Clustering Algorithm of Triple Refined Indeterminate Neutrosophic Set for Personality Grouping

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***Abstract*—Triple Refined Indeterminate Neutrosophic Set (TRINS) which is a case of the refined neutrosophic set was introduced in [27]. It provides the additional possibility to represent with sensitivity and accuracy the uncertain, imprecise, incomplete, and inconsistent information which are available in real world. More precision is provided in handling indeterminacy; by classifying indeterminacy (I) into three, based on membership; as indeterminacy leaning towards truth membership (I_T), indeterminacy membership (I) and indeterminacy leaning towards false membership (I_F). This kind of classification of indeterminacy is not feasible with the existing Single Valued Neutrosophic Set (SVNS). TRINS is better equipped at dealing indeterminate and inconsistent information, with more accuracy than SVNS and Double Refined Indeterminate Neutrosophic Set (DRINS), which fuzzy sets and Intuitionistic Fuzzy Sets (IFS) are incapable of. TRINS can be used in any place where the Likert scale is used, which is an advantage. Personality test usually make use of the Likert scale. Using the Open Extended Jung Type Scale test and TRINS, an indeterminacy based personality test was introduced and personality classification was done [27]. A generalized distance measure between TRINS and related distance matrix is defined, based on which a clustering algorithm is constructed. This article proposes a Triple Refined Indeterminate Neutrosophic Minimum Spanning Tree (TRIN-MST) clustering algorithm, to cluster the data represented by Triple Refined Indeterminate Neutrosophic information. Illustrative examples using the indeterminacy based personality test are given to exhibit the applications and effectiveness of the TRIN-MST clustering algorithm.**

Keywords*—Personality test; Personality grouping, Neutrosophic Set, Triple Refined Neutrosophic Set (TRINS), TRIN-MST clustering algorithm

I. Introduction

Carl Jung in his collected work [1] had theorized the eight psychological types based on two main attitude types: extroversion and introversion, two perceiving functions: sensation and intuition and two judging functions: thinking and feeling. The psychological types were Extraverted sensation, Introverted sensation, Extraverted intuition, Introverted intuition, Extraverted thinking, Introverted thinking, Extraverted feeling and Introverted feeling. The MyersBriggs Type Indicator (MBTI) [2], is based on the typological theory proposed by Carl Jung. It sorts some of these psychological differences into four opposite pairs, or "dichotomies", with a resulting 16 possible psychological types. The MBTI is an introspective self-report questionnaire structured to indicate psychological preferences of people's perception of the world and their decision making. These personality test are mostly objective in nature, where the test taker is forced to select a dominant choice. Quoting Carl Jung himself *"There is no such thing as a pure extrovert or a pure introvert. Such a man would be in the lunatic asylum."*, it is clear that there are degrees of variations, no person fits into a category 100%. Since it is not feasible for a person to put down his answer as single choice in reality, without ignoring the other degrees of variation. It necessitates a tool which can give more than one choice to represent their personality. The choice also depends highly on the situation and circumstance the individual faces at that time,

Fuzzy set theory introduced by Zadeh [3] provides a constructive analytic tool for soft division of sets. Zadeh's fuzzy set theory [3] was extended to intuitionistic fuzzy set (A-IFS), in which each element is assigned a membership degree and a non-membership degree by Atanassov [4]. A-IFS was further generalized into the notion of interval valued intuitionistic fuzzy set (IVIFS) by Atanassov and Gargov [5].

To represent uncertain, imprecise, incomplete, and inconsistent information that are present in real world, the concept of a neutrosophic set from philosophical point of view was proposed by Smarandache [6]. The neutrosophic set is a prevailing framework that generalizes the concept of the classic set, fuzzy set, intuitionistic fuzzy set, interval valued fuzzy set, interval valued intuitionistic fuzzy set, paraconsistent set, paradoxist set, and tautological set. Truth membership, indeterminacy membership, and falsity membership are represented independently in the neutrosophic set. However, the neutrosophic set generalizes the above mentioned sets from the philosophical point of view, and its functions $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, that is, $T_A(x) : X \to]^-0, 1^+[$, $I_A(x) : X \to]^-0, 1^+[$, and $F_A(x) : X \to]^-0, 1^+[$, respectively with the condition $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

It is difficult to apply neutrosophic set in this form in real scientific and engineering areas. To overcome this difficulty, Wang et al. [7] introduced a Single Valued Neutrosophic Set (SVNS), which is an instance of a neutrosophic set. SVNS can deal with indeterminate and inconsistent information, which fuzzy sets and intuitionistic fuzzy sets are incapable of.

Owing to the fuzziness, uncertainty and indeterminate nature of many practical problems in the real world, neutrosophy has found application in many fields including Social Network Analysis (Salama et al [8]), Decision-making problems (Ye [9]–[12]), Image Processing (Cheng and Guo [13], Sengur and Guo [14], Zhang et al [15]), Social problems (Vasantha and Smarandache [16], [17]) etc. Liu et al, have applied neutrosophy to group decision problems and multiple attribute decision making problems [18]–[24] etc.

To provide more accuracy and precision to indeterminacy, the indeterminacy value present in the neutrosophic set has been classified into two; based on membership; as indeterminacy leaning towards truth membership and as indeterminacy leaning towards false membership. They make the indeterminacy involved in the scenario to be more accurate and precise. This modified refined neutrosophic set was defined as Double Refined Indeterminacy Neutrosophic Set (DRINS) by Kandasamy [25] and Kandasamy and Florentin [26]. To increase the accuracy, precision and to fit in the Likert's scale which is usually used in personality test; here the indeterminacy concept is divided into three, as indeterminacy leaning towards truth membership, indeterminacy membership and indeterminacy leaning towards false membership. This refined neturosophic set known as the Triple Refined Indeterminate Neutrosophic Sets (TRINS) was defined by Kandasamy and Florentin [27].

Consider an example from a personality test "You tend to sympathize with others". The person need not be forced to opt for a single choice; cause it is natural that the behaviour is dependent on several external and internal factors, varying from the person's mood to surrounding. So a person might not always react in a particular way, in a particular scenario. There is always a degree to which the person will strongly agree to the statement (say 0.7), will just agree (0.1), neither agree or disagree (0.05), will agree (0.1) and will strongly disagree(0.05). When a person is taking a personality test he/she is forced to opt for a single choice, thereby the degrees of membership of others are completely lost. Whereas using TRINS this statement is represented as $\langle 07, 0.1, 0.05, 0.1, 0.05\rangle$, it can be evaluated accurately; thereby giving very useful necessary precision to the result. All the various choices are captures thereby avoiding the preferential choice that is executed in the classical method.

Section one is introductory in nature. The rest of the paper is organized as follows: Section two recalls some basic concepts about neutrosophy and The Open Extended Jungian Type Scales (OEJTS) personality test. Section three recalls TRINS, its related set theoretic concepts, distance measure over TRINS and the indeterminacy based personality test [27]. Section four introduces the clustering algorithm using the Minimum Spanning Tree known as the TRIN-MST clustering algorithm. Clustering of different people based on the personality test can be done using the TRIN-MST clustering algorithm. Illustrative examples are provided in the section five to show the working and efficiency of the algorithm. The conclusions and future direction of work based on personality testing and TRINS is given in the last section.

II. Preliminaries / Basic Concepts

A. Personality test

There are several types of personality tests, the most common type is the objective personality tests. It involves the administration of many questions/items to test-takers who respond by rating the degree to which each item reflects their behaviour and can be scored objectively. The term 'item' is used because many test questions are not actually questions; they are typically statements on questionnaires that allow respondents to indicate level of agreement. The most famously used personality test is the Myers-Briggs Type Indicator test. Most personality tests on the internet offer little information about how they were developed or how the results are calculated. A comparative study of different tests has not been carried out. There are currently no criteria for what makes a good Myers-Briggs/Jungian type. Of course, it could just be accepted that the Myers-Briggs Type Indicator (MBTI) defines Myers-Briggs/Jungian types and so that means that the measure of a test is just how similar it is to the MBTI.

The Open Extended Jungian Type Scales test [28] is an open source alternative to the Myers Briggs type indicator test. A comparative validity study of the Open Extended Jungian Type Scales was done using three other on-line tests that measured the same construct (Myers-Briggs Type Indicator alternatives). The ability of the OEJTS to differentiate among individuals of known type was significantly higher than other tests. This is good evidence for the OEJTS being the most accurate on-line Myers-Briggs/Jungian type test. There are innumerable on-line Myers-Briggs tests, only three were chosen due to limits on subjects. The three were chosen on the basis of their judged popularity within Myers-Briggs enthusiast communities. The three chosen were the Human Metrics Jung Typology Test, Similar Minds Jung Personality Test and 16-Personalities personality test. The OEJTS test alone is taken for future discussion in this paper.

B. The Open Extended Jungian Type Scales (OEJTS)

An extension of the Jung's Theory of psychological type casting is the Myers- Briggs Type Indicator (MBTI). It has four personality dichotomies that are combined to yield 16 personality types. The dichotomies are [28]

1) Introversion (I) vs. Extroversion (E); sometimes is described as a persons orientation, they either orient within themselves or to the outside world. Other times the focus is put more explicitly on social interaction, with some claiming that social interactions wears out introverts whereas social interaction raises the energy level in extroverts.
2) Sensing (S) vs. Intuition (N); defined as how a person takes in information by Myers and Briggs, who said that sensors pay attention to the five senses while intuitives pay attention to possibilities.
3) Feeling (F) vs. Thinking (T); has been defined as what a person values and what they base their decisions on: either interpersonal considerations or through dispassionate logic.
4) Judging (J) vs. Perceiving (P); was a dichotomy added by Myers and Briggs to pick between the second and third pair of functions. Individuals who prefer a structured lifestyle are supposed to use their judging functions (thinking and feeling) while individuals who prefer a flexible lifestyle are supposed to prefer the sensing functions (sensing and intuition).

The Open Extended Jungian Type Scales (OEJTS) measures four scales, each intended to produce a very large score differential along one dichotomy.

TABLE I. QUESTIONNAIRE

Q		Scale	
Q_1	makes lists	1 2 3 4 5	relies on memory
Q_2	sceptical	1 2 3 4 5	wants to believe
Q_3	bored by time alone	1 2 3 4 5	needs time alone
Q_7	energetic	1 2 3 4 5	mellow
Q_{11}	works best in groups	1 2 3 4 5	works best alone
Q_{15}	worn out by parties	1 2 3 4 5	gets fired up by parties
Q_{19}	talks more	1 2 3 4 5	listens more
Q_{23}	stays at home	1 2 3 4 5	goes out on the town
Q_{27}	finds it difficult to yell very loudly	1 2 3 4 5	yelling to others when they are far away comes naturally
Q_{31}	perform in public	1 2 3 4 5	avoids public speaking

The item format for the OEJTS has been chosen to be two statements that form a biploar scale (e.g. humble to arrogant), operationalized as a five point scale. A sample questionnaire is shown in Table I.

C. Working of the Open Extended Jungian Type Scales

The OEJTS personality test gives a result equivalent to the Myers-Briggs Type Indicator, although the test is not the MBTI and has no affiliation with it. There are 32 pairs of personality descriptions connected by a five point scale. Marking for each pair, is a choice where on the scale between them you think you are. For example, if the pair is angry versus calm, you should circle a 1 if you are always angry and never calm, a 3 if you are half and half, etc. Sample questions are as shown in Table I. Questions 3, 7, 11, 15, 19, 23, 17 and 31 are related to Extrovert Introvert.

The scoring instructions are as follows [28] :

$$IE = 30 - Q_3 - Q_7 - Q_{11} + Q_{15} - Q_{19} + Q_{23} + Q_{27} - Q_{31}$$

$$SN = 12 + Q_4 + Q_8 + Q_{12} + Q_{16}) + Q_{20} - Q_{24} - Q_{28} + Q_{32}$$

$$FT = 30 - Q_2 + Q_6 + Q_{10} - Q_{14} - Q_{18} + Q_{22} - Q_{26} - Q_{30}$$

$$JP = 18 + Q_1 + Q_5 - Q_9 + Q_{13} - Q_{17} + Q_{21} - Q_{25} + Q_{29}$$

If IE is more than 24, you are extrovert (E), otherwise you are introvert (I). If SN is more than 24, you are intuitive (N), otherwise you are sensing (S). If FT is more than 24, you are thinking (T), otherwise you are feeling (F). If JP is more than 24, you are perceiving (P), otherwise you are judging (J). The four letters are combined to obtain the personality type (e.g. I, S, F, P = ISFP).

D. Neutrosophy and Single Valued Neutrosophic Set (SVNS)

Neutrosophy is a branch of philosophy, introduced by Smarandache [6], which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. It considers a proposition, concept, theory, event, or entity, "A" in relation to its opposite, "Anti-A" and that which is not A, "Non-A", and that which is neither "A" nor "Anti-A", denoted by "Neut-A". Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics.

The concept of a neutrosophic set from philosophical point of view, introduced by Smarandache [6], is as follows.

Definition 1: [6] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non standard subsets of $]^-0, 1^+[$, that is, $T_A(x) : X \to]^-0, 1^+[$, $I_A(x) : X \to]^-0, 1^+[$, and $F_A(x) : X \to]^-0, 1^+[$, with the condition $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

This definition of neutrosophic set is difficult to apply in real world application of scientific and engineering fields. Therefore, the concept of Single Valued Neutrosophic Set (SVNS), which is an instance of a neutrosophic set was introduced by Wang et al. [7].

Definition 2: [7] Let X be a space of points (objects) with generic elements in X denoted by x. An Single Valued Neutrosophic Set (SVNS) A in X is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$, and falsity membership function $F_A(x)$. For each point x in X, there are $T_A(x), I_A(x), F_A(x) \in [0, 1]$, and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Therefore, an SVNS A can be represented by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$. The following expressions are defined in [7] for SVNSs A, B:

- $A \in B$ if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X.
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}$.

The refined neutrosophic logic defined by [29] is as follows:

Definition 3: T can be split into many types of truths: $T_1, T_2, \ldots, T_p$, and I into many types of indeterminacies: I_1, $I_2, \ldots, I_r$, and F into many types of falsities: $F_1, F_2, \ldots, F_s$, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$. In the same way, but all subcomponents T_j, I_k, F_l are not symbols, but subsets of [0, 1], for all $j \in \{1, 2, \ldots, p\}$ all $k \in \{1, 2, \ldots, r\}$ and all $l \in \{1, 2, \ldots, s\}$. If all sources of information that separately provide neutrosophic values for a specific subcomponent are independent sources, then in the general case we consider that each of the subcomponents T_j, I_k, F_l is independent with respect to the others and it is in the non-standard set $]^-0, 1^+[$.

The following section is recalled from [27] for the sake of completeness of this paper.

III. TRIPLE REFINED INDETERMINACY NEUTROSOPHIC SET (TRINS)

Here the indeterminacy concept is divided into three, as indeterminacy leaning towards truth membership, indeterminacy membership and indeterminacy leaning towards false membership. This division aids in increasing the accuracy and precision of the indeterminacy and to fit in the Likert's scale which is usually used in personality test. This refined neturosophic set is defined as the Triple Refined Indeterminate Neutrosophic Sets (TRINS) [27].

Definition 4: Let X be a space of points (objects) with generic elements in X denoted by x. A Triple Refined Indeterminate Neutrosophic Set (TRINS) A in X is characterized by truth membership function $T_A(x)$, indeterminacy leaning towards truth membership function $I_{TA}(x)$, indeterminacy membership function $I_A(x)$, indeterminacy leaning towards falsity membership function $I_{FA}(x)$, and falsity membership function $F_A(x)$. Each membership function has a weight $w_m \in [0,5]$ associated with it. For each generic element $x \in X$, there are

$$T_A(x),\ I_{TA}(x),\ I_A(x),\ I_{FA}(x),\ F_A(x) \in [0,1],$$

$$w_T(T_A(x)),\ w_{I_T}(I_{TA}(x)),\ w_I(I_A(x)),\ w_{I_F}(I_{FA}(x)),\\ w_F(F_A(x)) \in [0,5],$$

and

$$0 \le T_A(x) + I_{TA}(x) + I_A(x) + I_{FA}(x) + F_A(x) \le 5.$$

Therefore, a TRINS A can be represented by

$$A = \{\langle x, T_A(x), I_{TA}(x), I_A(x), I_{FA}(x), F_A(x)\rangle \mid x \in X\}.$$

A TRINS A is represented as

$$A = \int_X \{\langle T(x), I_T(x), I(x), I_F(x), F(x)\rangle / dx, x \in X\} \quad (1)$$

when X is continuous. It is represented as

$$A = \sum_{i=1}^{n} \{\langle T(x_i), I_T(x_i), I(x_i), I_F(x_i), F(x_i)\rangle \mid x_i, x_i \in X\} \quad (2)$$

when X is discrete.

Definition 1: Let $X = [x_1, x_2]$ where x_1 is question 1 and x_2 is question 2 from Table II. The values of x_1 and x_2 are in $[0,1]$ and when the weight of the membership is applied the values of $w_m(x_1)$ and $w_m(x_2)$ are in $[1,5]$. This is obtained from the questionnaire of the user.

Consider question 1, instead of a forced single choice; their option for question 1 would be a degree of "make list", a degree of indeterminacy choice towards "make list" , a degree of uncertain and indeterminate combination of making list and depending on memory, an degree of indeterminate choice more of replying on memory, and a degree of "relying on memory".

A is a TRINS of X defined by

$$A = \langle 0.0, 0.4, 0.1, 0.0, 0.5\rangle / x_1 + \langle 0.5, 0.1, 0.1, 0.1, 0.2\rangle / x_2.$$

The associated membership weights are $w_T = 1$, $w_{I_T} = 2$, $w_I = 3$, $w_{I_F} = 4$, $w_F = 5$. Then the weighted TRINS $w_T(T_A(x))$, $w_{I_T}(I_{TA}(x))$, $w_I(I_A(x))$, $w_{I_F}(I_{FA}(x))$, $w_F(F_A(x)) \in [0,5]$, will be

$$A = \langle 0.0, 0.8, 0.3, 0.0, 1.5\rangle / x_1 + \langle 0.5, 0.2, 0.3, 0.4, 1.0\rangle / x_2.$$

Definition 5: The complement of a TRINS A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{Tc(A)}(x) = 1 - I_{TA}(x)$, $I_{c(A)}(x) = 1 - I_A(x)$, $I_{Fc(A)}(x) = 1 - I_{FA}(x)$ and $F_{c(A)}(x) = T_A(x)$ for all x in X.

Definition 6: A TRINS A is contained in the other TRINS B, $A \subseteq B$, if and only if $T_A(x) \le T_B(x)$, $I_{TA}(x) \le I_{TB}(x)$, $I_A(x) \le I_B(x)$, $I_{FA}(x) \le I_{FB}(x)$ and $F_A(x) \ge F_B(x)$ for all x in X.

Note that by the definition of containment relation, X is a partially ordered set and not a totally ordered set.

For example, let A and B be the TRINSs as defined in Example 1, then A is not contained in B and B is not contained in A.

Definition 7: Two TRINSs A and B are equal, denoted as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

The union of two TRINSs A and B is a TRINS C, denoted as $C = A \cup B$, the intersection of two TRINSs A and B is a TRINS C, denoted as $C = A \cap B$, and the difference of two TRINSs D, written as $D = A \setminus B$, was defined in [27].

Three operators truth favourite ($\triangle$), falsity favourite ($\bigtriangledown$) and indeterminacy neutral (∇) are defined over TRINSs. Two operators truth favourite ($\triangle$) and falsity favourite ($\bigtriangledown$) are defined to remove the indeterminacy in the TRINSs and transform it into intuitionistic fuzzy sets or paraconsistent sets. Similarly the TRINS is transformed into a SVNS by operator indeterminacy neutral (∇) by combining the indeterminacy values of the TRINS. These three operators are unique on TRINS was defined in [27].

All set theoretic operators like commutativity, associativity, distributivity, idempotency, absorption and the DeMorgan's Laws were defined over TRINs [27]. The definition of complement, union and intersection of TRINS and TRINS itself satisfy most properties of classical set, fuzzy set, intuitionistic fuzzy set and SNVS. Similar to fuzzy set, intuitionistic fuzzy set and SNVS, it does not satisfy the principle of middle exclude.

A. Distance Measures of TRINS

The weight measures over TRINS is defined in the following:

Consider TRINS A in a universe of discourse, $X = \{x_l, x_2, \ldots, x_n\}$, which are denoted by $A = \{\langle x_i, T_A(x_i), I_{TA}(x_i), I_A(x_i), I_{FA}(x_i), F_A(x_i)\rangle \mid x_i \in X\}$, where $T_A(x_i), I_{TA}(x_i), I_A(x_i), I_{FA}(x_i), F_A(x_i), \in [0,1]$ for every $x_i \in X$. Let w_m be the weight of each membership, then $w_T(T_A(x))$, $w_{I_T}(I_{TA}(x))$, $w_I(I_A(x))$, $w_{I_F}(I_{FA}(x))$, $w_F(F_A(x)) \in [0,5]$. Hereafter by the membership $T_A(x_i)$, $I_{TA}(x_i)$, $I_A(x_i)$, $I_{FA}(x_i)$, $F_A(x_i)$, we mean the weight membership $w_T(T_A(x))$, $w_{I_T}(I_{TA}(x))$, $w_I(I_A(x))$, $w_{I_F}(I_{FA}(x))$, $w_F(F_A(x))$.

Then, the generalized Triple Refined Indeterminate Neutrosophic weight is defined as follows:

$$w(A) = \{\sum_{i=1}^{n} \{w_T(T_A(x_i)) + w_{I_T}(I_{TA}(x_i)) + \\ w_I(I_A(x_i)) + w_{I_F}(I_{FA}(x_i)) + w_F(F_A(x_i))\} \quad (3)$$

The distance measures over TRINs is defined in the following and the related algorithm for determining the distance is given:

Consider two TRINSs A and B in a universe of discourse, $X = x_l, x_2, \ldots, x_n$, which are denoted by

$$A = \{\langle x_i, T_A(x_i), I_{TA}(x_i), I_A(x_i), I_{FA}(x_i), F_A(x_i)\rangle \mid x_i \in X\}, \text{ and } B = \{\langle x_i, T_B(x_i), I_{TB}(x_i), I_B(x_i), I_{FB}(x_i), F_B(x_i)\rangle \mid x_i \in X\},$$

where $T_A(x_i), I_{TA}(x_i), I_A(x_i), I_{FA}(x_i), F_A(x_i), T_B(x_i), I_{TB}(x_i), I_B(x_i), I_{FB}(x_i), F_B(x_i) \in [0,5]$ for every $x_i \in X$. Let $w_i(i = 1,2,\dots,n)$ be the weight of an element $x_i(i = 1,2,\dots,n)$, with $w_i \geq 0(i = 1,2,\dots,n)$ and $\sum_{i=1}^{n} w_i = 1$.

Then, the generalized Triple Refined Indeterminate Neutrosophic weighted distance is defined as follows:

$$d_\lambda(A,B) = \{\frac{1}{5}\sum_{i=1}^{n} w_i[\mid T_A(x_i) - T_B(x_i)\mid^\lambda + \mid I_{TA}(x_i) - I_{TB}(x_i)\mid^\lambda + \mid I_A(x_i) - I_B(x_i)\mid^\lambda + \mid I_{FA}(x_i) - I_{FB}(x_i)\mid^\lambda + \mid F_A(x_i) - F_B(x_i)\mid^\lambda]\}^{1/\lambda} \tag{4}$$

where $\lambda > 0$.

Equation 4 reduces to the Triple Refined Indeterminate Neutrosophic weighted Hamming distance and the Triple Refined Indeterminate Neutrosophic weighted Euclidean distance, when $\lambda = 1, 2$, respectively. The Triple Refined Indeterminate Neutrosophic weighted Hamming distance is given as

$$d_\lambda(A,B) = \frac{1}{5}\sum_{i=1}^{n} w_i[\mid T_A(x_i) - T_B(x_i)\mid + \mid I_{TA}(x_i) - I_{TB}(x_i)\mid + \mid I_A(x_i) - I_B(x_i)\mid + \mid I_{FA}(x_i) - I_{FB}(x_i)\mid + \mid F_A(x_i) - F_B(x_i)\mid] \tag{5}$$

where λ = 1 in Equation 4.

The Triple Refined Indeterminate Neutrosophic weighted Euclidean distance is given as

$$d_\lambda(A,B) = \{\frac{1}{5}\sum_{i=1}^{n} w_i[\mid T_A(x_i) - T_B(x_i)\mid^2 + \mid I_{TA}(x_i) - I_{TB}(x_i)\mid^2 + \mid I_A(x_i) - I_B(x_i)\mid^2 \mid I_{FA}(x_i) - I_{FB}(x_i)\mid^2 + \mid F_A(x_i) - F_B(x_i)\mid^2]\}^{1/2} \tag{6}$$

where λ = 2 in Equation 4.

The algorithm to obtain the generalized Triple Refined Indeterminate Neutrosophic weighted distance $d_\lambda(A,B)$ is given in Algorithm 1.

The following proposition is given for the distance measure.

Definition 1: The generalized Triple Refined Indeterminate Neutrosophic weighted distance $d_\lambda(A,B)$ for $\lambda > 0$ satisfies the following properties:

1) (Property 1) $d_\lambda(A,B) \geq 0$;
2) (Property 2) $d_\lambda(A,B) = 0$ if and only if $A = B$;
3) (Property 3) $d_\lambda(A,B) = d_\lambda(B,A)$;
4) (Property 4) If $A \subseteq B \subseteq C, C$ is an TRINS in X, then $d_\lambda(A,C) \geq d_\lambda(A,B)$ and $d_\lambda(A,C) \geq d_\lambda(B,C)$.

It can be easily seen that $d_\lambda(A,B)$ satisfies the properties (Property 1) to (Property 4).

The Triple Refined Indeterminate Neutrosophic distance matrix D is defined in the following.

Definition 8: Let $A_j(j = 1,2,\dots,m)$ be a collection of m TRINs, then $D = (d_{ij})_{m\times m}$ is called a Triple valued neutrosophic distance matrix, where $d_{ij} = d_\lambda(A_i, A_j)$ is the generalized Triple distance valued neutrosophic between A_i and A_j, and its properties are as follows:

1) $0 \leq d_{ij} \leq 5$ for all $i,j = 1,2,\dots,m$;
2) $d_{ij} = 0$ if and only if $A_i = A_j$;
3) $d_{ij} = d_{ji}$ for all $i,j = 1,2,\dots,m$.

The algorithm to calculate the Triple Refined Indeterminate Neutrosophic weighted distance matrix D is given in Algorithm 2.

Algorithm 1 Generalized Triple Refined Indeterminate Neutrosophic weighted distance $d_\lambda(A,B)$

Input:
  $X \leftarrow x_l, x_2, \dots, x_n$
  $A \leftarrow \{\langle x_i, T_A(x_i), I_{TA}(x_i), I_A(x_i), I_{FA}(x_i), F_A(x_i)\rangle \mid x_i \in X\}$
  $B \leftarrow \{\langle x_i, T_B(x_i), I_{TB}(x_i), I_B(x_i), I_{FB}(x_i), F_A(x_i)\rangle \mid x_i \in X\}$, $w_i(i = 1,2,\dots,n)$
Output: $d_\lambda(A,B)$
  procedure DISTANCE $d_\lambda(A,B)$
    $d_\lambda \leftarrow 0$
    for $i \leftarrow 1, n$ **do**
      $d_\lambda \leftarrow d_\lambda + w_i[\mid T_A(x_i) - T_B(x_i)\mid^\lambda$
      $+ \mid I_{TA}(x_i) - I_{TB}(x_i)\mid^\lambda + + \mid I_A(x_i) - I_B(x_i)\mid^\lambda$
      $+ \mid I_{FA}(x_i) - I_{FB}(x_i)\mid^\lambda + \mid F_A(x_i) - F_B(x_i)\mid^\lambda]$
    end for
    $d_\lambda \leftarrow d_\lambda /4$
    $d_\lambda \leftarrow d_\lambda^{\{\frac{1}{\lambda}\}}$
  end procedure

Algorithm 2 Triple Refined Indeterminate Neutrosophic weighted distance matrix D

  procedure DISTANCE MATRIX $D(A_1, \dots\dots, A_m)$
    for $i \leftarrow 1, m$ **do**
      for $j \leftarrow 1, m$ **do**
        if $i = j$ **then**
          $d_{ij} \leftarrow 0$
        else
          $d_{ij} \leftarrow \{d_\lambda\ (A_i, A_j)\}$
        end if
      end for
    end for
  end procedure

B. The Indeterminacy Based Open Extended Jungian type Scales Using TRINS

1) Sample Questionnaire: A sample questionnaire of the indeterminacy based Open Extended Jungian Type Scales personality test using TRINS will be as given in table II.

The user is expected to fill the degree accordingly.

Definition 2: Consider question 1, the different options would be

1) a degree of "make list",
2) a degree of indeterminacy choice towards making list,
3) a degree of uncertain and indeterminate combination of making list and depending on memory,

TABLE II. SAMPLE QUESTIONNAIRE OF THE INDETERMINACY BASED OEJTS

Q		Scale weight 1 2 3 4 5	
Q_1	makes lists	□ □ □ □ □	relies on memory
Q_2	sceptical	□ □ □ □ □	wants to believe
Q_3	bored by time alone	□ □ □ □ □	needs time alone
Q_7	energetic	□ □ □ □ □	mellow
Q_{11}	works best in groups	□ □ □ □ □	works best alone
Q_{15}	worn out by parties	□ □ □ □ □	gets fired up by parties
Q_{19}	talks more	□ □ □ □ □	listens more
Q_{23}	stays at home	□ □ □ □ □	goes out on the town
Q_{27}	finds it difficult to yell very loudly	□ □ □ □ □	yelling to others . . . comes naturally
Q_{31}	perform in public	□ □ □ □ □	avoids public speaking

4) a degree of indeterminate choice more of relying on memory, and
5) a degree of "relying on memory".

Suppose the user thinks and marks a degree of "make list" is 0.0, a degree of indeterminate choice towards "make list" is 0.4 , a degree of uncertain and indeterminate combination of making list and depending on memory is 0.1, an degree of indeterminate choice more of relying on memory 0.3, and a degree of "relying on memory" is 0.2.

A is a TRINS of $Q = \{q_1\}$ defined by

$$A = \langle 0.0, 0.4, 0.1, 0.3, 0.2\rangle/q_1.$$

When the weight of each membership is applied, the TRINS A becomes

$$A = \langle 0.0, 0.8, 0.3, 1.2, 1.0\rangle/q_1 \ ; \ w(A) = 3.3.$$

In the general test, a whole number value from 1 to 5 will be obtained, whereas in the indeterminacy based OEJTS an accurate value is obtained. Thus the accuracy of the test is evident.

2) Calculating Results: Depending on the questionnaire the following grouping was carried out

TRINS E is defined in the discourse $Q_E = \{Q_{15}, Q_{23}, Q_{27}\}$ deals with the extrovert aspect and the introvert aspect is defined by TRINS I which is defined in the discourse $Q_I = \{Q_3, Q_7, Q_{11}, Q_{19}, Q_{31}\}$. The Sensing versus Intuition dichotomy is given by TRINSs S and N; S is defined in the discourse $Q_S = \{Q_{24}, Q_{28}\}$ and N is defined in the discourse $Q_N = \{Q_4, Q_8, Q_{12}, Q_{16}, Q_{20}, Q_{32}\}$. Similarly Feeling versus Thinking dichotomy is given by TRINSs F and T; F is defined in the discourse $Q_F = \{Q_2, Q_{14}, Q_{18}, Q_{26}, Q_{30}\}$ and T is defined the discourse $Q_T = \{Q_6, Q_{10}, Q_{22}\}$. TRINSs J and P are used to represent the Judging versus Perceiving dichotomy; J is defined in the discourse $Q_J = \{Q_{17}, Q_{25}\}$ and P is defined in the discourse $Q_P = \{Q_1, Q_5, Q_{13}, Q_{21}, Q_{29}\}$.

The weight of a TRINS E is given in Equations 3. The calculation for scoring is as follows:

$$IE = 30 - w(I) + w(E)$$

$$SN = 12 - w(S) + w(N)$$

$$FT = 30 - w(F) + w(T)$$

$$JP = 18 - w(J) + w(P).$$

The score results are based on the following rules:

1) If IE is more than 24, you are extrovert (E), otherwise you are introvert (I).
2) If SN is more than 24, you are intuitive (N), otherwise you are sensing (S).
3) If FT is more than 24, you are thinking (T), otherwise you are feeling (F).
4) If JP is more than 24, you are perceiving (P), otherwise you are judging (J).

3) Comparing results of two people: Consider this personality test is taken by a group of people. Using the distance measure given in Algorithm 1 is defined over TRINS the difference and similarity in two or more person's personality can be analysed along a particular dichotomy. They can be analysed along Extroversion (E), Introversion (I), Intuitive (N), Sensing (S), Thinking (T), Feeling (F), Perceiving (P) or judging (J) or any combination of the eight. Clustering of the results using the distance matrix given in Algorithm 2 is carried out. The following section provides the Triple Refined Indeterminate Neutrosophic Minimum Spanning Tree (TRINS-MST) clustering algorithm using the distance matrix.

IV. TRINS-MST CLUSTERING ALGORITHM

In this section, a Triple Refined Indeterminate Neutrosophic Minimum Spanning Tree (TRIN-MST) clustering algorithm is proposed as a generalization of the IFMST and SVN-MST clustering algorithms.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be an attribution space and the weight vector of an element $x_i (i = 1, 2, \ldots, n)$ be $w = \{w_1, w_2, \ldots, w_n\}$, with $w_i \geq 0 (i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$. Consider that $A_j (j = 1, 2, \ldots, m)$ is a collection of m TRINSs, which has m samples that need to be clustered. Then, they are represented in the following form: $A_j = \{\langle x_i, T_{A_j}(x_j), I_{TA_j}(x_j), I_{A_j}(x_j), I_{FA_j}(x_j), F_{A_j}(x_j)\rangle \mid x_j \in X\}$ Algorithm 3 provides the Triple Refined Indeterminate Neutrosophic Minimum Spanning Tree (TRIN-MST) clustering algorithm. The description of the algorithm is:

Step 1: Calculate the distance matrix $D = d_{ij} = d_\lambda(A_i, A_j)$ by Algorithm 2 (take $\lambda = 2$). The Triple Refined Indeterminate Neutrosophic distance matrix $D = (d_{ij})_{m\times m}$ obtained is:

$$D = \begin{bmatrix} 0 & d_{12} & \ldots & d_{1m} \\ \vdots & \vdots & & \vdots \\ d_{m1} & d_{m2} & \ldots & 0 \end{bmatrix}.$$

Step 2: The Triple Refined Indeterminate Neutrosophic graph $G(V, E)$ where every edge between A_i and $A_j (i, j = 1, 2, \ldots, m)$ is assigned the Triple Refined Indeterminate Neutrosophic weighted distance d_{ij}, it is an element of the Triple Refined Indeterminate Neutrosophic distance matrix $D = (d_{ij})_{m\times m}$, which represents the dissimilarity degree between the samples A_i and A_j. The Triple Refined Indeterminate Neutrosophic graph $G(V, E)$ is represented as a graph.

Step 3: Construct the MST of the Triple Refined Indeterminate Neutrosophic graph $G(V, E)$.

1) The sorted list of distances of edges of $G(V, E)$ in increasing order by weights is constructed.

Algorithm 3 Triple Refined Indeterminate Neutrosophic Minimum Spanning Tree (TRIN-MST) Clustering algorithm

Input: $D = (d_{ij})_{m \times m}$
Output: MST S and Clusters
procedure TRIN-MST CLUSTERING(D)
  Step 1: Calculate distance matrix D of $A_1, \ldots, A_m$
  $D(A_1, \ldots, A_m)$ ▷ Distance matrix D is from Algo 2

  Step 2: Create graph $G(V, E)$
  for $i \leftarrow 1, m$ **do**
    for $j \leftarrow 1, m$ **do**
      if i != j **then**
        Draw the edge between A_i and A_j with d_{ij}
      end if
    end for
  end for

  Step 3: Compute the MST of the Triple Refined Indeterminate Neutrosophic graph $G(V, E)$:
  ▷ Using Kruskal's algorithm
  Sort all the edges in increasing order of weight in E.
  while No. of edges in subgraph S of $G < (V - 1)$ **do**
    Select the smallest edge (v_i, v_j).
    Delete (v_i, v_j) from E
    if (v_i, v_j) forms a cycle with spanning tree S **then**
      Discard the edge v_i, v_j
    else
      Include the edge v_i, v_j in S
    end if
  end while
  S is the MST of the Triple Refined Indeterminate Neutrosophic graph $G(V, E)$.

  Step 4: Perform clustering
  for $i \leftarrow 1, m$ **do**
    for $j \leftarrow 1, m$ **do**
      if $d_{ij} \geq r$ **then** ▷ r is the threshold
        Disconnect edge
      else
        Edge is not disconnected
      end if
    end for
  end for
  Results in clusters automatically; it is tabulated
end procedure

2) Keep an empty subgraph S of $G(V, E)$ and select the edge e with the smallest weight to add in S, where the end points of e are disconnected.
3) The smallest edge e is added to S and deleted from the sorted list.
4) The next smallest edge is selected and if no cycle is formed in S it is added to S and deleted from the list.
5) Repeat the process (iv) until the subgraph S has $(m-1)$ edges.

Thus, the MST of the Triple Refined Indeterminate Neutrosophic graph $G(V, E)$ is obtained, and illustrated as a graph.

Step 4: Select a threshold r and disconnect all the edges of the MST with weights greater than r to obtain a certain number of clusters, list it as a table. The clustering results induced by the subtrees do not depend on some particular MST [30], [31].

TABLE III. SAMPLE QUESTIONNAIRE OF THE INDETERMINACY BASED OEJTS

Q		T I_T I I_F F	
Q_7	mellow	□ □ □ □ □	energetic
Q_{11}	works best alone	□ □ □ □ □	works best in groups
Q_{15}	worn out by parties	□ □ □ □ □	gets fired up by parties
Q_{19}	listens more	□ □ □ □ □	talks more
Q_{23}	stays at home	□ □ □ □ □	goes out on the town
Q_{27}	finds it difficult to yell very loudly	□ □ □ □ □	yelling to others comes naturally

V. ILLUSTRATIVE EXAMPLES

A descriptive example is presented and utilized to demonstrate the real world applications and the effectiveness of the proposed TRIN-MST clustering algorithm using the results of the indeterminacy based personality test conducted for eight different people.

Definition 3: Eight people $A_j (j = 1, 2, \ldots, 8)$ had given the personality test. For each person six evaluation questions (attributes) were used as given in Table III. Questions related to only the Extroversion (E) vs. Introversion (I) have been considered, the questionnaire has been altered according so as to enable the using of distance measures.

$$
\begin{aligned}
A_1 = \{&\langle x_1, 0.9, .02, .05, 0.03, 0.0\rangle, \langle x_2, 0.8, 0.1, .05, .05, 0.0\rangle,\\
&\langle x_3, 0.7, 0.1, 0.1, 0.0, 0.1\rangle, \langle x_4, 0.8, 0.0, 0.0, 0.1, 0.1\rangle,\\
&\langle x_5, 0.75, 0.05, 0.1, 0.1, 0.0\rangle, \langle x_6, 0.8, 0.1, 0.05, 0.0, 0.05\rangle\},\\
A_2 = \{&\langle x_1, 0.8, 0.15, 0.0, 0.0, 0.05\rangle, \langle x_2, 0.7, 0.2, 0.1, 0.0, 0.0\rangle,\\
&\langle x_3, 0.75, 0.05, 0.1, 0.0, 0.1\rangle, \langle x_4, 0.7, 0.2, 0.0, 0.0, 0.1\rangle,\\
&\langle x_5, 0.8, 0.1, 0.1, 0.0, 0.0\rangle, \langle x_6, 0.6, 0.3, 0.1, 0.0, 0.0\rangle\},\\
A_3 = \{&\langle x_1, 0.5, 0.2, 0.1, 0.1, 0.1\rangle, \langle x_2, 0.6, 0.2, 0.1, 0.05, 0.05\rangle,\\
&\langle x_3, 0.5, 0.3, 0.05, 0.05, 0.1\rangle, \langle x_4, 0.75, 0.1, 0.05, 0.1, 0.0\rangle,\\
&\langle x_5, 0.5, 0.1, 0.3, 0.0, 0.1\rangle, \langle x_6, 0.6, 0.2, 0.0, 0.0, 0.2\rangle\},\\
A_4 = \{&\langle x_1, 0.0, 0.1, 0.0, 0.0, 0.9\rangle, \langle x_2, 0.0, 0.0, 0.0, 0.1, 0.9\rangle,\\
&\langle x_3, 0.1, 0.0, 0.0, 0.0, 0.9\rangle, \langle x_4, 0.0, 0.0, 0.0, 0.1, 0.9\rangle,\\
&\langle x_5, 0.0, 0.0, 0.1, 0.0, 0.9\rangle, \langle x_6, 0.0, 0.0, 0.0, 0.1, 0.9\rangle\},\\
A_5 = \{&\langle x_1, 0.7, 0.3, 0.0, 0.0, 0.0\rangle, \langle x_2, 0.6, 0.2, 0.1, 0.0, 0.1\rangle,\\
&\langle x_3, 0.5, 0.1, 0.2, 0.1, 0.1\rangle, \langle x_4, 0.7, 0.0, 0.1, 0.1, 0.1\rangle,\\
&\langle x_5, 0.5, 0.2, 0.1, 0.1, 0.1\rangle, \langle x_6, 0.3, 0.2, 0.2, 0.2, 0.1\rangle\},\\
A_6 = \{&\langle x_1, 0.5, 0.2, 0.1, 0.0, 0.1\rangle, \langle x_2, 0.0, 0.1, 0.1, 0.7, 0.1\rangle,\\
&\langle x_3, 0.0, 0.1, 0.1, 0.5, 0.3\rangle, \langle x_4, 0.1, 0.0, 0.1, 0.1, 0.7\rangle,\\
&\langle x_5, 0.0, 0.1, 0.2, 0.6, 0.1\rangle, \langle x_6, 0.0, 0.1, 0.1, 0.7, 0.1\rangle\},\\
A_7 = \{&\langle x_1, 0.4, 0.3, 0.2, 0.0, 0.1\rangle, \langle x_2, 0.5, 0.1, 0.2, 0.1, 0.1\rangle,\\
&\langle x_3, 0.6, 0.1, 0.1, 0.1, 0.1\rangle, \langle x_4, 0.5, 0.2, 0.1, 0.1, 0.1\rangle,\\
&\langle x_5, 0.5, 0.1, 0.1, 0.2, 0.1\rangle, \langle x_6, 0.7, 0.2, 0.0, 0.0, 0.1\rangle\},\\
A_8 = \{&\langle x_1, 0.1, 0.1, 0.1, 0.2, 0.5\rangle, \langle x_2, 0.0, 0.1, 0.1, 0.1, 0.7\rangle,\\
&\langle x_3, 0.0, 0.0, 0.0, 0.3, 0.7\rangle, \langle x_4, 0.1, 0.0, 0.2, 0.4, 0.3\rangle,\\
&\langle x_5, 0.1, 0.0, 0.0, 0.1, 0.8\rangle, \langle x_6, 0.2, 0.0, 0.0, 0.1, 0.7\rangle\}
\end{aligned}
$$

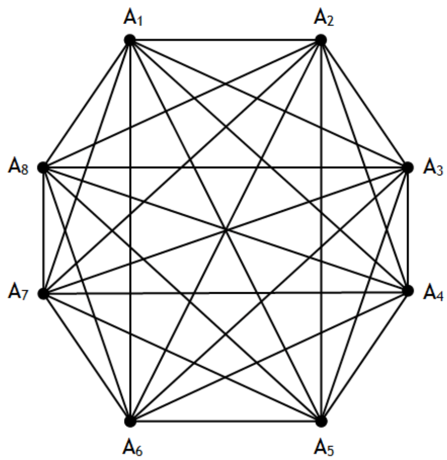

Fig. 1. Triple Refined Indeterminate Neutrosophic graph G

Let the weight vector of the attribute $x_i (i = 1, 2, \ldots, 6)$ be $w = (0.167, 0.167, 0.167, 0.167, 0.167, 0.167)^T$, then the TRIN-MST clustering algorithm given in Algorithm 3 is used to group the eight people of $A_j (j = 1, 2, \ldots, 8)$.

Step 1: Calculate the distance matrix $D = d_{ij} = d_\lambda(A_i, A_j)$ by Algorithm 2 (take $\lambda = 2$). The Triple Refined Indeterminate Neutrosophic distance matrix $D = (d_{ij})_{m\times m}$ is obtained as follows:

$$D = \begin{bmatrix} 0 & 0.0860 & 0.1384 & 0.5214 & 0.1512 & 0.4035 & 0.1588 & 0.4305 \\ 0.086 & 0 & 0.1284 & 0.507 & 0.1263 & 0.3938 & 0.139 & 0.4213 \\ 0.1384 & 0.1284 & 0 & 0.4534 & 0.1175 & 0.3539 & 0.1024 & 0.3536 \\ 0.5214 & 0.507 & 0.4534 & 0 & 0.453 & 0.3944 & 0.4394 & 0.186 \\ 0.1512 & 0.1263 & 0.1175 & 0.453 & 0 & 0.3155 & 0.134 & 0.3558 \\ 0.4035 & 0.3938 & 0.3539 & 0.3944 & 0.3155 & 0 & 0.3258 & 0.3238 \\ 0.1588 & 0.139 & 0.1024 & 0.4394 & 0.134 & 0.3258 & 0 & 0.3375 \\ 0.4305 & 0.4213 & 0.3536 & 0.186 & 0.3558 & 0.3238 & 0.3375 & 0 \end{bmatrix}.$$

Step 2: The Triple Refined Indeterminate Neutrosophic graph $G(V, E)$ where every edge between A_i and $A_j (i, j = 1, 2, \ldots, 8)$ is assigned the Triple Refined Indeterminate Neutrosophic weighted distance d_{ij}, it is an element of the Triple Refined Indeterminate Neutrosophic distance matrix $D = (d_{ij})_{m\times m}$, which represents the dissimilarity degree between the samples A_i and A_j. The Triple Refined Indeterminate Neutrosophic graph $G(V, E)$ is shown in Figure 1.

Step 3: Construct the MST of the Triple Refined Indeterminate Neutrosophic graph $G(V, E)$.

1) The sorted list of distances of edges of G in increasing order by weights is: $d_{12} \leq d_{37} \leq d_{35} \leq d_{25} \leq d_{23} \leq d_{57} \leq d_{72} \leq d_{51} \leq d_{71} \leq d_{84} \leq d_{65} \leq d_{68} \leq d_{67} \leq d_{87} \leq d_{83} \leq d_{63} \leq d_{86} \leq d_{62} \leq d_{64}$.
2) Keep an empty subgraph S of G and add the edge e with the smallest weight to S, where the end points of e are disconnected.
3) The edge between A_1 and A_2; $d_{12} = 0.086$, is the smallest, it is added to S and deleted from the sorted list.
4) The next smallest edge is selected from G and if no cycle is formed in S it is added to S and deleted from the list.
5) Repeat process (4) until the subgraph S has $(7 - 1)$ edges or spans eight nodes.

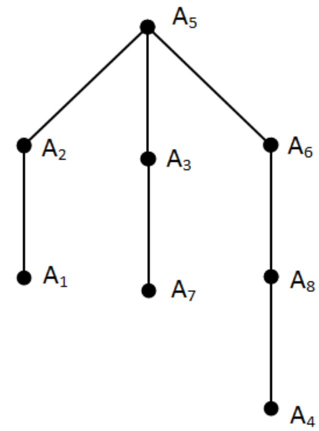

Fig. 2. Triple Refined Indeterminate Neutrosophic minimum spanning tree (TRIN-MST) S of graph G

TABLE IV. CLUSTERING RESULTS OF THE EIGHT DIFFERENT CARS USING TRIN-MST CLUSTERING ALGORITHM

Threshold r	Corresponding clustering result
$r = d_{68} = 0.3238$	$\{A_1, A_2, A_3, A_5, A_6, A_7\}, \{A_4, A_8\}$
$r = d_{56} = 0.3155$	$\{A_1, A_2, A_3, A_5, A_7\}, \{A_4, A_8\}, \{A_6\}$
$r = d_{84} = 0.186$	$\{A_1, A_2, A_3, A_5, A_7\}, \{A_4\}, \{A_8\}, \{A_6\}$
$r = d_{25} = 0.1263$	$\{A_1, A_2\}, \{A_3, A_5, A_7\}, \{A_4\}, \{A_6\}, \{A_8\}$
$r = d_{53} = 0.1175$	$\{A_1, A_2\}, \{A_3, A_7\}, \{A_4\}, \{A_5\}, \{A_6\}, \{A_8\}$
$r = d_{37} = 0.1024$	$\{A_1, A_2\}, \{A_3\}, \{A_4\}, \{A_5\}, \{A_6\}, \{A_7\}, \{A_8\}$
$r = d_{12} = 0.086$	$\{A_1\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_6, A_5\}, \{A_7\}, \{A_8\}$
$r = 0$	$\{A_1\}, \{A_2\}, \{A_3\}, \{A_4\}, \{A_5\}, \{A_6\}, \{A_7\}, \{A_8\}$

Thus, the MST of the Triple Refined Indeterminate Neutrosophic graph $G(V, E)$ is obtained, as illustrated in Figure 2.

Step 4: Select a threshold r and disconnect all the edges of the MST with weights greater than r to obtain a certain number of subtrees (clusters), as listed in Table IV.

The results of the clustering algorithm clearly shows when the threshold r is 0.3238 the clusters are of Extroversion (E) vs. Introversion (I), it is seen that A_4 and A_8 are introverts and the rest are extroverts.

VI. COMPARISON

The existing classic personality test force the test taker to select only one option and it is mostly what the user thinks he/she does often. The other options are lost to the test taker. It fails to capture the complete picture realistically. The dominant choice is selected, the selection might have very small margin. In such cases the accuracy of the test fails. Whereas when the indeterminacy based OEJTS Test is considered, it provides five different options to the test taker using TRINS for representing the choice. It is important to understand why TRINS makes the candidate for this kind of Personality test. The reason can be obtained by the following comparative analysis of the methods and their capacity to deal indeterminate, inconsistent and incomplete information. TRINS is an instance of a neutrosophic set, which approaches the problem more logically with accuracy and precision to represent the existing uncertainty, imprecise, incomplete, and inconsistent information. It has the additional feature of being able to describe with more sensitivity the indeterminate and inconsistent information. TRINS alone can give scope for a person to express accurately the exact realistic choices instead of opting for a dominant choice. While, the SVNS can handle indeterminate information and inconsistent information, it is cannot describe with accuracy about the existing indeterminacy. It is known that the connector in fuzzy set is defined with respect to T (membership only) so the information of indeterminacy and non membership is lost. The connectors in intuitionistic fuzzy set are defined with respect to truth membership and false membership only; here the indeterminacy is taken as what is left after the truth and false membership. Hence a personality test based on TRINS gives the most accurate and realistic result, cause it captures the complete scenario realistically. The TRIN-MST clustering algorithm is the only clustering algorithm that uses the existing uncertainty, imprecise, incomplete, and inconsistent information to capture the human responses in a personality test. The TRIN-MST clustering algorithm is capable of clustering people according to their personality with more accuracy and precision than the existing personality test.

VII. Conclusions

In objective type personality test like the MBTI or the OEJTS, the user is forced to selct an option, and mostly lands up selecting the most dominant choice. The rest of the options are lost. A person may not be in general capable to judge his/her behaviour very precisely and categorize it into a particular choice. Since it is the person doing self rating there is a lot of uncertain and indeterminate feelings involved. The results of the test depend on a number of internal and external factors. To provide a more accurate and realistic result, a personality test needs to provide more choices and a degree of acceptance with that particular choice. To represent the Likert scale using neutrosophy, the concept of Triple Refined Indeterminate Neutrosophic Set (TRINS) was utilized. More precision is provided in handling indeterminacy; by classifying indeterminacy (I) into three, based on membership; as indeterminacy leaning towards truth membership (I_T), indeterminacy membership (I) and indeterminacy leaning towards false membership (I_F). TRINS can be used in any place where the Likert scale is used like personality test. In this paper, the indeterminacy based personality test based on the OEJTS and TRINS was utilized and the TRIN-MST clustering algorithm was proposed. The calculation of results and personality grouping using the TRIN-MST clustering algorithm was discussed. The personality of a group of people was clustered using TRIN-MST clustering algorithm. An illustrative example using eight people was carried out and the cluster results of extroverts and introverts was clearly seen.

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