Classical Logic and Neutrosophic Logic.

Answers to K. Georgiev.

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Abstract.

In this paper we make distinctions between Classical Logic (where the propositions are 100% true, or 100 false) and the Neutrosophic Logic (where one deals with partially true, partially indeterminate and partially false propositions) in order to respond to K. Georgiev [1]'s criticism. We recall that if an axiom is true in a classical logic system, it is not necessarily that the axiom be valid in a modern (fuzzy, intuitionistic fuzzy, neutrosophic etc.) logic system.

1. Single Valued Neutrosophic Set.

We read with interest the paper [1] by K. Georgiev. The author asserts that he proposes "a general simplification of the Neutrosophic Sets a subclass of theirs, comprising of elements of R^3", but this was actually done before, since the first world publication on neutrosophics [2]. The simplification that Georgiev considers, is called single valued neutrosophc set.

The single valued neutrosophic set was introduced for the first time by us [Smarandache, [2], 1998].

Let $n = t + i + f$. (1)
In Section 3.7, "Generalizations and Comments", [pp. 129, last edition online], from this book [2], we wrote:

"Hence, the neutrosophic set generalizes:
- the intuitionistic set, which supports incomplete set theories (for $0 < n < 1$; $0 \leq t, i, f \leq 1$) and incomplete known elements belonging to a set;
- the fuzzy set (for $n = 1$ and $i = 0$, and $0 \leq t, i, f \leq 1$);
- the classical set (for $n = 1$ and $i = 0$, with t, f either 0 or 1);
- the paraconsistent set (for $n > 1$, with all $t, i, f < 1$);
- the faillibilist set ($i > 0$);
- the dialetheist set, a set M whose at least one of its elements also belongs to its complement C(M); thus, the intersection of some disjoint sets is not empty;
- the paradoxist set ($t = f = 1$);
- the pseudoparadoxist set ($0 < i < 1$; $t =1$ and $f > 0$ or $t > 0$ and $f = 1$);
- the tautological set ($i, f < 0$)."

It is clear that we have worked with single-valued neutrosophic sets, we mean that t, i, f were explicitly real numbers from [0, 1].
See also (Smarandache, [3], 2002, p. 426).

More generally, we have considered that: t varies in the set T, i varies in the set I, and f varies in the set F, but in the same way taking crisp numbers n = t + i + f, where all t, i, f are single (crisp) real numbers in the interval [0, 1]. See [2] pp. 123-124, and [4] pp. 418-419.

Similarly in *The Free Online Dictionary of Computing [FOLDOC]*, 1998, updated in 1999, edited by Denis Howe [3].

Unfortunately, Dr. Georgiev in 2005 took into consideration only the neutrosophic publication [6] from year 2003, and he was not aware of previous publications [2, 3, 4] on the neutrosophics from the years 1998 - 2002.

The misunderstanding was propagated to other authors on neutrosophic set and logic, which have considered that Haibin Wang, Florentin Smarandache, Yanqing Zhang, Rajshekhar Sunderraman (2010, [5]) have defined the single valued neutrosophic set.

2. Standard and Non-Standard Real Subsets.

Section 3 of paper [1] by Georgiev is called "Reducing Neutrosophic Sets to Subsets of R^3". But this was done already since 1998. In our Section 0.2, [2], p. 12, we wrote:

> *"Let T, I, F be standard or non-standard real subsets…".*

"Standard real subsets", which we talked about above, mean just the classical real subsets.

We have taken into consideration the non-standard analysis in our attempt to be able to describe the absolute truth as well [i.e. truth in all possible worlds, according to Leibniz's denomination, whose neutrosophic value is equal to 1^+], and relative truth [i.e. truth in at least one world, whose truth value is equal to *1*]. Similarly for absolute indeterminacy and absolute falsehood.

We tried to get a definition as general as possible for the neutrosophic logic (and neutrosophic set respectively), including the propositions from a philosophical point of [absolute or relative] view.

Of course, in technical and scientific applications we do not consider non-standard things, we take the classical unit interval [0, 1] only, while T, I, F are classical real subsets of it.

In Section 0.2, Definition of Neutrosophic Components [2], 1998, p. 12, we wrote:

> *"The sets T, I, F are not necessarily intervals, but may be any real sub-unitary subsets: discrete or continuous; single-element, finite, or (countable or uncountable) infinite; union or intersection of various subsets; etc.*
> *They may also overlap. The real subsets could represent the relative errors in determining t, i, f (in the case when the subsets T, I, F are reduced to points)."*

So, we have mentioned many possible real values for T, I, F. Such as: each of T, I, F can be "single-element" {as Georgiev proposes in paper [1]}, "interval" {developed later in [7], 2005, and called interval-neutrosophic set and interval-neutrosophic logic respectively}, "discrete" [called hesitant neutrosophic set and hesitant neutrosophic logic respectively] etc.

3. Degrees of Membership > 1 or < 0 of the Elements.

In Section 4 of paper [1], Georgiev says that: “Smarandache has adopted Leibniz’s ‘worlds’ in his work, but it seems to be more like a game of words.”

As we have explained above, “Leibniz’s worlds” are not simply a game of words, but they help making a distinction in philosophy between absolute and relative truth / indeterminacy / falsehood respectively. {In technics and science yes they are not needed.}

Besides absolute and relative, the non-standard values or hyper monads ($^-0$ and 1^+) have permitted us to introduce, study and show applications of the neutrosophic overset (when there are elements into a set whose real (standard) degree of membership is > 1), neutrosophic underset (when there are elements into a set whose real degree of membership is < 0), and neutrosophic offset (when there are both elements whose real degree of membership is > 1 and other elements whose real degree of membership is < 0). Check the references [8-11].

4. Neutrosophic Logic Negations.

In Section 4 of the same paper [1], Georgiev asserts that

“according to the neutrosophic operations we have

$$\neg\neg A = A \qquad (2)$$

and since

$$\neg\neg A \neq A \qquad (3)$$

is just the assumption that has brought intuitionism to life, the neutrosophic logic could not be a generalization of any Intuitionistic logic.”

First of all, Georgiev’s above assertation is partially true, partially false, and partially indeterminate (as in the neutrosophic logic).

In neutrosophic logic, there is a class of neutrosophic negation operators, not only one. For some neutrosophic negations the equality (2) holds, for others it is invalid, or indeterminate.

Let $A(t, i, f)$ be a neutrosophic proposition A whose neutrosophic truth value is (t, i, f), where t, i, f are single real numbers of $[0, 1]$. We consider the easiest case.

a) For examples, if the neutrosophic truth value of $\neg A$, the negation of A, is defined as:

$$(1-t, 1-i, 1-f) \text{ or } (f, i, t) \text{ or } (f, 1-i, t) \qquad (4)$$

then the equality (2) is valid.

b) Other examples, if the neutrosophic truth value of $\neg A$, the negation of A, is defined as:

$$(f, (t+i+f)/3, t) \text{ or } (1-t, (t+i+f)/3, 1-f) \qquad (5)$$

then the equality (2) is invalid, as in intuitionistic fuzzy logic, and as a consequence the inequality (3) holds.

c) For the future new to be designed/invented neutrosophic negations (needed/adjusted for new applications) we do not know {so (2) has also a percentage of indeterminacy.

5. Degree of Dependence and Independence between (Sub)Components.

In Section 4 of [1], Georgiev also asserts that "The neutrosophic logic is not capable of maintaining modal operators, since there is no normalization rule for the components T, I, F". This is also partially true, and partially false.

In our paper [12] about the dependence / independence between components, we wrote that:

> "For single valued neutrosophic logic, the sum of the components t+i+f is:
> $0 \leq t+i+f \leq 3$ when all three components are 100% independent;
> $0 \leq t+i+f \leq 2$ when two components are 100% dependent, while the third one is 100% independent from them;
> $0 \leq t+i+f \leq 1$ when all three components are 100% dependent.
> When three or two of the components t, i, f are 100% independent, one leaves room for incomplete information (therefore the sum $t+i+f < 1$), paraconsistent and contradictory information ($t+i+f > 1$), or complete information ($t+i+f = 1$).
> If all three components t, i, f are 100% dependent, then similarly one leaves room for incomplete information ($t+i+f < 1$), or complete information ($t+i+f = 1$)."

Therefore, for complete information the normalization to 1, 2, 3 or so respectively {see our paper [12] for the case when one has degrees of dependence between components or between subcomponents (for refined neutrosophic set respectively) which are different from 100% or 0%} is done.

But, for incomplete information and paraconsistent information, in general, the normalization is not done.

Neutrosophic logic is capable of maintaining modal operators. The connection between Neutrosophic Logic and Modal Logic will be shown in a separate paper, since it is much longer, called Neutrosophic Modal Logic (under press).

6) Definition of Neutrosophic Logic.

In Section 5, paper [1], it is said: "Apparently there isn't a clear definition of truth value of the neutrosophic formulas." The author is right that "apparently", but in reality the definition of neutrosophic logic is very simple and common sense:

In neutrosophic logic a proposition P has a degree of truth (T); a degree of indeterminacy (I) that means neither true nor false, or both true and false, or unknown, indeterminate; and a degree of falsehood (F); where T, I, F are subsets (either real numbers, or intervals, or any subsets) of the interval [0, 1].

What is unclear herein?

In a soccer game, as an easy example, between two teams, Bulgaria and Romania, there is a degree of truth about Bulgaria winning, degree of indeterminacy (or neutrality) of tie game, and degree of falsehood about Bulgaria being defeated.

7) Neutrosophic Logical Systems.

a) Next sentence of Georgiev is "in every meaningful logical system if A and B are sets (formulas) such that $A \subseteq B$ then B ‘ A, i.e. when B is true then A is true." (6)

In other words, when $B \rightarrow A$ (B implies A), and B is true, then A is true.

This is true for the Boolean logic where one deals with 100% truths, but in modern logics we work with partial truths.

If an axiom is true in the classical logic, it does not mean that that axiom has to be true in the modern logical system. Such counter-example has been provided by Georgiev himself, who pointed out that the law of double negation {equation (2)}, which is valid in the classical logic, is not valid any longer in intuitionistic fuzzy logic.

A similar response we have with respect to his above statement on the logical system axiom (6): it is partially true, partially false, and partially indeterminate. All depend on the types of chosen neutrosophic implication operators.

In neutrosophic logic, let's consider the neutrosophic propositions $A(t_A, i_A, f_A)$ and $B(t_B, i_B, f_B)$,

and the neutrosophic implication:

$$B(t_B, i_B, f_B) \rightarrow A(t_A, i_A, f_A), \qquad (7)$$

that has the neutrosophic truth value

$$(B \rightarrow A)(t_{B\rightarrow A}, i_{B\rightarrow A}, f_{B\rightarrow A}). \qquad (8)$$

Again, we have a class of many neutrosophic implication operators, not only one; see our publication [13], 2015, pp. 79-81.

Let's consider one such neutrosophic implication for single valued neutrosophic logic:

$(B \rightarrow A)(t_{B\rightarrow A}, i_{B\rightarrow A}, f_{B\rightarrow A})$ *is equivalent to* $B(t_B, i_B, f_B) \rightarrow A(t_A, i_A, f_A)$

which is equivalent to $\neg B(f_B, 1\text{-}i_B, t_B) \vee A(t_A, i_A, f_A)$

which is equivalent to $(\neg B \vee A)(max\{f_B, t_A\}, min\{1\text{-}i_B, i_A\}, min\{t_B, f_A\}).$ (9)

Or:

$$(t_{B\rightarrow A}, i_{B\rightarrow A}, f_{B\rightarrow A}) = (max\{f_B, t_A\}, min\{1\text{-}i_B, i_A\}, min\{t_B, f_A\}). \qquad (10)$$

Now, a question arises: what does "(B $\rightarrow$) A is true" mean in fuzzy logic, intuitionistic fuzzy logic, and respectively in neutrosophic logic?

Similarly for the “B is true”, what does it mean in these modern logics? Since in these logics we have infinitely many truth values $t(B) \in (0, 1)$; {we made abstraction of the truth values 0 and 1, which represent the classical logic}.

b) Theorem 1, by Georgiev, “Either A H k(A) [i.e. A is true if and only if k(A) is true] or the neutrosophic logic is contradictory.”

We prove that his theorem is a nonsense.

First at all, the author forgets that when he talks about neutrosophic logic he is referring to a modern logic, not to the classical (Boolean) logic. The logical propositions in neutrosophic logic are partially true, in the form of (t, i, f), not totally 100% true or (1, 0, 0). Similarly for the implications and equivalences, they are not classical (i.e. 100% true), but partially true {i.e. their neutrosophic truth values are in the form of (t, i, f) too}.

- The author starts using the previous classical logical system axiom (6), i.e.

“since $k(A) \subseteq A$ we have A ‘ *k(A)* ” meaning that

$A \rightarrow$ k(A) and when A is true, then k(A) is true.

- Next Georgiev’s sentence: “Let assume k(A) be true and assume that A is not true”.

The same comments as above:

What does it mean in fuzzy logic, intuitionistic fuzzy logic, and neutrosophic logic that a proposition is true? Since in these modern logics we have infinitely many values for the truth value of a given proposition. Does, for example, $t(k(A)) = 0.8$ {i.e. the truth value of *k(A)* is equal to 0.8}, mean that *k(A)* is true?

If one takes $t(k(A)) = 1$, then one falls in the classical logic.

Similarly, what does it mean that proposition A is not true? Does it mean that its truth value

$t(A) = 0.1$ or in general $t(A) < 1$? Since, if one takes $t(A) = 0$, then again we fall into the classical logic.

The author confuses the classical logic with modern logics.

- In his “proof” he states that “since the Neutrosophic logic is not an intuitionistic one, $\neg A$ should be true leading to the conclusion that $k(\neg A) = \neg k(A)$ is true”.

For the author an “intuitionistic logic” means a logic that invalidates the double negation law {equation (3)}. But we have proved before in Section 4, of this paper, that depending on the type of neutrosophic negation operator used, one has cases when neutrosophic logic invalidates the double negation law [hence it is “intuitionistic” in his words], cases when the neutrosophic logic does not invalidate the double negation law {formula (2)}, and indeterminate cases {depending on the new possible neutrosophic negation operators to be design in the future}.

- The author continues with “We found that $k(A) \wedge \neg k(A)$ is true which means that the simplified neutrosophic logic is contradictory.”

Georgiev messes up the classical logic with modern logic. In classical logic, indeed

$k(A) \wedge \neg k(A)$ is false, being a contradiction.

But we are surprised that Georgiev does not know that in modern logic we may have

$k(A) \wedge \neg k(A)$ that is not contradictory, but partially true and partially false.

For example, in fuzzy logic, let’s say that the truth value (t) of $k(A)$ is

$t(k(A)) = 0.4$, then the truth value of its negation, $\neg k(A)$, is $t(\neg k(A)) = 1 – 0.4 = 0.6$.

Now, we apply the t-norm “min” in order to do the fuzzy conjunction, and we obtain:

$t(k(A) \wedge \neg k(A)) = \min\{0.4, 0.6\} = 0.4 \neq 0$.

Hence, $k(A) \wedge \neg k(A)$ is not a contradiction, since its truth value is 0.4, not 0.

Similarly in intuitionistic fuzzy logic.

The same in neutrosophic logic, for example:

Let the neutrosophic truth value of $k(A)$ be (0.5, 0.4, 0.2), that we denote as:

$k(A)(0.5, 0.4, 0.2)$, then its negation $\neg k(A)$ will have the neutrosophic truth value:

$\neg k(A)(0.2, 1\text{-}0.4, 0.5) = \neg k(A)(0.2, 0.6, 0.5)$.

Let’s do now the neutrosophic conjunction:

$k(A)(0.5, 0.4, 0.2) \wedge \neg k(A)(0.2, 0.6, 0.5) = (k(A) \wedge \neg k(A))(\min\{0.5, 0.2\}, \max\{0.4, 0.6\}, \max\{0.2, 0.5\}) = (k(A) \wedge \neg k(A))(0.2, 0.6, 0.5)$.

In the same way, $k(A) \wedge \neg k(A)$ is not a contradiction in neutrosophic logic, since its neutrosophic truth value is (0.2, 0.6, 0.5), which is different from (0, 0, 1) or from (0, 1, 1).

Therefore, Georgiev’s “proof” that the simplified neutrosophic logic [= single valued neutrosophic logic] is a contradiction has been disproved!

His following sentence, “But since the simplified neutrosophic logic is only a subclass of the neutrosophic logic, then the neutrosophic logic is a contradiction” is false. Simplified neutrosophic logic is indeed a subclass of the neutrosophic logic, but he did not prove that the so-called simplified neutrosophic logic is contradictory (we have showed above that his “proof” was wrong).

8. Conclusion.

We have showed in this paper that Georgiev's critics on the neutrosophic logic are not founded. We made distinctions between the Boolean logic systems and the neutrosophic logic systems.

Neutrosophic logic is developing as a separate entity with its specific neutrosophic logical systems, neutrosophic proof theory and their applications.

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