Improved cosine similarity measures of simplified intuitionistic sets for medicine diagnoses

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Abstract: Similarity measures are an important tool in pattern recognition and medical diagnosis. To overcome some disadvantages of existing cosine similarity measures for simplified neutrosophic sets (SNSs) in vector space, this paper proposes improved cosine similarity measures for SNSs based on the cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures. Then, the weighted cosine similarity measures of SNSs are introduced by considering the importance of each element. Moreover, compared with existing cosine similarity measures of SNSs by numerical examples, the improved cosine similarity measures of SNSs demonstrate their effectiveness and rationality and can overcome some disadvantages of existing cosine similarity measures of SNSs in some cases. Finally, the medical diagnosis problems are given to show the applications and effectiveness of the improved cosine similarity measures.

Keywords: Simplified neutrosophic set; Single valued neutrosophic set; Interval neutrosophic set; Cosine similarity measure; Medical diagnosis

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1. Introduction

Due to the increased volume of information available to physicians from modem medical technologies, medicine diagnosis contains a lot of incomplete, uncertainty, and inconsistent information, which is important information of medical diagnosis problems. A symptom usually implies a lot of incomplete, uncertainty, and inconsistent information for a disease, hence the incomplete, uncertainty and inconsistent information characterizes a relation between symptoms and diseases. Thus we work with the uncertainties and inconsistencies to lead us to proper decision making in medicine. In most of the medical diagnosis problems, there exist some patterns, and the experts make decision based on the similarity between unknown sample and the basic diagnosis patterns. In some practical situations, there is the possibility of each element having different truth-membership, indeterminacy-membership, and falsity-membership functions. Therefore, Smarandache [1] originally proposed the concept of a neutrosophic set from philosophical point of view. A neutrosophic set A in a universal set X is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \rightarrow\,]^-0, 1^+[$, $I_A(x)$: $X \rightarrow\,]^-0, 1^+[$, and $F_A(x)$: $X \rightarrow\,]^-0, 1^+[$. However, the domain of the definition and range of the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a neutrosophic set A is the non-standard unit interval $]^-0, 1^+[$, it is only used for philosophical applications, especially when distinction is required between absolute and relative truth/falsehood/indeterminacy. To easily use in technical applications of the neutrosophic set, the domain of the definition and range of $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be restrained to the normal standard real unit interval [0, 1]. As a simplified form of the neutrosophic set, a simplified neutrosophic set [2] is the appropriate choice as it is easily expresses and deals with incomplete, uncertainty, and inconsistent information in real science and engineering fields. Simplified neutrosophic sets include single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs) and a generalization of classic sets, fuzzy sets (FSs) [3], intuitionistic fuzzy sets (IFSs) [4] and interval-valued intuitionistic fuzzy sets (IVIFSs) [5]. However, FSs, IFSs and IVIFSs cannot represent and handle uncertainty and inconsistent information [1]. Then, similarity measures are not only an important tool in pattern recognition, medicine diagnosis, and decision making but also an important research topic in the neutrosophic theory. Various similarity measures have been proposed by some researchers. Broumi and Smarandache [6] defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. Majumdar and Samanta [7] introduced several similarity measures of single valued neutrosophic sets (SVNSs) based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Ye [8] also presented the Hamming and Euclidean distances between interval neutrosophic sets (INSs) and their similarity measures and applied them to multiple attribute decision-making problems with interval neutrosophic information. Ye [9] further proposed the distance-based similarity measure of SVNSs and applied it to group decision making problems with single valued neutrosophic information. Furthermore, Ye [2] proposed three vector similarity measures for SNSs, including the Jaccard, Dice, and cosine similarity measures for SVNSs and INSs, and applied them to multicriteria decision-making problems with simplified neutrosophic information. Till now, existing similarity measures for neutrosophic sets are scarcely applied to medical diagnosis problems. However, the cosine similarity measures defined in vector space [2] have some drawbacks in some situations. For instance, they may produce no defined (unmeaningful) phenomena or some results calculated by the cosine similarity measures are unreasonable in some real cases (details given in Sections 3). Therefore, in the situations, it is difficult to apply them to pattern recognition and medicine diagnosis. To overcome some drawbacks of existing cosine measures in [2], this paper aims to propose improved cosine similarity measures for SNSs and apply them to medicine diagnosis. To do so, the rest of the article is organized as follows. In Section 2, we briefly introduce some basic concepts of SNSs. Section 3 reviews existing cosine similarity measures of SNSs in vector space and their drawbacks. Sections 4 proposes the improved cosine similarity measures of SNSs based on the cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures, and investigates their properties. In Section 5, by two numerical examples we give the comparative analysis between the improved cosine similarity measures and existing cosine similarity measures for SNSs to show the effectiveness and rationality of the improved cosine measures. In Section 6, the cosine similarity measures are applied to medicine diagnosis problems. Conclusions and further research are contained in Section 7.

2. Some basic concepts of SNSs

Smarandache [1] originally presented the concept of a neutrosophic set from philosophical point of view. In a neutrosophic set A in a universal set X, its characteristic functions are expressed by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, respectively. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x): X \rightarrow]^-0, 1^+[$, $I_A(x): X \rightarrow]^-0, 1^+[$, and $F_A(x): X$

$\rightarrow\ ]^{-}0, 1^{+}[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ is no restriction, i.e. $^{-}0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$.

To apply a neutrosophic set to science and engineering areas, Ye [2] introduced SNS, which is a subclass of the neutrosophic set, and gave the following definition of a SNS.

Definition 1 [2]: Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], such that $T_A(x)$: $X \rightarrow [0, 1]$, $I_A(x)$: $X \rightarrow [0, 1]$, and $F_A(x)$: $X \rightarrow [0, 1]$. Then, a simplification of the neutrosophic set A is denoted by

$$A = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \right\},$$

which is called a SNS. It is a subclass of the neutrosophic set and includes the concepts of INS and SVNS.

On the one hand, if we only use the SNS A whose $T_A(x)$, $I_A(x)$ and $F_A(x)$ values are single points in the real standard [0, 1] instead of subintervals/subsets in the real standard [0, 1], the SNS A can be described by three real numbers in the real unit interval [0, 1]. Therefore, the sum of $T_A(x), I_A(x), F_A(x) \in [0, 1]$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. In this case, the SNS A reduces to the SVNS A.

For two SVNSs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}$, there are the following relations [10]:

(1) Complement: $A^c = \left\{ \langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle \mid x \in X \right\}$;

(2) Inclusion: $A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X;

(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

On the other hand, if we only consider three membership degrees in a SNS A as the subunit interval of the real unit interval [0, 1], the SNS can be described by three interval numbers in the real unit interval [0, 1]. For each point x in X, we have that $T_A(x) = [\inf T_A(x), \sup T_A(x)]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)]$, $F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0, 1]$ and $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$ for any $x \in X$. In this case, the SNS A reduces to the INS A.

For two INSs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle | x \in X\}$, there are the following relations [11]:

(1) Complement:

$$A^c = \left\{\left\langle x, [\inf F_A(x), \sup F_A(x)], [1 - \sup I_A(x), 1 - \inf I_A(x)], [\inf T_A(x), \sup T_A(x)\right\rangle | x \in X\right\};$$

(2) Inclusion:

$A \subseteq B$ if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$, $\sup F_A(x) \geq \sup F_B(x)$, for any x in X;

(3) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Especially when The upper and lower ends of three interval numbers $T_A(x)$, $I_A(x)$, $F_A(x)$ in A are equal, the INS A degrade to the SVNS A. Therefore, the SVNS A is a special case of the INS A, and also both are the special cases of the SNS A.

3. Existing cosine similarity measures of SNSs and their drawbacks

In this section, we introduce existing cosine similarity measures for SNSs in the literature [2] and review their drawbacks.

Then, similarity measures have the following definition.

Definition 2. A real-valued function S: SNS(X)×SNS(X) → [0, 1] is called a similarity measure on SNS(X) if it satisfies the following axiomatic requirements for A, B, $C \in$ SNS(X):

(S1) $0 \le S(A, B) \le 1$;

(S2) $S(A, B) = 1$ if and only if $A = B$;

(S3) $S(A, B) = S(B, A)$;

(S4) If $A \subseteq B \subseteq C$, then $S(A, C) \le S(A, B)$ and $S(A, C) \le S(B, C)$.

3.1 Existing cosine similarity measure for SVNSs and its drawbacks

In this section, we only use SVNSs in SNSs. Assume that there are two SVNSs $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle | x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle | x_j \in X\}$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, where $T_A(x_j), I_A(x_j), F_A(x_j) \in [0, 1]$ for any $x_j \in X$ in A and $T_B(x_j), I_B(x_j), F_B(x_j) \in [0, 1]$ for any $x_j \in X$ in B. Then, Ye [2] presented the cosine similarity measure of SVNSs in vector space as follows:

$$C_1(A,B) = \frac{1}{n}\sum_{j=1}^{n}\frac{T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j)}{\sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)}\sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)}}, \qquad (1)$$

However, one can find some drawbacks of Eq. (1) as follows:

(1) For two SVNSs A and B, if $T_A(x_j) = I_A(x_j) = F_A(x_j) = 0$ and/or $T_B(x_j) = I_B(x_j) = F_B(x_j) = 0$ for any x_j in X ($j = 1, 2, \ldots, n$), Eq. (1) is undefined or unmeaningful. In this case, one cannot utilize it to calculate the cosine similarity measure between A and B.

(2) If $T_A(x_j) = 2T_B(x_j)$, $I_A(x_j) = 2I_B(x_j)$, and $F_A(x_j) = 2F_B(x_j)$ or $2T_A(x_j) = T_B(x_j)$, $2I_A(x_j) = I_B(x_j)$, and $2F_A(x_j) = F_B(x_j)$ for any x_j in X ($j = 1, 2, \ldots, n$). By applying Eq. (1), we have

$$\begin{aligned} C_1(A,B) &= \frac{1}{n}\sum_{j=1}^{n}\frac{2T_A(x_j)T_B(x_j) + 2I_A(x_j)I_B(x_j) + 2F_A(x_j)F_B(x_j)}{2\sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)}\sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)}} \\ &= \frac{1}{n}\sum_{j=1}^{n}\frac{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)}{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)} = 1 \end{aligned}.$$

Since $A \neq B$, the measure value of Eq. (1) is equal to 1. This means that it only satisfies the necessary condition of the property (S2) in Definition 2, but not the sufficient condition.

Therefore, in this case, it is unreasonable to apply it to pattern recognition and medicine diagnosis.

3.2 Existing cosine similarity measure for INSs and its drawbacks

In this section, we only use INSs in SNSs. Assume that there are two INSs $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle | x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle | x_j \in X\}$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, where $T_A(x_j) = [\inf T_A(x_j), \sup T_A(x_j)]$, $I_A(x_j) = [\inf I_A(x_j), \sup I_A(x_j)]$, $F_A(x_j) = [\inf F_A(x_j), \sup F_A(x_j)] \subseteq [0, 1]$ for any $x_j \in X$ in A and $T_B(x_j) = [\inf T_B(x_j), \sup T_B(x_j)]$, $I_A(x_j) = [\inf I_B(x_j), \sup I_B(x_j)]$, $F_B(x_j) = [\inf F_B(x_j), \sup F_B(x_j)] \subseteq [0, 1]$ for any $x_j \in X$ in B. Then, Ye [2] presented the cosine similarity measure of INSs in vector space as follows:

$$C_2(A,B) = \frac{1}{n}\sum_{j=1}^{n} \frac{\begin{pmatrix} \inf T_A(x_j)\inf T_B(x_j) + \inf I_A(x_j)\inf I_B(x_j) \\ + \inf F_A(x_j)\inf F_B(x_j) + \sup T_A(x_j)\sup T_B(x_j) \\ + \sup I_A(x_j)\sup I_B(x_j) + \sup F_A(x_j)\sup F_B(x_j) \end{pmatrix}}{\begin{pmatrix} \sqrt{\begin{matrix} [\inf T_A(x_j)]^2 + [\inf I_A(x_j)]^2 + [\inf F_A(x_j)]^2 \\ + [\sup T_A(x_j)]^2 + [\sup I_A(x_j)]^2 + [\sup F_A(x_j)]^2 \end{matrix}} \\ \sqrt{\begin{matrix} [\inf T_B(x_j)]^2 + [\inf I_B(x_j)]^2 + [\inf F_B(x_j)]^2 \\ + [\sup T_B(x_j)]^2 + [\sup I_B(x_j)]^2 + [\sup F_B(x_j)]^2 \end{matrix}} \end{pmatrix}}. \quad (2)$$

Similarly, one can find some drawbacks of Eq. (2) as follows:

(1) For two INSs A and B, if $T_A(x_j) = I_A(x_j) = F_A(x_j) = [0, 0]$ and/or $T_B(x_j) = I_B(x_j) = F_B(x_j) = [0, 0]$ for any x_j in X ($j = 1, 2, \ldots, n$), Eq. (2) is undefined or unmeaningful. In this case, one cannot calculate the cosine similarity measure between A and B.

(2) If $T_A(x_j) = [2\inf T_B(x_j), 2\sup T_B(x_j)]$, $I_A(x_j) = [2\inf I_B(x_j), 2\sup I_B(x_j)]$, and $F_A(x_j) = [2\inf F_B(x_j), 2\sup F_B(x_j)]$ or $T_B(x_j) = [2\inf T_A(x_j), 2\sup T_A(x_j)]$, $I_B(x_j) = [2\inf I_A(x_j), 2\sup I_A(x_j)]$, and $F_B(x_j) =$

[2inf $F_A(x_j)$, 2sup $F_A(x_j)$] for any x_j in X (j = 1, 2, …, n). By using Eq. (2), we have

$$C_2(A,B)=\frac{1}{n}\sum_{j=1}^{n}\frac{2\begin{pmatrix}\inf T_A(x_j)\inf T_B(x_j)+\inf I_A(x_j)\inf I_B(x_j)\\+\inf F_A(x_j)\inf F_B(x_j)+\sup T_A(x_j)\sup T_B(x_j)\\+\sup I_A(x_j)\sup I_B(x_j)+\sup F_A(x_j)\sup F_B(x_j)\end{pmatrix}}{\begin{pmatrix}2\sqrt{\begin{matrix}[\inf T_A(x_j)]^2+[\inf I_A(x_j)]^2+[\inf F_A(x_j)]^2\\+[\sup T_A(x_j)]^2+[\sup I_A(x_j)]^2+[\sup F_A(x_j)]^2\end{matrix}}\\ \sqrt{\begin{matrix}[\inf T_B(x_j)]^2+[\inf I_B(x_j)]^2+[\inf F_B(x_j)]^2\\+[\sup T_B(x_j)]^2+[\sup I_B(x_j)]^2+[\sup F_B(x_j)]^2\end{matrix}}\end{pmatrix}}$$

$$=\frac{1}{n}\sum_{j=1}^{n}\frac{\begin{pmatrix}[\inf T_A(x_j)]^2+[\inf I_A(x_j)]^2+[\inf F_A(x_j)]^2\\+[\sup T_A(x_j)]^2+[\sup I_A(x_j)]^2+[\sup F_A(x_j)]^2\end{pmatrix}}{\begin{pmatrix}[\inf T_A(x_j)]^2+[\inf I_A(x_j)]^2+[\inf F_A(x_j)]^2\\+[\sup T_A(x_j)]^2+[\sup I_A(x_j)]^2+[\sup F_A(x_j)]^2\end{pmatrix}}=1.$$

Since $A \neq B$, the measure value of Eq. (2) is equal to 1. This means that it only satisfies the necessary condition of the property (S2) in Definition 2, but not the sufficient condition.

Therefore, in this case, the cosine similarity measure is unreasonable in the application of pattern recognition and medicine diagnosis.

In order to overcome the above mentioned disadvantages, we shall improve the cosine similarity measures of SNSs in the following section.

4. Improved cosine similarity measures for SNSs

4.1 Improved cosine similarity measures for SVNSs

Based on the cosine function, we propose two improved cosine similarity measures between SVNSs and investigate their properties.

Let $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle | x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle | x_j \in X\}$ be any two SVNSs in $X = \{x_1, x_2, …, x_n\}$, where $T_A(x_j)$, $I_A(x_j)$, $F_A(x_j) \in [0, 1]$ for any $x_j \in X$ in A and $T_B(x_j)$,

$I_B(x_j)$, $F_B(x_j) \in [0, 1]$ for any $x_j \in X$ in B. Then, based on the cosine function, we propose two improved cosine similarity measures between A and B, respectively, as follows:

$$SC_1(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi\left(\left|T_A(x_j) - T_B(x_j)\right| \vee \left|I_A(x_j) - I_B(x_j)\right| \vee \left|F_A(x_j) - F_A(x_j)\right|\right)}{2}\right], \quad (3)$$

$$SC_2(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi\left(\left|T_A(x_j) - T_B(x_j)\right| + \left|I_A(x_j) - I_B(x_j)\right| + \left|F_A(x_j) - F_A(x_j)\right|\right)}{6}\right], \quad (4)$$

where the symbol "$\vee$" is maximum operation. Then, the two improved cosine similarity measures satisfy the axiomatic requirements of similarity measures.

Proposition 1. For two SVNSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the cosine similarity measure $SC_k(A, B)$ (k =1, 2) should satisfy the following properties (S1-S4):

(S1) $0 \le SC_k(A, B) \le 1$;

(S2) $SC_k(A, B) = 1$ if and only if $A = B$;

(S3) $SC_k(A, B) = SC_k(B, A)$;

(S4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$.

Proof:

(S1) Since the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree in SVNS and the value of the cosine function are within [0, 1], the similarity measure based on the cosine function also is within [0, 1]. Hence $0 \le SC_k(A, B) \le 1$ for $k = 1, 2$.

(S2) For any two SVNSs A and B, if $A = B$, this implies $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Hence $\left|T_A(x_j) - T_B(x_j)\right| = 0$, $\left|I_A(x_j) - I_B(x_j)\right| = 0$, and $\left|F_A(x_j) - F_B(x_j)\right| = 0$. Thus $SC_k(A, B) = 1$ for $k = 1, 2$.

If $SC_k(A, B) = 1$ for $k = 1, 2$, this implies $\left|T_A(x_j) - T_B(x_j)\right| = 0$, $\left|I_A(x_j) - I_B(x_j)\right| = 0$, and $\left|F_A(x_j) - F_B(x_j)\right| = 0$ since $cos(0) = 1$. Then, these equalities indicate $T_A(x_j) = T_B(x_j)$,

$I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Hence $A = B$.

(S3) Proof is straightforward.

(S4) If $A \subseteq B \subseteq C$, then there are $T_A(x_j) \le T_B(x_j) \le T_C(x_j)$, $I_A(x_j) \ge I_B(x_j) \ge I_C(x_j)$, and $F_A(x_j) \ge F_B(x_j) \ge F_C(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Then, we have the following inequalities:

$$\left|T_A(x_j) - T_B(x_j)\right| \le \left|T_A(x_j) - T_C(x_j)\right|, \ \left|T_B(x_j) - T_C(x_j)\right| \le \left|T_A(x_j) - T_C(x_j)\right|,$$

$$\left|I_A(x_j) - I_B(x_j)\right| \le \left|I_A(x_j) - I_C(x_j)\right|, \ \left|I_B(x_j) - I_C(x_j)\right| \le \left|I_A(x_j) - I_C(x_j)\right|,$$

$$\left|F_A(x_j) - F_B(x_j)\right| \le \left|F_A(x_j) - F_C(x_j)\right|, \ \left|F_B(x_j) - F_C(x_j)\right| \le \left|F_A(x_j) - F_C(x_j)\right|.$$

Hence, $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$ for $k = 1, 2$ since the cosine function is a decreasing function within the interval $[0, \pi/2]$.

Therefore, we complete the proofs of these properties.

Usually, one takes the weight of each element x_j for $x_j \in X$ into account and assumes that the weight of a element x_j is w_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Thus we can introduce the following weighted cosine similarity measures between SVNSs:

$$WSC_1(A,B) = \sum_{j=1}^{n} w_j \cos\left[\frac{\pi\left(\left|T_A(x_j) - T_B(x_j)\right| \vee \left|I_A(x_j) - I_B(x_j)\right| \vee \left|F_A(x_j) - F_A(x_j)\right|\right)}{2}\right], \quad (5)$$

$$WSC_2(A,B) = \sum_{j=1}^{n} w_j \cos\left[\frac{\pi\left(\left|T_A(x_j) - T_B(x_j)\right| + \left|I_A(x_j) - I_B(x_j)\right| + \left|F_A(x_j) - F_A(x_j)\right|\right)}{6}\right], \quad (6)$$

Especially when $w_j = 1/n$ for $j = 1, 2, \ldots, n$, Eqs. (5) and (6) reduce to Eqs. (3) and (4).

4.2 Improved cosine similarity measures for INSs

Similarly, we propose two improved cosine similarity measures between INSs and investigate their properties.

Let $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle \mid x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle \mid x_j \in X\}$ be any two

INSs in $X = \{x_1, x_2, \ldots, x_n\}$, where $T_A(x_j) = [\inf T_A(x_j), \sup T_A(x_j)]$, $I_A(x_j) = [\inf I_A(x_j), \sup I_A(x_j)]$, $F_A(x_j) = [\inf F_A(x_j), \sup F_A(x_j)] \subseteq [0, 1]$ for any $x_j \in X$ in A and $T_B(x_j) = [\inf T_B(x_j), \sup T_B(x_j)]$, $I_A(x_j) = [\inf I_B(x_j), \sup I_B(x_j)]$, $F_B(x_j) = [\inf F_B(x_j), \sup F_B(x_j)] \subseteq [0, 1]$ for any $x_j \in X$ in B. Then, based on the cosine function, we propose two improved cosine similarity measures between A and B, respectively, as follows:

$$SC_3(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi}{4}\left(\begin{array}{l}\left|\inf T_A(x_j)-\inf T_B(x_j)\right| \vee \left|\inf I_A(x_j)-\inf I_B(x_j)\right| \vee \left|\inf F_A(x_j)-\inf F_A(x_j)\right| \\ +\left|\sup T_A(x_j)-\sup T_B(x_j)\right| \vee \left|\sup I_A(x_j)-\sup I_B(x_j)\right| \vee \left|\sup F_A(x_j)-\sup F_A(x_j)\right|\end{array}\right)\right], \tag{7}$$

$$SC_4(A,B) = \frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi}{12}\left(\begin{array}{l}\left|\inf T_A(x_j)-\inf T_B(x_j)\right| + \left|\inf I_A(x_j)-\inf I_B(x_j)\right| + \left|\inf F_A(x_j)-\inf F_A(x_j)\right| \\ +\left|\sup T_A(x_j)-\sup T_B(x_j)\right| + \left|\sup I_A(x_j)-\sup I_B(x_j)\right| + \left|\sup F_A(x_j)-\sup F_A(x_j)\right|\end{array}\right)\right], \tag{8}$$

where the symbol "$\vee$" is maximum operation. Then, the two improved cosine similarity measures of INSs satisfy the axiomatic requirements in Definition 2.

Proposition 2. For two INSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the cosine similarity measure $SC_k(A, B)$ (k =3, 4) should satisfy the following properties (S1-S4):

(S1) $0 \le SC_k(A, B) \le 1$;

(S2) $SC_k(A, B) = 1$ if and only if $A = B$;

(S3) $SC_k(A, B) = SC_k(B, A)$;

(S4) If C is an INS in X and $A \subseteq B \subseteq C$, then $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$.

Proof:

(S1) Since the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree in an INS and the value of the cosine function are within [0, 1], the similarity measure value based on the cosine function also is within [0, 1]. Thus $0 \le SC_k(A, B) \le 1$ for k = 3, 4.

(S2) For any two INSs A and B, if $A = B$, this implies $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for j = 1, 2, …, n and $x_j \in X$. Hence $\left|\inf T_A(x_j) - \inf T_B(x_j)\right| = 0$, $\left|\inf I_A(x_j) - \inf I_B(x_j)\right| = 0$, $\left|\inf F_A(x_j) - \inf F_B(x_j)\right| = 0$, $\left|\sup T_A(x_j) - \sup T_B(x_j)\right| = 0$, $\left|\sup I_A(x_j) - \sup I_B(x_j)\right| = 0$, and $\left|\sup F_A(x_j) - \sup F_B(x_j)\right| = 0$. Thus $SC_k(A, B) = 1$ for k = 3, 4.

If $SC_k(A, B) = 1$ for k = 3, 4, this implies $\left|\inf T_A(x_j) - \inf T_B(x_j)\right| = 0$, $\left|\inf I_A(x_j) - \inf I_B(x_j)\right| = 0$, $\left|\inf F_A(x_j) - \inf F_B(x_j)\right| = 0$, $\left|\sup T_A(x_j) - \sup T_B(x_j)\right| = 0$, $\left|\sup I_A(x_j) - \sup I_B(x_j)\right| = 0$, and $\left|\sup F_A(x_j) - \sup F_B(x_j)\right| = 0$ since $cos(0) = 1$. Then, these equalities indicate $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for j = 1, 2, …, n and $x_j \in X$. Hence $A = B$.

(S3) Proof is straightforward.

(S4) If $A \subseteq B \subseteq C$, then there are $\inf T_A(x_j) \le \inf T_B(x_j) \le \inf T_C(x_j)$, $\sup T_A(x_j) \le \sup T_B(x_j) \le \sup T_C(x_j)$, $\inf I_A(x_j) \ge \inf I_B(x_j) \ge \inf I_C(x_j)$, $\sup I_A(x_j) \ge \sup I_B(x_j) \ge \sup I_C(x_j)$, $\inf F_A(x_j) \ge \inf F_B(x_j) \ge \inf F_C(x_j)$, and $\sup F_A(x_j) \ge \sup F_B(x_j) \ge \sup F_C(x_j)$ for j = 1, 2, …, n and $x_j \in X$. Then, we have the following inequalities:

$$\left|\inf T_A(x_j) - \inf T_B(x_j)\right| \le \left|\inf T_A(x_j) - \inf T_C(x_j)\right|,$$

$$\left|\inf T_B(x_j) - \inf T_C(x_j)\right| \le \left|\inf T_A(x_j) - \inf T_C(x_j)\right|,$$

$$\left|\sup T_A(x_j) - \sup T_B(x_j)\right| \le \left|\sup T_A(x_j) - \sup T_C(x_j)\right|,$$

$$\left|\sup T_B(x_j) - \sup T_C(x_j)\right| \le \left|\sup T_A(x_j) - \sup T_C(x_j)\right|,$$

$$\left|\inf I_A(x_j) - \inf I_B(x_j)\right| \le \left|\inf I_A(x_j) - \inf I_C(x_j)\right|,$$

$$\left|\inf I_B(x_j)-\inf I_C(x_j)\right|\le\left|\inf I_A(x_j)-\inf I_C(x_j)\right|,$$

$$\left|\sup I_A(x_j)-\sup I_B(x_j)\right|\le\left|\sup I_A(x_j)-\sup I_C(x_j)\right|,$$

$$\left|\sup I_B(x_j)-\sup I_C(x_j)\right|\le\left|\sup I_A(x_j)-\sup I_C(x_j)\right|,$$

$$\left|\inf F_A(x_j)-\inf F_B(x_j)\right|\le\left|\inf F_A(x_j)-\inf F_C(x_j)\right|,$$

$$\left|\inf F_B(x_j)-\inf F_C(x_j)\right|\le\left|\inf F_A(x_j)-\inf F_C(x_j)\right|,$$

$$\left|\sup F_A(x_j)-\sup F_B(x_j)\right|\le\left|\sup F_A(x_j)-\sup F_C(x_j)\right|,$$

$$\left|\sup F_B(x_j)-\sup F_C(x_j)\right|\le\left|\sup F_A(x_j)-\sup F_C(x_j)\right|.$$

Since the cosine function is a decreasing function within the interval [0, π/2], hence $SC_k(A, C) \le SC_k(A, B)$ and $SC_k(A, C) \le SC_k(B, C)$ for $k = 3, 4$.

Thus, we complete the proofs of these properties.

When one takes the weight of each element x_j for $x_j \in X$ into account and assumes that the weight of a element x_j is w_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, we can introduce the following weighted cosine similarity measures between INSs A and B:

$$WSC_3(A,B)=\sum_{j=1}^{n} w_j \cos\left[\frac{\pi}{4}\left(\begin{array}{l}\left|\inf T_A(x_j)-\inf T_B(x_j)\right|\vee\left|\inf I_A(x_j)-\inf I_B(x_j)\right|\vee\left|\inf F_A(x_j)-\inf F_A(x_j)\right|\\+\left|\sup T_A(x_j)-\sup T_B(x_j)\right|\vee\left|\sup I_A(x_j)-\sup I_B(x_j)\right|\vee\left|\sup F_A(x_j)-\sup F_A(x_j)\right|\end{array}\right)\right],\tag{9}$$

$$WSC_4(A,B)=\sum_{j=1}^{n} w_j \cos\left[\frac{\pi}{12}\left(\begin{array}{l}\left|\inf T_A(x_j)-\inf T_B(x_j)\right|+\left|\inf I_A(x_j)-\inf I_B(x_j)\right|+\left|\inf F_A(x_j)-\inf F_A(x_j)\right|\\+\left|\sup T_A(x_j)-\sup T_B(x_j)\right|+\left|\sup I_A(x_j)-\sup I_B(x_j)\right|+\left|\sup F_A(x_j)-\sup F_A(x_j)\right|\end{array}\right)\right].\tag{10}$$

Especially when $w_j = 1/n$ for $j = 1, 2, \ldots, n$, Eqs. (9) and (10) reduce to Eqs. (7) and (8). Then, when $T_A(x_j) = \inf T_A(x_j) = \sup T_A(x_j$, $I_A(x_j) = \inf I_A(x_j) = \sup I_A(x_j)$, and $F_A(x_j) = \inf F_A(x_j) = \sup F_A(x_j)$ for any $x_j \in X$ in A and $T_B(x_j) = \inf T_B(x_j) = \sup T_B(x_j)$, $I_B(x_j) = \inf I_B(x_j) = \sup I_B(x_j)$, $F_B(x_j)$

= inf $F_B(x_j)$ = sup $F_B(x_j)$ for any $x_j \in X$ in *B*, the INSs *A* and *B* reduce to the SVNSs *A* and *B*, and then Eqs. (7)-(10) reduce to Eqs. (3)-(6), respectively.

5. Comparative analyses of various cosine similarity measures

To compare the improved cosine measures with existing cosine measures [2] in simplified neutrosophic setting, we provide two numerical examples to demonstrate the effectiveness and rationality of the improved cosine similarity measures of SNSs.

Example 1. We consider two SVNSs *A* and *B* in *X* = {*x*} and compare the improved cosine similarity measures with existing cosine similarity measure in [2]. By applying Eqs. (1), (3) and (4) the comparison of pattern recognitions is illustrated for the numerical example. These similarity measure results are shown in Table 1.

Table 1. Similarity measure values of Eqs. (1), (3) and (4)

	Case 1	Case 2	Case 3	Case 4	Case 5
A	$\langle x$,0.2,0.3,0.4$\rangle$	$\langle x$,0.3,0.2,0.4$\rangle$	$\langle x$,1,0,0$\rangle$	$\langle x$,1,0,0$\rangle$	$\langle x$,0.4,0.2,0.6$\rangle$
B	$\langle x$,0.2,0.3,0.4$\rangle$	$\langle x$,0.4,0.2,0.3$\rangle$	$\langle x$,0,1,1$\rangle$	$\langle x$,0,0,0$\rangle$	$\langle x$,0.2,0.1, 0.3$\rangle$
$C_1(A, B)$[2]	**1**	0.9655	0	**null**	**1**
$SC_1(A, B)$	1	0.9877	**0**	**0**	0.8910
$SC_2(A, B)$	1	0.9945	0	0.8660	0.9511

Example 2. Let us consider two INSs *A* and *B* in *X* = {*x*} and compare the improved cosine similarity measures with existing cosine similarity measure in [2]. The comparison of pattern recognitions for the numerical example is demonstrated by using Eqs. (2), (7), (8). These similarity measure results are shown in Table 2.

Table 2. Similarity measure values of Eqs. (2), (7) and (8)

	Case 1	Case 2	Case 3	Case 4	Case 5
A	⟨x,[0.3,0.5], [0.2,0.4], [0,0.1]⟩	⟨x,[0.3,0.5], [0.2,0.4], [0.4, 0.5]⟩	⟨x,[1,1], [0,0], [0,0]⟩	⟨x,[1,1], [0,0], [0,0]⟩	⟨x,[0.3,0.4], [0.2,0.3], [0.4,0.5]⟩
B	⟨x,[0.3,0.5], [0.2,0.4], [0,0.1]⟩	⟨x,[0.4,0.5], [0.2,0.4], [0.3, 0.5]⟩	⟨x,[0,0], [1,1], [1,1]⟩	⟨x,[0,0], [0,0], [0,0]⟩	⟨x,[0.6,0.8], [0.4,0.6], [0.8,1]⟩
$C_2(A, B)$ [2]	**1**	0.9895	0	**null**	**1**
$SC_3(A, B)$	1	0.9969	**0**	**0**	0.7604
$SC_4(A, B)$	1	0.9986	0	0.8660	0.8526

The results of Tables 1 and 2 show that the existing cosine similarity measure [2] not only cannot carry out the recognition between Case 1 and Case 5 but also produces an unreasonable phenomenon for Case 5 and an undefined (unmeaningful) phenomenon for Case 4. This will get the decision maker into trouble in practical applications. However, the improved cosine similarity measure SC_1 cannot also carry out the recognition between Case 3 and Case 4, but does not produces an undefined (unmeaningful) phenomenon. Then, the improved cosine similarity measure SC_2 demonstrates stronger discrimination among them. Obviously, the improved cosine similarity measures are superior to the existing cosine similarity measure in [2]. Then, the cosine similarity measure SC_2 is superior to the cosine similarity measure SC_1.

The two examples all demonstrate that in some cases the improved cosine similarity measures of SNSs based on the cosine function can overcome the disadvantages of the existing cosine similarity measures between two vectors.

6. Medicine diagnoses

Due to the increased volume of information available to physicians from modem medical technologies, medicine diagnosis contains a lot of incomplete, uncertainty, and inconsistent information. In some practical situations, there is the possibility of each element having different truth-membership, indeterminacy-membership, and falsity-membership degrees, by which an SNS is expressed. Hence, similarity measures for SNSs are a suitable tool to cope with it. Therefore, we apply the improved cosine similarity measures of SNSs to medicine diagnosis. In this section, we shall discuss the medical diagnosis problems adapted from [12].

Let us consider a set of diagnoses $Q = \{Q_1$(Viral fever), Q_2(Malaria), Q_3(Typhoid), Q_4(Stomach problem), Q_5(Chest problem)$\}$, and a set of symptoms $S = \{s_1$(Temperature), s_2(Headache), s_3(Stomach pain), s_4(Cough), s_5(Chest pain)$\}$.

Then each diagnosis Q_i (i = 1, 2, 3, 4, 5) can be indicated by SNSs with respect to all the symptoms as follows:

Q_1(Viral fever) = $\{\langle s_1, 0.4, 0.6, 0.0\rangle, \langle s_2, 0.3, 0.2, 0.5\rangle, \langle s_3, 0.1, 0.3, 0.7\rangle, \langle s_4, 0.4, 0.3, 0.3\rangle, \langle s_5, 0.1, 0.2, 0.7\rangle\}$,

Q_2(Malaria) = $\{\langle s_1, 0.7, 0.3, 0.0\rangle, \langle s_2, 0.2, 0.2, 0.6\rangle, \langle s_3, 0.0, 0.1, 0.9\rangle, \langle s_4, 0.7, 0.3, 0.0\rangle\}, \langle s_5, 0.1, 0.1, 0.8\rangle\}$,

Q_3(Typhoid) = $\{\langle s_1, 0.3, 0.4, 0.3\rangle, \langle s_2, 0.6, 0.3, 0.1\rangle, \langle s_3, 0.2, 0.1, 0.7\rangle, \langle s_4, 0.2, 0.2, 0.6\rangle, \langle s_5, 0.1, 0.0, 0.9\rangle\}$,

Q_4(Stomach problem) = $\{\langle s_1, 0.1, 0.2, 0.7\rangle, \langle s_2, 0.2, 0.4, 0.4\rangle, \langle s_3, 0.8, 0.2, 0.0\rangle, \langle s_4, 0.2, 0.1, 0.7\rangle, \langle s_5, 0.2, 0.1, 0.7\rangle\}$,

Q_5(Chest problem) = $\{\langle s_1, 0.1, 0.1, 0.8\rangle, \langle s_2, 0.0, 0.2, 0.8\rangle, \langle s_3, 0.2, 0.0, 0.8\rangle, \langle s_4, 0.2, 0.0, 0.8\rangle, \langle s_5, 0.8, 0.1, 0.1\rangle\}$.

Suppose a patient P_1 with all the symptoms can be represented by the following SVNS information:

P_1(Patient) = $\{\langle s_1, 0.8, 0.2, 0.1\rangle, \langle s_2, 0.6, 0.3, 0.1\rangle, \langle s_3, 0.2, 0.1, 0.8\rangle, \langle s_4, 0.6, 0.5, 0.1\rangle, \langle s_5, 0.1,$

0.4, 0.6⟩}.

To find a proper diagnosis, we can calculate the cosine measure $SC_k(P_1, Q_i)$ for k = 1 or 2 and i = 1, 2, 3, 4, 5. The proper diagnosis Q_{i*} for the patient P_1 is derived by

$$i^* = \arg\max_{1 \le i \le 5}\{SC_k(P, Q_i)\}.$$

For convenient comparison, we utilize the existing cosine measure [2] and the two improved cosine measures to handle the diagnosis problem. By applying Eqs. (1), (3) and (4), we can obtain the results of the three similarity measures between the patient P_1 and the considered disease Q_i (i = 1, 2, 3, 4, 5), as shown in Table 3.

Table 3. Various similarity measure values for SVNS information

	Viral fever (Q_1)	Malaria (Q_2)	Typhoid (Q_3)	Stomach problem (Q_4)	Chest problem (Q_5)
$C_1(P_1, Q_i)$ [2]	0.8505	**0.8661**	0.8185	0.5148	0.4244
$SC_1(P_1, Q_i)$	0.8942	**0.8976**	0.8422	0.6102	0.5607
$SC_2(P_1, Q_i)$	0.9443	**0.9571**	0.9264	0.8214	0.7650

In Table 3, the largest similarity measure indicates the proper diagnosis. Therefore, Patient P_1 suffers from malaria. We can see that the medicine diagnoses using various similarity measures indicate the same diagnosis results and demonstrate the effectiveness of these diagnoses. However, as mentioned above, the improved cosine measures can overcome some drawbacks of the existing cosine measure in [2] in some cases. Hence, the improved cosine measures are superior to the existing cosine measure.

Compared with the diagnosis results in [12], the diagnosis results of P_1 are different. The reason is that the diagnosis method in [12] is on the basis of the cosine measure of IFSs, while the diagnosis methods in this paper are based on the improved cosine measures of SVNSs. Therefore different measure methods with different kinds of information represented by IFSs and SVNSs may give different diagnosis results. Furthermore, the diagnosis method in [12] cannot handle the diagnosis problem with single valued neutrosophic information, while the diagnosis methods in this paper can deal with the diagnosis problems with intuitionistic fuzzy information and simplified neutrosophic information. Hence, the improved cosine measures of SVNSs are superior to the cosine measure of IFSs [12].

However, by only taking one time inspection, we wonder whether one can obtain a conclusion from a particular person with a particular decease or not. Hence, we have to examine the patient at different time intervals (e.g. two or three times a day) and can obtain that data drawn from multiple time inspections for the patient are interval values rather than single values. In this case, the improved cosine measures of INSs are a better tool to find a proper disease diagnosis.

Suppose a patient P_2 with all the symptoms can be represented by the following INS information:

P_2(Patient) = $\{\langle s_1, [0.3, 0.5], [0.2, 0.3], [0.4, 0.5]\rangle, \langle s_2, [0.7, 0.9], [0.1, 0.2], [0.1, 0.2]\rangle, \langle s_3, [0.4, 0.6], [0.2, 0.3], [0.3, 0.4]\rangle, \langle s_4, [0.3, 0.6], [0.1, 0.3], [0.4, 0.7]\rangle, \langle s_5, [0.5, 0.8], [0.1, 0.4], [0.1, 0.3]\rangle\}$.

Similarly, we utilize the existing cosine measure [2] and the two improved cosine measures of INSs to handle the diagnosis problem. By applying Eqs. (2), (7) and (8), we can obtain the results of various similarity measures between the patient P_2 and the considered disease Q_i (i = 1, 2, 3, 4, 5), as shown in Table 4.

Table 4. Various similarity measure values for INS information

	Viral fever (Q_1)	Malaria (Q_2)	Typhoid (Q_3)	Stomach problem (Q_4)	Chest problem (Q_5)
$C_2(P_2, Q_i)$ [2]	0.6775	0.5613	**0.7741**	0.7198	0.6872
$SC_3(P_2, Q_i)$	0.7283	0.6079	**0.7915**	0.7380	0.7157
$SC_4(P_2, Q_i)$	0.8941	0.8459	**0.9086**	0.9056	0.8797

In Table 4, the largest similarity measure indicates the proper diagnosis. Therefore, Patient P_2 suffers from typhoid. We can see that the medicine diagnoses using various similarity measures indicate the same diagnosis results and demonstrate the effectiveness of these diagnoses. However, as mentioned above, the improved cosine measures can overcome some drawbacks of the existing cosine measure in [2] in some cases. Hence, the improved cosine measures are superior to the existing cosine measure.

7. Conclusion

This paper proposed the improved cosine similarity measures for SNSs based on the cosine function, including single valued neutrosophic cosine similarity measures and interval neutrosophic cosine similarity measures. Then, the weighted cosine similarity measures of SNSs are proposed by considering the importance of each element. Compared with existing cosine similarity measures under simplified neutrosophic environment, the improved cosine measures of SNSs demonstrate their effectiveness and rationality and can overcome some drawbacks of existing cosine similarity measures of SNSs. Finally, medical diagnosis problems with simplified neutrosophic information are provided to demonstrate the applications and effectiveness of the improved cosine similarity measures of SNSs.

In further work, it is necessary to apply the cosine similarity measures of SNSs to other areas such as decision making, Image processing, and clustering analysis.

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Table(1)

Table 1. Similarity measure values of Eqs. (1), (3) and (4)

	Case 1	Case 2	Case 3	Case 4	Case 5
A	$\langle x,0.2,0.3,0.4\rangle$	$\langle x,0.3,0.2,0.4\rangle$	$\langle x,1,0,0\rangle$	$\langle x,1,0,0\rangle$	$\langle x,0.4,0.2,0.6\rangle$
B	$\langle x,0.2,0.3,0.4\rangle$	$\langle x,0.4,0.2,0.3\rangle$	$\langle x,0,1,1\rangle$	$\langle x,0,0,0\rangle$	$\langle x,0.2,0.1, 0.3\rangle$
$C_1(A, B)$[2]	**1**	0.9655	0	**null**	**1**
$SC_1(A, B)$	1	0.9877	**0**	**0**	0.8910
$SC_2(A, B)$	1	0.9945	0	0.8660	0.9511

Table(2)

Table 2. Similarity measure values of Eqs. (2), (7) and (8)

	Case 1	Case 2	Case 3	Case 4	Case 5
A	⟨*x*,[0.3,0.5], [0.2,0.4], [0,0.1]⟩	⟨*x*,[0.3,0.5], [0.2,0.4], [0.4, 0.5]⟩	⟨*x*,[1,1], [0,0], [0,0]⟩	⟨*x*,[1,1], [0,0], [0,0]⟩	⟨*x*,[0.3,0.4], [0.2,0.3], [0.4,0.5]⟩
B	⟨*x*,[0.3,0.5], [0.2,0.4], [0,0.1]⟩	⟨*x*,[0.4,0.5], [0.2,0.4], [0.3, 0.5]⟩	⟨*x*,[0,0], [1,1], [1,1]⟩	⟨*x*,[0,0], [0,0], [0,0]⟩	⟨*x*,[0.6,0.8], [0.4,0.6], [0.8,1]⟩
$C_2(A, B)$ [2]	**1**	0.9895	0	**null**	**1**
$SC_3(A, B)$	1	0.9969	**0**	**0**	0.7604
$SC_4(A, B)$	1	0.9986	0	0.8660	0.8526

Table(3)

Table 3. Various similarity measure values for SVNS information

	Viral fever (Q_1)	Malaria (Q_2)	Typhoid (Q_3)	Stomach problem (Q_4)	Chest problem (Q_5)
$C_1(P_1, Q_i)$ [2]	0.8505	**0.8661**	0.8185	0.5148	0.4244
$SC_1(P_1, Q_i)$	0.8942	**0.8976**	0.8422	0.6102	0.5607
$SC_2(P_1, Q_i)$	0.9443	**0.9571**	0.9264	0.8214	0.7650

Table(4)

Table 4. Various similarity measure values for INS information

	Viral fever (Q_1)	Malaria (Q_2)	Typhoid (Q_3)	Stomach problem (Q_4)	Chest problem (Q_5)
$C_2(P_2, Q_i)$ [2]	0.6775	0.5613	**0.7741**	0.7198	0.6872
$SC_3(P_2, Q_i)$	0.7283	0.6079	**0.7915**	0.7380	0.7157
$SC_4(P_2, Q_i)$	0.8941	0.8459	**0.9086**	0.9056	0.8797