introduction of DSmT. Section 3 applies DSmT to trust representation. Section 4 introduces the inverse problem in DSmT and attempts to give its solution. In section 5, we present our experimental results.

2 Brief Introduction of DSmT

We only give a brief review of DSmT here, more details can be found in [9, 10].

2.1 Notion of Hype-Power Set

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Let $\Theta = \{\theta_1, \cdots, \theta_n\}$ be the general frame of discernment (a finite set of n exhaustive elements), the hyper-power set D^Θ is defined as the set of all compositions built from elements of Θ with $\cup$ and $\cap$ operators such that

(1). $\Phi, \theta_1, \cdots, \theta_n \in D^\Theta$.

(2). If $A, B \in D^\Theta$, then $A \cap B \in D^\Theta$ and $A \cup B \in D^\Theta$.

(3). No other elements belong to D^Θ, except those obtained by using rules (1) and (2).

2.2 Notion of GBBA

Let Θ be a general frame of discernment of the problem under consideration. A map $m(.): D^\Theta \to [0,1]$ associated to a given body of evidence B is defined as

$$\begin{cases} m(\phi) = 0 \\ \sum_{A \subseteq D^\Theta} m(A) = 1 \end{cases} \quad (1)$$

The quantity $m(A)$ is called A 's general basic belief mass (GBBA).

2.3 Classic DSm Rule of Combination

Let $\mathrm{M}^f(\Theta)$ be a free DSm model [5]. m_1 and m_2 are two independent GBBAs over the same frame Θ, the classic DSm rule of combination is given as follows:

$$m_1 \oplus m_2 = m(C) = \sum_{\substack{A,B \subseteq D^\Theta \\ A \cap B = C}} m_1(A) m_2(B) \quad (2)$$

2.4 Generalized Belief Functions

The generalized belief and plausibility functions are defined as follows:

$$Bel(A) = \sum_{\substack{B \subseteq A \\ B \in D^\Theta}} m(B) \quad , \quad Pl(A) = \sum_{\substack{B \cap A \neq \Phi \\ B \in D^\Theta}} m(B) \quad (3)$$

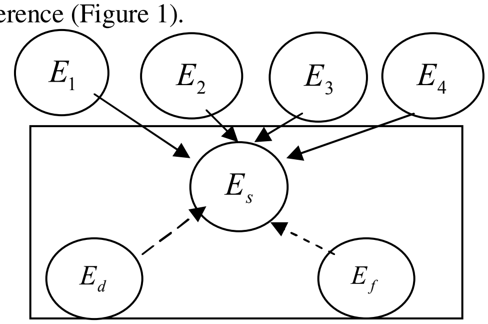
3 Applications of DSmT to Trust Management

3.1 Applications of DSmT to Trust Representation

In this section we apply DSmT to trust representation. Let $\Theta = \{T, \neg T\}$. $\{T\}$, $\{\neg T\}$, Θ and $\{T \cap \neg T\}$ represent trust, distrust, uncertainty and contradiction, respectively. In particular, $\{T \cap \neg T\}$ is not always an empty set, which means that contradiction (a feeling of both trust and distrust) may exist. And it is coincident with the real world to adopt contradiction in trust representation.

3.2 Disturbing Model

We suppose a scenario as follows: A wants to evaluate the trustworthiness of a service provider SP but A has little knowledge about SP in the beginning, so A consults four believable experts. They give the evidential trust evaluation E_1, E_2, E_3 and E_4, respectively. Then A combine E_1, E_2, E_3 and E_4, and gets E_s, which is called the second-hand evidential trust evaluation. As the interaction between A and SP increases, A get a first-hand evidential trust evaluation of SP which is denoted E_f. However, E_s and E_f are not always the same, instead, they sometimes differ from each other for the most part. So we propose the Disturbing Model in order to explain the difference (Figure 1).



E_f : *First-Hand Evidential Trust Evaluation,*

E_s : *Second-Hand Evidential Trust Evaluation,*

E_d : *Disturbing Evidential Trust Evaluation*

Figure 1. Disturbing model

In the Disturbing Model, we assume that there is a Disturbing Evidential Trust Evaluation E_d and E_s is a combination of E_f and E_d as it is showed in Figure 1. If E_d is an empty evidence, E_s is equivalent to E_f, otherwise, E_s differs from E_f.

4 The Inverse Problem in DSmT and Its Solution

4.1 Definition of the Inverse Problem in DSmT

Based on the Disturbing Model, it is possible for us to modify E_s with E_d to get a modifying evidential trust evaluation E_m that is more coincident with E_f, which will benefit decision-making. So we propose the inverse problem in DSmT for solution of E_d.

Definition 1 Suppose $m_1 \oplus m_2 = m$ and we have m_1 and the combination m, the solution of m_2 is called the inverse problem in DSmT, which is denoted $m_2 = m / m_1$.

4.2 Solution of the Inverse Problem

Theorem 1 In a free DSm model, let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, $|\Theta| = n$ for any GBBA m and m_1. If the inverse problem has exact solutions, there exists a unique solution m_2 such that $m = m_1 \oplus m_2$ iff $m_1(\Theta) \neq 0$ (i.e. $m_1(\bigcup_{i=1}^{n} \theta_i) \neq 0$).

We define a notion of Incremental Sequence before the proof of Theorem 1.

Definition 2 Incremental Sequence is a sequence that arranged in the following way.

We divide all the elements of $D^\Theta - \Phi$ into t classes: $M_1, M_2, \cdots, M_t$.

(1). $M_1 = \{\bigcap_{i=1}^{n} \theta_i\}$.

(2). M_i ($i > 1$) : Assume a set $S_i = D^\Theta - \bigcup_{j=1}^{i-1} M_j$, for any element x of S_i, if $\forall y \in S_i$, $y \not\subset x$, then $x \in M_i$. So $\forall a, b \in M_i$, $a \not\subset b$ and $b \not\subset a$.

(3). Follow step (2) and we have $M_2, M_3, \cdots, M_t$. Here $M_t = \Theta$.

(4). Suppose $M =$ {elements of M_1, elements of M_2, ⋯, elements of M_t} and elements of the same class are disordered, which is denoted $M = \{x_1, x_2, \cdots, x_n\}$. We call it Incremental Sequence as the sequence of $x_1, x_2, \cdots, x_n$.

Lemma 1 $\forall a \in M_j, b \in M_i (j < i), b \not\subset a$.

Proof of Lemma 1: $b \in M_i$, by Definition 2 we have $\exists b' \in M_j, b' \subset b$.

$\because a \in M_j, b' \in M_j$, $\therefore b' \not\subset a$. And $\because b' \subset b, \therefore b \not\subset a$.

Proof of Theorem 1: Suppose $m_2(\Phi) = 0$. We arrange the elements of $D^\Theta - \Phi$ to an Incremental Sequence, which is denoted $D^\Theta - \Phi = \{x_1, x_2, \cdots, x_n\}$. By Classic DSm rule of combination, we have

$$T \begin{bmatrix} m_2(x_1) \\ m_2(x_2) \\ \vdots \\ m_2(x_n) \end{bmatrix} = \begin{bmatrix} m(x_1) \\ m(x_2) \\ \vdots \\ m(x_n) \end{bmatrix} \quad (4)$$

($T = (t_{i,j})_{n\times n}, 1 \le i, j \le n$), where the i th equation is:

$$t_{i1}m_2(x_1) + t_{i2}m_2(x_2) + \cdots + t_{ii}m_2(x_i) + \cdots + t_{in}m_2(x_n) = m(x_i) \quad (5)$$

Suppose $x_k \in M_p, x_i \in M_q (k < i)$, by Definition 2 we have $p \le q$. If $p < q$, by Lemma 1 we have $x_i \not\subset x_k$, and if $p = q$, by Definition 2 we also have $x_i \not\subset x_k$. $\therefore x_i \neq a \cap x_k, a \in D^\Theta$, $\therefore t_{ik} = \sum_{\substack{x_i = a \cap x_k \\ a \in D^\Theta}} m_1(a) = 0$, $\therefore T$ is an upper triangular matrix, $\therefore \det(T) = \prod_{i=1}^{n} t_{ii}$. For t_{ii}, $t_{ii} = \sum_{\substack{x_i = a \cap x_i \\ a \in D^\Theta}} m_1(a)$, when $a = x_n = \Theta$, $x_i = a \cap x_i$, $\therefore t_{ii} \ge m_1(x_n)$ (if and only if $i = n$, $t_{ii} = m_1(x_n) = m_1(\Theta)$), $\therefore \det(T) = t_{11} \cdot t_{22} \cdots t_{n-1n-1} \cdot t_{nn} = t_{11} \cdot t_{22} \cdots t_{n-1n-1} \cdot m_1(\Theta)$, $\therefore \det(T) \neq 0 \iff m_1(\Theta) \neq 0 \iff$ System of equations (5) has the unique solution.

4.3 Approximate Solution of the Inverse Problem in DSmT by Means of PSO

If there is not a unique exact solution to the inverse problem in DSmT, we apply PSO arithmetic [11] for the approximate solution of the inverse problem. The dimension of the approximate solution is 4. We have 10 particles and the max iteration is set to be 500, and the permissible error is 0.0001. Suppose the approximate solution is m_2, we use MSE (Mean Squared Error) between $m_1 \oplus m_2$ and m to estimate it, denoted by MSE. And the fitness function is defined as

$$f(m_2) = MSE \quad (6)$$

Learning factors $c_1 = c_2 = 2$.

Example 1: Suppose $m_1 \oplus m_2 = m$, solve m_2, where $m(\{T \cap \neg T\}) = 0.7$, $m(\{T\}) = 0.1$, $m(\{\neg T\}) = 0.1$, $m(\Theta) = 0.1$, $m_1(\{T \cap \neg T\}) = 0.73$, $m_1(\{T\}) = 0.11$, $m_1(\{\neg T\}) = 0.11$, $m_1(\Theta) = 0.05$.

First we try to find the exact solution of this problem, and a negative appears in the result as is indicated in the second column of Table 1. Since it denies solution, we find an approximate solution as it is shown in the third column of Table 1 by the method mentioned above

Table 1. An example of solution of the inverse problem in DSmT.

	m_2	
	Analytic Solution	Approximate Solution
$\{T \cap \neg T\}$	-0.1	0.0000
$\{T\}$	0.3	0.2000
$\{\neg T\}$	0.3	0.2000
Θ	0.5	0.6000

5 Simulation

5.1 Simulation Setup

Suppose that a society can be divided into some small communities. In the Simulation, we have a small community C of 100 members who have pretty similar interests. There are 20 service providers and 4 experts in it. We assume that the experts are familiar with every service provider and they can provide any user with their trust evaluation of any service provider, while the members are utterly ignorant of these service providers in the beginning.

In each cycle, we randomly designate a member to be the requester (e.g. A request SP_i for service). First, A sends queries to the four experts. When an expert receives a query, it will give its evidential trust evaluation of SP_i to A. Second, A combines all collected evidences to the second-hand evidential trust evaluation E_s. Then A sends the service request based on its interests by perturbing to SP_i and SP_i replies with a service based on its expertise by perturbing. After receiving the service, A estimates it and C keep it as a history record. When the interactions between C and SP_i increases to 10 times, C get a first evidential trust evaluation of SP_i. And by the Disturbing Model, we can solve the disturbing evidential evaluation E_d of SP_i. After 200 cycles, C establishes all the disturbing evidential trust evaluations which can be used to modify the second-hand evidential trust evaluations in the next 2000 cycles.

In the simulation, each expert has an expertise of 5-demension vector and each member in C has an interest of 5-demension vector. The way of trust acquisition refers to [12], where threshold of high QOS is 0.7, threshold of low QOS is 0.2, perturbing by 0.05. In addition, we have a modification to it that $m(\Theta)$ is not less than about 0.1 since there is no absolute trust in the society.

5.2 Simulation Results

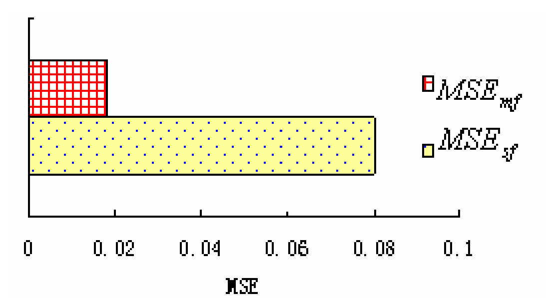


Figure 2. Simulation results

MSE_{sf} is the MSE between E_s and E_f, and MSE_{mf} is the MSE between E_m and E_f. The effectiveness of the Disturbing Model could be estimated by the compare of MSE_{sf} and MSE_{mf}. We run the simulation 40 times and get average results as $MSE_{sf} = 0.0802$, $MSE_{mf} = 0.0182$. The simulation results indicate that the difference between the second evidential trust evaluation and the first evidential trust evaluation is greatly reduced by some modification based on the Disturbing Model. So we argue that the Disturbing Model can explain this difference to a great extent.

Conclusion

The main contribution of this paper includes two areas. On the one hand, in trust management area, we introduce a new trust representation with DSmT which is an extension of DST and can handle both uncertainty and contradiction effectively. In addition, the Disturbing Model we proposed makes it possible to modify the second evidential trust evaluation, which has been proved by the simulation. And further applying DSmT into trust management will be present in another work. On the other hand, our contribution is also for DSmT. We propose the inverse problem in DSmT and attempt to give Theorem 1 for whose unique exact solution. When it denies solution, we adopt PSO arithmetic for its approximate solution. As DSmT cannot handle dependent evidences, we assume that two dependent evidences are resulted from orthogonal sum of one dependent original evidence and two independent original evidences, respectively. Combining the two dependent evidences can be reduced to an orthogonal sum of the two independent original evidences and the dependent original evidence. Identifying the independent original evidences is again an inverse problem, by which we are finding a way to discard the strong constraint of independence in DSmT. Thus, our work is a complementary of DSmT. Moreover, although DSmT is more powerful than DST in representation, it is greatly challenged that the computational complexity increases sharply as the size of Θ increases. Fortunately, it does not matter as it applies to trust management since the size of Θ is only 2. So we present an appropriate example for its application when it is in trouble.

The research developed here is still in progress. Much remains to be done to establish an effective and robust trust model based on the trust representational framework presented in this paper. We are currently working in the direction of designing a trust model and implementing it in a decentralized environment.

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