

CORRELATION MEASURE FOR NEUTROSOPHIC REFINED SETS AND ITS APPLICATION IN MEDICAL DIAGNOSIS

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Abstract In this paper, the correlation measure of neutrosophic refined(multi-) sets is proposed. The concept of this correlation measure of neutrosophic refined sets is the extension of correlation measure of neutrosophic sets and intuitionistic fuzzy multi-sets. Finally, using the correlation of neutrosophic refined set measure, the application of medical diagnosis and pattern recognition are presented.

1 Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness such as; probability set theory, fuzzy set theory [60], intuitionistic fuzzy set theory [8], interval intuitionistic fuzzy set theory [7], soft set theory [38], FP-soft set theory [21, 22], IFP-soft set theory [25] etc. But, all these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system, therefore, Smarandache [46] developed a new concept called neutrosophic set (NS) which generalizes fuzzy sets and intuitionistic fuzzy sets and so on. Then, Wang et al. [53] introduced an instance of neutrosophic sets known as single valued neutrosophic sets (SVNS), which were motivated from the practical point of view and that can be used in real scientific and engineering application, and provide the set theoretic operators and various properties of SVNSs. This theory and their hybrid structures have proven useful in many different fields such as control theory [1], databases [4, 5], medical diagnosis problem [2], decision making problem [20, 34, 36, 58], physics [40], topology [35], etc. The works on neutrosophic set, in theories and applications, have been progressing rapidly (e.g. [3, 6, 12, 16, 17, 24, 55, 56]).

Combining neutrosophic set models with other mathematical models has attracted the attention of many researchers. Maji et al. [37] presented the concept of neutrosophic soft sets which is based on a combination of the neutrosophic set and soft set [38] models, Broumi and Smarandache [9, 13] introduced the concept of the intuitionistic neutrosophic soft set by combining the intuitionistic neutrosophic sets and soft sets. Also, Broumi et al. presented the concept of rough neutrosophic set [18] which is based on a combination of neutrosophic sets and rough set models. The works on neutrosophic sets combining with soft sets, in theories and applications, have been progressing rapidly (e.g. [10, 14, 15, 27, 28, 29, 30]).

The multi-set theory was formulated first in [54] by Yager as generalization of the concept of set theory and then the multi-set was developed in [19] by Calude et al. Several authors from time to time made a number of generalizations of the multi-set theory. For example, Sebastian and Ramakrishnan [49, 50] introduced a new notion called multi fuzzy sets which is a generalization of the multi-set. Since then, several researchers [39, 48, 52] discussed more properties on multi fuzzy set. And they [31, 51] made an extension of the concept of fuzzy multi-sets to an intuitionistic fuzzy set which was called intuitionistic fuzzy multi-sets (IFMS). Since then in the study on IFMS, a lot of excellent results have been achieved by researchers [23, 41, 42, 43, 44, 45]. An element of a multi fuzzy set can occur more than once with possibly the same or different membership values whereas an element of intuitionistic fuzzy multi-set allows the repeated occurrences of membership and non membership values. The concepts of FMS and IFMS fail to deal with indeterminacy. In 2013 Smarandache [47] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively T_1, T_2, \dots, T_m and I_1, I_2, \dots, I_p , and F_1, F_2, \dots, F_r . Recently, Deli et al.[26]and Ye and Ye [59] used the concept of neutrosophic refined(multi-set) sets and studied some of their

basic properties. The concept of neutrosophic the refined set (NRS) is a generalization of fuzzy multi-sets and intuitionistic fuzzy multi-sets.

Rajarajeswari and Uma [45] put forward the correlation measure for IFMS. Recently, Broumi and Smarandache defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance such as; set theoretic approach and matching function to calculate the similarity degree between neutrosophic sets. In the same year, Broumi and Smarandache [11] also proposed the correlation coefficient between interval neutrosophic sets. In other research, Ye [57] proposed three vector similarity measure for SNSs, an instance of SVNS and INS, including the Jaccard, Dice, and cosine similarity measures for SVNS and INSs, and applied them to multi-criteria decision-making problems with simplified neutrosophic information. Hanafy et al. [32] proposed the correlation coefficients of neutrosophic sets and studied some of their basic properties. Based on centroid method, Hanafy et al. [33], introduced and studied the concepts of correlation and correlation coefficient of neutrosophic sets and studied some of their properties.

The purpose of this paper is an attempt to extend the correlation measure of neutrosophic sets to neutrosophic refined(multi) sets (NRS). This paper is arranged in the following manner. In section 2, we present some definitions and notion about neutrosophic set and neutrosophic refined (multi-) set theory which help us in later section. In section 3, we study the concept of correlation measure of neutrosophic refined set. In section 4, we present an application of correlation measure of neutrosophic refined set to medical diagnosis problem. Finally, we conclude the paper.

2 PRELIMINARIES

In this section, we present the basic definitions and results of intuitionistic fuzzy set theory [8], intuitionistic fuzzy multi-set theory [41], neutrosophic set theory [46, 53], neutrosophic refined (multi-) set theory [26, 59] and correlation measure of intuitionistic fuzzy multi-sets [44] that are useful for subsequent discussions. See especially [2, 3, 4, 5, 6, 12, 20, 26, 27, 34, 35, 40] for further details and background.

Definition 2.1. [8] Let E be a universe. An intuitionistic fuzzy set I on E can be defined as follows:

$$I = \{ \langle x, \mu_I(x), \gamma_I(x) \rangle : x \in E \}$$

where, $\mu_I : E \rightarrow [0, 1]$ and $\gamma_I : E \rightarrow [0, 1]$ such that $0 \leq \mu_I(x) + \gamma_I(x) \leq 1$ for any $x \in E$.

Here, $\mu_I(x)$ and $\gamma_I(x)$ is the degree of membership and degree of non-membership of the element x , respectively.

Definition 2.2. [41] Let E be a universe. An intuitionistic fuzzy multi-set K on E can be defined as follows:

$$K = \{ \langle x, (\mu_K^1(x), \mu_K^2(x), \dots, \mu_K^P(x)), (\gamma_K^1(x), \gamma_K^2(x), \dots, \gamma_K^P(x)) \rangle : x \in E \}$$

where, $\mu_K^1(x), \mu_K^2(x), \dots, \mu_K^P(x) : E \rightarrow [0, 1]$ and $\gamma_K^1(x), \gamma_K^2(x), \dots, \gamma_K^P(x) : E \rightarrow [0, 1]$ such that $0 \leq \mu_K^i(x) + \gamma_K^i(x) \leq 1 (i = 1, 2, \dots, P)$ and $\mu_K^1(x) \leq \mu_K^2(x) \leq \dots \leq \mu_K^P(x)$ for any $x \in E$.

Here, $(\mu_K^1(x), \mu_K^2(x), \dots, \mu_K^P(x))$ and $(\gamma_K^1(x), \gamma_K^2(x), \dots, \gamma_K^P(x))$ is the membership sequence and non-membership sequence of the element x , respectively.

We arrange the membership sequence in decreasing order but the corresponding non membership sequence may not be in decreasing or increasing order.

Definition 2.3. [45] Let E be a universe and $K = \{ \langle x, (\mu_K^1(x), \mu_K^2(x), \dots, \mu_K^P(x)), (\gamma_K^1(x), \gamma_K^2(x), \dots, \gamma_K^P(x)) \rangle : x \in E \}$, $L = \{ \langle x, (\mu_L^1(x), \mu_L^2(x), \dots, \mu_L^P(x)), (\gamma_L^1(x), \gamma_L^2(x), \dots, \gamma_L^P(x)) \rangle : x \in E \}$ be two intuitionistic fuzzy multi-sets consisting of the membership and non membership functions, then the correlation coefficient of K and L defined as follows:

$$\rho_{IFMS}(K, L) = \frac{C_{IFMS}(K, L)}{\sqrt{C_{IFMS}(K, K) * C_{IFMS}(L, L)}}$$

where

$$C_{IFMS}(K, L) = \sum_{j=1}^P \left(\sum_{i=1}^n (\mu_K^j(x_i) \mu_L^j(x_i) + \gamma_K^j(x_i) \gamma_L^j(x_i)) \right)$$

$$C_{IFMS}(K, K) = \sum_{j=1}^P \left(\sum_{i=1}^n (\mu_K^j(x_i) \mu_K^j(x_i) + \gamma_K^j(x_i) \gamma_K^j(x_i)) \right)$$

and

$$C_{IFMS}(L, L) = \sum_{j=1}^P \left(\sum_{i=1}^n (\mu_L^j(x_i) \mu_L^j(x_i) + \gamma_L^j(x_i) \gamma_L^j(x_i)) \right)$$

Expresses the so-called informational energy of neutrosophic sets A and B.

Definition 2.4. [46] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set(N-set) A in U is characterized by a truth-membership function T_A , an indeterminacy-membership function I_A and a falsity-membership function F_A . $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$.

It can be written as

$$A = \{ \langle x, (T_A(x), I_A(x), F_A(x)) \rangle : x \in U, T_A(x), I_A(x), F_A(x) \subseteq [0, 1] \}.$$

There is no restriction on the sum of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so $-0 \leq \sup T_A(u) + \sup I_A(u) + \sup F_A(u) \leq 3^{+}$.

Here, $1^{+} = 1 + \varepsilon$, where 1 is its standard part and ε its non-standard part. Similarly, $-0 = 1 + \varepsilon$, where 0 is its standard part and ε its non-standard part.

For application in real scientific and engineering areas, Wang et al.[53] proposed the concept of an SVNS, which is an instance of neutrosophic set. In the following, we introduce the definition of SVNS.

Definition 2.5. [53] Let U be a space of points (objects), with a generic element in U denoted by u. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ belongs to $[0, 1]$ for each point u in U. Then, an SVNS A can be expressed as

$$A = \{ \langle u, (T_A(x), I_A(x), F_A(x)) \rangle : x \in E, T_A(x), I_A(x), F_A(x) \in [0, 1] \}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

Definition 2.6. [26, 59] Let E be a universe. A neutrosophic refined (multi-) set(NRs) A on E can be defined as follows:

$$A = \{ \langle x, (T_A^1(x), T_A^2(x), \dots, T_A^P(x)), (I_A^1(x), I_A^2(x), \dots, I_A^P(x)), (F_A^1(x), F_A^2(x), \dots, F_A^P(x)) \rangle : x \in E \}$$

where,

$$T_A^1(x), T_A^2(x), \dots, T_A^P(x) : E \rightarrow [0, 1],$$

$$I_A^1(x), I_A^2(x), \dots, I_A^P(x) : E \rightarrow [0, 1],$$

and

$$F_A^1(x), F_A^2(x), \dots, F_A^P(x) : E \rightarrow [0, 1]$$

such that

$$0 \leq \sup T_A^i(x) + \sup I_A^i(x) + \sup F_A^i(x) \leq 3$$

($i = 1, 2, \dots, P$) and

$$T_A^1(x) \leq T_A^2(x) \leq \dots \leq T_A^P(x)$$

for any $x \in E$.

$(T_A^1(x), T_A^2(x), \dots, T_A^P(x))$, $(I_A^1(x), I_A^2(x), \dots, I_A^P(x))$ and $(F_A^1(x), F_A^2(x), \dots, F_A^P(x))$ is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x , respectively. Also, P is called the dimension(cardinality) of Nms A, denoted $d(A)$. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.

The set of all Neutrosophic neutrosophic (multi-)sets on E is denoted by NRS(E).

Definition 2.7. [26, 59] Let $A, B \in NRS(E)$. Then,

- (i) A is said to be Nm-subset of B is denoted by $A \widetilde{\subseteq} B$ if $T_A^i(x) \leq T_B^i(x)$, $I_A^i(x) \geq I_B^i(x)$, $F_A^i(x) \geq F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, \dots, P$.
- (ii) A is said to be neutrosophic equal of B is denoted by $A = B$ if $T_A^i(x) = T_B^i(x)$, $I_A^i(x) = I_B^i(x)$, $F_A^i(x) = F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, \dots, P$.
- (iii) The union of A and B is denoted by $A \widetilde{\cup} B = C$ and is defined by

$$C = \{ \langle x, (T_C^1(x), T_C^2(x), \dots, T_C^P(x)), (I_C^1(x), I_C^2(x), \dots, I_C^P(x)), (F_C^1(x), F_C^2(x), \dots, F_C^P(x)) \rangle : x \in E \}$$

where $T_C^i = T_A^i(x) \vee T_B^i(x)$, $I_C^i = I_A^i(x) \wedge I_B^i(x)$, $F_C^i = F_A^i(x) \wedge F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, \dots, P$.

- (iv) The intersection of A and B is denoted by $A \widetilde{\cap} B = D$ and is defined by

$$D = \{ \langle x, (T_D^1(x), T_D^2(x), \dots, T_D^P(x)), (I_D^1(x), I_D^2(x), \dots, I_D^P(x)), (F_D^1(x), F_D^2(x), \dots, F_D^P(x)) \rangle : x \in E \}$$

where $T_D^i = T_A^i(x) \wedge T_B^i(x)$, $I_D^i = I_A^i(x) \vee I_B^i(x)$, $F_D^i = F_A^i(x) \vee F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, \dots, P$.

3 Correlation Measure of two Neutrosophic refined sets

In this section, we give correlation measure of two neutrosophic refined sets. Some of it is quoted from [32, 33, 44, 45, 58].

Following the correlation measure of two intuitionistic fuzzy multi-sets defined by Rajarajeswari and Uma in [45]. In this section, we extend these measures to neutrosophic refined sets.

Definition 3.1. Let $X = \{x_1, x_2, x_3, \dots, x_n\}$ be the finite universe of discourse and $A = \{ \langle T_A^j(x_i), I_A^j(x_i), F_A^j(x_i) \rangle : x_i \in X \}$, $B = \{ \langle T_B^j(x_i), I_B^j(x_i), F_B^j(x_i) \rangle : x_i \in X \}$ be two neutrosophic refined sets consisting of the membership, indeterminate and non-membership functions. Then the correlation coefficient of A and B

$$\rho_{NRS}(A, B) = \frac{C_{NRS}(A, B)}{\sqrt{C_{NRS}(A, A) * C_{NRS}(B, B)}}$$

where

$$C_{NRS}(A, B) = \frac{1}{p} \sum_{j=1}^p \sum_{i=1}^n \{ T_A^j(x_i) T_B^j(x_i) + I_A^j(x_i) I_B^j(x_i) + F_A^j(x_i) F_B^j(x_i) \},$$

$$C_{NRS}(A, A) = \frac{1}{p} \sum_{j=1}^p \sum_{i=1}^n \{ T_A^j(x_i) T_A^j(x_i) + I_A^j(x_i) I_A^j(x_i) + F_A^j(x_i) F_A^j(x_i) \}$$

and

$$C_{NRS}(B, B) = \frac{1}{p} \sum_{j=1}^p \sum_{i=1}^n \{ T_B^j(x_i) T_B^j(x_i) + I_B^j(x_i) I_B^j(x_i) + F_B^j(x_i) F_B^j(x_i) \}$$

Proposition 3.2. The defined correlation measure between NRS A and NRS B satisfies the following properties

- (i) $0 \leq \rho_{NRS}(A, B) \leq 1$
- (ii) $\rho_{NRS}(A, B) = 1$ if and only if $A = B$
- (iii) $\rho_{NRS}(A, B) = \rho_{NRS}(B, A)$.

Proof

- (i) $0 \leq \rho_{NRS(A, B)}(A, B) \leq 1$

As the membership, indeterminate and non-membership functions of the NRS lies between 0 and 1, $\rho_{NRS}(A, B)$ also lies between 0 and

(ii) $\rho_{NRS}(A,B)=1$ if and only if $A=B$

a. Let the two NRS A and B be equal (i.e $A=B$). Hence for any

$$T_A^j(x_i) = T_B^j(x_i), I_B^j(x_i) = I_A^j(x_i) \text{ and } F_A^j(x_i) = F_B^j(x_i),$$

then

$$C_{NRS}(A, A) = C_{NRS}(B, B) = \frac{1}{p} \sum_{j=1}^p \sum_{i=1}^n \left\{ T_A^j(x_i) T_A^j(x_i) + I_A^j(x_i) I_A^j(x_i) + F_A^j(x_i) F_A^j(x_i) \right\}$$

and

$$\begin{aligned} C_{NRS}(A, B) &= \frac{1}{p} \sum_{j=1}^p \sum_{i=1}^n \left\{ T_A^j(x_i) T_B^j(x_i) + I_A^j(x_i) I_B^j(x_i) + F_A^j(x_i) F_B^j(x_i) \right\} \\ &= \frac{1}{p} \sum_{i=1}^p \sum_{i=1}^n \left\{ T_A^j(x_i) T_A^j(x_i) + I_A^j(x_i) I_A^j(x_i) + F_A^j(x_i) F_A^j(x_i) \right\} \\ &= C_{NRS}(A, A) \end{aligned}$$

Hence

$$\rho_{NRS}(A, B) = \frac{C_{NRS}(A, B)}{\sqrt{C_{NRS}(A, A) * C_{NRS}(B, B)}} = \frac{C_{NRS}(A, A)}{\sqrt{C_{NRS}(A, A) * C_{NRS}(A, A)}} = 1$$

b. Let the $\rho_{NRS}(A,B)=1$. Then, the unite measure is possible only if

$$\frac{C_{NRS}(A, B)}{\sqrt{C_{NRS}(A, A) * C_{NRS}(B, B)}} = 1$$

this refers that

$$T_A^j(x_i) = T_B^j(x_i), I_B^j(x_i) = I_A^j(x_i) \text{ and } F_A^j(x_i) = F_B^j(x_i)$$

for all i, j values. Hence $A=B$

(iii) If $\rho_{NRS}(A,B)=\rho_{NRS}(B,A)$, it obvious that

$$\frac{C_{NRS}(A, B)}{\sqrt{C_{NRS}(A, A) * C_{NRS}(B, B)}} = \frac{C_{NRS}(B, A)}{\sqrt{C_{NRS}(A, A) * C_{NRS}(B, B)}} = \rho_{NRS}(B, A)$$

as

$$\begin{aligned} C_{NRS}(A, B) &= \frac{1}{p} \sum_{j=1}^p \sum_{i=1}^n \left\{ T_A^j(x_i) T_B^j(x_i) + I_A^j(x_i) I_B^j(x_i) + F_A^j(x_i) F_B^j(x_i) \right\} \\ &= \frac{1}{p} \sum_{j=1}^p \sum_{i=1}^n \left\{ T_B^j(x_i) T_A^j(x_i) + I_B^j(x_i) I_A^j(x_i) + F_B^j(x_i) F_A^j(x_i) \right\} \\ &= C_{NRS}(B, A) \end{aligned}$$

4 Application

In this section, we give some applications of NRS in medical diagnosis and pattern recognition problems using the correlation measure. Some of it is quoted from [44, 45, 51].

From now on, we use

$$A = \{ \langle x, (T_A^1(x), I_A^1(x), F_A^1(x)), (T_A^2(x), I_A^2(x), F_A^2(x)), \dots, (T_A^P(x), I_A^P(x), F_A^P(x)) \rangle : x \in E \}$$

instead of

$$A = \{ \langle x, (T_A^1(x), T_A^2(x), \dots, T_A^P(x)), (I_A^1(x), I_A^2(x), \dots, I_A^P(x)), (F_A^1(x), F_A^2(x), \dots, F_A^P(x)) \rangle : x \in E \}$$

4.1 Medical Diagnosis via NRS Theory

In what follows, let us consider an illustrative example adopted from Rajarajeswari and Uma [44] and typically considered in [45, 51]. Obviously, the application is an extension of intuitionistic fuzzy multi sets [44].

"As Medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different set of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having different truth membership, indeterminate and false membership functions. The proposed correlation measure among the patients Vs symptoms and symptoms Vs diseases gives the proper medical diagnosis. The unique feature of this proposed method is that it considers multi truth membership, indeterminate and false membership. By taking one time inspection, there may be error in diagnosis. Hence, this multi time inspection, by taking the samples of the same patient at different times gives best diagnosis" [44].

Now, an example of a medical diagnosis will be presented.

Example 4.1. Let $P = \{P_1, P_2, P_3\}$ be a set of patients, $D = \{Viral\ Fever, Tuberculosis, Typhoid, Throat\ disease\}$ be a set of diseases and $S = \{Temperature, cough, throat\ pain, headache, bodypain\}$ be a set of symptoms. Our solution is to examine the patient at different time intervals (three times a day), which in turn give arise to different truth membership, indeterminate and false membership function for each patient.

Table I: Q (the relation Between Patient and Symptoms)

| Q | Temperature | Cough | Throat pain | Headache | Body Pain |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| P ₁ | (0.4, 0.3, 0.4) | (0.5, 0.4, 0.4) | (0.3, 0.5, 0.5) | (0.5, 0.3, 0.4) | (0.5, 0.2, 0.4) |
| | (0.3, 0.4, 0.6) | (0.4, 0.1, 0.3) | (0.2, 0.6, 0.4) | (0.5, 0.4, 0.7) | (0.2, 0.3, 0.5) |
| | (0.2, 0.5, 0.5) | (0.3, 0.4, 0.5) | (0.1, 0.6, 0.3) | (0.3, 0.3, 0.6) | (0.1, 0.4, 0.3) |
| P ₂ | (0.6, 0.3, 0.5) | (0.6, 0.3, 0.7) | (0.6, 0.3, 0.3) | (0.6, 0.3, 0.1) | (0.4, 0.4, 0.5) |
| | (0.5, 0.5, 0.2) | (0.4, 0.4, 0.2) | (0.3, 0.5, 0.4) | (0.4, 0.5, 0.8) | (0.3, 0.2, 0.7) |
| | (0.4, 0.4, 0.5) | (0.2, 0.4, 0.5) | (0.1, 0.4, 0.5) | (0.2, 0.4, 0.3) | (0.1, 0.5, 0.5) |
| P ₃ | (0.8, 0.3, 0.5) | (0.5, 0.5, 0.3) | (0.3, 0.3, 0.6) | (0.6, 0.2, 0.5) | (0.6, 0.4, 0.5) |
| | (0.7, 0.5, 0.4) | (0.3, 0.4, 0.3) | (0.2, 0.5, 0.7) | (0.5, 0.3, 0.6) | (0.3, 0.3, 0.4) |
| | (0.6, 0.4, 0.4) | (0.1, 0.6, 0.4) | (0.1, 0.4, 0.5) | (0.2, 0.2, 0.6) | (0.2, 0.2, 0.6) |

Let the samples be taken at three different timings in a day (in 08:00,16:00,24:00)

Table II: R (the relation among Symptoms and Diseases)

| R | Viral Fever | Tuberculosis | Typhoid | Throat diseases |
|-------------|---------------|---------------|---------------|-----------------|
| Temperature | (0.2,0.5,0.6) | (0.4,0.6,0.5) | (0.6,0.4,0.5) | (0.3,0.7,0.8) |
| Cough | (0.6,0.4,0.6) | (0.8,0.2,0.3) | (0.3,0.2,0.6) | (0.2,0.4,0.1) |
| Throat Pain | (0.5,0.2,0.3) | (0.4,0.5,0.3) | (0.4,0.5,0.5) | (0.2,0.6,0.2) |
| Headache | (0.6,0.8,0.2) | (0.2,0.3,0.6) | (0.1,0.6,0.3) | (0.2,0.5,0.5) |
| Body Pain | (0.7,0.4,0.4) | (0.2,0.3,0.4) | (0.2,0.3,0.4) | (0.2,0.2,0.3) |

Table III: The Correlation Measure between NRS Q and R

| Correlation measure | Viral Fever | Tuberculosis | Typhoid | Throat diseases |
|---------------------|-------------|--------------|--------------|-----------------|
| P ₁ | 0.846 | 0.910 | 0.884 | 0.880 |
| P ₂ | 0.849 | 0.868 | 0.892 | 0.809 |
| P ₃ | 0.792 | 0.853 | 0.872 | 0.822 |

The highest correlation measure from the Table III gives the proper medical diagnosis. Therefore, patient P₁ suffers from Tuberculosis, patient P₂ and P₃ suffers from Typhoid.

4.2 Pattern Recognition of NRS using proposed correlation measure

In what follows, let us consider an illustrative example adopted from Rajarajeswari and Uma [44] and typically considered in [45, 51]. Obviously, the application is an extension of intuitionistic fuzzy multi sets [44].

Example 4.2. Let $X = \{A_1, A_2, A_3, \dots, A_n\}$ with $A = \{A_1, A_2, A_3, A_4, A_5\}$ and $B = \{A_2, A_5, A_7, A_8, A_9\}$ are the NRS defined as

$$\begin{aligned} \text{Pattern I} = \{ & \langle A_1, (0.4, 0.5, 0.6), (0.2, 0.3, 0.5) \rangle, \langle A_2, (0.5, 0.5, 0.2), (0.3, 0.2, 0.7) \rangle, \\ & \langle A_3, (0.6, 0.3, 0.4), (0.6, 0.5, 0.3) \rangle, \langle A_4, (0.7, 0.4, 0.5), (0.5, 0.4, 0.6) \rangle, \\ & \langle A_5 : (0.3, 0.7, 0.2), (0.3, 0.2, 0.5) \rangle \} \end{aligned}$$

and

$$\begin{aligned} \text{Pattern II} = \{ & \langle A_2, (0.5, 0.2, 0.4), (0.3, 0.4, 0.6) \rangle, \langle A_5, (0.7, 0.3, 0.1), (0.6, 0.1, 0.4) \rangle, \\ & \langle A_7 : (0.7, 0.2, 0.4), (0.4, 0.5, 0.3) \rangle, \langle A_8, (0.8, 0.1, 0.4), (0.3, 0.5, 0.7) \rangle, \\ & A_9, (0.6, 0.3, 0.1), (0.2, 0.6, 0.1) \rangle \} \end{aligned}$$

Then the testing NRS pattern II be $\{A_6, A_7, A_8, A_9, A_{10}\}$ such that

$$\begin{aligned} \text{Pattern III} = \{ & \langle A_6, (0.6, 0.4, 0.2), (0.4, 0.3, 0.7) \rangle, \langle A_7, (0.9, 0.1, 0.1), (0.8, 0.3, 0.3) \rangle, \\ & \langle A_8, (0.6, 0.7, 0.1), (0.3, 0.8, 0.2) \rangle, \langle A_9, (0.3, 0.8, 0.5), (0.2, 0.7, 0.2) \rangle, \\ & A_{10}, (0.4, 0.5, 0.6), (0.3, 0.7, 0.2) \rangle \} \end{aligned}$$

Then, the correlation measure between pattern I and III is 0.8404, pattern II and III is 0.8286. Therefore; the testing pattern III belongs to pattern I type.

5 Conclusion

In this paper, we have firstly defined the correlation measure of neutrosophic refined sets and proved some of their basic properties. We have present an application of correlation measure of neutrosophic refined sets in medical diagnosis and pattern recognition. In The future work, we will extend this correlation measure to the case of interval neutrosophic refined sets.

References

- [1] S. Aggarwal, R. Biswas and A. Q. Ansari, Neutrosophic Modeling and Control, Computer and Communication Technology, 718–723, 2010.
- [2] A. Q. Ansari, R. Biswas and S. Aggarwal, Proposal for Applicability of Neutrosophic Set Theory in Medical AI, International Journal of Computer Applications, 27(5), 5–11, 2011.
- [3] A. Q. Ansari, R. Biswas and S. Aggarwal, Neutrosophic classifier: An extension of fuzzy classifier, Applied Soft Computing, 13, 563–573, 2013.
- [4] M. Arora and R. Biswas, Deployment of Neutrosophic Technology to Retrieve Answers for Queries Posed in Natural Language, 3. Int. Conf. on Comp. Sci. and Inform. Tech., 435–439, 2010.
- [5] M. Arora, R. Biswas and U. S. Pandey, Neutrosophic Relational Database Decomposition, International Journal of Advanced Computer Science and Applications, 2(8), 121–125, 2011.
- [6] C. Ashbacher, Introduction to Neutrosophic Logic, American Research Press, Rehoboth, 2002.
- [7] K. Atanassov, Gargov, interval $\tilde{\mu}$ -valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31, 343–349, 1989.
- [8] K. T. Atanassov, Intuitionistic Fuzzy Set. Fuzzy Sets and Systems, 20(1), 87–86, 1986.
- [9] S. Broumi and F. Smarandache, Intuitionistic Neutrosophic Soft Set, Journal of Information and Computing Science, 8(2), 130–140, 2013.
- [10] S. Broumi, Generalized Neutrosophic Soft Set, International Journal of Computer Science, Engineering and Information Technology, 3(2) 17–30, 2013.
- [11] S. Broumi, F. Smarandache, Correlation Coefficient of Interval Neutrosophic set, Proceedings of the International Conference ICMERA, Bucharest, October 2013.
- [12] S. Broumi, F. Smarandache, Several Similarity Measures of Neutrosophic Sets, Neutrosophic Sets and Systems, 1, 54–62, 2013.

- [13] S. Broumi, F. Smarandache, More on Intuitionistic Neutrosophic Soft Sets, *Computer Science and Information Technology*, 1(4), 257–268, 2013.
- [14] S. Broumi, I. Deli and F. Smarandache, Relations on Interval Valued Neutrosophic Soft Sets, *Journal of New Results in Science*, 5, 1–20, 2014.
- [15] S. Broumi, I. Deli, F. Smarandache, Neutrosophic Parametrized Soft Set theory and its decision making problem, *International Frontier Science Letters*, 1 (1), 01–11, 2014.
- [16] S Broumi and F Smarandache, On Neutrosophic Implications. *Neutrosophic Sets and Systems*, 2, 9–17, 2014.
- [17] S. Broumi, F. Smarandache, Lower and Upper Soft Interval Valued Neutrosophic Rough Approximations of An IVNSS-Relation, *Proceedings of SISOM ACOUSTICS, International Conference, Bucharest*, 22-23, 204–211, 2014.
- [18] S. Broumi, M. Dhar, F.Smarandache, Rough neutrosophic sets, *Italian journal of pure and applied mathematics*, 32, 493–502, 2014.
- [19] C. S. Calude, G. Paun, G. Rozenberg, A. Saloma, *Lecture notes in computer science: Multiset Processing Mathematical, Computer Science, and Molecular Computing Points of View*, 2235, Springer, New York, 2001.
- [20] P. Chi and L. Peide, An Extended TOPSIS Method for the Multiple Attribute Decision Making Problems Based on Interval Neutrosophic, *Neutrosophic Sets and Systems*, 1, 63–70, 2013.
- [21] N. Çađman and I. Deli, Products of FP-soft sets and their applications, *Hacettepe Journal of Mathematics and Statistics*, 41 (3) (2012), 365– 374.
- [22] N. Çađman and I. Deli, Means of FP-soft sets and their applications, *Hacettepe Journal of Mathematics and Statistics* 41 (5) (2012), 615 –625.
- [23] S. Das, M. B. Kar and S. Kar, Group multi-criteria decision making using intuitionistic multi-fuzzy sets, *Journal of Uncertainty Analysis and Applications*, 10(1), 1- 16, 2013.
- [24] M. Dhar, S. Broumi and F. Smarandache, A Note on Square Neutrosophic Fuzzy Matrices, *Neutrosophic sets and systems*, 3, 37–41, 2014.
- [25] I. Deli, N. Çađman, Intuitionistic fuzzy parameterized soft set theory and its decision making, *Applied Soft Computing* 28 (2015) 109–113.
- [26] I. Deli, S. Broumi and F. Smarandache Neutrosophic multi-sets and its application in medical diagnosis, (submitted), 2014.
- [27] I. Deli and S. Broumi, Neutrosophic soft relations and some properties, *Annals of Fuzzy Mathematics and Informatics* 9(1), 169–182, 2015.
- [28] I. Deli, Interval-valued neutrosophic soft sets and its decision making, <http://arxiv.org/abs/1402.3130>.
- [29] I. Deli, S. Broumi, Neutrosophic Soft Matrices and NSM-decision Making, *Journal of Intelligent and Fuzzy Systems*, Doi: 10.3233/IFS-141505.
- [30] I. Deli, Y. Toktas, and S. Broumi. Neutrosophic Parameterized Soft relations and Their Application, *Neutrosophic Sets and Systems*, Vol. 4, 25–34, 2014.
- [31] P. A. Ejegwa, J. A. Awolola, Intuitionistic Fuzzy Multiset (IFMS) In Binomial Distributions, *International Journal Of Scientific and Technology Research*, 3(4), 335- 337, 2014.
- [32] I. M. Hanafy, A. A. Salama and K. Mahfouz, Correlation of neutrosophic Data, *International Refereed Journal of Engineering and Science*, 1(2), 39–43, 2012.
- [33] I. M. Hanafy, A. A. Salama and K. Mahfouz, Correlation Coefficients of Neutrosophic Sets by Centroid Method, *International Journal of Probability and Statistics*, 2(1), 9–12, 2013.
- [34] A. Kharal, A Neutrosophic Multicriteria Decision Making Method, *New Mathematics and Natural Computation*, Creighton University, USA, 2013.
- [35] F. G. Lupiá ñ ez, On neutrosophic topology, *Kybernetes*, 37(6), 797–800, 2008.
- [36] P. Liu, Y.Wang ,Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean, *Neural Computing and Applications*, 2014.
- [37] P. K. Maji, Neutrosophic soft set, *Annals of Fuzzy Mathematics and Informatics*, 5(1), 157–168, 2013.
- [38] D.A. Molodtsov, Soft set theory-first results, *Computers and Mathematics with Applications*, 37, 19-31, 1999.
- [39] R. Muthuraj and S. Balamurugan, Multi-Fuzzy Group and its Level Subgroups, *Gen. Math. Notes*, 17(1), 74- 81, 2013.
- [40] D. Rabounski F. Smarandache L. Borissova, Neutrosophic Methods in General Relativity, *Hexis*, 10, 2005.
- [41] P. Rajarajeswari and N. Uma, On Distance and Similarity Measures of Intuitionistic Fuzzy Multi Set, *IOSR Journal of Mathematics*, 5(4), 19–23, 2013.
- [42] P. Rajarajeswari and N. Uma, A Study of Normalized Geometric and Normalized Hamming Distance Measures in Intuitionistic Fuzzy Multi Sets, *International Journal of Science and Research, Engineering and Technology*, 2(11), 76–80, 2013.
- [43] P. Rajarajeswari, N. Uma, Intuitionistic Fuzzy Multi Relations, *International Journal of Mathematical Archives*, 4(10), 244- 249, 2013.

- [44] P. Rajarajeswari and N. Uma, Zhang and Fu's Similarity Measure on Intuitionistic Fuzzy Multi Sets, *International Journal of Innovative Research in Science, Engineering and Technology*, 3(5), 12309–12317, 2014.
- [45] P. Rajarajeswari, N. Uma, Correlation Measure For Intuitionistic Fuzzy Multi Sets, *International Journal of Research in Engineering and Technology*, 3(1) 611- 617, 2014.
- [46] F. Smarandache, *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*, Rehoboth: American Research Press, 1998.
- [47] F. Smarandache, n-Valued Refined Neutrosophic Logic and Its Applications in Physics, *Progress in Physics*, 4, 143–146, 2013.
- [48] S. Sebastian and T. V. Ramakrishnan, Multi-fuzzy Subgroups, *Int. J. Contemp. Math. Sciences*, 6(8), 365–372, 2011.
- [49] S. Sebastian and T. V. Ramakrishnan, Multi-fuzzy extension of crisp functions using bridge functions, *Annals of Fuzzy Mathematics and Informatics*, 2(1), 1–8, 2011.
- [50] S. Sebastian and T. V. Ramakrishnan, Multi-Fuzzy Sets, *International Mathematical Forum*, 5(50), 2471–2476, 2010.
- [51] T. K. Shinoj and S. J. John, Intuitionistic fuzzy multi-sets and its application in medical diagnosis, *World Academy of Science, Engineering and Technology*, 6, 01–28, 2012.
- [52] A. Syropoulos, On generalized fuzzy multi-sets and their use in computation, *Iranian Journal Of Fuzzy Systems*, 9(2), 113–125, 2012.
- [53] H. Wang, F. Smarandache, Y. Q. Zhang and R. Sunderraman, Single valued neutrosophic sets, *Multispace and Multistructure* 4 410–413, 2010.
- [54] R. R. Yager, On the theory of bags (Multi sets), *Int. Joun. Of General System*, 13, 23–37, 1986.
- [55] J. Ye, Similarity measure between interval neutrosophic sets and their applications in multicriteria decision making, *journal of intelligent and fuzzy systems* 26,165–172, 2014.
- [56] J. Ye, single valued neutrosophic cross-entropy for multicriteria decision making problems, *Applied Mathematical Modelling*, 38, 1170–1175, 2014.
- [57] J. Ye, Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria Decision Making *International Journal of Fuzzy Systems*, Vol. 16, No. 2, 204–215, 2014.
- [58] J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, *International Journal of General Systems*, 42(4), 386–394, 2013.
- [59] S. Ye, J. Ye, Dice Similarity Measure between Single Valued Neutrosophic Multisets and Its Application in Medical Diagnosis, *Neutrosophic Sets and Systems*, 6, 48–53, 2014.
- [60] L. A. Zadeh, Fuzzy Sets, *Inform. and Control*, 8, 338- 353, 1965.

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