CHAPTER «ENGINEERING SCIENCES»

SYNTHESIS OF INFORMATION DECISION-SUPPORT TECHNOLOGIES UNDER COMPLEX FORMS OF IGNORANCE

Igor Kovalenko[1]
Alyona Shved[2]

DOI: https://doi.org/10.30525/978-9934-26-001-8-2-6

Abstract. The analysis of multi-criteria techniques showed that at present, methods based on the mechanism of pairwise comparison are widely used. This may be due to the fact that it is easier for experts to compare objects in pairs than, for example, to give them some ordering (ranking). In turn, such methods have a number of disadvantages, for example, a limitation on the number of elements compared in pairs, the need to evaluate all available elements (objects, alternatives), a high level of consistency of expert assessments, etc. A modification of the analytic hierarchy process (AHP) method based on the mathematical apparatus of a mathematical theory of evidence and a theory of plausible and paradoxical reasoning has been considered as approach that allows obtaining more effective results of pairwise comparison, as well as taking into account various forms of interaction of expert judgments expressed on the same set of initial data and factors such as uncertainty, inaccuracy, fuzziness and incompleteness of expert information. As part of solving the problem of ranking group expert assessments, mathematical models of expert judgments (evidence, assessments) have been developed that allow processing the results of an expert survey in order to construct final rank-orderings of group expert assessments under multi-criteria, multi-alternative, uncertainty and

[1] Doctor of Technical Sciences, Professor,
Professor at Department of Software Engineering,
Petro Mohyla Black Sea National University, Ukraine
[2] Candidate of Technical Sciences,
Associate Professor at Department of Software Engineering,
Petro Mohyla Black Sea National University, Ukraine

conflicting (contradictory) expert judgments. Alternative approaches of evidence combination in the framework of the Dempster-Shafer model are investigated and a method for constructing a final ranking is proposed, which is based on the complex use of the conjunctive consensus evidence combination rules (Dempster's, Yager's, Zhang's, Inagaki's, Smets' rules). The proposed technique takes into account the degree of difference and the structure of individual groups of evidence to choose the order of expert judgments combination. This allows to make full use of the original expert information and exclude situations in which part of the expert information may be lost (for example, when trying to combine contradictory evidence). An adaptive algorithm for choosing the optimal combination rule has been proposed. The adaptability of the algorithm lies in the fact that, depending on the formed set of criteria, one of the considered combination rules is selected for each pair of expert judgments that are combined. As criteria for choosing the rules, the following were considered: information about data sources (experts), their competence, the nature of the analyzed data (information about conflicts and consensus; information about the degree of interaction and structure of expert evidence, etc.). A methodology for synthesis of information technology and a generalized structure of information technology for decision-making are proposed for solving the problem of structuring expert assessments under multi-criteria and complex forms of ignorance based on the methods of the theory of evidence and the theory of plausible and paradoxical reasoning. The practical implementation of the proposed information technology synthesis technique for construction of the final rank-ordering of the analyzed objects is considered on the example of solving the problem of choosing a geographic information systems.

1. Introduction

Most of the choice problems that arise in practice are semi-structured, i.e. those in which qualitative, non-formalized factors prevail. At the same time, the decision-making problems become much more complicated, they are characterized by a large number of features, indicators, which are called criteria for alternatives evaluation, and the information obtained from sources (experts) is inaccurate, incomplete and subjective.

However, in practice, there may be situations in which various forms of ignorance are simultaneously present, for example, a combination of uncertainty and imprecision. For example, during the examination, the methods of identifying and (or) analyzing expert information were not reasonably chosen (the factor generates a situation of inaccuracy of the obtained data); during the analysis of expert information, the information about the competence of experts was not taken into account (the factor generates a situation of uncertainty regarding the obtained data). At the same time, it is necessary to take into account the fact that in the process of expert assessment, judgments received from experts can interact with each other to one degree or another, regarding the information that they can give about a set of initial data. For the analysis and processing of expert judgments, which may in some way combine or intersect, it becomes necessary to apply new approaches that allow to analyze some specific types of uncertainties, such approaches include the mathe-matical theory of evidence (Dempster-Shafer, evidence theory, DS theory, DST), theory of plausible and paradoxical reasoning (Dezert-Smarandache theory, DSm theory, DSmT) [1; 3, p. 39–42; 6; 12; 15, p. 11–18; 16, p. 91–94].

In this regard, it is actual to develop new procedures, methods and algorithms, which allow the assessment of expert judgments, which are characterized by unac-ceptable, and possibly contradictory expert judgments, to take into account possible ways of interaction of expert judgments obtained under multi-criteria, and various forms of ignorance (uncertainty, inconsistency, inaccuracy and their possible combinations), and the developing on their basis the modern information technologies for decision-support.

The purpose of the research is to present and formalize the main ideas and conceptions of the information technology for decision support in conditions of complex forms of ignorance, which is based on a systematic approach using the instrumental expert methods for scenario analysis and scenario construction based on the mathematical apparatus of the theory of evidence and the theory of plausible and paradoxical reasoning.

2. The basic concepts of DS theory

The mathematical apparatus of the DS theory is based on the modeling of spe-cific forms of ignorance, caused by combinations, for example, uncertainty and con-flict, which are detected in the course of different possible interaction of expert judgments.

Let $\Omega = \{\omega_i \mid i = 1, \ldots, n\}$ be a set, which in DS theory notation is called a frame of discernment [1, p. 21; 3, p. 39–42; 6; 12]. The basis of this set (set of analyzed elements) is a set of mutually exclusive (uniquely defined and different from others) and exhaustive (all possible) elements. In this case, it is known that only one element $\omega_0 \in \Omega$ is true in each case.

Based on Ω, arbitrary subsets of the elements $X_j \subseteq \Omega$ can be formed, provided that the ω_0 can belong to each of them. The number of possible subsets of Ω is $\left|2^{\Omega}\right|$, where 2^{Ω} is the exponent set, which is the union of all subsets of Ω, taking into account the empty subset $\varnothing$.

Evidence is any source of information on the basis of which degrees of proba-bility can be obtained that the element ω_0 belongs to the subset $X_j \subseteq \Omega$.

Subsets Xj can be formed on the basis of a system of rules of the next form:

1. $X_j = \{\varnothing\}$;
2. $X_j = \{\omega_i\}$ – the expert has evaluated one element $\omega_i \in \Omega$.
3. $X_j = \{\omega_i \mid i = \overline{1, k}\}$, $\underline{k <}$ n – the expert evaluated k elements $\omega_i \in \Omega$. (1)
4. $X_j = \Omega = \{\omega_i \mid i = \overline{1, n}\}$ – the expert had it difficult to evaluate any elements of Ω.

There are three main functions that form the basis of the DS theory ($\forall X \subseteq \Omega$):

– the basic probability assignment (*bpa*) function $m : \Lambda \to [0,1]$:

$$0 \le m(X_j) \le 1, \quad \forall (X_j \in \Lambda), \quad m(\varnothing) = 0, \quad \sum_{X_j \in \Lambda} m(X_j) = 1 . \tag{2}$$

– the belief function $Bel : \Lambda \to [0,1]$:

$$Bel(B) = \sum_{X_j \subseteq B,\, X_j \in \Lambda} m(X_j) . \tag{3}$$

– the plausibility function $Pl : \Lambda \to [0,1]$:

$$Pl(B) = \sum_{X_j \cap B \ne \varnothing,\, X_j \in \Lambda} m(X_j) , \tag{4}$$

where Λ corresponds to 2^{Ω}.

The value of the *m*(*X*) function determines the subjective certainty that the element ω_0 is in the subset $X \subset \Omega$.

The value of the belief function $Bel(\cdot)$ determines the total degree of support provided to each of the subsets $X \subset \Omega$.

The value of the plausibility function $Pl(\cdot)$ determines the full degree of potential support that can be provided to each of the subsets $X \subset \Omega$.

The values of the functions $Bel(\cdot)$ and $Pl(\cdot)$ determine the upper and lower limits of the interval, which contains the exact probability p(X) of the subset X $\subset \Omega$:

$$Bel(X) \le p(X) \le Pl(X).$$

The main procedure underlying the theory of evidence is the combination of different groups of expert evidence, which are characterized by different structures of interaction.

3. Evidence combination rules

The rules of combination allow to obtain aggregated expert evidence obtained from different (multiple) sources. Each such source of evidence is assumed to be in-dependent. The combination of expert assessments in the theory of evidence is based on the Dempster's combination rule, but this rule is not able to properly operate with conflicting evidence. As a result, a whole class of alternative combination rules are appeared [1, p. 43, 72–91; 2, p. 73–81; 9; 11, p. 13–23; 13, p. 4–13; 15, p. 5–9; 16, p. 8–9, 36–50; 17; 18; 19; etc.].

The combined *bpa* in the Dempster's combination rule is defined as follows [1, p. 43; 13, p. 4; 15, p. 6]:

$$m_{DS}(X) = \frac{1}{1-k_{12}} \cdot \sum_{X_1, X_2 \in 2^{\mathrm{A}},\ X_1 \cap X_2 = X} m_1(X_1) m_2(X_2)\,, \tag{5}$$

where X_1, X_2 are groups of evidence obtained from 1st and 2nd independent sources; k_{12} is a degree of conflict:

$$k_{12} = \sum_{X_1, X_2 \in 2^{\mathrm{A}},\ X_1 \cap X_2 = \varnothing} m_1(X_1) m_2(X_2)\,. \tag{6}$$

The combined *bpa* in the Yager's rule is defined as follows [1, p. 72; 13, p. 7; 18]:

$$m_Y(X) = \sum_{X_1, X_2 \in 2^{\mathrm{A}}, X_1 \cap X_2 = X} m_1(X_1) m_2(X_2)\,. \tag{7}$$

In the case when $X = \mathrm{A}$, the combined *bpa* of the universal set (frame of dis-cernment) is defined in the following way:

$$m_Y(\mathrm{A}) = q(\mathrm{A}) + q(\varnothing) = m_1(\mathrm{A}) m_2(\mathrm{A}) + \sum_{X_1, X_2 \in 2^{\mathrm{A}}, X_1 \cap X_2 = \varnothing} m_1(X_1) m_2(X_2)\,. \tag{8}$$

where $q(\mathrm{A})$ and $q(\varnothing)$ are combined *bpa* of the universal and the null set, respectively.

The combined *bpa* in the Inagaki's combination rule for any non-empty subset $X = X_1 \cap X_2$ is defined as follows [1, p. 91; 9; 13, p. 7]:

$$m_k^U(X) = \left[1 + kq(\varnothing)\right] \cdot q(X), \;\; X \neq \mathrm{A}, \varnothing \, . \tag{9}$$

where $q(X) = \sum_{X_1, X_2 \in 2^{\mathrm{A}}, X_1 \cap X_2 = X} m_1(X_1) m_2(X_2)$; $q(\varnothing)$ is the combined *bpa* over all empty intersections of marginal focal elements; the parameter k is used to normalization:

$$0 \leq k \leq \frac{1}{1 - q(\varnothing) - q(\mathrm{A})} \, .$$

For $X = \mathrm{A}$ Inagaki suggests the following equation

$$m_k^U(\mathrm{A}) = \left[1 + kq(\varnothing)\right] \cdot q(\mathrm{A}) + \left[1 + kq(\varnothing) - k\right] \cdot q(\varnothing) \, . \tag{10}$$

where $q(\mathrm{A})$ is combined *bpa* of the universal set.

In the case when $k = 0$ Inagaki's combination rule is equal to Yager's rule; with $k = 1/(1 - q(\varnothing))$ Dempster's rule is obtained.

Zhang's combination rule takes into account the degree of intersection of the selected subsets [1, p. 83; 13, p. 16; 19]:

$$r(X_1, X_2) = \frac{|X|}{|X_1||X_2|} = \frac{|X_1 \cap X_2|}{|X_1||X_2|} \, . \tag{11}$$

where $X_1 \cap X_2 = X$; $|\cdot|$ is the cardinality of the corresponding focal elements.

The combined *bpa* in the Zhang's rule is defined as follows:

$$m_Z(X) = k \cdot \sum_{X_1, X_2 \in 2^{\mathrm{A}}, X_1 \cap X_2 = X} \left[r(X_1, X_2) m_1(X_1) m_2(X_2)\right], \tag{12}$$

where k is a normalization coefficient.

The combined *bpa* in the Smets rule $\forall\, (\mathrm{X} \neq \varnothing) \in 2\mathrm{A}$, is defined in the next way [1, p. 135; 13, p. 6; 17]:

$$m_S(X) = \sum_{X_1, X_2 \in 2^{\mathrm{A}}, X_1 \cap X_2 = X} m_1(X_1) m_2(X_2) \, , \tag{13}$$

The combined *bpa* of the null set can be determined as follows:

$$m_S(\varnothing) \equiv k_{12} = \sum_{X_1, X_2 \in 2^{\mathrm{A}}, X_1 \cap X_2 = \varnothing} m_1(X_1) m_2(X_2) \, , \tag{14}$$

If the conflict level is significant (even equal to 1), then the Proportional Conflict Redistribution Rule (e.g. PCR5) can be applied to combine *bpa's*.

The combined *bpa* according to PCR5 rule for two sources of evidence, provided that $\forall\,(X \neq \varnothing) \in 2A$, $m(\varnothing)=0$, is defined as follows [13, p. 12; 16, p. 36]:

$$m_{PCR5}(X) = \sum_{\substack{X_1, X_2 \in 2^{A} \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2) + \sum_{\substack{Y \in 2^{A} \\ Y \cap C = \varnothing}} \left[\frac{m_1(C)^2 \cdot m_2(Y)}{m_1(C) + m_2(Y)} + \frac{m_2(C)^2 \cdot m_1(Y)}{m_2(C) + m_1(Y)} \right]. \quad (15)$$

This rule allows the reallocation of partial conflicting mass to the elements in-volved in conflict and proportionally to their individual *bpa's*.

4. Information technology for expert assessments rank-ordering based on Dempster-Shafer conception

The developed conception of information technology is intended to obtain the final rank-ordering of group expert assessments formed under of multi-criteria and complex forms of ignorance. Information technology is based on an approach of complex use of different evidence combination rules in the DST and DSmT frameworks.

This approach allows to select and group various combinations of initial op-tions (objects of expertise, alternatives) into clusters, according to individual choice of an expert, to analyze them, and to obtain a resulting rank-ordering of group expert assessments of the objects under consideration (alternatives, resulting subsets of alternatives). At the same time, the restriction on the number of analyzed objects (alternatives) and the condition for the consistency of expert preferences (estimates) has been removed.

Let us consider the main provisions of information technology for rank-ordering of group expert assessments (Figure 1).

The expert group $\mathrm{E} = \{E_k \mid k = \overline{1,m}\}$ is presented with a variety of objects (alternatives) $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$ in order to obtain the resulting rank-ordering of the studied objects of the form:

$$R_{\mathrm{rez}} : \{A_1 \succ A_2 \succ \ldots \succ A_j \succ \ldots \succ A_n\} \vee \{A_1 \succ A_2 \sim A_3 \succ \ldots \succ A_{j-1} \sim A_j \succ \ldots \succ A_n\}. \quad (16)$$

The procedure for generating the resulting rank-ordering can be represented in the form of next sequential steps:

1. Determination of the set of criteria $\mathrm{K} = \{K_j \mid j = \overline{1,s}\}$ against which the choice is made.

2. Calculation of the priority vector $\Omega=\{\omega_i \mid i=\overline{1,s}\}$ of criteria, for example, using one of the methods given in [10, p. 53]. The elements of the vector Ω must meet the conditions:

$$0 \le \omega_j \le 1, \quad \forall\, j=\overline{1,s}; \ \sum_{j=1}^{s}\omega_j = 1. \tag{17}$$

The priorities between the criteria are set according to the degree of their influ-ence on the choice of the option (alternative).

3. Revealing the expert preferences. The expert E_k forms for each criterion K_j, $j=\overline{1,s}$, a system of subsets $\mathrm{P}_j^{(k)}=\{X_i^{(k)} \mid i=\overline{1,d}\}$, $d \le 2^{|\mathrm{A}|}-1$, $j=\overline{1,s}$, in accordance with (1). Thus, the expert E_k will form s sets $\mathrm{P}_j^{(k)}$, reflecting his choice for each of the criteria $K_j \subset \mathrm{K}$, $j=\overline{1,s}$.

4. Revealing the degree of superiority of the selected groups of elements (alter-natives) $X_i^{(k)}$ in the values of a given scale of preferences for each of the criteria.

Thus, for each system of subsets $\mathrm{P}_j^{(k)}=\{X_i^{(k)} \mid i=\overline{1,d}\}$, a vector $\mathrm{B}_j^{(k)}=\{b_i^{(k)} \mid i=\overline{1,d}\}$ will be generated that contains the numerical values of the degrees of preference bi ($X_i^{(k)} \succeq \mathrm{A}$, $X_i^{(k)} \subseteq \mathrm{P}_j^{(k)}$), identified by the expert E_k by the criterion K_j.

5. Calculation of the *bpa's* corresponding to the selected subsets (groups of elements) X_i. For each generated system of subsets $\mathrm{P}_j^{(k)}=\{X_i^{(k)} \mid i=\overline{1,d}\}$, a vector $\mathrm{M}_j^{(k)}=\{m_i^{(k)} \mid i=\overline{1,d+1}\}$ will be obtained whose elements are calculated by the formulas [4; 5, p. 155]:

$$m_i^{(k)}(X_i^{(k)})=\frac{b_i^{(k)}\cdot\omega_j}{\sum_{i=1}^{d} b_i^{(k)}\cdot\omega_j+\sqrt{d}}, \text{ (i=}\overline{1,d}\text{), } m_{d+1}^{(k)}(\mathrm{A})=\frac{\sqrt{d}}{\sum_{i=1}^{d} b_i^{(k)}\cdot\omega_j+\sqrt{d}}, \tag{18}$$

where d is the total number of selected subsets (groups of elements) $X_i^{(k)}$ formed by the expert E_k by the criterion K_j; $b_i^{(k)}$ is the degree of preference $X_i^{(k)}$ ($X_i^{(k)} \succeq \mathrm{A}$), assigned by the expert E_k according to the criterion K_j; ω_j is the weight coefficient of the criterion K_j; $m_i^{(k)}(X_i^{(k)})$ is the *bpa* assigned to the set $X_i^{(k)}$; $m_{d+1}^{(k)}(\mathrm{A})$ is the *bpa* assigned to the frame of discernment A. The value $m_{d+1}^{(k)}(\mathrm{A})$ reflects the degree of complete igno-rance of the criterion K_j.

6. Selection the order of evidence combination. For this, the degree of difference between $\mathrm{P}_i^{(k)}$ and $\mathrm{P}_j^{(k)}$ is calculated based on the measure [7, p. 95; 8]:

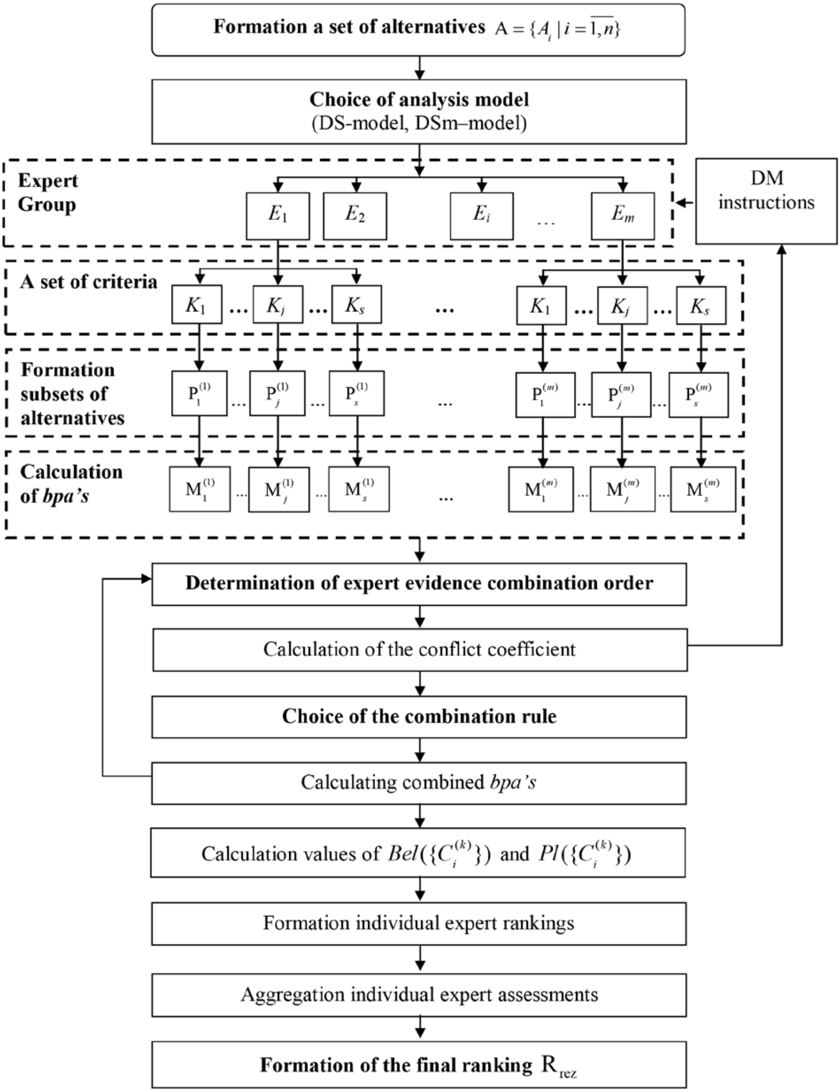

Figure 1. Structure of information technology for expert assessments rank-ordering

$$d(\mathrm{M}_1, \mathrm{M}_2) = \sqrt{\frac{1}{2}(\mathrm{M}_1 - \mathrm{M}_2)^T D(\mathrm{M}_1 - \mathrm{M}_2)}\,, \tag{19}$$

where D is a matrix $2^{|A|} \times 2^{|A|}$ whose elements $d_{ij} = \left|X_i \cap X_j\right| / \left|X_i \cup X_j\right|$ are a measure of the difference between elements (subsets) X_i and X_j; $|\cdot|$ is the cardinality of the corresponding subsets. Thus, $d(\mathrm{M}_1, \mathrm{M}_2) \in [0;1]$ is the distance that represents the difference between $\mathrm{P}_i^{(k)}$ and $\mathrm{P}_j^{(k)}$; M_i corresponds to $\mathrm{M}_i^{(k)}$.

7. Selection of evidence combination rule.

At the preliminary stage it is necessary to highlight a number of criteria against which the combination rule will be evaluated. A set of criteria for choosing a combination rule includes factors such as information about conflicts between individual judgments of experts (evidence), information about data sources (experts), information about the degree of interaction and the structure of expert judgments (evidence), the value of the *bpa* connected with the total ignorance, etc.

Based on the analysis of [2, p. 94–95; 11, p 45–46; 13, p. 22–24; 14, p. 4–6; 15, p. 21–27], the next classification procedure for choosing an effective evidence combination rule has been proposed:

1. In the case when the sources of information are recognized as reliable (as a measure of reliability, a competence of experts can be considered) and the conflict level is acceptable (not significant; less than 0.5), the conjunctive consensus combination rules are applied. These rules include the Dempster's, Yager's, Zhang's, Inagaki's rules.

1.1 Dempster's rule applies in cases when the sum of *bpa* connected with all the focal elements is much greater than the *bpa* connected with the frame of discernment (which characterizes total ignorance).

1.2 The Zhang's rule applies in cases when the sum of *bpa* connected with all the focal elements is much greater or comparable to the *bpa* connected with the frame of discernment.

1.3 The Inagaki's rule applies in cases when the *bpa* associated with the frame of discernment is greater than or comparable to the *bpa* connected with all the focal elements.

1.4 Yager's rule applies in cases when the sum of all *bpa* connected with focal elements is comparable to the *bpa* associated with frame of discernment.

2. The PCR5 conflict redistribution rule can be applied in situations of high level of conflict between individual groups of evidence.

3. In the case when it is known that some of the sources are recognized as unreliable, but it is not known a priori which ones, then it makes sense to apply disjunctive consensus rules, for example, the disjunctive rule of Dubois and Prade.

4. In the case when it is known which of the sources are unreliable, such sources are excluded from consideration. If all sources are found to be unreliable, it is necessary to supplement information from other sources that can be considered reliable. In this case, the composition of the expert commission changes, and the examination is repeated.

5. In the case when among the set of initial data, there are no elements that are truly the best choice, it is possible that some elements can be added to the basis of analysis based on new evidence, the Smets' rule is applied.

A generalized algorithm for choosing a combination rule is shown in Figure 2.

8. Aggregation of expert assessments by combining the obtained *bpa's* $\mathrm{M}_j^{(k)} = \{m_i^{(k)} \mid i = \overline{1, d+1}\}$ and $\mathrm{P}_j^{(k)} = \{X_i^{(k)} \mid i = \overline{1, d}\}$, formed by an expert E_k according to all criteria K_j, $(j = \overline{1, s})$. Thus, s–1 combination operations will be performed.

In this case, the assessments of all experts are taken as an independent source of information; one of the conjunctive consensus combination rules (5) – (13) or (15) is used. The combination rule is selected for each pair $\mathrm{P}_i^{(k)}$ and $\mathrm{P}_j^{(k)}$ that are combined.

At each stage, $\mathrm{P}_i^{(k)}$ and $\mathrm{P}_j^{(k)}$ are combined with a minimum value of measure (19) $\min(d(\mathrm{M}_i^{(k)}, \mathrm{M}_j^{(k)}))$. The result of the combination, in accordance with the selected rule, is a set $\mathrm{P}_{\mathrm{rez}}^{(k)} = \{C_1^{(k)}, C_2^{(k)}, ..., C_i^{(k)}, ..., C_t^{(k)}\}$, $t \le 2^{|A|} - 1$, with resulting subsets $C_i^{(k)}$, obtained by combining $\mathrm{P}_i^{(k)}$ and $\mathrm{P}_j^{(k)}$, $\forall\, i, j = \overline{1, s}$ and a vector $\mathrm{M}_{\mathrm{rez}}^{(k)}$, that contains the *bpa's* $m_{\mathrm{rez}}(C_i^{(k)})$ of the resulting subsets $C_i^{(k)}$ of expert E_k.

9. Calculation of the values of the belief $Bel(\cdot)$ and plausibility $Pl(\cdot)$ functions. The functions $Bel(\cdot)$ and $Pl(\cdot)$ are calculated for each subset $C_i^{(k)}$ using formulas (3) and (4). The subset $C_i^{(k)}$ priority is established by comparing the obtained intervals $[Bel(\{C_i^{(k)}\}), Pl(\{C_i^{(k)}\})]$ formed by the belief and plausibility functions. The best element is that element

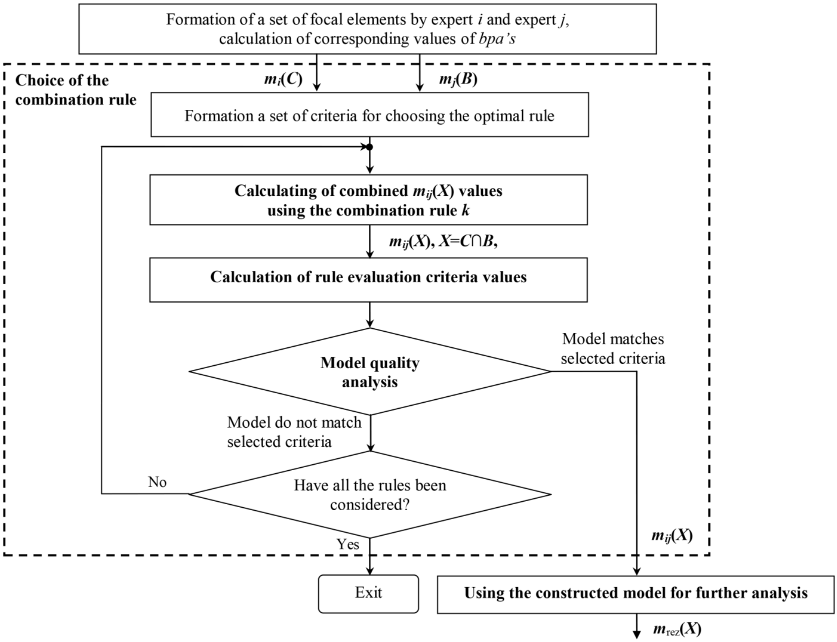

Figure 2. A generalized algorithm for choosing a combination rule

(a subgroup of elements), for which the values of the belief and plausibility functions reaches the highest values among analogous values of all other intervals. In the case when the intervals are nested, then their comparison is performed using the expression:

$$\gamma \cdot Bel(\{C_i^{(k)}\}) + (1-\gamma) \cdot Pl(\{C_i^{(k)}\}) \, , \tag{20}$$

where $\gamma \in [0,1]$ is the optimism coefficient.

The result of model construction is an individual rank-ordering that reflects the expert's E_k , $k = \overline{1,m}$ choice:

$$\mathrm{R}^{(k)} : \{C_1^{(k)} \succ C_2^{(k)} \succ \ldots \succ C_i^{(k)} \succ \ldots \succ C_t^{(k)}\} \, , \tag{21}$$

or

$$\mathrm{R}^{(k)} : \{C_1^{(k)} \succ C_2^{(k)} \sim C_3^{(k)} \succ \ldots \succ C_{i-1}^{(k)} \sim C_i^{(k)} \succ \ldots \succ C_t^{(k)}\} \, , \tag{22}$$

If the transitivity condition is satisfied, then the experts' assessments are taken consistent.

10. Building the final rank-ordering $\mathrm{R}_{\mathrm{rez}}$. The resulting subsets $\mathrm{P}_{\mathrm{rez}} = \{C_i \mid i = \overline{1,t}\}$, $t \le 2^{|\mathrm{A}|} - 1$ and $\mathrm{M}_{\mathrm{rez}}$ are calculated by combining the corresponding values $\mathrm{P}_{\mathrm{rez}}^{(k)}$ and $\mathrm{M}_{\mathrm{rez}}^{(k)}$, $k = \overline{1,m}$:

$$\mathrm{P}_{\mathrm{rez}} = \mathrm{P}_{\mathrm{rez}}^{(1)} \cap \mathrm{P}_{\mathrm{rez}}^{(2)} \cap \ldots \cap \mathrm{P}_{\mathrm{rez}}^{(k)} \cap \ldots \cap \mathrm{P}_{\mathrm{rez}}^{(m)}$$

and

$$\mathrm{M}_{\mathrm{rez}} = \mathrm{M}_{\mathrm{rez}}^{(1)} \oplus \mathrm{M}_{\mathrm{rez}}^{(2)} \oplus \ldots \oplus \mathrm{M}_{\mathrm{rez}}^{(k)} \oplus \ldots \oplus \mathrm{M}_{\mathrm{rez}}^{(m)}$$

Aggregation of individual expert preferences into a collective expert assessment is carried out in accordance with the procedure given in clauses 6–8. The values of the belief and plausibility functions are calculated for all the resulting C_i, in accordance with the procedure given in clause 9. By comparing the intervals obtained $[Bel(\{C_i\}), Pl(\{C_i\})]$, a final rank-ordering $\mathrm{R}_{\mathrm{rez}}$ of the form (21) or (22) is constructed, which reflects the collective opinion of the expert group.

The main advantage of the proposed methodology is that, in contrast to the existing methods of multi-criteria assessment of alternatives, it allows to process the assessments of experts with mismatched, and possibly contradictory preferences for many different criteria, to take into account various types of interaction of judgments (preferences, assessments) expressed by several experts.

In contrast to the modified AHP method proposed by Beynon [3, p. 42–46; 4; 5, p. 155–156], which uses the mathematical apparatus of the evidence theory, the proposed method uses a number of an alternative combination rules (a rule is selected for each pair of expert evidence that is combined) and takes into account the measure of difference between individual groups of evidence for determination the order of combination.

5. Methodology for the synthesis of IT for solving the problem of ranking group expert assessments on the example of the of GIS technology choice

Modern trends in the development of information technologies the volume and nature of the processed spatial (geographical) data and related information about the objects under consideration, contribute to the increasing demand for geographic information system (GIS) technologies for information and cartographic support of projects, for example, in urban planning; when studying issues related to the sustainability of river systems;

for evaluative mapping of vegetation, in order to study the ecological potential, etc.

GIS should be understood as a hardware-software man-machine complex that provides processes connected with collection, processing, mapping and dissemination of spatially coordinated data, integration of data and knowledge about the territory for their effective using in solving scientific and applied geographic problems related to inventory, analysis, modeling, forecasting and management of the environment and the territorial organization of society.

Currently, there are many GIS designed for geoinformation mapping, in this regard, there is a problem of choosing a GIS mapping technology that meets the re-quirements.

Let us consider the problem of choosing a GIS for mapping and presenting land cadastral data via the Internet. The mechanism of expert assessment based on the theory of evidence and the theory of plausible and paradoxical reasoning was used as an analysis technologies.

The problem under consideration can be divided into next subtasks:

1. Formation of an expert group.

Selection of experts is carried out by the decision maker in accordance with their competence.

2. Formation of a set (list) of analyzed GIS.

3. Selection of the range of indicators (characteristics, criteria) of GIS quality.

The choice of quality indicators allows to establish a list of GIS quality criteria, in accordance with the selected quality model, that provides the assessment of the quality level of the analyzed GIS.

4. Formation of individual expert GIS rankings in accordance with a given set of quality criteria.

The expert group evaluates the GIS under consideration in accordance with a given set of quality criteria. As a result, the decision maker is presented with a set of individual rankings of experts for their subsequent analysis and development of a final decision.

5. Construction of a generalized ranking, reflecting the collective opinion of the expert group on the choice of GIS.

To solve the assigned task, the decision-maker formed a group of experts, and a list of GIS to be analyzed was determined, such as: ArcIMS, AspMap, Autodesk MapGuide, GeoMedia Web Map, GIS WebServer, Internet CSI-

MAP Server, LiveMapGIS, MapInfo MapXtreme, MapObjects Internet Map Server, MOSMAP-GIS, WebMap, "ГеоКонструктор Web-Сервер".

Thus, there is a set of analyzed objects (GIS) $A = \{A_i \mid i = \overline{1,12}\}$ and a set of experts $E = \{E_i \mid i = \overline{1,10}\}$.

5.1. Determination of the level of competence of experts.

The expert group includes 10 experts – employees of a firm engaged in the analysis of geological and geophysical information using modern GIS technologies. Of course, not only the firm's employees can act as experts, but also involved specialists, if it is deemed appropriate.

To determine the level of competence of experts, the decision maker (head of the firm) has formed a list of 6 professional and personal competencies: professional competence (K_1); scientific intuition (K_2); interest in the objective results of the examination (K_3); composure (K_4); communicativeness (K_5); independence of judg-ment (K_6).

Based on the paired comparison method a vector of priorities (weighting coef-ficients of competencies) was obtained $P = \{p_i \mid i = \overline{1,6}\} = \{0.42891$: 0.2499; 0.13513; 0.0558; 0.04983; 0.08043$\}$. The values of the priority vector P were ob-tained using the geometric mean estimate [10, p. 53].

In accordance with the approved set of competencies, the decision maker (DM) has assessed the professional and personal qualities of the expert group and expressed the degree of his preferences according to the rule [10, p. 53]: 3 – weak superiority, 5 – strong superiority, 7 – significant superiority, 9 – absolute superiority, values 4, 6, 8 – correspond to intermediate judgments between each consecutive pair of considered values. The results are shown in Table 1.

During the analysis, for example, for competence – professional competence, the decision maker formed a ranking of the following form:

$$\{E_1 \sim E_5\} \succ \{E_7 \sim E_9\} \succ E_2 \succ \{E_3 \sim E_4\} \succ \{E_6 \sim E_7\}.$$

For each selected subset, the *bpa's* was calculated by (2).

Figure 3 shows a graph of the dependence of the value of *bpa's* on the value of the weight of competence (professional competence). The value ranges from 0 to 1.

Figure 3 shows that with an increase of p_1 value, the value given by (the value of complete ignorance, uncertainty) smoothly decreases, when $p_1 = 1$ the value of complete ignorance reaches its minimum and corresponds to

Table 1

Estimates (Degrees of Preference) of the Decision Maker

№	Experts		Competencies					
			1	**2**	**3**	**4**	**5**	**6**
			K_1	K_2	K_3	K_4	K_5	K_6
1	E_1	Expert 1	9	8	7	5	9	5
2	E_2	Expert 2	7	9	9	7	5	6
3	E_3	Expert 3	6	7	5	5	8	5
4	E_4	Expert 4	6	8	8	8	9	9
5	E_5	Expert 5	9	5	6	8	5	8
6	E_6	Expert 6	5	9	5	7	7	7
7	E_7	Expert 7	8	7	9	8	9	9
8	E_8	Expert 8	4	5	8	7	8	8
9	E_9	Expert 9	8	5	5	8	7	8
10	E_{10}	Expert 10	5	8	7	6	9	4

the m_1 (E) = 0.059. The value reaches a maximum (i.e. equal to 1), when $p_1 = 0$, that is, the less the significance of the competence (criterion), the greater the amount of ignorance (the uncertainty increases).

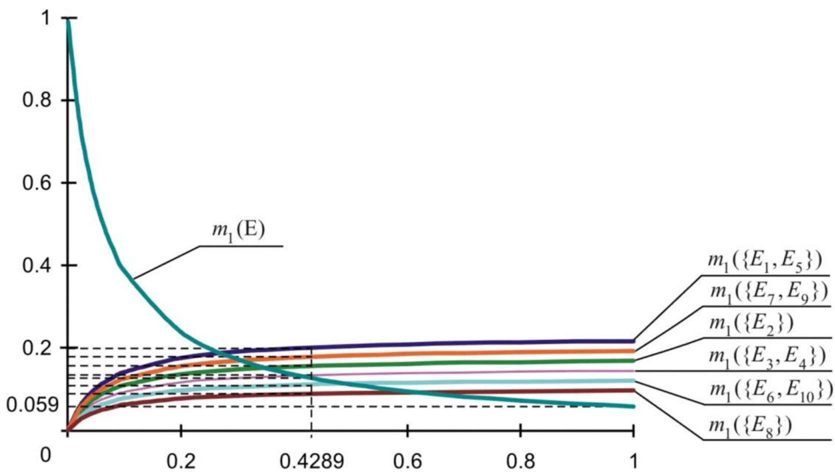

Figure 3. Graphical representation of the dependence of m(·) on the value for competence (professional competence)

With an increase of p_1, the values $m_1(\{E_1, E_5\})$, $m_1(\{E_2\})$, $m_1(\{E_3, E_4\})$, $m_1(\{E_6, E_{10}\})$, $m_1(\{E_7, E_9\})$, $m_1(\{E_8\})$ are gradually increase. The probability mass $m_1(\{E_1, E_5\})$ has the maximum value, since the choice $\{E_1, E_5\}$ was assigned a maximum degree of superiority equal to 9, which determines its preference over other objects of analysis.

For $p_1 = 0.4289$ the values of $m_1(\cdot)$ correspond to the calculated *bpa's* values:

$m_1(\{E_1, E_5\}) = 0.201$; $m_1(\{E_3, E_4\}) = 0.134$; $m_1(\mathrm{E}) = 0.128$.

$m_1(\{E_6, E_{10}\}) = 0.112$; $m_1(\{E_7, E_9\}) = 0.179$;

$m_1(\{E_2\}) = 0.157$;

$m_1(\{E_8\}) = 0.089$;

To obtain a generalized assessment, it is necessary to combine the values of *bpa's* obtained for each competence. To determine the order of evidence combination, the measure of difference between individual expert evidence was calculated based on eq. (19), while the condition $d_J(m_i, m_j) = d_J(m_j, m_i)$ is true.

Let's calculate the measures $d_J(m_i, m_j)$ for the original set of evidence:

$d_J(m_1, m_2) = 0.258$;
$d_J(m_1, m_3) = 0.239$;
$d_J(m_1, m_4) = 0.358$;
$d_J(m_1, m_5) = 0.358$;
$d_J(m_1, m_6) = 0.265$;
$d_J(m_2, m_3) = 0.225$;
$d_J(m_2, m_4) = 0.292$;
$d_J(m_2, m_5) = 0.287$;
$d_J(m_2, m_6) = 0.215$;
$d_J(m_3, m_4) = 0.246$;
$d_J(m_3, m_5) = 0.239$;
$d_J(m_3, m_6) = 0.184$;
$d_J(m_4, m_5) = \mathbf{0.138}$;
$d_J(m_4, m_6) = 0.165$;
$d_J(m_5, m_6) = 0.176$.

The smallest value $d_J(m_i, m_j)$ of the distance between groups of evidence $m_i(\cdot)$ and $m_j(\cdot)$ was ob-tained for $d_J(m_4, m_5)$. Therefore, the first to combine the groups of evidence $m_4(\cdot)$ and $m_5(\cdot)$. Next, we find the measure of the difference between the calculated $m_{45}(\cdot)$ and the original $m_1(\cdot)$, $m_2(\cdot)$, $m_3(\cdot)$ and $m_6(\cdot)$.

Find the minimum value and calculate the resulting (combined) subsets and corresponding $m_{ij}(\cdot)$, thus determining the order of combination the remaining groups of evidence.

Thus the following order of combining evidence was obtained:

1. $m_{45} = m_{45} \oplus m_{45}$; $d_J(m_4, m_5) = 0.138$;
2. $m_{456} = m_{45} \oplus m_6$; $d_J(m_{45}, m_6) = 0.122$;
3. $m_{1456} = m_1 \oplus m_{456}$; $d_J(m_1, m_{456}) = 0.181$;
4. $m_{23} = m_2 \oplus m_3$; $d_J(m_2, m_3) = 0.225$;
5. $m_{123456} = m_{23} \oplus m_{1456}$; $d_J(m_{23}, m_{1456}) = 0.146$.

Where $\oplus$ is the evidence combination rule for based on conjunctive consensus.

Table 2 shows the resulting subsets obtained by combination of formed by the DM subsets for competencies K_1 and K_2. Taking into account the fact that the sources are taken as independent and reliable, calculate the resulting values of *bpa's* for the given by DM subsets using the combination rules based on the conjunctive consensus (5) – (13).

Table 2

Intersections of Selected Subsets by Competencies K_1 and K_2

		Competence K_1						
		$\{E_1, E_5\}$	$\{E_2\}$	$\{E_3, E_4\}$	$\{E_6, E_{10}\}$	$\{E_7, E_9\}$	$\{E_8\}$	{E}
Competence K_2	$\{E_1, E_4, E_{10}\}$	$\{E_1\}$	$\varnothing$	$\{E_4\}$	$\{E_{10}\}$	$\varnothing$	$\varnothing$	$\{E_1, E_4, E_{10}\}$
	$\{E_2, E_6\}$	$\varnothing$	$\{E_2\}$	$\varnothing$	$\{E_6\}$	$\varnothing$	$\varnothing$	$\{E_2, E_6\}$
	$\{E_3, E_7\}$	$\varnothing$	$\varnothing$	$\{E_3\}$	$\varnothing$	$\{E_7\}$	$\varnothing$	$\{E_3, E_7\}$
	$\{E_5, E_8, E_9\}$	$\{E_5\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\{E_9\}$	$\{E_8\}$	$\{E_5, E_8, E_9\}$
	{E}	$\{E_1, E_5\}$	$\{E_2\}$	$\{E_3, E_4\}$	$\{E_6, E_{10}\}$	$\{E_7, E_9\}$	$\{E_8\}$	{E}

The calculated values of belief $Bel(\cdot)$ and plausibility $Pl(\cdot)$ functions by eq. (3) and (4), for each element of the $\mathrm{E} = \{E_i \mid i = \overline{1,10}\}$ set, are given in Table. 3.

The differences between the obtained results for different combination rules are associated with the use of different approaches when combining the *bpa's* of the original focal elements (selected subgroups of the analyzed objects), mainly in dealing with the combined probability masses for empty intersections of the original focal elements.

Formed intervals $[Bel(E_i); Pl(E_i)]$ characterize the range of uncertainty associated with the choice. Such uncertainty may be a reflection of the uncertainty in the initial assessments of the experts.

The situation in which the uncertainty of the resulting estimates is greater than the uncertainty of the initial expert estimates, greatly complicates the

Table 3

Intervals Formed by *Bel*ief and *Pl*ausibility Measures

Expert	Evidence combination rule				
	Dempster	**Inagaki**	**Zhang**	**Smets**	**Yager**
E_1	[0.088; 0.161]	[0.094; 0.150]	[0.069; 0.079]	[0.009; 0.016]	[0.018;0.503]
E_2	[0.115; 0.159]	[0.120; 0.147]	[0.119; 0.126]	[0.011; 0.015]	[0.056;0.505]
E_3	[0.057; 0.108]	[0.060; 0.094]	[0.051; 0.056]	[0.006; 0.010]	[0.014;0.491]
E_4	[0.096; 0.160]	[0.102; 0.144]	[0.068; 0.075]	[0.01; 0.016]	[0.018;0.541]
E_5	[0.077; 0.134]	[0.079; 0.120]	[0.074; 0.080]	[0.008; 0.013]	[0.025;0.512]
E_6	[0.068; 0.121]	[0.068; 0.107]	[0.09; 0.096]	[0.007; 0.012]	[0.021;0.545]
E_7	[0.114; 0.183]	[0.120; 0.168]	[0.326; 0.330]	[0.011; 0.018]	[0.056;0.507]
E_8	[0.061; 0.101]	[0.062; 0.081]	[0.064; 0.069]	[0.006; 0.010]	[0.024;0.522]
E_9	[0.060; 0.125]	[0.063; 0.109]	[0.031; 0.039]	[0.006; 0.010]	[0.017;0.517]
E_{10}	[0.058; 0.120]	[0.060; 0.102]	[0.086; 0.093]	[0.006; 0.011]	[0.047;0.495]
E	**0.0135**	**0.0013**	**0.00126**	**0.0013**	**0.412**

interpretation of the results obtained and makes it problematic to form well-grounded conclusions.

The approximation the value of the function $Bel(E_i)$ to the value of the function $Pl(E_i)$ is characterized by a decrease in the level of uncertainty.

Based on the data in Table 3, it is impossible to unambiguously determine the resulting ranking, since the obtained intervals overlap and some of them are nested. To convert interval estimates to crisp estimates, the eq. (20) was used with the value of the coefficient γ equal to 0.6.

Aggregated estimates of the analyzed objects (elements of the set E), obtained based on considered rules, are shown in Table 4.

Calculations made on the basis of various evidence combination rules allow to draw the following conclusions:

1. According to the results presented in Table 3 and Table 4, the highest values of belief and plausibility functions belong to the choice $\{E_7\}$, regardless of the applied combination rule, but the degree of confidence calculated on the basis of different rules is different.

2. The degree of confidence assigned to the choice (expert) $\{E_7\}$ is in the range from 0.011 to 0.326.

Table 4

Aggregated Estimates

Expert	Evidence combination rule				
	Dempster	**Inagaki**	**Zhang**	**Smets**	**Yager**
E_1	0.11714	0.11638	0.072771	0.011347	0.21162
E_2	0.13268	0.13074	0.12133	0.012851	0.23534
E_3	0.07716	0.073603	0.052526	0.0083242	0.2047
E_4	0.12133	0.11836	0.070566	0.011752	0.2271
E_5	0.09968	0.095425	0.07592	0.0096551	0.21967
E_6	0.089034	0.083767	0.091858	0.0086239	0.23534
E_7	**0.14099**	**0.13922**	**0.32753**	**0.013656**	**0.23646**
E_8	0.076467	0.069596	0.066113	0.0083242	0.22317
E_9	0.08594	0.081751	0.034316	0.0083242	0.21679
E_{10}	0.08123	0.07533	0.088355	0.0083242	0.22584

3. The conflict level varies from 0.25 to 0.46, depending on the combination rule, which indicates the presence of some conflict between individual groups of evidence.

4. The total value of all probability masses of the selected focal elements is greater than the probability mass related to the frame of discernment $(\sum_{i=1}^{p} m_j(X_i) > m_j(\mathrm{E}),\ j=\overline{1,10})$.

5. The decision maker's judgments can be considered as consistent.

All of the above considerations allow to conclude that the Inagaki's rule can be considered the most effective rule for the considered example.

As a result of the calculations, the normalized values of the experts' compe-tence coefficients were obtained, which are respectively equal to $\Omega = \{\omega_i \mid i=\overline{1,10}\}$ = {0.118; 0.133; 0.075; 0.12; 0.097; 0.085; 0.142; 0.071; 0.083; 0.076}.

Based on the values of the vector Ω, it is possible to obtain the ranking of experts, based on the calculated values of the competence coefficients in the form of:

$$E_7 \succ E_2 \succ E_4 \succ E_1 \succ E_5 \succ E_6 \succ E_9 \succ E_{10} \succ E_3 \succ E_8$$

The expert E_7 is recognized as the most competent in the expert group (for solving the problem under consideration), the expert E_8 is recognized as the least competent in the group.

5.2. Selection of the range of characteristics of GIS software quality. To solve this problem, the hierarchical quality model regulated by the ISO 9126-1 standard has been considered. The quality model contains 6 basic quality characteristics: functionality, reliability, efficiency, usability, maintainability, and portability. Table 5 gives the names of the characteristics of software product quality mode according to the ISO/IEC 9126–1 standard.

According to the condition of the problem, there is a set of alternatives $K = \{C_j \mid i = \overline{1,21}\}$, which are a list of properties (or attributes) characterizing the quality of the GIS software (Table 5). Ten experts need to evaluate and select the nomenclature of GIS software quality indicators, taking into account the purpose of GIS, requirements and scope, as well as assign appropriate quantitative (numerical) weight indicators to the selected sub-characteristics of the GIS software quality.

Experts need to evaluate the available alternatives (subcharacteristics of the GIS software quality) $C_i \in \mathbf{K}$, $i = \overline{1,21}$, or select the preferred groups of alternatives, $X_k = \{C_j \mid i = \overline{1,s}\}$, s ≤ 21, $X_k \subseteq \mathbf{K}$, and determine the degree of their preference within a given scale in relation to all re-maining alternatives (set K). As a result of the expert survey, the groups of alternatives $X_k \subseteq$ were

Table 5

Characteristics of Software Product Quality Model

№	Specification name	№	Subcharacteristics of software product quality model	№	Specification name	№	Subcharacteristics of software product quality model
1	Functionality	1.1	suitability				
		1.2	recoverability	4	Efficiency	4.1	time behaviour
		1.3	interoperability			4.2	resource utilization
		1.4	security	5	Maintainability	5.1	analyzability
2	Reliability	2.1	maturity			5.2	changeability
		2.2	fault tolerance			5.3	stability
		2.3	accuracy			5.4	testability
3	Usability	3.1	understandability	6	Portability	6.1	adaptability
		3.2	learnability			6.2	installability
		3.3	operability			6.3	co-existence
		3.4	attractiveness			6.4	replaceability

identified and the degrees of preference for the selected groups of alternatives (Table 6) were determined in a given scale of relations 3÷9 [10, p. 53].

For each group of expert evidence, the main *bpa's* of the selected subsets of alternatives (quality criteria) has been calculated. For example, the subsets identified by the expert and the calculated *bpa's* obtained by eq. (2) can be presented as follows:

$m_1(\{C_1, C_8, C_{20}\}) = 0.151$; $m_1(\{C_2, C_6\}) = 0.117$;

$m_1(\{C_3\}) = 0.134$; $m_1(\{C_9, C_{12}, C_{15}, C_{16}\}) = 0.084$;

$m_1(\{C_{10}, C_{18}\}) = 0.067$; $m_1(\{C_{13}\}) = 0.1$; $m_1(\mathrm{K}) = 0.347$.

Table 6

Expert's Estimates

№	**Subcharacteristics name**		**Expert's estimates**									
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
			E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}
1	C_1	suitability	9	9	8	9	9	9	9	9	9	9
2	C_2	recoverability	7	5	5	6	7	8	5	8	9	9
3	C_3	interoperability	8	7	6	9	8	7	9	9		8
4	C_4	security										
5	C_5	maturity										
6	C_6	fault tolerance	7	6	7	7	4	9	8	8	7	6
7	C_7	accuracy										
8	C_8	understandability	9	7	9	9	4	5	4	6	8	5
9	C_9	learnability	5	3	5	4	5	3	4	5	2	3
10	C_{10}	operability	4	5	7	6	7	8	8	8	7	7
11	C_{11}	attractiveness										
12	C_{12}	time behaviour	5	4	8	5	8	7	8	7	6	6
13	C_{13}	resource utilization	6	8	5	4	8	4	6	7	5	9
14	C_{14}	analyzability										
15	C_{15}	changeability	5	5	3	3	5	5		3	3	3
16	C_{16}	stability	5	6	8	6	7	8	6	9	8	7
17	C_{17}	testability										
18	C_{18}	adaptability	4	6	3	7	5	5	4	5	3	3
19	C_{19}	installability										
20	C_{20}	co-existence	9	9	9	8	6	9	5	3	5	5
21	C_{21}	replaceability										

The aggregation of subsets identified by experts 1 ÷ 10 was performed on the basis of the combination mechanism

The combination was performed based on the rules of Dempster, Yager, Zhang, Inagaki and Smets. The calculated values of the belief $Bel(\{C_i\})$ and the plausibility $Pl(\{C_i\})$ functions for each initial alternative are given in Table. 7.

Table 7

Intervals Formed by *Bel*ief and *Pl*ausibility Measures

Ci	Evidence combination rule				
	Dempster	**Inagaki**	**Zhang**	**Smets**	**Yager**
C_1	**[0.155; 0.214]**	**[0.173; 0.209]**	**[0.222; 0.223]**	**[0.006; 0.002]**	**[0.023;0.547]**
C_2	[0.060; 0.109]	[0.062; 0.086]	[0.017; 0.018]	[0.0006; 0.0011]	[0.015;0.532]
C_3	[0.085; 0.128]	[0.099; 0.115]	[0.164; 0.165]	[0.0008; 0.0013]	[0.057;0.498]
C_4	[0;0.0101]	[0; 0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
C_5	[0; 0.0101]	[0; 0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
C_6	[0.069; 0.115]	[0.076; 0.097]	[0.04; 0.041]	[0.0007; 0.001]	[0.012;0.508]
C_7	[0; 0.0101]	[0; 0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
C_8	[0.061; 0.102]	[0.068; 0.095]	[0.042; 0.043]	[0.0006; 0.001]	[0.018;0.492]
C_9	[0.018; 0.043]	[0.019; 0.028]	[0.002; 0.003]	[0.0002; 0.0004]	[0.008;0.455]
C_{10}	[0.051; 0.105]	[0.052; 0.078]	[0.018; 0.0182]	[0.0005; 0.001]	[0.013;0.516]
C_{11}	[0; 0.0101]	[0;0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
C_{12}	[0.089; 0.133]	[0.086;0.099]	[0.169; 0.170]	[0.0009; 0.001]	[0.037;0.518]
C_{13}	[0.064; 0.092]	[0.071;0.077]	[0.127; 0.1271]	[0.0006; 0.0009]	[0.015;0.514]
C_{14}	[0; 0.0101]	[0;0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
C_{15}	[0.010; 0.036]	[0.012;0.024]	[0.0003; 0.0004]	[0.0001; 0.0004]	[0.001;0.451]
C_{16}	[0.068; 0.112]	[0.071;0.090]	[0.016; 0.017]	[0.0007; 0.001]	[0.019;0.518]
C_{17}	[0; 0.010]	[0;0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
C_{18}	[0.021; 0.055]	[0.024;0.039]	[0.0019; 0.0020]	[0.0002; 0.0006]	[0.002;0.459]
C_{19}	[0; 0.010]	[0;0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
C_{20}	[0.074; 0.114]	[0.082;0.110]	[0.175; 0.176]	[0.0007; 0.001]	[0.010;0.480]
C_{21}	[0; 0.010]	[0;0.000099]	[0; 0.0000043]	[0; 0.000099]	[0;0.418]
K	0.0101	0.000099	0.0000043	0.000099	0.4178

Calculations performed on the basis of various rules of evidence combination allow to draw the following conclusions:

1. According to the results presented in Table. 7, the highest values of the be-lief and plausibility functions belong to the choice $\{C_1\}$, regardless of the applied combination rule, but the degree of belief assigned by different rules differs.

2. The degree of belief assigned to the quality criterion $\{C_1\}$ based on expert as-sessments is in the range from 0.0015 to 0.2228.

3. The smallest value of complete ignorance and the largest value of the ex-pressed belief to the quality criterion $\{C_1\}$ occurs when applying the Zhang combination rule, which indicates a fairly fast convergence when using this rule.

4. Yager's rule is pessimistic about the expressed trust (the lowest value) and, as a result, the level of complete ignorance is very high (the highest value).

5. The level of trust gradually increases in the order in which Yager's, Dempster's, Inagaki's and Zhang's rules are applied.

6. There is a decrease in the degree of complete ignorance, in the same order (clause 5), which indicates the presence of an inversely proportional relationship between the level of trust and complete ignorance.

7. There are quality criteria (C_4, C_5, C_7, C_{11}, C_{14}, C_{17}, C_{19}, C_{21}), the degree of confidence of which, regardless of the applied combination rule, is equal to 0. These criteria are excluded from further consideration.

8. The level of conflict varies from 0.22 to 0.43, depending on the rule, which indicates the presence of some conflict between individual groups of evidence.

9. The total value of all *bpa's* of the identified focal elements is greater than the *bpa's* related to the frame of discernment ($\sum_{i=1}^{p} m_j(X_i) > m_j(\mathrm{K})$, $j = \overline{1,21}$).

10. Experts' judgments can be considered as arbitrary.

All of the above considerations allow to conclude that the Inagaki's rule can be considered the most effective rule for the considered example.

As a result of the analysis and based on the data presented in Table 7, the following ranking of GIS software quality criteria was obtained:

$$C_1 \succ C_3 \succ C_{20} \succ C_{12} \succ C_6 \succ C_{16} \succ C_8 \succ C_{13} \succ C_2 \succ C_{10} \succ C_{18} \succ C_9 \succ C_{15}$$

As can be seen from the above results that the quality criterion C_1 has the highest values of the belief and plausibility functions, which corresponds to the subcharacteristic of quality model such as functional suitability (the properties if software that determine its ability to provide an appropriate set of functions for solving specified problems and achieving user goals).

The quality criteria C_4, C_5, C_7, C_{11}, C_{14}, C_{17}, C_{19}, C_{21}, corresponding to the sub-characteristics of security, maturity, accuracy, attractiveness, analyzability, testability, installability and replaceability, were excluded by experts from further analysis. The list of quality sub-characteristics, as well as their weight coefficients, are given in Table 8.

Table 8

Quality Subcharacteristic Weight Coefficients

№	Specification name	weight	№	Specification name	weight
1	suitability (C'_1)	0.188	8	time behaviour (C'_8)	0.091
2	recoverability (C'_2)	0.072	9	resource utilization (C'_9)	0.074
3	interoperability (C'_3)	0.106	10	changeability (C'_{10})	0.017
4	fault tolerance (C'_4)	0.085	11	stability (C'_{11})	0.079
5	understandability (C'_5)	0.076	12	adaptability (C'_{12})	0.03
6	learnability (C'_6)	0.022	13	co-existence (C'_{13})	0.094
7	operability (C'_7)	0.063			

5.3. Formation of individual expert GIS rankings in accordance with a given set of quality criteria

At this stage, experts need to rank the provided list of software tools (GIS) in accordance with the formed set of quality criteria.

There is a set of analyzed objects (GIS) A = $\{A_i \mid i = \overline{1,12}\}$ and a set of experts E = $\{E_i \mid i = \overline{1,10}\}$, performed analysis (assessment) of elements of set A in accordance with a given set of quality criteria.

Each expert for each of the subcharacteristics of quality ($C_1 \div C_{13}$) has expressed his preferences in a given scale of relations ($3 \div 9$). Degrees of preference of expert 1 (assessments of the considered GIS in relation to quality criteria) are summarized in Table 9.

Based on the obtained data (Table 9), the *bpa's* of the selected subsets of alternatives (GIS) by expert 1 can be calculated, using (2), as follows:

Table 9

Assessments (Degrees of Preference) of an Expert 1

№		GIS name	Quality subcharacteristic												
			1	2	3	4	5	6	7	8	9	10	11	12	13
			C'_1	C'_2	C'_3	C'_4	C'_5	C'_6	C'_7	C'_8	C'_9	C'_{10}	C'_{11}	C'_{12}	C'_{13}
1	A_1	ArcIMS	9	9	5	7	7	7	9	9	7	9	7	8	9
2	A_2	LiveMapGIS	7	6	6	6	8	6	6	6	8	6	6	5	
3	A_3	Autodesk MapGuide	8	9	7	9	9	7	7	9	7	7	7	6	7
4	A_4	GeoMedia Web Map		3	6	7	6	8	9	7	5	7	8		9
5	A_5	MapInfo MapXtreme		8	8	8	7	8	7	8	8	7	8	6	7
6	A_6	MapObjects Internet	6	5	6	8	9		9	9		5	5	7	6
7	A_7	MOSMAP-GIS		8		6		5	8	6	7	5	7		
8	A_8	AspMap	9			7	7	6	6	7		8	6	6	9
9	A_9	GIS WebServer	7	5	5	9	8	7		5	6	8	4	7	6
10	A_{10}	ГеоКонструктор	6	5	7	7		6	6	8	8	6	5	5	
11	A_{11}	Internet CSI-MAP	5	3	5	5	6	4	7	7	6		5	5	6
12	A_{12}	WebMap Резидент			5	5		5	5	6	5	5	6		4

according to C_1 (suitability):

$m^1_{C1}(\{A_1, A_8\}) = 0.192;$ $m^1_{C1}(\{A_6, A_{10}\}) = 0.128;$ $m^1_{C1}(\{A_{11}\}) = 0.107;$

$m^1_{C1}(\{A_2, A_9\}) = 0.149;$ $m^1_{C1}(\{A_3\}) = 0.171;$ $m^1_{C1}(\mathrm{A}) = 0.253;$

according to C_2 (recoverability):

$m^1_{C2}(\{A_5, A_7\}) = 0.129;$ $m^1_{C2}(\{A_6, A_9, A_{10}\}) = 0.081;$ $m^1_{C2}(\{A_2\}) = 0.097;$

$m^1_{C2}(\{A_1, A_3\}) = 0.145;$ $m^1_{C2}(\{A_4, A_{11}\}) = 0.048;$ $m^1_{C2}(\mathrm{A}) = 0.5;$

according to C_3 (interoperability):

$m^1_{C3}(\{A_3, A_{10}\}) = 0.156;$ $m^1_{C3}(\{A_2, A_4, A_6\}) = 0.134;$ $m^1_{C3}(\mathrm{A}) = 0.421;$

$m^1_{C3}(\{A_5\}) = 0.178;$ $m^1_{C3}(\{A_1, A_9, A_{11}, A_{12}\}) = 0.111;$

according to C_4 (fault tolerance):

$m^1_{C4}(\{A_2, A_7\}) = 0.098;$ $m^1_{C4}(\{A_5, A_6\}) = 0.13;$ $m^1_{C4}(\{A_{11}, A_{12}\}) = 0.0$

$m^1_{C4}(\{A_3, A_9\}) = 0.147;$ $m^1_{C4}(\{A_1, A_4, A_8, A_{10}\}) = 0.114;$ $m^1_{C4}(\mathrm{A}) = 0.43;$

according to C_5 (understandability):

$m^1_{C5}(\{A_1, A_5, A_8\}) = 0.126;$ $m^1_{C5}(\{A_3, A_6\}) = 0.163;$ $m^1_{C5}(\mathrm{A}) = 0.458;$

$m^1_{C5}(\{A_2, A_9\}) = 0.145;$ $m^1_{C5}(\{A_4, A_{11}\}) = 0.108;$

according to C_6 (learnability):

$m^1_{C6}(\{A_7, A_{12}\}) = 0.038;$ $m^1_{C6}(\{A_1, A_3, A_9\}) = 0.054;$ $m^1_{C6}(\{A_{11}\}) = 0.03;$

$m^1_{C6}(\{A_4, A_5\}) = 0.062;$ $m^1_{C6}(\{A_2, A_8, A_{10}\}) = 0.046;$ $m^1_{C6}(\mathrm{A}) = 0.77;$

according to C_7 (operability):

$m^1_{C7}(\{A_1, A_4, A_6\}) = 0.127;$ $m^1_{C7}(\{A_3, A_5, A_{11}\}) = 0.099;$ $m^1_{C7}(\{A_{12}\}) = 0.071;$

$m^1_{C7}(\{A_2, A_8, A_{10}\}) = 0.08;$ $m^1_{C7}(\{A_7\}) = 0.113;$ $m^1_{C7}(\mathrm{A}) = 0.51;$

according to C_8 (time behaviour):

$m^1_{C8}(\{A_5, A_{10}\}) = 0.135;$ $m^1_{C8}(\{A_1, A_3, A_6\}) = 0.151;$ $m^1_{C8}(\{A_9\}) = 0.084;$

$m^1_{C8}(\{A_2, A_7, A_{12}\}) = 0.1;$ $m^1_{C8}(\{A_4, A_8, A_{11}\}) = 0.118;$ $m^1_{C8}(\mathrm{A}) = 0.412;$

according to C_9 (resource utilization):

$m^1_{C9}(\{A_9, A_{11}\}) = 0.113;$ $m^1_{C9}(\{A_1, A_3, A_7\}) = 0.132;$

$m^1_{C9}(\{A_4, A_{12}\}) = 0.094;$ $m^1_{C9}(\{A_2, A_5, A_{10}\}) = 0.151;$ $m^1_{C9}(\mathrm{A}) = 0.51;$

according to C_{10} (changeability):

$m^1_{C10}(\{A_2, A_{10}\}) = 0.036;$ $m^1_{C6}(\{A_3, A_4, A_5\}) = 0.042;$ $m^1_{C6}(\{A_1\}) = 0.054;$

$m^1_{C10}(\{A_8, A_9\}) = 0.048;$ $m^1_{C6}(\{A_6, A_7, A_{12}\}) = 0.03;$ $m^1_{C10}(\mathrm{A}) = 0.79;$

according to C_{11} (stability):

$m^1_{C11}(\{A_1, A_3, A_7\}) = 0.12;$ $m^1_{C11}(\{A_2, A_8, A_{12}\}) = 0.103;$ $m^1_{C11}(\{A_9\}) = 0.07;$

$m^1_{C11}(\{A_4, A_5\}) = 0.137;$ $m^1_{C11}(\{A_6, A_{10}, A_{11}\}) = 0.09;$ $m^1_{C11}(\mathrm{A}) = 0.48;$

according to C_{12} (adaptability):

$m^1_{C12}(\{A_1\}) = 0.087;$ $m^1_{C12}(\{A_3, A_5, A_8\}) = 0.065;$

$m^1_{C12}(\{A_6, A_9\}) = 0.076;$ $m^1_{C12}(\{A_2, A_{10}, A_{11}\}) = 0.055;$ $m^1_{C12}(\mathrm{A}) = 0.717;$

according to C_{13} (co-existence):

$m^1_{C13}(\{A_1, A_4, A_8\}) = 0.19;$ $m^1_{C13}(\{A_3, A_5\}) = 0.148;$

$m^1_{C13}(\{A_{12}\}) = 0.085;$ $m^1_{C13}(\{A_6, A_9, A_{11}\}) = 0.127;$ $m^1_{C13}(\mathrm{A}) = 0.45.$

The data from Table 9 can be presented in the form of a diagram, Figure 4.

Using eq. (2), the *bpa's* of subsets identified by experts 2 ÷ 10 are calculated. Table 10 shows the resulting subsets obtained by intersection of the subsets selected by expert 1 according to the quality criteria and .

The results of expert assessments combination according to the Inagaki's rule (9), in accordance with a given set of quality criteria (÷), are presented in Table 11.

As a result of the analysis, the following individual rankings of the evaluated objects (GIS) has been obtained.

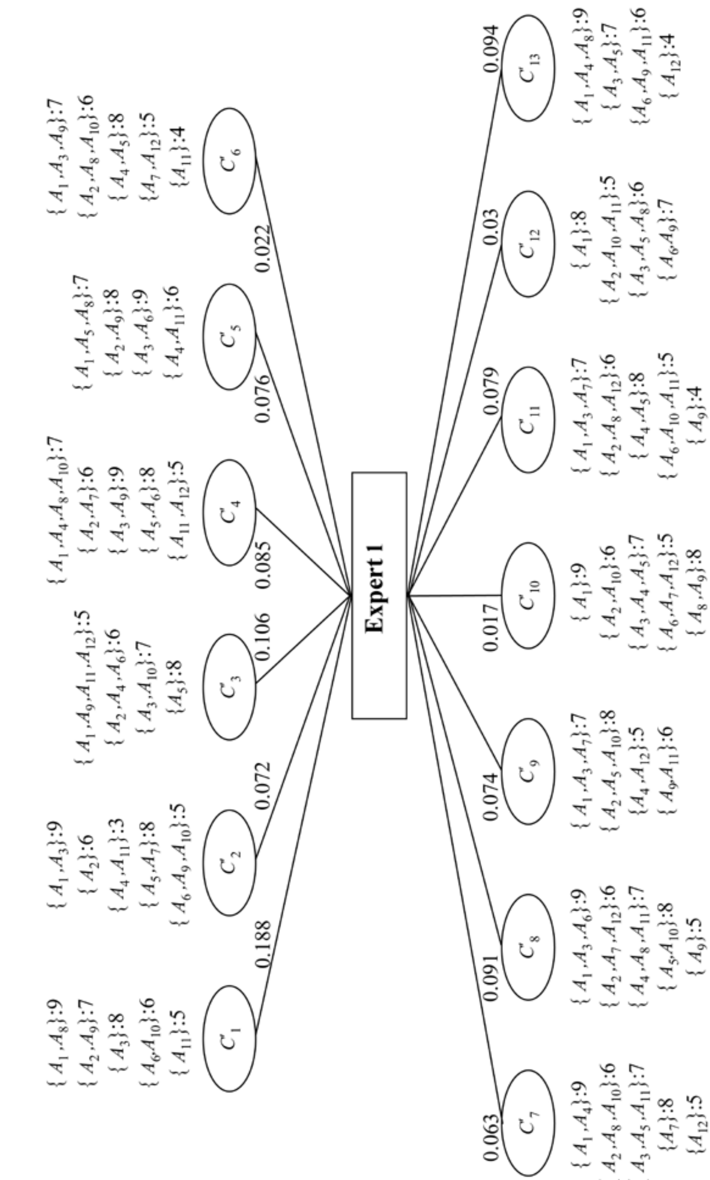

Figure 4. Schema (data model) generated by expert 1

Table 10

Intersections of the Subsets of Alternatives Selected by Expert 1 by C_1' and C_2'

	Specification C_1						
	X_j	$\{A_1, A_8\}$	$\{A_2, A_9\}$	$\{A_3\}$	$\{A_6, A_{10}\}$	$\{A_{11}\}$	$\{A\}$
Specification C_2	$\{A_1, A_3\}$	$\{A_1\}$	$\varnothing$	$\{A_3\}$	$\varnothing$	$\varnothing$	$\{A_1, A_3\}$
	$\{A_2\}$	$\varnothing$	$\{A_2\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\{A_2\}$
	$\{A_4, A_{11}\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{A_{11}\}$	$\{A_4, A_{11}\}$
	$\{A_5, A_7\}$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{A_5, A_7\}$
	$\{A_6, A_9, A_{10}\}$	$\varnothing$	$\{A_9\}$	$\varnothing$	$\{A_6, A_{10}\}$	$\varnothing$	$\{A_6, A_9, A_{10}\}$
	$\{A\}$	$\{A_1, A_8\}$	$\{A_2, A_9\}$	$\{A_3\}$	$\{A_6, A_{10}\}$	$\{A_{11}\}$	$\{A\}$

Table 11

Combined Values of Probability Mass Function

	Experts									
GIS	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}
A_1	0.198	0.169	0.195	0.171	0.089	0.102	0.08	0.097	0.09	0.084
A_2	0.089	0.068	0.069	0.093	0.086	0.072	0.077	0.076	0.071	0.074
A_3	0.214	0.188	0.232	0.243	0.196	0.089	0.088	0.099	0.0933	0.088
A_4	0.04	0.086	0.145	0.033	0.1088	0.0881	0.1	0.107	0.085	0.099
A_5	0.083	0.119	0.046	0.046	0.018	0.091	0.085	0.091	0.0927	0.093
A_6	0.092	0.04	0.077	0.091	0.1326	0.081	0.084	0.085	0.083	0.08
A_7	0.014	0.033	0.017	0.022	0.064	0.069	0.071	0.079	0.082	0.085
A_8	0.05	0.061	0.007	0.0834	0.079	0.0879	0.102	0.07	0.086	0.085
A_9	0.085	0.11	0.11	0.0825	0.1096	0.085	0.081	0.087	0.086	0.082
A_{10}	0.061	0.055	0.038	0.071	0.041	0.071	0.084	0.072	0.072	0.071
A_{11}	0.064	0.033	0.05	0.0521	0.063	0.081	0.07	0.069	0.079	0.079
A_{12}	0.01	0.038	0.014	0.012	0.013	0.083	0.078	0.068	0.08	0.08

Expert 1 (E_1):

$R_1 : \{A_3 \succ A_1 \succ A_6 \succ A_2 \succ A_9 \succ A_5 \succ A_{11} \succ A_{10} \succ A_8 \succ A_4 \succ A_7 \succ A_{12}\}$.

Expert 2 (E_2):

$R_2 : \{A_3 \succ A_1 \succ A_5 \succ A_9 \succ A_4 \succ A_2 \succ A_8 \succ A_{10} \succ A_6 \succ A_{12} \succ A_7 \succ A_{11}\}$.

Expert 3 (E_3):

$R_3 : \{A_3 \succ A_1 \succ A_4 \succ A_9 \succ A_6 \succ A_2 \succ A_{11} \succ A_5 \succ A_{10} \succ A_7 \succ A_{12} \succ A_8\}$.

Expert 4 (E_4):

$R_4 : \{A_3 \succ A_1 \succ A_2 \succ A_6 \succ A_8 \succ A_9 \succ A_{10} \succ A_{11} \succ A_5 \succ A_4 \succ A_7 \succ A_{12}\}$.

Expert 5 (E_5):

$R_5 : \{A_3 \succ A_6 \succ A_9 \succ A_4 \succ A_1 \succ A_2 \succ A_8 \succ A_7 \succ A_{11} \succ A_{10} \succ A_5 \succ A_{12}\}$.

Expert 6 (E_6):

$R_6 : \{A_1 \succ A_5 \succ A_3 \succ A_4 \succ A_8 \succ A_9 \succ A_{12} \succ A_{11} \succ A_6 \succ A_2 \succ A_{10} \succ A_7\}$.

Expert 7 (E_7):

$R_7 : \{A_8 \succ A_4 \succ A_3 \succ A_5 \succ A_{10} \succ A_6 \succ A_9 \succ A_1 \succ A_{12} \succ A_2 \succ A_7 \succ A_{11}\}$.

Expert 8 (E_8):

$R_8 : \{A_4 \succ A_3 \succ A_1 \succ A_5 \succ A_9 \succ A_6 \succ A_7 \succ A_2 \succ A_{10} \succ A_8 \succ A_{11} \succ A_{12}\}$.

Expert 9 (E_9):

$R_9 : \{A_3 \succ A_5 \succ A_1 \succ \{A_8 \sim A_9\} \succ A_4 \succ A_6 \succ A_7 \succ A_{12} \succ A_{11} \succ A_{10} \succ A_2\}$.

Expert 10 (E_{10}):

$R_{10} : \{A_4 \succ A_5 \succ A_3 \succ \{A_7 \sim A_8\} \succ A_1 \succ A_9 \succ \{A_6 \sim A_{12}\} \succ A_{11} \succ A_2 \succ A_{10}\}$.

5.4 Construction of a collective (generalized) ranking of GIS technologies.

Aggregation of individual expert preferences is carried out by combining the *bpa's* of alternatives identified by experts and given in Table 11. The coefficients of the competence of experts $\Omega = \{\omega_i \mid i = \overline{1,10}\}$ have been taken into account. Based on the fact that the values of the coefficient of conflict vary from 0.67 to 0.8, which indicates the presence of a significant conflict between individual groups of evidence, the PCR5 conflict redistribution rule was applied according to eq. (15) to obtain the combined *bpa's*. The combined values of the *bpa's* are summarized in Table 12.

Table 12

Combined *bpa's* of Initial Alternatives

	Alternatives					
GIS	A_1	A_2	A_3	A_4	A_5	A_6
m_{PCR5}	0.1363	0.0474	0.261	0.086	0.073	0.0629
GIS	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
m_{PCR5}	0.0573	0.0627	0.0661	0.0443	0.051	0.052

Based on the values of the evaluated objects (GIS) from Table 12, the resulting ratio (generalized ranking) is built, reflecting the collective opinion of the expert group:

$$A_3 \succ A_1 \succ A_4 \succ A_5 \succ A_9 \succ A_6 \succ A_8 \succ A_7 \succ A_{12} \succ A_{11} \succ A_2 \succ A_{10}.$$

It can be seen from the given results that the alternative A_3 has the greatest m_{PCR5} value, which corresponds to the AutoDesk Map GIS software.

6. Conclusions

The main provisions of IT for decision support for solving the problem of analyzing (structuring) group expert assessments, which are formed under multi-criteria, multi-alternativeness and complex forms of ignorance generated by combinations of uncertainty, inconsistency, fuzziness, etc. has been proposed.

A methodology of practical application of the developed information technology of analysis and structuring of group expert assessments has been proposed on the example of solving the problem of ranking group expert assessments for GIS technology choice.

The models, algorithms and information technology proposed in the work for solving a multi-criteria decision-making problem under uncertainty are implemented as a package of software modules and are used to analyze the results of an expert survey.

References:

1. Uzhga-Rebrov O.I. (2010). Upravlenie neopredelennostyami. Sovremennye neveroyatnostnye metody. Rezekne: RA Izdevnieciba. (in Russian)

2. Annamdas K.K. (2009). Evidence based uncertainty models and particles swarm optimization for multiobjective optimization of engineering systems (PhD Thesis). Florida: University of Miami.

3. Beynon M.J., Curry B., Morgan P. (2000). The Dempster-Shafer theory of evidence: an alternative approach to milticriteria decision modeling. *Omega,* vol. 28, no. 1, pp. 37–50. doi: 10.1016/S0305-0483(99)00033-X

4. Beynon M.J. (2005). A method of aggregation in DS/AHP for group decision-making with the non-equivalent importance of individuals in the group. *Computers & Operations Research*, vol. 32, pp. 1881–1896.

5. Beynon M.J. (2002). DS/AHP method: A mathematical analysis, including an understanding of uncertainty. *European Journal of Operational Research*, vol. 140, pp. 148–164.

6. Dempster A.P. (1967). Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, vol. 38, no. 2, pp. 325–339. doi: 10.1214/aoms/1177698950

7. Jousselme A.L., Grenier D., Boss´e E. (2001). A new distance between two bodies of evidence. *Information Fusion*, vol. 2, no. 2, pp. 91–101. doi: 10.1016/S1566-2535(01)00026-4

8. Jousselme A.L., Grenier D., Boss´e E. (2002). Analyzing approximation algorithms in the theory of evidence. *Sensor Fusion: Architecture, Algorithms and Applications VI,* vol. 4731, pp. 65–74.

9. Inagaki T. (1991). Interdepence between safety-control policy and multiple-sensor schemes via Dempster-Shafer theory. *Transaction on Reliability*, vol. 40, no. 2, pp. 182–188.

10. Saaty T. (1980). The Analytic Hierarchy Process: panning, priority setting, resource allocation. New York: McGraw Hill.

11. Sentz K., Ferson S. (2002). Combination of evidence in Dempster-Shafer theory: tech. report SAND 2002-0835. Albuquerque: Sandia National Laboratories. 94 p.

12. Shafer G. (1976). A mathematical theory of evidence. Princeton: Princeton University Press.

13. Smarandache F. (2004). An in-depth look at information fusion rules and the unification of fusion theories. *Computing Research Repository (CoRR),* vol. cs.OH/0410033, 22 p.

14. Smarandache F. (2004). Unification of Fusion Theories (UFT). *International Journal of Applied Mathematics and Statistics*, vol. 2, pp. 1–14.

15. Smarandache F., Dezert J. (2004). Representation of DSmT. *Advances and Applications of DSmT for Information Fusion*. Rehoboth: American Research Press, vol. 1, pp. 3–35.

16. Smarandache F., Dezert J. (2006). Advances and applications of DSmT for information fusion. *Collected works*. Rehoboth: American Research Press, vol. 2.

17. Smets Ph. (1990). The combination of evidence in the transferable belief model. *Pattern analysis and Machine Intelligence*, vol. 12, no. 5, pp. 447–458. doi: 10.1109/34.55104

18. Yager R.R. (1987). On the Dempster-Shafer framework and new combination rules. *Information Science*, vol. 41, no. 2, pp. 93–137. doi: 10.1016/0020-0255(87)90007-7

19. Zhang L. (1994). Representation, independence and combination of evidence in the Dempster-Shafer Theory of Evidence. *Advances in the Dempster-Shafer Theory of Evidence*. New York: John Wiley & Sons, pp. 51–69.