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GRA Method of Multiple Attribute Decision Making with Single Valued Neutrosophic Hesitant Fuzzy Set Information

Abstract

Single valued neutrosophic hesitant fuzzy set has three independent parts, namely the truth membership hesitancy function, indeterminacy membership hesitancy function, and falsity membership hesitancy function, which are in the form of sets that assume values in the unit interval [0, 1]. Single valued neutrosophic hesitant fuzzy set is considered as a powerful tool to express uncertain, incomplete, indeterminate and inconsistent information in the process of multi attribute decision making problems. In this paper we study multi attribute decision making problems in which the rating values are expressed with single valued neutrosophic hesitant fuzzy set information. Firstly, we define score value and accuracy value to compare single valued neutrosophic hesitant fuzzy sets and then define normalised Hamming distance between the single valued neutrosophic hesitant fuzzy sets. Secondly, we propose the grey relational analysis method for multi attribute decision making under single valued neutrosophic hesitant fuzzy set environment. Finally, we provide an illustrative example to demonstrate the validity and effectiveness of the proposed method.

Keywords

Hesitant fuzzy sets, single-valued neutrosophic hesitant fuzzy sets, score and accuracy function, grey relational analysis method, multi-attribute decision making.

1. Introduction

Multi-attribute decision making (MADM) used in human activities is a useful process for selecting the best alternative that has the highest degree of satisfaction from a set of feasible alternatives with respect to the attributes. Because the real world is fuzzy rather than precise in nature, the rating values of alternative with respect to attribute considered in MADM problems are often imprecise or incomplete in nature. This has led to the development of the fuzzy set theory proposed by Zadeh [1]. Fuzzy set theory has been proved to be an effective tool in MADM process [2-6]. However, fuzzy set can represent imprecise information with membership degree only. The intuitionistic fuzzy set (IFS) proposed by Attanasov [7], a generalisation of fuzzy sets, is characterized by membership and non-membership functions where non-membership is independent. Recently, IFS has been successfully applied in many decision making problems, especially in MADM problems [8-12].

However IFS can handle incomplete information and but it cannot express indeterminate and inconsistent information with membership and non-membership functions. Smarandache [13] introduced the neutrosophic set (NS) from philosophical point of view to deal with uncertain, imprecise, incomplete and inconsistent information that exist in real world. NS is characterised with truth membership, indeterminacy and falsity membership degree, which are independent in nature. This set generalises the concept of crisp set, fuzzy set, intuitionistic fuzzy set, paraconsistent set, dialetheist set, paradoxist set, and tautological set. Since the introduction of NS and single-valued neutrosophic set proposed by Wang et al. [14] in 2010, the model of decision making under neutrosophic environment has been received much attention to the researchers. Many methods of MADM such as TOPSIS method [15, 16], grey relational analysis (GRA) method [17,18], distance and similarity measure method [19-23], and outranking method [24] were developed under neutrosophic environment.

However, in a decision making process sometimes decision maker may feel hesitate to take decision among the set of possible values instead of single value. Tora [25], Tora and Narukawa [26] introduced the hesitant fuzzy set (HF), which permits the membership degree of an element to a given set to be represented by the set of possible numerical values in [0,1]. HF, an extension of fuzzy set, is useful to deal uncertain information in the process of MADM. Xia and Xu [27] proposed some aggregation operators for hesitant fuzzy information and applied them to MADM problem in hesitant fuzzy environment. Wei [28] studied some models for hesitant fuzzy MADM problem by developing some prioritized aggregation operators for hesitant fuzzy information. Xu and Zhang [29] developed TOPSIS method for hesitant fuzzy MADM with incomplete weight information.

Decision maker does not consider the non-membership degrees of rating values in hesitant fuzzy MADM. However, non-membership degrees play an important role to express incomplete information. Zhu et al. [30] gave the idea of the dual hesitant fuzzy set (DHFS), in which membership degrees and non-membership degrees are in the form of sets of values in [0,1]. DHFS generalizes the HF sets and expresses incomplete information effectively. Ye [31] and Chen et al.[32] proposed co-relation method between DHFSs and applied the method to MADM with hesitant fuzzy information. Singh [33] defined and applied distance and similarity measure between DHFSs in MADM. However in a decision making process, indeterminate type information cannot be captured with DHFS.

In 2014, Ye [34] introduced single-valued neutrosophic hesitant fuzzy set (SVNHFS) by coordinating HFS and SVNS. SVNHFS generalises the FS, IFS, HFS, DHFS and SVNS, and can represent uncertain, imprecise, incomplete and inconsistent information. SVNHFSs are characterized by truth hesitancy, indeterminacy hesitancy and falsity-hesitancy membership functions which are independent. Therefore SVNHFS can express the three kinds of hesitancy information that exist in MADM in real situations. Ye [34] developed single valued neutrosophic hesitant fuzzy weighted averaging and single valued neutrosophic hesitant fuzzy weighted geometric operators for SVNHFS information and applied these two operators in MADM. Liu and Shi [35] proposed hybrid weighted average operator for interval neutrosophic hesitant fuzzy set in which the truth hesitancy, indeterminacy hesitancy and falsity-hesitancy membership functions are in the form of sets of interval values contained in [0, 1]. Sahin and Liu [36] defined co-relation co-efficient between SVNHFSs and used it for MADM.

Grey relational analysis (GRA)[37], a part of grey system theory, is successfully applied in solving a variety of MADM problems in intuitionistic fuzzy environment [38-42], neutrosophic environment [43], interval neutrosophic environment [44, 45, 46], neutrosophic soft set environment [47-49], rough neutrosophic environment [50] respectively. However, literature review reflects that GRA method of MADM with SVNHFS has not been studied in the literature. Therefore we need attention for this issue. The aim of the paper is to extend the concept of GRA method for solving MADM problem in which the rating values of the alternatives over the attributes are considered with SVNHFSs.

The rest of the paper is organised as follows: Section 2 presents some basic concept related to SVNHFSs. In Section 3, we propose GRA method for MADM problems, where rating values are considered with SVNHFSs. In Section 4, we illustrate our proposed method with an example. Section 5 presents concluding remarks of the study.

2. Preliminaries

In this section we recall some basic definitions of hesitant fuzzy set, single valued neutrosophic hesitant fuzzy set, score function accuracy function of triangular fuzzy intuitionistic fuzzy numbers.

Definition 1. [25]Let X be a fixed set, then a hesitant fuzzy set (HFS) A on X is in terms of a function that when applied to X returns a subset of $[0,1]$, i.e.,

$A=\{\langle x,h_A(x)\rangle \mid x\in X\}$, where, $h_A(x)$ is a set of some different values in $[0,1]$, representing the possible membership degrees of the element $x\in X$ to A. For convenience, $h_A(x)$ is called a hesitant fuzzy element (HFE).

Definition 2. [34] Let X be fixed set, then a single valued hesitant fuzzy element (SVHFE) N on X is defined as $N=\{\langle x,t(x),i(x),f(x)\rangle \mid x\in X\}$ (1)

where $t(x)$, $i(x)$ and $f(x)$ represent three sets of values in $[0,1]$, denoting respectively the possible truth, indeterminacy and falsity membership degree of the element $x\in X$ to the set N. The membership degrees $t(x)$, $i(x)$ and $f(x)$ satisfy the following conditions:

$$0\le \delta,\gamma,\eta \le 1;\ 0\le \delta^+ +\gamma^+ +\eta^+ \le 3 \tag{2}$$

where, $\delta\in t(x),\gamma\in i(x),\eta\in f(x)$, $\delta^+\in t^+(x)=\bigcup_{\delta\in t(x)}\max t(x)$, $\gamma^+\in i^+(x)=\bigcup_{\delta\in t(x)}\max i(x)$, $\eta^+\in f^+(x)=\bigcup_{\delta\in t(x)}\max f(x)$ for all $x\in X$.

For convenience, the triplet $n(x)=\langle t(x),i(x),f(x)\rangle$ is called a SVNHFE denoted by $n=\langle t,i,f\rangle$. Note that the number of values for possible truth, indeterminacy and falsity membership degrees of the element in different SVNHFEs may be different.

Definition 3. [34] Let $n_1=\langle t_1,i_1,f_1\rangle$ and $n_2=\langle t_2,i_2,f_2\rangle$ be two SVNHFEs, the following operational rules are defined as follows:

7. $n_1\oplus n_2=\left\langle \bigcup_{\delta_1\in t_1,\gamma_1\in i_1,\eta_1\in f_1,\ \delta_2\in t_2,\gamma_2\in i_2,\eta_2\in f_2} \{\{t_1+t_2-t_1t_2\},\{i_1i_2\},\{f_1,f_2\}\}\right\rangle;$

8. $n_1\otimes n_2=\left\langle \bigcup_{\delta_1\in t_1,\gamma_1\in i_1,\eta_1\in f_1,\ \delta_2\in t_2,\gamma_2\in i_2,\eta_2\in f_2} \{\{t_1t_2\},\{i_1+i_2-i_1i_2\},\{f_1+f_2-f_1f_2\}\}\right\rangle;$

9. $\lambda n_1=\left\langle \bigcup_{\delta_1\in t_1,\gamma_1\in i_1,\eta_1\in f_1} \{\{1-(1-t_1)^\lambda\},\{i_1^\lambda\},\{f_1^\lambda\}\}\right\rangle,\lambda>0$;

10. $n_1^\lambda=\left\langle \bigcup_{\delta_1\in t_1,\gamma_1\in i_1,\eta_1\in f_1} \{\{t_1^\lambda\},\{1-(1-i_1)^\lambda\},\{1-(1-f_1)^\lambda\}\}\right\rangle,\lambda>0.$

Definition 4. Let $n_i = \langle t_i, i_i, f_i \rangle$ $(i = 1, 2, ..., n)$ be a collection of SVNHFEs, then the score function $S(n_i)$, and accuracy function $A(n_i)$ of $n_i (i = 1, 2, ..., n)$ can be defined as follows:

1. $$S(n_i) = \frac{1}{3}\left[2 + \frac{1}{l_t}\sum_{\delta \in t}\delta - \frac{1}{l_i}\sum_{\gamma \in i}\gamma - \frac{1}{l_f}\sum_{\eta \in f}\eta\right] \quad (3)$$
2. $$A(n_i) = \frac{1}{l_t}\sum_{\delta \in t}\delta - \frac{1}{l_f}\sum_{\eta \in f}\eta; \quad (4)$$

where, l_t, l_i, and l_f, are the numbers of values of t_i, i_i, and f_i respectively in n_i.

Definition 5. Let $n_1 = \langle t_1, i_1, f_1 \rangle$ and $n_2 = \langle t_2, i_2, f_2 \rangle$ be two SVNHFEs, the following rules can be defined for comparison purposes:

1. If $S(n_1) > S(n_2)$, then n_1 is greater than n_2 and denoted by $n_1 \succ n_2$;
2. If $S(n_1) = S(n_2)$ and $A(n_1) > A(n_2)$, then $n_1 \succ n_2$;
3. If $S(n_1) = S(n_2)$ and $A(n_1) = A(n_2)$, then $n_1 \approx n_2$.

Definition 6. Let $n_1 = \langle t_1, i_1, f_1 \rangle$ and $n_2 = \langle t_2, i_2, f_2 \rangle$ be two SVNHFEs, the normalised Hamming distance is defined as

$$D(n_1, n_2) = \frac{1}{3}\left(\left|\frac{1}{l_{t_1}}\sum_{\delta_1 \in t_1}\delta_1 - \frac{1}{l_{t_2}}\sum_{\delta_2 \in t_2}\delta_2\right| + \left|\frac{1}{l_{i_1}}\sum_{\gamma_1 \in i_1}\gamma_1 - \frac{1}{l_{i_2}}\sum_{\gamma_2 \in i_2}\gamma_2\right| + \left|\frac{1}{l_{f_1}}\sum_{\eta_1 \in f_1}\eta_1 - \frac{1}{l_{f_2}}\sum_{\eta_2 \in f_2}\eta_2\right|\right) \quad (5)$$

where l_{t_k}, l_{i_k}, and l_{f_k} are the possible membership values in n_k for $k = 1, 2$, respectively.

The distance function $D(n_1, n_2)$ of two SVNHFEs n_1 and n_2 satisfies the following properties:

1. $0 \le D(n_1, n_2) \le 1$;
2. $D(n_1, n_2) = 0$ if and only if $n_1 = n_2$;
3. $D(n_1, n_2) = D(n_2, n_1)$;
4. If $n_1 \le n_2 \le n_3$, and n_3 is an SVNHFE on X, then $D(n_1, n_2) \le D(n_1, n_3)$ and $D(n_2, n_3) \le D(n_1, n_3)$.

3. GRA method for multi-attribute decision making with SVNHFS information

In this section, we propose GRA based approach to find out the best alternative in multi-attribute decision making problem in SVNHFS environment. Assume that $A = \{A_1, A_2, ..., A_m\}$ be the discrete set of m alternatives and $C = \{C_1, C_2, ..., C_n\}$ be the set of n attributes for a multi-attribute decision making problem. Suppose that the rating values of the $i-$th alternative $A_i (i = 1, 2, ..., m)$ over the attribute $C_j (j = 1, 2, ..., n)$ are expressed in terms of SVNHFSs $x_{ij} = \langle t_{ij}, i_{ij}, f_{ij} \rangle$, where $t_{ij} = \{\delta_{ij} \mid \delta_{ij} \in t_{ij}, 0 \le \delta_{ij} \le 1\}$, $i_{ij} = \{\gamma_{ij} \mid \gamma_{ij} \in i_{ij}, 0 \le \gamma_{ij} \le 1\}$, and $f_{ij} = \{\eta_{ij} \mid \eta_{ij} \in f_{ij}, 0 \le \eta_{ij} \le 1\}$ are the possible truth, indeterminacy and falsity membership degrees, respectively. With these rating values, we can construct a decision matrix $X = (x_{ij})_{m \times n}$, where the entries of this matrix are SVNHFSs. The decision matrix can be presented as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & \dots x_{1n} \\ x_{21} & x_{22} & \dots x_{2n} \\ \vdots & \vdots & \ddots \vdots \\ x_{m1} & x_{m2} & \dots x_{mn} \end{bmatrix} \quad (6)$$

We develop the GRA method using the following steps by considering the weight vector $W = (w_1, w_2, ..., w_n)^T$ of attributes where $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Step 1. Determine the single valued neutrosophic hesitant fuzzy positive ideal solution (SVNHFPIS) A^+ and

the single valued neutrosophic hesitant fuzzy negative ideal solution (SVNHFNIS) A^- of alternatives in the decision matrix X by the following equations, respectively:

$$A^+ = \begin{cases} \max\limits_{1\le i\le m}(x_{i1}), \max\limits_{1\le i\le m}(x_{i2}), ..., \max\limits_{1\le i\le m}(x_{in}) \text{ for benefit type attribute;} \\ \min\limits_{1\le i\le m}(x_{i1}), \min\limits_{1\le i\le m}(x_{i2}), ..., \min\limits_{1\le i\le m}(x_{in}) \text{ for cost type attribute} \end{cases} \quad (7)$$
$$= \left(A_{i1}{}^+, A_{i2}{}^+, ..., A_{in}{}^+\right)$$

$$A^- = \begin{cases} \min\limits_{1\le i\le m}(x_{i1}), \min\limits_{1\le i\le m}(x_{i2}), ..., \min\limits_{1\le i\le m}(x_{in}) \text{ for benefit type attribute;} \\ \max\limits_{1\le i\le m}(x_{i1}), \max\limits_{1\le i\le m}(x_{i2}), ..., \max\limits_{1\le i\le m}(x_{in}) \text{ for cost type attribute} \end{cases} \quad (8)$$
$$= \left(A_{i1}{}^-, A_{i2}{}^-, ..., A_{in}{}^-\right)$$

The rating values x_{ij} can be compared by the score function $S(x_{ij})$ and accuracy function $A(x_{ij})$ defined in Definition 3.

Step 2. Determine the grey relational co-efficient of each alternative from A^+ and A^- by the following equations:

$$\xi_{ij}^+ = \frac{\min\limits_{1\le i\le m}\min\limits_{1\le i\le m} D(x_{ij}, A_j^+) + \max\limits_{1\le i\le m}\max\limits_{1\le i\le m} D(x_{ij}, A_j^+)}{D(x_{ij}, A_j^+) + \rho \max\limits_{1\le i\le m}\max\limits_{1\le i\le m} D(x_{ij}, A_j^+)} \quad (9)$$

$$\xi_{ij}^- = \frac{\min\limits_{1\le i\le m}\min\limits_{1\le i\le m} D(x_{ij}, A_j^-) + \max\limits_{1\le i\le m}\max\limits_{1\le i\le m} D(x_{ij}, A_j^-)}{D(x_{ij}, A_j^-) + \rho \max\limits_{1\le i\le m}\max\limits_{1\le i\le m} D(x_{ij}, A_j^-)} \quad (10)$$

where the identification co-efficient is considered as $\rho = 0.5$.

Step 3. Calculate the degree of grey relational coefficient of each alternative $A_i (i = 1, 2, ..., m)$ from A^+ and A^- by the following equations:

$$\xi_i^+ = \sum_{j=1}^{n} w_j \xi_{ij}^+ \quad (11)$$

$$\xi_i^- = \sum_{j=1}^{n} w_j \xi_{ij}^- \quad (12)$$

Step 4. Calculate the relative closeness co-efficient ξ_i for each alternative $A_i (i = 1, 2, .., m)$ with respect to the positive ideal solution A^+ as

$$\xi_i = \frac{\xi_i^+}{\xi_i^+ + \xi_i^-} \text{ for } i = 1, 2, .., m. \quad (13)$$

Step 5. Rank the alternative according the relative closeness co-efficient $\xi_i (i = 1, 2, .., m)$.

4. A Numerical Example

In this section we consider the example adopted from Ye [34] to illustrate the application of the proposed GRA method for MADM proposed in Section 4. Consider an investment company that wants to invest a sum of money in the best option. The following four possible alternatives are considered to invest the money:

1. A_1 is the car company;
2. A_2 is the food company;
3. A_3 is the computer company;
4. A_4 is the arms company.

The investment company must take a decision according to the following three attributes:

1. C_1 is the risk analysis;
2. C_2 is the growth analysis;
3. C_3 is the environmental impact analysis.

The attribute weight vector is given as $W=(0.35,0.25.0.40)^T$. The four possible alternatives $\{A_1,A_2,A_3,A_4\}$ are evaluated using SVNHFEs under three attributes $C_j(j=1,2,3)$. We can arrange the rating values in a matrix form called a SVNHF decision matrix $X=(x_{ij})_{4\times 3}$ (see Table-1).

Table 1. *Single valued neutrosophic hesitant fuzzy decision matrix*

C_1	C_2	C_3
$\{\{0.3,0.4,0.5\},\{0.1\},\{0.3,0.4\}\}$	$\{\{0.5,0.6\},\{0.2,0.3\},\{0.3,0.4\}\}$	$\{\{0.3,0.4,0.5\},\{0.1\},\{0.3,0.4\}\}$
$\{\{0.6,0.7\},\{0.1,0.2\},\{0.2,0.3\}\}$	$\{\{0.6,0.7\},\{0.1\},\{0.3\}\}$	$\{\{0.3,0.4,0.5\},\{0.1\},\{0.3,0.4\}\}$
$\{\{0.5,0.6\},\{0.4\},\{0.2,0.3\}\}$	$\{\{0.6\},\{0.3\},\{0.4\}\}$	$\{\{0.5,0.6\},\{0.1\},\{0.3\}\}$
$\{\{0.7,0.8\},\{0.1\},\{0.1,0.2\}\}$	$\{\{0.6,0.7\},\{0.1\},\{0.2\}\}$	$\{\{0.3,0.5\},\{0.2\},\{0.1,0.2,0.3\}\}$

Now we apply the proposed method to find out the best alternative, which can be described as follows:

Step 1. Comparing the attribute values by score function and accuracy function of SVNHFEs, we can determine the neutrosophic hesitant fuzzy positive ideal solution (SVNHFPIS) A^+ by the Eq.(7) as follows:

$$\begin{aligned} A^+ &= \left[\{\{0.7,0.8\},\{0.1\},\{0.1,0.2\}\},\{\{0.6,0.7\},\{0.1\},\{0.2\}\},\{\{0.6,0.7\},\{0.1,0.2\},\{0.1,0.2\}\}\right] \\ &= \left[A_1^+,A_2^+,A_3^+\right] \end{aligned} \tag{14}$$

Similarly, we can determine the neutrosophic hesitant fuzzy negative ideal solution (SVNHFPIS) A^- by the Eq.(8) as follows:

$$\begin{aligned} A^- &= \left[\{\{0.5,0.6\},\{0.4\},\{0.2,0.3\}\},\{\{0.6\},\{0.3\},\{0.4\}\},\{\{0.2,0.3\},\{0.1,0.2\},\{0.5,0.6\}\}\right] \\ &= \left[A_1^-,A_2^-,A_3^-\right] \end{aligned} \tag{15}$$

Step 2. Calculate the grey relational co-efficient of each alternative from positive ideal solutions A^+ and negative ideal solutions A^- by equations (9) and (10) for $\rho=0.5$, respectively.

$$\xi_{ij}^+ = \begin{bmatrix} 0.4218 & 0.5010 & 0.3333 \\ 0.6166 & 0.8018 & 1.0000 \\ 0.4003 & 0.4709 & 0.5717 \\ 1.0000 & 1.0000 & 0.5350 \end{bmatrix} \tag{16}$$

$$\xi_{ij}^+ = \begin{bmatrix} 0.4218 & 0.7275 & 1.0000 \\ 0.5329 & 0.5329 & 0.3333 \\ 1.0000 & 1.0000 & 0.4218 \\ 0.4003 & 0.4709 & 0.4218 \end{bmatrix}$$

(17)

Here, we consider $i=1,2,3,4$ and $j=1,2,3.$

Step 3. Calculate the degree of grey relational co-efficient of each alternative from A^+ and A^- by Eqs. (11) and (12), respectively.

$\xi_1^+ = 0.4062 \qquad \xi_2^+ = 0.8162 \qquad \xi_3^+ = 0.4865 \qquad \xi_4^+ = 0.8140 \qquad (18)$

$\xi_1^- = 0.7295 \qquad \xi_2^- = 0.4531 \qquad \xi_3^- = 0.7687 \qquad \xi_4^- = 0.4265 \qquad (19)$

Step 4. Calculate the relative closeness coefficient ξ_i for each alternative $A_i (i = 1,2,3,4)$ by Eq.(13). $\xi_1 = 0.3577,\ \xi_2 = 0.6430,\ \xi_3 = 0.3875,$ and $\xi_4 = 0.6561.$

Step 5. Rank the alternative according to the relative closeness coefficient $\xi_i (i = 1,2,3,4)$.

Therefore $A_4 \succ A_2 \succ A_3 \succ A_1$ indicates that the most desirable alternative is A_4.

We notice that the ranking order obtained by the proposed method is indifferent with the ranking of the alternative obtained by Ye's method [34].

5. Conclusions

In general, the information of rating values considered in MADM problems is imprecise, indeterminate, incomplete and inconsistent in nature. SVNHFS is a useful tool that can capture all these type of information in MADM process. In this paper we investigate MADM problem in which rating values are considered with SVNHFSs. To extend the GRA method for MADM, we first define score value, accuracy value, certainty value, and normalised Hamming distance of SVNHFS. Having defined the positive ideal solution (PIS) and the negative ideal solution (NIS) by score value and accuracy value, we calculate the grey relational degree between each alternative and ideal alternatives (PIS and NIS). Then we determine a relative relational degree to obtain the ranking order of all alternatives by calculating the degree of grey relation to both the positive and negative ideal solution simultaneously. Finally, we provide an illustrative example to show the validity and effectiveness of the proposed approach. The proposed approach is compared with other existing methods to show that our approach is straightforward and can be applied effectively with other decision making problems under SVNHF environment. In future, we will extend the proposed approach to MADM under SVNHFS environment with unknown weight information and MADM with interval valued neutrosophic hesitant fuzzy environment.

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