Complex Neutrosophic Similarity Measures in Medical Diagnosis

Kalyan Mondal¹, Mumtaz Ali², Surapati Pramanik³*, Florentin Smarandache⁴

¹Department of Mathematics, Jadavpur University, West Bengal, India  Email: kalyanmathematic@gmail.com
²Department of Mathematics, Quaid-i-Azam University, Islamabad, 44000, Pakistan. E-mail: bloomy_boy2006@yahoo.com
³Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, PO - Narayanpur, and District: North 24 Parganas, Pin Code: 743126, West Bengal, India. Email: sura_pati@yahoo.co.in
⁴Department of Mathematics and Science, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA. E-mail: fsmarandache@gmail.com
³*Corresponding author’s email: sura_pati@yahoo.co.in

Abstract

This paper presents some similarity measures between complex neutrosophic sets. A complex neutrosophic set is a generalization of neutrosophic set whose complex-valued truth membership function, complex-valued indeterminacy membership function, and complex valued falsity membership functions are the combinations of real-valued truth amplitude term in association with phase term, real-valued indeterminate amplitude term with phase term, and real-valued false amplitude term with phase term respectively. In the present study, we have proposed neutrosophic complex cosine, Dice and Jaccard similarity measures and investigated some of their properties. Finally, complex neutrosophic cosine, Dice and Jaccard similarity measures have been applied to a medical diagnosis problem with complex neutrosophic information.

Key Words: neutrosophic set; single valued neutrosophic set; complex neutrosophic sets; medical diagnosis; similarity measures

1. Introduction

It is avowed that uncertainty plays an important role in modeling real world problems. So it is necessary to bridge the gap between mathematical models and uncertainty and their explorative explanations. This gap can be found in problems of mathematics, operations research, biological and social sciences, modern technology and other applied sciences. In 1965, Zadeh [1] proposed the new concept of mathematics namely fuzzy sets (FS). In fuzzy set theory, the sum of membership and non-membership degrees of an element of a fuzzy set is equal to one. However, there exist some situations where the sum of membership and non-membership degrees are not equal to one. In order to handle such situations Atanassov [2] introduced intuitionistic fuzzy set (IFS). Each element of an intuitionistic fuzzy set is assigned by membership and non-membership degrees, where the sum of the two degrees is less than one.

The concept of intuitionistic fuzzy set has been widely studied and applied in many areas such as decision-making problems [3, 4, 5], selection problem [6, 7], educational problem [8], medical diagnosis [9, 10, 11] etc. Smarandache [12] introduced the degree of indeterminacy as independent component and defined the neutrosophic set to deal with uncertainty, indeterminacy and inconsistency. To use the concept of neutrosophic set in practical fields such as real scientific and engineering applications, Wang et al.[13] restricted the concept of neutrosophic set to single valued neutrosophic set since single value is an instance of set value. Similarity measures play an important role in the analysis and research of medical diagnosis [14], pattern recognition [15], decision making [16, 17], and clustering analysis [18] in uncertain, indeterminate and inconsistent environment. Various similarity measures of SVNSs have been proposed in the literature. Majumdar and Samanta [19] introduced the similarity

Ramot et al. [30] introduced a concept of complex fuzzy sets (CFS). It is an extension of fuzzy sets. Here, membership function \( z = r_i e^{iw(s)} \) where, \( i = (-1)^{0.5} \) which ranges in a unit circle. The membership function is defined for the complex fuzzy set as \( r_i e^{iw(s)}(x) \). Here, \( r_i(x) \) is the amplitude term and \( w_i(x) \) is the phase term. \( r_i(x) \) ranges in the interval \([0, 1]\) and \( w_i(x) \) is a periodic function. Ramot et al. [31] also proposed different complex fuzzy operations like union, intersection, complement etc. The amplitude term explains the idea of “fuzziness” and phase term implies declaration of complex fuzzy set. Chen et al. [32] proposed a neuro-fuzzy system architecture rule as a practical application of complex fuzzy logic.

Alkouri and Salleh [33] introduced complex intuitionistic fuzzy set (CIFS). CIFS is a generalization of complex fuzzy set. Complex fuzzy set is transformed into complex intuitionistic fuzzy set by adding complex-valued non-membership grade.

The complex intuitionistic fuzzy sets can deal the problems involving uncertainty and periodicity simultaneously. The concept of phase term is extended in complex intuitionistic fuzzy set which appears in several prominent concepts such as distance measure, Cartesian products, projections, relations, and so on. The complex fuzzy set has one additional phase. Complex intuitionistic fuzzy set has two additional phase terms. Recently Ali and Smarandache [34] proposed the concept of complex neutrosophic set. It seems to be very powerful. In this paper an attempt has been made to establish some similarity measures namely, cosine, Dice and Jaccard similarity measures in complex neutrosophic environment and their applications in medical diagnosis.

Rest of the paper is structured as follows: Section 2 presents neutrosophic and complex neutrosophic preliminaries. In Section 3 we introduce complex Cosine, Dice and Jaccard similarity measure for complex neutrosophic sets and establish some of their properties. Section 4 is devoted to present new method of medical diagnosis based on complex Dice and Jaccard similarity measures. Section 5 presents an application of complex Cosine, Dice and Jaccard similarity measures in medical diagnosis. Section 6 presents the concluding remarks and future scope of research.

2. Mathematical Preliminaries

Definition 2.1

Let \( G \) be a space of points with generic element in \( E \) denoted by \( y \). Then a neutrosophic set \( P \) in \( G \) is characterized by a truth membership function \( T_p \), an indeterminacy membership function \( I_p \) and a falsity membership function \( F_p \). The functions \( T_p \) and \( F_p \) are real standard or non-standard subsets of \([0, 1]^+\) [that is \( T_p: G \rightarrow [0, 1]^+; I_p: G \rightarrow [0, 1]^+; F_p: G \rightarrow [0, 1]^+\)]. The sum of \( T_p(y), I_p(y), F_p(y) \) is given by \( 0 \leq \sup T_p(y) + \sup I_p(y) + \sup F_p(y) \leq 3 \)

Definition 2.2
The complement of a neutrosophic set $P$ is denoted by $P^c$ and is defined as follows: $\text{T}_{P^c}(y) = \{1^\prime\} - \text{T}_P(y)$; $\text{I}_{P^c}(y) = \{1^\prime\} - \text{I}_P(y)$; $\text{F}_{P^c}(y) = \{1^\prime\} - \text{F}_P(y)$.

**Definition 2.3**

A neutrosophic set $P$ is contained in the other neutrosophic set $Q$, $P \subseteq Q$ if and only if the following result holds.

\[
\begin{align*}
\inf \text{T}_P(y) & \leq \inf \text{T}_Q(y), \quad \sup \text{T}_P(y) \leq \sup \text{T}_Q(y) ; \\
\inf \text{I}_P(y) & \geq \inf \text{I}_Q(y), \quad \sup \text{I}_P(y) \geq \sup \text{I}_Q(y) ; \\
\inf \text{F}_P(y) & \geq \inf \text{F}_Q(y), \quad \sup \text{F}_P(y) \geq \sup \text{F}_Q(y), \quad \text{for all } y \in G.
\end{align*}
\]

**Definition 2.4 Single-valued neutrosophic set**

Let $G$ be a universal space of points with generic element of $G$ denoted by $y$. A single valued neutrosophic set $S$ is characterized by a truth membership function $\text{T}_S(y)$, a falsity membership function $\text{F}_S(y)$ and indeterminacy function $\text{I}_S(y)$ such that $\text{T}_S(y)$, $\text{F}_S(y)$, $\text{I}_S(y) \in [0, 1]$ for all $y \in G$.

When $G$ is continuous, a SNVS $S$ can be written as follows:

\[
S = \{\{\text{T}_S(y), \text{F}_S(y), \text{I}_S(y)\} / y, \forall y \in G
\]

and when $G$ is discrete, a SVNS $S$ can be written as follows:

\[
S = \sum \{\text{T}_S(y), \text{F}_S(y), \text{I}_S(y)\} / y, \forall y \in G
\]

It should be noted that for a SVNS $S$, $0 \leq \sup \text{T}_S(y) + \sup \text{F}_S(y) + \sup \text{I}_S(y) \leq 3$, $\forall y \in G$.

**Definition 2.5**

The complement of a single valued neutrosophic set $S$ is denoted by $S^c$ and is defined as follows:

\[
\text{T}^c_S(y) = \text{F}_S(y), \quad \text{I}^c_S(y) = 1 - \text{I}_S(y), \quad \text{F}^c_S(y) = \text{T}_S(y)
\]

**Definition 2.6**

A SVNS $S_1$ is contained in the other SVNS $S_2$, denoted as $S_1 \subseteq S_2$ if and only if $\text{T}_{S_1}(y) \leq \text{T}_{S_2}(y)$; $\text{I}_{S_1}(y) \geq \text{I}_{S_2}(y)$; $\text{F}_{S_1}(y) \geq \text{F}_{S_2}(y)$, $\forall y \in G$.

**Definition 2.7**

Two single valued neutrosophic sets $S_1$ and $S_2$ are equal, i.e. $S_1 = S_2$, if and only if, $S_1 \subseteq S_2$ and $S_1 \supseteq S_2$.

**Definition 2.8**

The union of two SVNSs $S_1$ and $S_2$ is a SVNS $S_3$, written as $S_3 = S_1 \cup S_2$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to $S_1$ and $S_2$ by the following equations.
\[ T_{s_3}(y) = \max(T_{s_1}(y), T_{s_2}(y)) \; ; \]
\[ I_{s_3}(y) = \max(I_{s_1}(y), I_{s_2}(y)) \; ; \]
\[ F_{s_3}(y) = \min(F_{s_1}(y), F_{s_2}(y)) \; \text{for all } y \in G. \]

**Definition 2.9**

The intersection of two SVNSs \( S_1 \) and \( M \) is a SVNS \( S_2 \), written as \( S_1 \cap S_2 \). The truth membership, indeterminacy membership and falsity membership functions can be defined as follows:

\[ T_{s_3}(y) = \min(T_{s_1}(y), T_{s_2}(y)) \; ; \]
\[ I_{s_3}(y) = \max(I_{s_1}(y), I_{s_2}(y)) \; ; \]
\[ F_{s_3}(y) = \max(F_{s_1}(y), F_{s_2}(y)), \forall y \in G. \]

**Definition 2.10 Distance between two neutrosophic sets**

The general SVNS can be presented in the following form as follows:

\[ S = \{(y/(T_S(y), I_S(y), F_S(y)) : y \in G\} \]

Finite SVNSs can be represented as follows:

\[ S = \{(y_1/(T_{s_1}(y_1), I_{s_1}(y_1), F_{s_1}(y_1)), \ldots, (y_m/(T_{s_m}(y_m), I_{s_m}(y_m), F_{s_m}(y_m))), \forall y \in G \}

**Definition 2.11**

\[ S_1 = \{(y_1/(T_{s_1}(y_1), I_{s_1}(y_1), F_{s_1}(y_1)), \ldots, (y_n/(T_{s_n}(y_n), I_{s_n}(y_n), F_{s_n}(y_n))) \}
\]

\[ S_2 = \{(y_1/(T_{s_2}(y_1), I_{s_2}(y_1), F_{s_2}(y_1)), \ldots, (y_n/(T_{s_n}(y_n), I_{s_n}(y_n), F_{s_n}(y_n))) \}

be two single-valued neutrosophic sets, then the Hamming distance between two SNVS \( S_1 \) and \( S_2 \) can be defined as follows:

\[ d(S_1, S_2) = \frac{1}{n} \sum_{i=1}^{n} \left| T_{s_1}(y) - T_{s_2}(y) \right| + \left| I_{s_1}(y) - I_{s_2}(y) \right| + \left| F_{s_1}(y) - F_{s_2}(y) \right| \]

and normalized Hamming distance between two SNVS \( S_1 \) and \( S_2 \) can be defined as follows:

\[ \frac{1}{3n} \sum_{i=1}^{n} \left| T_{s_1}(y) - T_{s_2}(y) \right| + \left| I_{s_1}(y) - I_{s_2}(y) \right| + \left| F_{s_1}(y) - F_{s_2}(y) \right| \]

with the following properties

1. \( 0 \leq d(S_1, S_2) \leq 3n \)
2. \( 0 \leq N d(S_1, S_2) \leq 1 \)

**2.1 Complex fuzzy set [30]**

A complex fuzzy set \( S \), defined on a universe of discourse \( X \), is characterized by a membership function \( \eta_k(x) \) that assigns any element \( x \in X \) a complex-valued grade of membership in \( S \). The values \( \eta_k(x) \) all lie within the unit
circle in the complex plane, and thus all of the form \( p_\delta(x) e^{i\mu(x)} \) where, \( p_\delta(x) \) and \( \mu_\delta(x) \) are both real valued and \( p_\delta(x) \in [0, 1] \). Here, \( p_\delta(x) \) is termed as amplitude term and \( e^{i\mu(x)} \) is termed as phase term. The complex fuzzy set may be represented in the set form as \( \{x, \eta_\delta(x)\}_x \in X \).

The complex fuzzy set is denoted as CFS. We now present some set operations of complex fuzzy sets.

**Definition 2.1.1**

Let \( S \) be a complex fuzzy set on \( X \) and \( \eta_\delta(x) = p_\delta(x) e^{i\mu_\delta(x)} \) its complex-valued membership function. The complement of \( S \), denoted as \( c(S) \) and is specified by a function

\[
\eta_{c(S)}(x) = p_{c(S)}(x) e^{i\mu_{c(S)}(x)} = (1 - p_\delta(x)) e^{i(2\pi - \mu_\delta(x))}
\]

**Definition 2.1.2**

Let \( A \) and \( B \) be two complex fuzzy sets on \( X \), and \( \eta_A(x) = p_A(x) e^{i\mu_A(x)} \) and \( \eta_B(x) = p_B(x) e^{i\mu_B(x)} \) be their membership functions respectively. The union of \( A \) and \( B \) is denoted as \( A \cup B \) which is characterized by a function

\[
\eta_{A \cup B}(x) = p_{A \cup B}(x) e^{i\mu_{A \cup B}(x)} = \max(p_A(x), p_B(x)) e^{i\max(\mu_A(x), \mu_B(x))}
\]

**Definition 2.1.3**

Let \( A \) and \( B \) be two complex fuzzy sets on \( X \), and \( \eta_A(x) = p_A(x) e^{i\mu_A(x)} \) and \( \eta_B(x) = p_B(x) e^{i\mu_B(x)} \) be their membership functions respectively. The intersection of \( A \) and \( B \) is denoted as \( A \cap B \) which is characterized by a function

\[
\eta_{A \cap B}(x) = p_{A \cap B}(x) e^{i\mu_{A \cap B}(x)} = \min(p_A(x), p_B(x)) e^{i\min(\mu_A(x), \mu_B(x))}
\]

**Definition 2.1.4**

Let \( A \) and \( B \) be two complex fuzzy sets on \( X \), and \( \eta_A(x) = p_A(x) e^{i\mu_A(x)} \) and \( \eta_B(x) = p_B(x) e^{i\mu_B(x)} \) be their membership functions respectively. The complex fuzzy product of \( A \) and \( B \) is denoted as \( A \odot B \) which is characterized by a function

\[
\eta_{A \odot B}(x) = p_{A \odot B}(x) e^{i\mu_{A \odot B}(x)} = (p_A(x), p_B(x)) e^{i\mu_A(x) + \mu_B(x)}
\]

**Definition 2.1.5** \( \delta \) equality of Complex Fuzzy sets [30]

Let \( A \) and \( B \) be two complex fuzzy sets on \( X \), and \( \eta_A(x) = p_A(x) e^{i\mu_A(x)} \) and \( \eta_B(x) = p_B(x) e^{i\mu_B(x)} \) be their membership functions respectively. Now, \( A \) and \( B \) are \( \delta \) equal if and only if

\[
d(A, B) = 1 - \delta \]

where \( 0 \leq \delta \leq 1 \).

**2.2 Complex intuitionistic fuzzy set [33]**

A complex intuitionistic fuzzy set \( S \) defined on a universe of discourse \( X \), is characterized by a membership function \( \eta_\delta(x) \) and a non-membership function \( \psi_\delta(x) \) that assigns any element \( x \in X \) a complex-valued grade of membership in \( S \). The values \( \eta_\delta(x) \) and \( \psi_\delta(x) \) lie within the unit circle in the complex plane, and thus all of the form \( p_\delta(x) e^{i\mu_\delta(x)} \) and \( q_\delta(x) e^{i\phi_\delta(x)} \) where, \( p_\delta(x), q_\delta(x), \mu_\delta(x), \phi_\delta(x) \) are both real valued and \( p_\delta(x), q_\delta(x) \in [0, 1] \). Here, \( p_\delta(x) \) and \( q_\delta(x) \) are expressed as amplitude terms and \( e^{i\mu_\delta(x)} \) and \( e^{i\phi_\delta(x)} \) expressed as phase terms. The complex intuitionistic fuzzy set is represented in the set form as \( S = \{x, \eta_\delta(x), \psi_\delta(x)\}_x \in X \).

The complex intuitionistic fuzzy set is denoted as CIFS. Some set operations of complex intuitionistic fuzzy sets are given below.

**Definition 2.2.1** Complement of Complex Intuitionistic Fuzzy set

Let \( S \) be a complex intuitionistic fuzzy set on \( X \) and \( \eta_\delta(x) = p_\delta(x) e^{i\mu_\delta(x)} \) and \( \eta_\delta(x) = q_\delta(x) e^{i\phi_\delta(x)} \) be it complex-valued membership function and non-membership function respectively. The complement of \( S \) denoted as \( c(S) \) and is expressed by \( \eta_{c(S)}(x) \) as follows.

\[
\eta_{c(S)}(x) = p_{c(S)}(x) e^{i\mu_{c(S)}(x)} = (p_{c(S)}(x)) e^{i(2\pi - \mu_\delta(x))} \quad \text{and} \quad \psi_{c(S)}(x) = q_{c(S)}(x) e^{i\phi_{c(S)}(x)} = (q_{c(S)}(x)) e^{i(2\pi - \phi_\delta(x))}
\]

**Definition 2.2.2** Union of Complex intuitionistic Fuzzy sets
Let A and B be two complex intuitionistic fuzzy sets on X, and $\eta(x) = e^{i\mu(x)}$ and $\eta(x) = e^{i\nu(x)}$ and $\eta(x) = e^{i\mu(x)}$ and $\eta(x) = e^{i\nu(x)}$ be their membership functions and non membership function respectively. The union of A and B is denoted as $A \cup B$ which is expressed by $\eta_{A \cup B}(x)$ as follows.

$$\eta_{A \cup B}(x) = e^{i\mu_{A \cup B}(x)} = \max(p_A(x), p_B(x)) e^{i\max(\mu_A(x), \mu_B(x))}$$

$$\eta_{A \cup B}(x) = e^{i\mu_{A \cup B}(x)} = \min(p_A(x), p_B(x)) e^{i\min(\mu_A(x), \mu_B(x))}.$$  

Definition 2.2.3 Intersection of Complex intuitionistic Fuzzy sets
Let A and B be two complex intuitionistic fuzzy sets on X, and $\eta(x) = e^{i\mu(x)}$ and $\eta(x) = e^{i\nu(x)}$ and $\eta(x) = e^{i\mu(x)}$ and $\eta(x) = e^{i\nu(x)}$ be their membership functions and non membership function respectively. The intersection of A and B is denoted as $A \cap B$ and can be defined as follows:

$$\eta_{A \cap B}(x) = e^{i\mu_{A \cap B}(x)} = \min(p_A(x), p_B(x)) e^{i\min(\mu_A(x), \mu_B(x))}$$

$$\eta_{A \cap B}(x) = e^{i\mu_{A \cap B}(x)} = \max(p_A(x), p_B(x)) e^{i\max(\mu_A(x), \mu_B(x))}.$$  

2.3 Complex Neutrosophic Set [34]
A complex neutrosophic set $S$ on a universe of discourse $X$, which is characterized by a truth membership function $T_S(x)$, an indeterminacy membership function $I_S(x)$, and a falsity membership function $F_S(x)$ that identifies a complex-valued grade of $T_S(x)$, $I_S(x)$, $F_S(x)$ in $S$ for all $x \in X$. The values $T_S(x)$, $I_S(x)$, $F_S(x)$. Their sum is within the unit circle in the complex plane. So it can be expressed as follows.

$$T_S(x) = p_S(x) e^{i\delta_S(x)}, I_S(x) = q_S(x) e^{i\phi_S(x)}, F_S(x) = r_S(x) e^{i\omega_S(x)}$$

Where, $p_S(x)$, $q_S(x)$, $r_S(x)$ and $\delta_S(x)$, $\phi_S(x)$, $\omega_S(x)$ are respectively real valued and $p_S(x)$, $q_S(x)$, $r_S(x) \in [0,1]$ such that $0 \leq p_S(x) + q_S(x) + r_S(x) \leq 3$.

Definition 2.3.1
A complex neutrosophic set $CN_1$ is contained in the other complex neutrosophic set $CN_2$ denoted as $CN_1 \subseteq CN_2$ if and only if $p_{CN_1}(x) \leq p_{CN_2}(x)$, $q_{CN_1}(x) \leq q_{CN_2}(x)$, $r_{CN_1}(x) \leq r_{CN_2}(x)$, and $\mu_{CN_1}(x) \leq \mu_{CN_2}(x)$, $\delta_{CN_1}(x) \leq \delta_{CN_2}(x)$, $\omega_{CN_1}(x) \leq \omega_{CN_2}(x)$.

Definition 2.3.2
Two complex neutrosophic sets $CN_1$ and $CN_2$ are equal i.e. $CN_1 = CN_2$ if and only if $p_{CN_1}(x) = p_{CN_2}(x)$, $q_{CN_1}(x) = q_{CN_2}(x)$, $r_{CN_1}(x) = r_{CN_2}(x)$, $\mu_{CN_1}(x) = \mu_{CN_2}(x)$, $\delta_{CN_1}(x) = \delta_{CN_2}(x)$, and $\omega_{CN_1}(x) = \omega_{CN_2}(x)$.

Definition 2.3.3 Complex Neutrosophic number (CNN)
A complex neutrosophic number (CNN) in a complex neutrosophic set $S$, can be defined as three complex components. It can be expressed as $(T_S(x), I_S(x), F_S(x))$. Here, $T_S(x) = (p_x e^{i\mu(x)}, I_S(x) = q_S(x) e^{i\phi_S(x)}, F_S(x) = r_S(x) e^{i\omega_S(x)}$ and $p_S(x)$, $q_S(x)$, $r_S(x)$, $\mu_S(x)$, $\phi_S(x)$, $\omega_S(x)$ are respectively real valued and $p_S(x)$, $q_S(x)$, $r_S(x) \in [0,1]$ such that $0 \leq p_S(x) + q_S(x) + r_S(x) \leq 3$.

3. Complex neutrosophic similarity measures
3.1 Complex neutrosophic cosine similarity measure (CNCSM)
The complex cosine similarity measure is defined as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two complex neutrosophic sets. Literature review suggests that cosine similarity measure with complex neutrosophic sets has not been defined. Therefore, a new cosine similarity measure between complex neutrosophic sets is proposed in 3-D vector space.
Definition 3.1.1 Assume that there are two complex neutrosophic sets namely, 
\[ CN_1 = \left\{ p_{S_1}(x)e^{i\mu_{S_1}(x)}, q_{S_1}(x)e^{i\delta_{S_1}(x)}, r_{S_1}(x)e^{i\omega_{S_1}(x)} \right\} \quad \text{and} \quad CN_2 = \left\{ p_{S_2}(x)e^{i\mu_{S_2}(x)}, q_{S_2}(x)e^{i\delta_{S_2}(x)}, r_{S_2}(x)e^{i\omega_{S_2}(x)} \right\} \] 
in \( S \) for all \( x \) belongs to \( X \). A complex cosine similarity measure between complex neutrosophic sets \( CN_1 \) and \( CN_2 \) is defined as follows:

\[ C_{\text{CNS}} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{a_1 b_1 a_2 b_2 + c_1 d_1 c_2 d_2 + e_1 f_1 e_2 f_2}{(a_1 b_1 + c_1 d_1 + e_1 f_1)(a_2 b_2 + c_2 d_2 + e_2 f_2)} \right)^{0.5} \]

where, 
\[ a_1 = \text{Re} \left[ p_{S_1}(x)e^{i\mu_{S_1}(x)} \right], \quad b_1 = \text{Im} \left[ p_{S_1}(x)e^{i\mu_{S_1}(x)} \right], \quad a_2 = \text{Re} \left[ p_{S_2}(x)e^{i\mu_{S_2}(x)} \right], \quad b_2 = \text{Im} \left[ p_{S_2}(x)e^{i\mu_{S_2}(x)} \right], \]
\[ c_1 = \text{Re} \left[ q_{S_1}(x)e^{i\delta_{S_1}(x)} \right], \quad d_1 = \text{Im} \left[ q_{S_1}(x)e^{i\delta_{S_1}(x)} \right], \quad c_2 = \text{Re} \left[ q_{S_2}(x)e^{i\delta_{S_2}(x)} \right], \quad d_2 = \text{Im} \left[ q_{S_2}(x)e^{i\delta_{S_2}(x)} \right], \]
\[ e_1 = \text{Re} \left[ r_{S_1}(x)e^{i\omega_{S_1}(x)} \right], \quad f_1 = \text{Im} \left[ r_{S_1}(x)e^{i\omega_{S_1}(x)} \right], \quad e_2 = \text{Re} \left[ r_{S_2}(x)e^{i\omega_{S_2}(x)} \right], \quad f_2 = \text{Im} \left[ r_{S_2}(x)e^{i\omega_{S_2}(x)} \right]. \]

Where, “Re” indicates real part and “Im” indicates imaginary part of corresponding complex number.

Let \( CN_1 \) and \( CN_2 \) be complex neutrosophic sets then,

I. \( 0 \leq C_{\text{CNS}}(CN_1, CN_2) \leq 1 \)
II. \( C_{\text{CNS}}(CN_1, CN_2) = C_{\text{CNS}}(CN_2, CN_1) \)
III. \( C_{\text{CNS}}(CN_1, CN_2) = 1 \) if and only if \( CN_1 = CN_2 \)
IV. If \( CN \) is a CNS in \( S \) and \( CN_1 \subseteq CN_2 \subseteq CN \) then, \( C_{\text{CNS}}(CN_1, CN) \leq C_{\text{CNS}}(CN_1, CN_2) \), and \( C_{\text{CNS}}(CN_1, CN) \leq C_{\text{CNS}}(CN_2, CN) \).

Proof:
I. It is obvious because all positive values of cosine function are within \([0, 1]\).
II. It is obvious that the proposition is true.
III. When \( CN_1 = CN_2 \), then obviously \( C_{\text{CNS}}(CN_1, CN_2) = 1 \). On the other hand if \( C_{\text{CNS}}(CN_1, CN_2) = 1 \) then, \( a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2, f_1 = f_2 \).

This implies that \( CN_1 = CN_2 \).
IV. Let, \( CN = \left\{ p_S(x)e^{i\mu_S(x)}, q_S(x)e^{i\delta_S(x)}, r_S(x)e^{i\omega_S(x)} \right\} \) and also assume that \( l_1 = \text{Re} \left[ p_S(x)e^{i\mu_S(x)} \right], l_2 = \text{Im} \left[ p_S(x)e^{i\mu_S(x)} \right], m_1 = \text{Re} \left[ q_S(x)e^{i\delta_S(x)} \right], m_2 = \text{Im} \left[ q_S(x)e^{i\delta_S(x)} \right], n_1 = \text{Re} \left[ r_S(x)e^{i\omega_S(x)} \right], n_2 = \text{Im} \left[ r_S(x)e^{i\omega_S(x)} \right] \)

If \( CN_1 \subseteq CN_2 \subseteq CN \) then we can write \( a_1 b_1 \leq a_2 b_2 \leq l_1 l_2, c_1 d_1 \geq c_2 d_2 \geq m_1 m_2, e_1 f_1 \geq e_2 f_2 \geq n_1 n_2 \).

The cosine function is decreasing function within the interval \([0, \pi/2] \). Hence we can write \( C_{\text{CNS}}(CN_1, CN) \leq C_{\text{CNS}}(CN_2, CN) \), and \( C_{\text{CNS}}(CN_1, CN) \leq C_{\text{CNS}}(CN_2, CN) \).

3.2 Weighted Complex neutrosophic Cosine similarity measure (WCNCSM)

Definition 3.2.1 Assume that there are two complex neutrosophic sets namely, 
\[ CN_1 = \left\{ p_{S_1}(x)e^{i\mu_{S_1}(x)}, q_{S_1}(x)e^{i\delta_{S_1}(x)}, r_{S_1}(x)e^{i\omega_{S_1}(x)} \right\} \quad \text{and} \quad CN_2 = \left\{ p_{S_2}(x)e^{i\mu_{S_2}(x)}, q_{S_2}(x)e^{i\delta_{S_2}(x)}, r_{S_2}(x)e^{i\omega_{S_2}(x)} \right\} \] 
in \( S \) for all \( x \) belongs to \( X \). A weighted complex cosine similarity measure between complex neutrosophic sets \( CN_1 \) and \( CN_2 \) can be defined as follows:

\[ C_{\text{WCNS}} = \sum_{i=1}^{n} \left( \frac{a_1 b_1 a_2 b_2 + c_1 d_1 c_2 d_2 + e_1 f_1 e_2 f_2}{(a_1 b_1 + c_1 d_1 + e_1 f_1)(a_2 b_2 + c_2 d_2 + e_2 f_2)} \right)^{0.5} \]

(7)
Where, \( \sum_{i=1}^{n} w_i = 1 \)

Let \( CN_1 \) and \( CN_2 \) be complex neutrosophic sets then,

I. \( 0 \leq C_{WCNS}(CN_1, CN_2) \leq 1 \)

II. \( C_{WCNS}(CN_1, CN_2) = C_{WCNS}(CN_2, CN_1) \)

III. \( C_{WCNS}(CN_1, CN_2) = 1 \) if and only if \( CN_1 = CN_2 \)

IV. If \( CN \) is a CNS in \( S \) and \( CN_1 \subset CN_2 \subset CN \) then, \( C_{WCNS}(CN_1, CN) \leq C_{WCNS}(CN_1, CN_2) \), and \( C_{WCNS}(CN_1, CN) \leq C_{WCNS}(CN_2, CN) \)

Proofs:

I. Since \( \sum_{i=1}^{n} w_i = 1 \) and all positive values of cosine function are within 0 and 1, it can be written as \( 0 \leq C_{WCNS}(CN_1, CN_2) \leq 1 \).

II. It is obvious that the proposition is true.

III. When \( CN_1 = CN_2 \), then \( C_{WCNS}(CN_1, CN_2) = 1 \). On the other hand if \( C_{WCNS}(CN_1, CN_2) = 1 \) then, \( a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2, f_1 = f_2 \).

This implies that \( CN_1 = CN_2 \).

IV. Let, \( CN = \left\{ p_S(x)e^{i \theta_1(x)}, q_S(x)e^{i \theta_2(x)}, r_S(x)e^{i \phi(x)} \right\} \) and also assume that \( l_1 = \text{Re} \left[ p_S(x)e^{i \theta_1(x)} \right], l_2 = \text{Im} \left[ p_S(x)e^{i \theta_1(x)} \right], m_1 = \text{Re} \left[ q_S(x)e^{i \theta_2(x)} \right], m_2 = \text{Im} \left[ q_S(x)e^{i \theta_2(x)} \right], n_1 = \text{Re} \left[ r_S(x)e^{i \phi(x)} \right], n_2 = \text{Im} \left[ r_S(x)e^{i \phi(x)} \right] \)

If \( CN_1 \subset CN_2 \subset CN \) then we can write \( a_1 b_1 \leq a_2 b_2 \leq l_1 l_2, c_1 d_1 \geq c_2 d_2 \geq m_1 m_2, e_1 f_1 \geq e_2 f_2 \geq n_1 n_2 \).

The cosine function is decreasing function within the interval \([0, \pi/2]\). Hence we can write \( C_{WCNS}(CN_1, CN) \leq C_{WCNS}(CN, CN_2) \), and \( C_{WCNS}(CN_1, CN) \leq C_{WCNS}(CN_2, CN) \).

3.3 Complex neutrosophic Dice similarity measure (CNDMS)

Definition 3.3.1

Assume that there are two complex neutrosophic sets namely, \( CN_1 = \left\{ p_{S_1}(x)e^{i \theta_1(x)}, q_{S_1}(x)e^{i \theta_2(x)}, r_{S_1}(x)e^{i \phi_1(x)} \right\} \) and \( CN_2 = \left\{ p_{S_2}(x)e^{i \theta_2(x)}, q_{S_2}(x)e^{i \theta_1(x)}, r_{S_2}(x)e^{i \phi_1(x)} \right\} \) in \( S \) for all \( x \) belongs to \( X \). A complex Dice similarity measure between complex neutrosophic sets \( CN_1 \) and \( CN_2 \) can be defined as follows:

\[
D_{CN} = \sum_{i=1}^{n} \frac{2(a_1 b_1 + a_2 b_2 + c_1 d_1 + c_2 d_2 + e_1 f_1 + e_2 f_2)}{(a_1 b_1 + a_2 b_2 + c_1 d_1 + c_2 d_2 + e_1 f_1 + e_2 f_2) + (a_2 b_2 + c_1 d_1 + c_2 d_2 + e_1 f_1 + e_2 f_2)}
\]

(8)

Let \( CN_1 \) and \( CN_2 \) be complex neutrosophic sets then,

III. \( 0 \leq D_{CN}(CN_1, CN_2) \leq 1 \)

III. \( D_{CN}(CN_1, CN_2) = D_{CN}(CN_2, CN_1) \)
III. $\text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}_2) = 1$, iff $\text{CN}_1 = \text{CN}_2$

IV. If $\text{CN}$ is a CNS in $S$ and $\text{CN}_1 \subset \text{CN}_2 \subset \text{CN}$ then, $\text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}) \leq \text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}_2)$, and $\text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}) \leq \text{D}_{\text{CNS}}(\text{CN}_2, \text{CN})$.

Proofs:

I. Since, $2(a_1 b_1 a_2 b_2)^{0.5} + (c_1 d_1 c_2 d_2)^{0.5} + (e_1 f_1 e_2 f_2)^{0.5} \leq (a_1 b_1 + c_1 d_1 + e_1 f_1) + (a_2 b_2 + c_2 d_2 + e_2 f_2)$ it can be written as $0 \leq \text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}_2) \leq 1$.

II. It is obvious that the proposition is true.

III. When $\text{CN}_1 = \text{CN}_2$, then obviously $\text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}_2) = 1$. On the other hand if $\text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}_2) = 1$ then, $a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2, f_1 = f_2$.

This implies that $\text{CN}_1 = \text{CN}_2$.

IV. Let, $\text{CN} = \left\{ p_S(x) e^{i\alpha(x)}, q_S(x) e^{i\beta(x)}, r_S(x) e^{i\gamma(x)} \right\}$ and also assume that $l_1 = \text{Re} \left[ p_S(x) e^{i\alpha(x)} \right]$, $l_2 = \text{Im} \left[ p_S(x) e^{i\alpha(x)} \right]$, $m_1 = \text{Re} \left[ q_S(x) e^{i\beta(x)} \right]$, $m_2 = \text{Im} \left[ q_S(x) e^{i\beta(x)} \right]$, $n_1 = \text{Re} \left[ r_S(x) e^{i\gamma(x)} \right]$, $n_2 = \text{Im} \left[ r_S(x) e^{i\gamma(x)} \right]$.

If $\text{CN}_1 \subset \text{CN}_2 \subset \text{CN}$ then we can write $a_1 b_1 \leq a_2 b_2 \leq l_1 l_2$, $c_1 d_1 \geq c_2 d_2 \geq m_1 m_2$, $e_1 f_1 \geq e_2 f_2 \geq n_1 n_2$.

Hence we can write $\text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}) \leq \text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}_2)$, and $\text{D}_{\text{CNS}}(\text{CN}_1, \text{CN}) \leq \text{D}_{\text{CNS}}(\text{CN}_2, \text{CN})$.

3.4 Weighted Complex neutrosophic Dice similarity measure (WCNDSM)

Definition 3.4.1

Assume that there are two complex neutrosophic sets namely, $\text{CN}_1 = \left\{ p_{S_1}(x) e^{i\alpha_1(x)}, q_{S_1}(x) e^{i\beta_1(x)}, r_{S_1}(x) e^{i\gamma_1(x)} \right\}$ and $\text{CN}_2 = \left\{ p_{S_2}(x) e^{i\alpha_2(x)}, q_{S_2}(x) e^{i\beta_2(x)}, r_{S_2}(x) e^{i\gamma_2(x)} \right\}$ in $S$ for all $x$ belongs to $X$. A weighted complex Dice similarity measure between complex neutrosophic sets $\text{CN}_1$ and $\text{CN}_2$ can be defined as follows:

$$D_{\text{WCNS}} = \sum_{i=1}^{n} w_i \left( 2 \left( (a_1 b_1 a_2 b_2)^{0.5} + (c_1 d_1 c_2 d_2)^{0.5} + (e_1 f_1 e_2 f_2)^{0.5} \right) \right) \left( (a_1 b_1 + c_1 d_1 + e_1 f_1) + (a_2 b_2 + c_2 d_2 + e_2 f_2) \right)^{0.5} \right)$$

(9)

Where, $\sum_{i=1}^{n} w_i = 1$

Let $\text{CN}_1$ and $\text{CN}_2$ be complex neutrosophic sets then,

I. $0 \leq D_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) \leq 1$

II. $D_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = D_{\text{WCNS}}(\text{CN}_2, \text{CN}_1)$

III. $D_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = 1$, iff $\text{CN}_1 = \text{CN}_2$

IV. If $\text{CN}$ is a CNS in $S$ and $\text{CN}_1 \subset \text{CN}_2 \subset \text{CN}$ then, $D_{\text{WCNS}}(\text{CN}_1, \text{CN}) \leq D_{\text{WCNS}}(\text{CN}_1, \text{CN}_2)$, and $D_{\text{WCNS}}(\text{CN}_1, \text{CN}) \leq D_{\text{WCNS}}(\text{CN}_2, \text{CN})$

Proofs:

I. Since, $\sum_{i=1}^{n} w_i = 1$ and $2 \left( (a_1 b_1 a_2 b_2)^{0.5} + (c_1 d_1 c_2 d_2)^{0.5} + (e_1 f_1 e_2 f_2)^{0.5} \right) \leq (a_1 b_1 + c_1 d_1 + e_1 f_1) + (a_2 b_2 + c_2 d_2 + e_2 f_2)$ it can be written as $0 \leq D_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) \leq 1$.

II. It is obvious that the proposition is true.

III. When $\text{CN}_1 = \text{CN}_2$, then obviously $D_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = 1$. On the other hand if $D_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = 1$ then, $a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2, f_1 = f_2$. 
This implies that \( CN_1 = CN_2 \).

IV. Let, \( CN = \left\{ p_S(x)e^{i\theta_1(x)}, q_S(x)e^{i\theta_2(x)}, r_S(x)e^{i\omega(x)} \right\} \) and also assume that \( l_1 = \text{Re} \left[ p_S(x)e^{i\theta_1(x)} \right], l_2 = \text{Im} \left[ p_S(x)e^{i\theta_2(x)} \right], m_1 = \text{Re} \left[ q_S(x)e^{i\theta_2(x)} \right], m_2 = \text{Im} \left[ q_S(x)e^{i\theta_2(x)} \right], n_1 = \text{Re} \left[ r_S(x)e^{i\omega(x)} \right], n_2 = \text{Im} \left[ r_S(x)e^{i\omega(x)} \right] \)

If \( CN_1 \subset CN_2 \subset CN \) then we can write \( a_1b_1 \leq a_2b_2 \leq l_1l_2, c_1d_1 \geq c_2d_2 \geq m_1m_2, e_1f_1 \geq e_2f_2 \geq n_1n_2 \).

Hence we can write \( D_{WCSNS}(CN_1, CN) \leq D_{WCSNS}(CN_2, CN) \), and \( D_{WCSNS}(CN_1, CN) \leq D_{WCSNS}(CN_2, CN) \).

3.5 Complex neutrosophic Jaccard similarity measure (CNJSM)

Definition 3.5.1

Assume that there are two complex neutrosophic sets namely, \( CN_1 = \left\{ p_S(x)e^{i\theta_1(x)}, q_S(x)e^{i\theta_2(x)}, r_S(x)e^{i\omega(x)} \right\} \) and \( CN_2 = \left\{ p_S(x)e^{i\theta_1(x)}, q_S(x)e^{i\theta_2(x)}, r_S(x)e^{i\omega(x)} \right\} \) in \( S \) for all \( x \) belongs to \( X \). A complex Jaccard similarity measure between complex neutrosophic sets \( CN_1 \) and \( CN_2 \) can be defined as follows:

\[
J_{\text{CNJSM}} = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{(a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5}}{(a_1b_1+c_1d_1+e_1f_1) + (a_2b_2+c_2d_2+e_2f_2)} \right] \tag{10}
\]

\[
a_1 = \text{Re} \left[ p_S(x)e^{i\theta_1(x)} \right], b_1 = \text{Im} \left[ p_S(x)e^{i\theta_1(x)} \right], a_2 = \text{Re} \left[ p_S(x)e^{i\theta_2(x)} \right], b_2 = \text{Im} \left[ p_S(x)e^{i\theta_2(x)} \right],
\]

\[
c_1 = \text{Re} \left[ q_S(x)e^{i\theta_1(x)} \right], d_1 = \text{Im} \left[ q_S(x)e^{i\theta_1(x)} \right], c_2 = \text{Re} \left[ q_S(x)e^{i\theta_2(x)} \right], d_2 = \text{Im} \left[ q_S(x)e^{i\theta_2(x)} \right],
\]

\[
e_1 = \text{Re} \left[ r_S(x)e^{i\omega(x)} \right], f_1 = \text{Im} \left[ r_S(x)e^{i\omega(x)} \right], e_2 = \text{Re} \left[ r_S(x)e^{i\omega(x)} \right], f_2 = \text{Im} \left[ r_S(x)e^{i\omega(x)} \right].
\]

Where, “Re” indicates real part and “Im” indicates imaginary part of corresponding complex number.

Let \( CN_1 \) and \( CN_2 \) be complex neutrosophic sets then,

I. \( 0 \leq J_{\text{CNJSM}}(CN_1, CN_2) \leq 1 \)

II. \( J_{\text{CNJSM}}(CN_1, CN_2) = J_{\text{CNJSM}}(CN_2, CN_1) \)

III. \( J_{\text{CNJSM}}(CN_1, CN_2) = 1 \), iff \( CN_1 = CN_2 \)

IV. If \( CN \) is a CNS in \( S \) and \( CN_1 \subset CN_2 \subset CN \) then, \( J_{\text{CNJSM}}(CN_1, CN) \leq J_{\text{CNJSM}}(CN_1, CN_2) \), and \( J_{\text{CNJSM}}(CN_1, CN) \leq J_{\text{CNJSM}}(CN_2, CN) \).

Proofs:

I. Since, \( (a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5} \leq (a_1b_1+c_1d_1+e_1f_1) + (a_2b_2+c_2d_2+e_2f_2) \)

\[-(a_1b_1a_2b_2)^{0.5} + (c_1d_1c_2d_2)^{0.5} + (e_1f_1e_2f_2)^{0.5} \leq (a_1b_1+c_1d_1+e_1f_1) + (a_2b_2+c_2d_2+e_2f_2) \]

it can be written as \( 0 \leq J_{\text{CNJSM}}(CN_1, CN_2) \leq 1 \).

II. It is obvious that the proposition is true.

III. When \( CN_1 = CN_2 \), then obviously \( J_{\text{CNJSM}}(CN_1, CN_2) = 1 \). On the other hand if \( J_{\text{CNJSM}}(CN_1, CN_2) = 1 \) then, \( a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2, f_1 = f_2 \).

This implies that \( CN_1 = CN_2 \).

IV. Let, \( CN = \left\{ p_S(x)e^{i\theta_1(x)}, q_S(x)e^{i\theta_2(x)}, r_S(x)e^{i\omega(x)} \right\} \) and also assume that \( l_1 = \text{Re} \left[ p_S(x)e^{i\theta_1(x)} \right], l_2 = \text{Im} \left[ p_S(x)e^{i\theta_2(x)} \right], m_1 = \text{Re} \left[ q_S(x)e^{i\theta_2(x)} \right], m_2 = \text{Im} \left[ q_S(x)e^{i\theta_2(x)} \right], n_1 = \text{Re} \left[ r_S(x)e^{i\omega(x)} \right], n_2 = \text{Im} \left[ r_S(x)e^{i\omega(x)} \right].\)
If \( \text{CN}_1 \subseteq \text{CN}_2 \subseteq \text{CN} \) then we can write \( a_1 b_1 \leq a_2 b_2 \leq l_1 l_2, c_1 d_1 \geq c_2 d_2 \geq m_1 m_2, e_1 f_1 \geq e_2 f_2 \geq n_1 n_2 \).

Hence we can write \( J_{\text{CNS}}(\text{CN}_1, \text{CN}) \leq J_{\text{CNS}}(\text{CN}, \text{CN}_2) \), and \( J_{\text{CNS}}(\text{CN}_1, \text{CN}) \leq J_{\text{CNS}}(\text{CN}_2, \text{CN}) \).

### 3.6 Weighted Complex neutrosophic Jaccard similarity measure (WCNJSM)

**Definition 3.6.1**

Assume that there are two complex neutrosophic sets namely, \( \text{CN}_1 = \left\{ p_{\text{S}_1}(x)e^{i\theta_{\text{S}_1}(x)}, q_{\text{S}_1}(x)e^{i\theta_{\text{S}_1}(x)}, r_{\text{S}_1}(x)e^{i\theta_{\text{S}_1}(x)} \right\} \)
and \( \text{CN}_2 = \left\{ p_{\text{S}_2}(x)e^{i\theta_{\text{S}_2}(x)}, q_{\text{S}_2}(x)e^{i\theta_{\text{S}_2}(x)}, r_{\text{S}_2}(x)e^{i\theta_{\text{S}_2}(x)} \right\} \)
in \( S \) for all \( x \) belongs to \( X \). A weighted complex Jaccard similarity measure between complex neutrosophic sets \( \text{CN}_1 \) and \( \text{CN}_2 \) can defined as follows:

\[
J_{\text{WCNS}} = \frac{\sum_{i=1}^{n} w_i (a_1 b_1 a_2 b_2)^{0.5} + (c_1 d_1 c_2 d_2)^{0.5} + (e_1 f_1 e_2 f_2)^{0.5}}{(a_1 b_1 + c_1 d_1 + e_1 f_1) + (a_2 b_2 + c_2 d_2 + e_2 f_2) - (a_1 b_1 a_2 b_2)^{0.5} + (c_1 d_1 c_2 d_2)^{0.5} + (e_1 f_1 e_2 f_2)^{0.5}}
\]

(11)

Where, \( \sum_{i=1}^{n} w_i = 1 \)

Let \( \text{CN}_1 \) and \( \text{CN}_2 \) be complex neutrosophic sets then

I. \( 0 \leq J_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) \leq 1 \)

II. \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = J_{\text{WCNS}}(\text{CN}_2, \text{CN}_1) \)

III. \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = 1 \), iff \( \text{CN}_1 = \text{CN}_2 \)

IV. If \( \text{CN} \) is a CNS in \( S \) and \( \text{CN}_1 \subseteq \text{CN}_2 \subseteq \text{CN} \) then, \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}) \leq J_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) \), and \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}) \leq J_{\text{WCNS}}(\text{CN}_2, \text{CN}) \).

**Proofs:**

I. Since \( \sum_{i=1}^{n} w_i = 1 \) and \( (a_1 b_1 a_2 b_2)^{0.5} + (c_1 d_1 c_2 d_2)^{0.5} + (e_1 f_1 e_2 f_2)^{0.5} \leq (a_1 b_1 + c_1 d_1 + e_1 f_1) + (a_2 b_2 + c_2 d_2 + e_2 f_2) \)

- \( (a_1 b_1 a_2 b_2)^{0.5} + (c_1 d_1 c_2 d_2)^{0.5} + (e_1 f_1 e_2 f_2)^{0.5} \) it can be written as \( 0 \leq J_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) \leq 1 \).

II. It is obvious that the proposition is true.

III. When \( \text{CN}_1 = \text{CN}_2 \), then obviously \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = 1 \). On the other hand if \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}_2) = 1 \) then, \( a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, e_1 = e_2, f_1 = f_2 \).

This implies that \( \text{CN}_1 = \text{CN}_2 \).

IV. Let, \( \text{CN} = \left\{ p_{\text{S}}(x)e^{i\theta_{\text{S}}(x)}, q_{\text{S}}(x)e^{i\theta_{\text{S}}(x)}, r_{\text{S}}(x)e^{i\theta_{\text{S}}(x)} \right\} \) and also assume that \( l_1 = \text{Re} \left[ p_{\text{S}}(x)e^{i\theta_{\text{S}}(x)} \right] \), \( l_2 = \text{Im} \left[ p_{\text{S}}(x)e^{i\theta_{\text{S}}(x)} \right] \), \( m_1 = \text{Re} \left[ q_{\text{S}}(x)e^{i\theta_{\text{S}}(x)} \right] \), \( m_2 = \text{Im} \left[ q_{\text{S}}(x)e^{i\theta_{\text{S}}(x)} \right] \), \( n_1 = \text{Re} \left[ r_{\text{S}}(x)e^{i\theta_{\text{S}}(x)} \right] \), and \( n_2 = \text{Im} \left[ r_{\text{S}}(x)e^{i\theta_{\text{S}}(x)} \right] \).

If \( \text{CN}_1 \subseteq \text{CN}_2 \subseteq \text{CN} \) then we can write \( a_1 b_1 \leq a_2 b_2 \leq l_1 l_2, c_1 d_1 \geq c_2 d_2 \geq m_1 m_2, e_1 f_1 \geq e_2 f_2 \geq n_1 n_2 \).

Hence we can write \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}) \leq J_{\text{WCNS}}(\text{CN}, \text{CN}_2) \), and \( J_{\text{WCNS}}(\text{CN}_1, \text{CN}) \leq J_{\text{WCNS}}(\text{CN}_2, \text{CN}) \).

### 4. Methodology of medical diagnosis

Assume that, \( H_1, H_2, \ldots, H_m \) be a discrete set of patients, \( D_1, D_2, \ldots, D_b \) be the set of diseases, and \( A_1, A_2, \ldots, A_k \) be a set of symptoms. The decision-maker provides the ranking of diseases with respect to each symptom. Medical diagnosis procedure under complex neutrosophic environment based on Cosine, Dice and Jaccard similarity measure can be presented using the following steps.
Step 1: Determination the relation between patients and symptoms
The ranking presents the performances of patients $H_i$ ($i = 1, 2, ..., m$) against the symptoms $A_j$ ($j = 1, 2, ..., k$). The complex neutrosophic values associated with the patients and their symptoms for diagnosis problem can be presented in the decision matrix (see the table 1).

<table>
<thead>
<tr>
<th>Table 1: The relation between Patients and Symptoms (R-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>$H_1$</td>
</tr>
<tr>
<td>$H_2$</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>$H_m$</td>
</tr>
</tbody>
</table>

Here $\langle T_{ij}, I_{ij}, F_{ij} \rangle$ ($i = 1, 2, ..., m; j = 1, 2, ..., k$) is the complex neutrosophic number associated to the i-th patient and the j-th symptom.

Step 2: Determination of the relation between symptoms and diseases
The relation between symptoms $A_j$ ($j = 1, 2, ..., k$) and diseases $D_t$ ($t = 1, 2, ..., n$) in terms of complex neutrosophic numbers can be presented in the decision matrix (see the table 2).

<table>
<thead>
<tr>
<th>Table 2: The relation between symptoms and diseases (R-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>$A_1$</td>
</tr>
<tr>
<td>$A_2$</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>$A_k$</td>
</tr>
</tbody>
</table>

Here $\langle \xi_{ij}, \eta_{ij}, \zeta_{ij} \rangle$ ($i = 1, 2, ..., k; j = 1, 2, ..., n$) is the complex neutrosophic number associated to the i-th symptom and the j-th disease.

Step 3: Determination of the similarity measures
Determine the complex cosine, Dice and Jaccard similarity measures $C_{\text{CNS}}$, $D_{\text{CNS}}$ and $J_{\text{CNS}}$ between the table 1 and the table 2 using equation (6), equation (8) and equation (10).

Step 4: Ranking the alternatives
Ranking of diseases can be prepared based on the descending order of complex cosine, Dice and Jaccard similarity measures. The disease corresponding to highest similarity value reflects that patient $H_i$ ($i = 1, 2, ..., m$) suffering from that disease.

Step 5: End
5. Example on medical diagnosis

We consider a medical diagnosis problem for illustration of the proposed approach. Medical diagnosis comprises uncertainties and increased volume of information available to physicians from new medical technologies. So, all collected information may be in complex neutrosophic form. The three components of a complex neutrosophic set are the combinations of real-valued truth amplitude term in association with phase term, real-valued indeterminate amplitude term with phase term, and real-valued false amplitude term with phase term respectively. So, to deal more indeterminacy situations in medical diagnosis complex neutrosophic environment is more acceptable.

The process of classifying different set of symptoms under a single name of a disease is very difficult. In some practical situations, there exists possibility of each element within a periodic form of neutrosophic sets. So, medical diagnosis involves more indeterminacy. Complex neutrosophic sets handle this situation. Actually this approach is more flexible, dealing with more indeterminacy areas and easy to use. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will provide the proper medical diagnosis in complex neutrosophic environment.

The main feature of this proposed approach is that it considers complex truth membership, complex indeterminate and complex false membership of each element taking periodic form of neutrosophic sets.

Now, consider an example of a medical diagnosis. Assume that \( H = \{H_1, H_2, H_3\} \) be a set of patients, \( D = \{\text{Viral Fever, Malaria, Stomach problem, Chest problem}\} \) be a set of diseases and \( A = \{\text{Temperature, Headache, Stomach pain, cough, Chest pain}\} \) be a set of symptoms. Our investigation is to examine the patient and to determine the disease of the patient in complex neutrosophic environment.

**Step 1: Determination the relation between patients and symptoms**

In the diagnosis process the relation between Patients and Symptoms in complex neutrosophic form has been presented in the decision matrix as follows (see table 3).

**Table 3: Relation between Patients and Symptoms in complex neutrosophic form (R-1)**

<table>
<thead>
<tr>
<th>R-1</th>
<th>Temperature</th>
<th>Headache</th>
<th>Stomach pain</th>
<th>cough</th>
<th>Chest pain</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₁</td>
<td>(0.6 e^{1.0i}, 0.4 e^{1.2i}, 0.2 e^{0.8i})</td>
<td>(0.4 e^{1.2i}, 0.4 e^{1.4i}, 0.3 e^{0.7i})</td>
<td>(0.3 e^{1.0i}, 0.4 e^{1.0i}, 0.4 e^{0.6i})</td>
<td>(0.6 e^{1.0i}, 0.5 e^{1.2i}, 0.3 e^{0.8i})</td>
<td>(0.4 e^{1.0i}, 0.3 e^{1.0i}, 0.2 e^{0.5i})</td>
</tr>
<tr>
<td>H₂</td>
<td>(0.7 e^{1.3i}, 0.4 e^{1.2i}, 0.5 e^{0.9i})</td>
<td>(0.4 e^{1.5i}, 0.6 e^{1.8i}, 0.3 e^{0.5i})</td>
<td>(0.5 e^{1.4i}, 0.4 e^{1.2i}, 0.4 e^{0.1i})</td>
<td>(0.6 e^{1.0i}, 0.4 e^{1.0i}, 0.4 e^{0.6i})</td>
<td>(0.3 e^{1.5i}, 0.4 e^{1.0i}, 0.5 e^{0.8i})</td>
</tr>
<tr>
<td>H₃</td>
<td>(0.5 e^{0.6i}, 0.5 e^{1.2i}, 0.5 e^{0.9i})</td>
<td>(0.5 e^{1.3i}, 0.4 e^{1.2i}, 0.4 e^{0.4i})</td>
<td>(0.4 e^{1.0i}, 0.4 e^{1.0i}, 0.2 e^{0.6i})</td>
<td>(0.4 e^{1.0i}, 0.5 e^{1.1i}, 0.2 e^{1.2i})</td>
<td>(0.5 e^{1.2i}, 0.2 e^{1.2i}, 0.2 e^{1.4i})</td>
</tr>
</tbody>
</table>

Numerical values of \((a₁b₁)^{0.5}, (c₁d₁)^{0.5}\) and \((e₁f₁)^{0.5}\) corresponding to each CNN (from table 3) has been presented in the following matrix (see table 4).

**Table 4: Numerical values of \((a₁b₁)^{0.5}, (c₁d₁)^{0.5}\) and \((e₁f₁)^{0.5}\) corresponding to each CNN (from table 3)**

<table>
<thead>
<tr>
<th>Patients</th>
<th>Temperature</th>
<th>Headache</th>
<th>Stomach pain</th>
<th>cough</th>
<th>Chest pain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([a₁b₁]^{0.5}, (c₁d₁)^{0.5}, (e₁f₁)^{0.5})</td>
<td>([a₁b₁]^{0.5}, (c₁d₁)^{0.5}, (e₁f₁)^{0.5})</td>
<td>([a₁b₁]^{0.5}, (c₁d₁)^{0.5}, (e₁f₁)^{0.5})</td>
<td>([a₁b₁]^{0.5}, (c₁d₁)^{0.5}, (e₁f₁)^{0.5})</td>
<td>([a₁b₁]^{0.5}, (c₁d₁)^{0.5}, (e₁f₁)^{0.5})</td>
</tr>
</tbody>
</table>
Step 2: Determination of the relation between symptoms and diseases

The relation between symptoms namely, temperature, headache, stomach pain, cough and diseases namely, viral fever, malaria, stomach pain, chest pain in terms of complex neutrosophic numbers has been presented in the following decision matrix (see the table 5).

<table>
<thead>
<tr>
<th></th>
<th>R-2</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td></td>
<td>0.6 e^{1.3i}, 0.4 e^{1.4i}, 0.2 e^{-0.6i}</td>
<td>0.5 e^{1.4i}, 0.5 e^{1.5i}, 0.2 e^{-0.6i}</td>
<td>0.6 e^{1.5i}, 0.4 e^{1.6i}, 0.5 e^{0.7i}</td>
<td></td>
</tr>
<tr>
<td>Headache</td>
<td></td>
<td>0.4 e^{0.7i}, 0.2 e^{-0.8i}</td>
<td>0.5 e^{0.8i}, 0.4 e^{-0.9i}, 0.2 e^{-0.6i}</td>
<td>0.5 e^{0.9i}, 0.4 e^{1.0i}, 0.5 e^{0.8i}</td>
<td></td>
</tr>
<tr>
<td>Stomach pain</td>
<td></td>
<td>0.4 e^{-1.2i}, 0.2 e^{-1.3i}, 0.5 e^{1.3i}</td>
<td>0.4 e^{1.2i}, 0.4 e^{1.3i}, 0.5 e^{1.4i}</td>
<td>0.4 e^{1.3i}, 0.4 e^{1.4i}, 0.3 e^{1.5i}</td>
<td></td>
</tr>
<tr>
<td>Cough</td>
<td></td>
<td>0.3 e^{1.8i}, 0.4 e^{1.5i}, 0.5 e^{0.6i}</td>
<td>0.4 e^{1.5i}, 0.5 e^{0.6i}, 0.3 e^{0.7i}</td>
<td>0.4 e^{0.7i}, 0.4 e^{0.8i}, 0.4 e^{0.9i}</td>
<td></td>
</tr>
</tbody>
</table>

Numerical values of \((a_2b_2)^{0.5}, (c_2d_2)^{0.5}\) and \((e_2f_2)^{0.5}\) corresponding to each CNN (from table 5) is presented in the following matrix (see the table 6).

<table>
<thead>
<tr>
<th>Symtoms</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>[0.232, 0.205]</td>
<td>0.161, 0.134</td>
<td>0.205, 0.138</td>
<td>[0.158, 0.274, 0.351]</td>
</tr>
<tr>
<td>Headache</td>
<td>[0.345, 0.141]</td>
<td>0.281, 0.354</td>
<td>0.279, 0.354</td>
<td>[0.349, 0.270, 0.354]</td>
</tr>
</tbody>
</table>
Step 3: Determination of the similarity measures

The complex cosine, Dice and Jaccard similarity measures $C_{\text{CNS}}$, $D_{\text{CNS}}$ and $J_{\text{CNS}}$ between the table 3 and the table 5 using equation (6), equation (8) and equation (10) have been presented in the table 7, the table 8 and the table 9.

**Table 7: Complex neutrosophic cosine similarity measure between R-1 and R-2**

<table>
<thead>
<tr>
<th>CNCSM</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>0.9303</td>
<td>0.9272</td>
<td>0.8662</td>
<td>0.8442</td>
</tr>
<tr>
<td>$H_2$</td>
<td>0.8581</td>
<td>0.7512</td>
<td>0.8148</td>
<td>0.8681</td>
</tr>
<tr>
<td>$H_3$</td>
<td>0.9267</td>
<td>0.8602</td>
<td>0.8409</td>
<td>0.7864</td>
</tr>
</tbody>
</table>

**Table 8: Complex neutrosophic Dice similarity measure between R-1 and R-2**

<table>
<thead>
<tr>
<th>CNDSM</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>0.8623</td>
<td>0.8281</td>
<td>0.8596</td>
<td>0.8451</td>
</tr>
<tr>
<td>$H_2$</td>
<td>0.8024</td>
<td>0.7320</td>
<td>0.7935</td>
<td>0.8307</td>
</tr>
<tr>
<td>$H_3$</td>
<td>0.9005</td>
<td>0.8473</td>
<td>0.8187</td>
<td>0.7672</td>
</tr>
</tbody>
</table>

**Table 9: Complex neutrosophic Jaccard similarity measure between R-1 and R-2**

<table>
<thead>
<tr>
<th>CNJSM</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>0.8595</td>
<td>0.8114</td>
<td>0.8498</td>
<td>0.8443</td>
</tr>
<tr>
<td>$H_2$</td>
<td>0.8201</td>
<td>0.8019</td>
<td>0.7911</td>
<td>0.8502</td>
</tr>
<tr>
<td>$H_3$</td>
<td>0.8708</td>
<td>0.8147</td>
<td>0.8469</td>
<td>0.7425</td>
</tr>
</tbody>
</table>

**Step 4: Ranking the alternatives**

The highest correlation measure from the table 7, table 8 and table 9 reflects the proper medical diagnosis. Therefore, patients $H_1$ and $H_3$ suffer from viral fever and patient $H_2$ suffers from chest problem.

**Step 4: End.**
Conclusion

In this paper, we have proposed three similarity measures namely, Cosine, Dice and Jaccard similarity measures based on complex neutrosophic sets. We have also proved some of their basic properties. We have presented their applications in a medical diagnosis problem. The concept presented in this paper can be applied to multiple attribute decision making problems, pattern recognition, personnel selection, artificial intelligence in complex neutrosophic environment.

References


Highlights

- We propose complex neutrosophic cosine, Dice and Jaccard similarity measures.
- We establish some of the properties of complex neutrosophic cosine, Dice and Jaccard similarity measures.
- We present an application of neutrosophic complex cosine, Dice and Jaccard similarity measures have to medical diagnosis problem with complex neutrosophic information.
- We conclude that the proposed similarity measures can be applied in multi attribute decision making, pattern recognition, personnel selection, etc problems.
Table 1: The relation between Patients and Symptoms (R-1)

<table>
<thead>
<tr>
<th>R − 1</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>⋯</th>
<th>$A_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>$\langle T_{11}, I_{11}, F_{11} \rangle$</td>
<td>$\langle T_{12}, I_{12}, F_{12} \rangle$</td>
<td>⋯</td>
<td>$\langle T_{1k}, I_{1k}, F_{1k} \rangle$</td>
</tr>
<tr>
<td>$H_2$</td>
<td>$\langle T_{21}, I_{21}, F_{21} \rangle$</td>
<td>$\langle T_{22}, I_{22}, F_{22} \rangle$</td>
<td>⋯</td>
<td>$\langle T_{2k}, I_{2k}, F_{2k} \rangle$</td>
</tr>
<tr>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
</tr>
<tr>
<td>$H_m$</td>
<td>$\langle T_{m1}, I_{m1}, F_{m1} \rangle$</td>
<td>$\langle T_{m2}, I_{m2}, F_{m2} \rangle$</td>
<td>⋯</td>
<td>$\langle T_{mk}, I_{mk}, F_{mk} \rangle$</td>
</tr>
</tbody>
</table>
Table 2: The relation between symptoms and diseases (R-2)

<table>
<thead>
<tr>
<th>R = 2</th>
<th>D₁</th>
<th>D₂</th>
<th>...</th>
<th>Dₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>⟨ξ₁₁, η₁₁, ζ₁₁⟩</td>
<td>⟨ξ₁₂, η₁₂, ζ₁₂⟩</td>
<td>...</td>
<td>⟨ξ₁ₙ, η₁ₙ, ζ₁ₙ⟩</td>
</tr>
<tr>
<td>A₂</td>
<td>⟨ξ₂₁, η₂₁, ζ₂₁⟩</td>
<td>⟨ξ₂₂, η₂₂, ζ₂₂⟩</td>
<td>...</td>
<td>⟨ξ₂ₙ, η₂ₙ, ζ₂ₙ⟩</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Aₖ</td>
<td>⟨ξₖ₁, ηₖ₁, ζₖ₁⟩</td>
<td>⟨ξₖ₂, ηₖ₂, ζₖ₂⟩</td>
<td>...</td>
<td>⟨ξₖₙ, ηₖₙ, ζₖₙ⟩</td>
</tr>
</tbody>
</table>
Table 3: Relation between Patients and Symptoms in complex neutrosophic form (R-1)

<table>
<thead>
<tr>
<th>R-1</th>
<th>Temperature</th>
<th>Headache</th>
<th>Stomach pain</th>
<th>cough</th>
<th>Chest pain</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>(0.6e^{0.1i}, 0.4e^{1.2i})</td>
<td>(0.4e^{1.2i}, 0.4e^{1.3i})</td>
<td>(0.3e^{1.1i}, 0.4e^{1.0i})</td>
<td>(0.6e^{1.0i}, 0.5e^{2.2i})</td>
<td>(0.4e^{1.0i}, 0.3e^{1.0i})</td>
</tr>
<tr>
<td></td>
<td>(0.2e^{0.8i})</td>
<td>(0.3e^{0.7i})</td>
<td>(0.4e^{0.6i})</td>
<td>(0.3e^{0.8i})</td>
<td>(0.2e^{0.5i})</td>
</tr>
<tr>
<td>H2</td>
<td>(0.7e^{1.3i}, 0.4e^{1.2i})</td>
<td>(0.4e^{1.5i}, 0.6e^{2.3i})</td>
<td>(0.5e^{1.4i}, 0.4e^{1.2i})</td>
<td>(0.6e^{1.0i}, 0.4e^{1.0i})</td>
<td>(0.3e^{1.5i}, 0.4e^{1.0i})</td>
</tr>
<tr>
<td></td>
<td>(0.5e^{0.9i})</td>
<td>(0.3e^{0.5i})</td>
<td>(0.4e^{0.6i})</td>
<td>(0.4e^{0.6i})</td>
<td>(0.5e^{1.0i})</td>
</tr>
<tr>
<td>H3</td>
<td>(0.5e^{0.6i}, 0.5e^{1.2i})</td>
<td>(0.5e^{1.3i}, 0.4e^{1.2i})</td>
<td>(0.4e^{1.0i}, 0.4e^{1.0i})</td>
<td>(0.4e^{1.0i}, 0.5e^{1.3i})</td>
<td>(0.5e^{1.2i}, 0.2e^{1.2i})</td>
</tr>
<tr>
<td></td>
<td>(0.5e^{0.9i})</td>
<td>(0.4e^{0.4i})</td>
<td>(0.2e^{0.6i})</td>
<td>(0.2e^{1.2i})</td>
<td>(0.2e^{1.4i})</td>
</tr>
<tr>
<td>R-1</td>
<td>Temperature</td>
<td>Headache</td>
<td>Stomach pain</td>
<td>cough</td>
<td>Chest pain</td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
<td>----------</td>
<td>--------------</td>
<td>-------</td>
<td>------------</td>
</tr>
<tr>
<td>H1</td>
<td>(0.6e^{0.01}, 0.4e^{1.21})</td>
<td>(0.4e^{1.21}, 0.4e^{1.31})</td>
<td>(0.3e^{0.71}, 0.4e^{0.61})</td>
<td>(0.6e^{1.01}, 0.5e^{1.21})</td>
<td>(0.4e^{1.01}, 0.3e^{1.01})</td>
</tr>
<tr>
<td></td>
<td>(0.2e^{0.31})</td>
<td>(0.3e^{0.71})</td>
<td>(0.4e^{0.61})</td>
<td>(0.3e^{0.81})</td>
<td>(0.2e^{0.51})</td>
</tr>
<tr>
<td>H2</td>
<td>(0.7e^{1.31}, 0.4e^{1.21})</td>
<td>(0.4e^{1.51}, 0.6e^{1.21})</td>
<td>(0.5e^{1.41}, 0.4e^{1.21})</td>
<td>(0.6e^{1.21}, 0.4e^{1.01})</td>
<td>(0.3e^{1.51}, 0.4e^{1.01})</td>
</tr>
<tr>
<td></td>
<td>(0.5e^{0.91})</td>
<td>(0.3e^{0.51})</td>
<td>(0.4e^{0.61})</td>
<td>(0.4e^{0.61})</td>
<td>(0.5e^{1.01})</td>
</tr>
<tr>
<td>H3</td>
<td>(0.5e^{0.61}, 0.5e^{1.21})</td>
<td>(0.5e^{1.31}, 0.4e^{1.21})</td>
<td>(0.4e^{1.01}, 0.4e^{1.01})</td>
<td>(0.4e^{1.01}, 0.5e^{1.31})</td>
<td>(0.5e^{1.21}, 0.2e^{1.21})</td>
</tr>
<tr>
<td></td>
<td>(0.5e^{0.91})</td>
<td>(0.4e^{0.41})</td>
<td>(0.2e^{0.61})</td>
<td>(0.2e^{1.21})</td>
<td>(0.2e^{1.41})</td>
</tr>
<tr>
<td>R-2</td>
<td>Viral Fever</td>
<td>Malaria</td>
<td>Stomach problem</td>
<td>Chest problem</td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
<td>---------</td>
<td>-----------------</td>
<td>---------------</td>
<td></td>
</tr>
<tr>
<td><strong>Temperature</strong></td>
<td>(\langle 0.4 \epsilon^{1.21}, 0.4 \epsilon^{1.41}, 0.3 \epsilon^{0.61} \rangle)</td>
<td>(\langle 0.6 \epsilon^{1.31}, 0.4 \epsilon^{1.41}, 0.2 \epsilon^{1.51} \rangle)</td>
<td>(\langle 0.5 \epsilon^{1.41}, 0.5 \epsilon^{1.51}, 0.2 \epsilon^{0.61} \rangle)</td>
<td>(\langle 0.6 \epsilon^{1.51}, 0.4 \epsilon^{0.61}, 0.5 \epsilon^{0.71} \rangle)</td>
<td></td>
</tr>
<tr>
<td><strong>Headache</strong></td>
<td>(\langle 0.5 \epsilon^{0.61}, 0.4 \epsilon^{0.71}, 0.2 \epsilon^{0.81} \rangle)</td>
<td>(\langle 0.4 \epsilon^{0.71}, 0.4 \epsilon^{0.81}, 0.3 \epsilon^{0.91} \rangle)</td>
<td>(\langle 0.5 \epsilon^{0.81}, 0.4 \epsilon^{0.91}, 0.2 \epsilon^{1.01} \rangle)</td>
<td>(\langle 0.5 \epsilon^{0.91}, 0.4 \epsilon^{1.01}, 0.5 \epsilon^{0.81} \rangle)</td>
<td></td>
</tr>
<tr>
<td><strong>Stomach pain</strong></td>
<td>(\langle 0.4 \epsilon^{1.01}, 0.4 \epsilon^{1.11}, 0.4 \epsilon^{1.21} \rangle)</td>
<td>(\langle 0.5 \epsilon^{1.11}, 0.2 \epsilon^{1.21}, 0.2 \epsilon^{1.31} \rangle)</td>
<td>(\langle 0.4 \epsilon^{1.21}, 0.4 \epsilon^{1.31}, 0.5 \epsilon^{1.41} \rangle)</td>
<td>(\langle 0.4 \epsilon^{1.31}, 0.4 \epsilon^{1.41}, 0.3 \epsilon^{1.51} \rangle)</td>
<td></td>
</tr>
<tr>
<td><strong>Cough</strong></td>
<td>(\langle 0.3 \epsilon^{1.41}, 0.4 \epsilon^{1.51}, 0.5 \epsilon^{0.61} \rangle)</td>
<td>(\langle 0.4 \epsilon^{1.51}, 0.5 \epsilon^{0.61}, 0.3 \epsilon^{0.71} \rangle)</td>
<td>(\langle 0.5 \epsilon^{0.61}, 0.4 \epsilon^{0.71}, 0.3 \epsilon^{0.81} \rangle)</td>
<td>(\langle 0.3 \epsilon^{0.71}, 0.4 \epsilon^{0.81}, 0.4 \epsilon^{0.91} \rangle)</td>
<td></td>
</tr>
<tr>
<td><strong>Chest pain</strong></td>
<td>(\langle 0.4 \epsilon^{0.81}, 0.4 \epsilon^{0.91}, 0.5 \epsilon^{1.01} \rangle)</td>
<td>(\langle 0.6 \epsilon^{1.01}, 0.4 \epsilon^{1.21}, 0.3 \epsilon^{1.41} \rangle)</td>
<td>(\langle 0.4 \epsilon^{1.21}, 0.4 \epsilon^{1.41}, 0.5 \epsilon^{0.61} \rangle)</td>
<td>(\langle 0.4 \epsilon^{1.41}, 0.3 \epsilon^{0.61}, 0.2 \epsilon^{0.81} \rangle)</td>
<td></td>
</tr>
</tbody>
</table>
Table 6: Numerical values of \((a_2b_2)^{0.5}, (c_2d_2)^{0.5}\) and \((e_2f_2)^{0.5}\) corresponding to each CNN (from table 5)

<table>
<thead>
<tr>
<th>Symptoms</th>
<th>Viral Fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([a_2b_2]^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5})</td>
<td>([a_2b_2]^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5})</td>
<td>([a_2b_2]^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5})</td>
<td>([a_2b_2]^{0.5}, (c_2d_2)^{0.5}, (e_2f_2)^{0.5})</td>
</tr>
<tr>
<td>Temperature</td>
<td>[0.232, 0.161, 0.205]</td>
<td>[0.581, 0.702]</td>
<td>[0.205, 0.134, 0.138]</td>
<td>[0.158, 0.274, 0.351]</td>
</tr>
<tr>
<td>Headache</td>
<td>[0.345, 0.141]</td>
<td>[0.281, 0.210]</td>
<td>[0.354, 0.134]</td>
<td>[0.349, 0.270, 0.354]</td>
</tr>
<tr>
<td>Stomach pain</td>
<td>[0.425, 0.232]</td>
<td>[0.319, 0.100]</td>
<td>[0.232, 0.205]</td>
<td>[0.202, 0.164, 0.077]</td>
</tr>
<tr>
<td>Cough</td>
<td>[0.122, 0.342]</td>
<td>[0.105, 0.210]</td>
<td>[0.342, 0.212]</td>
<td>[0.210, 0.283, 0.279]</td>
</tr>
<tr>
<td>Chest pain</td>
<td>[0.283, 0.338]</td>
<td>[0.313, 0.122]</td>
<td>[0.236, 0.342]</td>
<td>[0.164, 0.141]</td>
</tr>
<tr>
<td>CNCSM</td>
<td>Viral Fever</td>
<td>Malaria</td>
<td>Stomach problem</td>
<td>Chest problem</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>---------</td>
<td>----------------</td>
<td>---------------</td>
</tr>
<tr>
<td>H₁</td>
<td>0.9303</td>
<td>0.9272</td>
<td>0.8662</td>
<td>0.8442</td>
</tr>
<tr>
<td>H₂</td>
<td>0.8581</td>
<td>0.7512</td>
<td>0.8148</td>
<td>0.8681</td>
</tr>
<tr>
<td>H₃</td>
<td>0.9267</td>
<td>0.8602</td>
<td>0.8409</td>
<td>0.7864</td>
</tr>
<tr>
<td>CNDSM</td>
<td>Viral Fever</td>
<td>Malaria</td>
<td>Stomach problem</td>
<td>Chest problem</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>---------</td>
<td>-----------------</td>
<td>---------------</td>
</tr>
<tr>
<td>H₁</td>
<td><strong>0.8623</strong></td>
<td>0.8281</td>
<td>0.8596</td>
<td>0.8451</td>
</tr>
<tr>
<td>H₂</td>
<td>0.8024</td>
<td>0.7320</td>
<td>0.7935</td>
<td><strong>0.8307</strong></td>
</tr>
<tr>
<td>H₃</td>
<td><strong>0.9005</strong></td>
<td>0.8473</td>
<td>0.8187</td>
<td>0.7672</td>
</tr>
<tr>
<td>CNJSM</td>
<td>Viral Fever</td>
<td>Malaria</td>
<td>Stomach problem</td>
<td>Chest problem</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>---------</td>
<td>-----------------</td>
<td>---------------</td>
</tr>
<tr>
<td>H₁</td>
<td><strong>0.8595</strong></td>
<td>0.8114</td>
<td>0.8498</td>
<td>0.8443</td>
</tr>
<tr>
<td>H₂</td>
<td>0.8201</td>
<td>0.8019</td>
<td>0.7911</td>
<td><strong>0.8502</strong></td>
</tr>
<tr>
<td>H₃</td>
<td><strong>0.8708</strong></td>
<td>0.8147</td>
<td>0.8469</td>
<td>0.7425</td>
</tr>
</tbody>
</table>