# Smarandache数列几个渐近公式*

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摘 要: 通过对数列 $a(n)$的研究,利用 Euler求和公式及解析方法,得出几个有趣的渐近公式.

关键词: 平方根;数论函数; Euler渐近公式

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1993年,罗马尼亚数论专家 F. Smarandache教授在文献[1]中提出了 100个尚未解决的问题,引起了广大学者的极大兴趣,其中第 80个问题为:

平方根序列: 0, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 8, 8,…

对于上述数列,我们定义

$a(n)=[\sqrt{n}]$　$n=0, 1, 2, 3\cdots$;其中 $[n]$为不超过 $n$的最大整数.

关于这个数列, F. Smarandache教授要求我们研究它的性质,作者在对已有成果研究的基础上,对文献[2]作了进一步的推广,得出几个有趣的渐近公式.

## 1 相关定理及其证明

### 1.1 $a(n)^{\frac{1}{3}}$平均值

**定理 1** 设 $n$是正整数,$a(n)=[\sqrt{n}]$,则我们有:

$$\sum_{n\le x} a(n)^{\frac{1}{3}} = \frac{6}{7}x^{\frac{7}{6}} + \frac{3}{4}x^{\frac{4}{3}} + C(x^{\frac{2}{3}})$$

证明: 因对于任意正整数 $x$,一定存在正整数 $N$,使得 $N^2\le x<(N+1)^2$,于是,我们有

$$\sum_{n\le x} a(n)^{\frac{1}{3}} = \sum_{n\le x}[\sqrt{n}]^{\frac{1}{3}} = \sum_{1^2\le i<2^2}[\sqrt{i}]^{\frac{1}{3}} + \sum_{2^2\le i<3^2}[\sqrt{i}]^{\frac{1}{3}} + \cdots + \sum_{N^2\le i\le x<(N+1)^2}[\sqrt{i}]^{\frac{1}{3}} + O(N^{\frac{1}{3}}) =$$

$$3*1^{\frac{1}{3}} + 5*2^{\frac{1}{3}} + \cdots + [(N+1)^2-N^2]N^{\frac{1}{3}} + O(N^{\frac{1}{3}}) =$$

$$\sum_{j\le N}(2j+1)j^{\frac{1}{3}} + O(N^{\frac{1}{3}}) = 2\sum_{j\le N} j^{\frac{4}{3}} + \sum_{j\le N} j^{\frac{1}{3}} + O(N^{\frac{1}{3}}) =$$

$$\frac{6}{7}N^{\frac{7}{3}} + \frac{3}{4}N^{\frac{4}{3}} + O(N^{\frac{4}{3}}) = \frac{6}{7}x^{\frac{7}{6}} + \frac{3}{4}x^{\frac{2}{3}} + O(x^{\frac{2}{3}})$$

**定理 2** 设 $n$是正整数,$a(n)=[n^{\frac{1}{3}}]$,则我们有:

$$\sum_{n\le x} a(n)^{\frac{1}{3}} = \frac{9}{10}x^{\frac{10}{9}} + O(x^{\frac{7}{9}})$$

证明 因对于任意整数 $x$,一定存在正整数 $N$,使得 $N^2\le x<(N+1)^2$,于是,我们有

$$\sum_{n\le x} a(n)^{\frac{1}{3}} = \sum_{n\le x}[n^{\frac{1}{3}}] = \sum_{1^3\le i<2^3}[i^{\frac{1}{3}}]^{\frac{1}{3}} + \sum_{2^3\le i<3^3}[i^{\frac{1}{3}}]^{\frac{1}{3}} + \cdots + \sum_{N^3\le i\le x<(N+1)^3}[i^{\frac{1}{3}}]^{\frac{1}{3}} =$$

$$7*1^{\frac{1}{3}} + 19*2^{\frac{1}{3}} + \cdots + [(N+1)^3-N^3]N^{\frac{1}{3}} + O(N^{\frac{1}{3}}) =$$

$$\sum_{j\le N}(3j^2+3j+1)j^{\frac{1}{3}} + O(N^{\frac{1}{3}}) = 3\sum_{j\le N} j^{\frac{7}{3}} + 3\sum_{j\le N} j^{\frac{4}{3}} + \sum_{j\le N} j^{\frac{1}{3}} + O(N^{\frac{1}{3}}) =$$

$$\frac{9}{10}N^{\frac{10}{3}} + O(N^{\frac{7}{3}}) = \frac{9}{10}x^{\frac{10}{9}} + O(x^{\frac{9}{7}})$$

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## 1.2　$a(n)^{\frac{1}{2}}$ 平均值

**定理 3**　设 $n,k$是正整数,$a(n)=[n^{\frac{1}{k}}]$,则我们有:

$$\sum_{n\le x} a(n)^{\frac{1}{2}}=\frac{2k}{2k+1}x^{\frac{2k+1}{2k}}+O(x^{\frac{2k-1}{2k}})$$

证明:

$$\sum_{n\le x}(a(n))^{\frac{1}{2}}=\sum_{n\le x}[n^{\frac{1}{k}}]^{\frac{1}{2}}=$$
$$\sum_{1^k\le i<2^k}[i^{\frac{1}{k}}]^{\frac{1}{2}}+\sum_{2^k\le i<3^k}[i^{\frac{1}{k}}]^{\frac{1}{2}}+\cdots+$$
$$\sum_{N^k\le i\le x<(N+1)^k}[i^{\frac{1}{k}}]^{\frac{1}{2}}+O(N^{\frac{1}{2}})=$$
$$\sum_{j\le N}[(j+1)^k-j^k]j^{\frac{1}{2}}+O(N^{\frac{1}{2}})=$$
$$\sum_{j\le N}(C_k^1 j^{k-1+\frac{1}{2}}+C_k^2 j^{k-2+\frac{1}{2}}+\cdots+C_k^k j^{\frac{1}{2}})+$$
$$O(N^{\frac{1}{2}})=k\frac{N^{k+\frac{1}{2}}}{k+\frac{1}{2}}+O(N^{k-\frac{1}{2}})=$$
$$\frac{2k}{2k+1}x^{\frac{2k+1}{2k}}+O(x^{\frac{2k-1}{2k}})$$

## 1.3　$a(n)^{\frac{1}{k}}$ 平均值

**定理 4**　设 $n,k$是正整数,$a(n)=[n^{\frac{1}{2}}]$,则我们有:

$$\sum_{n\le x} a(n)^{\frac{1}{2}}=\frac{2k}{2k+1}x^{\frac{2k+1}{2k}}+O(x^{\frac{k+1}{2k}})$$

证明:

$$\sum_{n\le x}(a(n))^{\frac{1}{k}}=\sum_{n\le x}[n^{\frac{1}{2}}]^{\frac{1}{k}}=$$
$$\sum_{1^2\le i<2^2}[i^{\frac{1}{2}}]^{\frac{1}{k}}+\sum_{2^2\le i<3^2}[i^{\frac{1}{2}}]^{\frac{1}{k}}+\cdots+$$
$$\sum_{N^2\le i\le x<(N+1)^2}[i^{\frac{1}{2}}]^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$3*1^{\frac{1}{k}}+5*2^{\frac{1}{k}}+\cdots+$$
$$[(N+1)^2-N^2]N^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$\sum_{j\le N}(2j+1)j^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$2\sum_{j\le N}j^{\frac{k+1}{k}}+\sum_{j\le N}j^{\frac{1}{k}}+$$
$$O(N^{\frac{1}{k}})=\frac{2k}{2k+1}N^{\frac{2k+1}{2k}}+O(N^{\frac{k+1}{k}})$$
$$\frac{2k}{2k+1}x^{\frac{2k+1}{2k}}+O(x^{\frac{k+1}{2k}})$$

**定理 5**　设 $n,k$是正整数,$a(n)=[n^{\frac{1}{3}}]$,则我们有:

$$\sum_{n\le x} a(n)^{\frac{1}{k}}=\frac{3k}{3k+1}x^{\frac{3k+1}{3k}}+O(x^{\frac{2k+1}{3k}})$$

证明:

$$\sum_{n\le x}(a(n))^{\frac{1}{k}}=\sum_{n\le x}[n^{\frac{1}{3}}]^{\frac{1}{k}}=$$
$$\sum_{1^3\le i<2^3}[i^{\frac{1}{3}}]^{\frac{1}{k}}+\sum_{2^3\le i<3^3}[i^{\frac{1}{3}}]^{\frac{1}{k}}+\cdots+$$
$$\sum_{N^3\le i\le x<(N+1)^3}[i^{\frac{1}{3}}]^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$7*1^{\frac{1}{k}}+19*2^{\frac{1}{k}}+\cdots+$$
$$[(N+1)^3-N^3]N^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$\sum_{j\le N}(3j^2+3j+1)j^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$3\sum_{j\le N}j^{\frac{2k+1}{k}}+3\sum_{j\le N}j^{\frac{k+1}{k}}+\sum_{j\le N}j^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$\frac{3k}{3k+1}N^{\frac{3k+1}{k}}+O(N^{\frac{2k+1}{k}})=$$
$$\frac{3k}{3k+1}x^{\frac{3k+1}{3k}}+O(x^{\frac{2k+1}{3k}})$$

**定理 6**　设 $n,k$是正整数,$a(n)=[n^{\frac{1}{k}}]$,则我们有:

$$\sum_{n\le x}(a(n))^{\frac{1}{k}}=\frac{k^2}{k^2+1}x^{\frac{k^2+1}{k^2}}+O(x^{\frac{k^2-k+1}{k^2}})$$

证明:

$$\sum_{n\le x}(a(n))^{\frac{1}{k}}=\sum_{n\le x}[n^{\frac{1}{k}}]^{\frac{1}{k}}=$$
$$\sum_{1^k\le i<2^k}[n^{\frac{1}{k}}]^{\frac{1}{k}}+\sum_{2^k\le i<3^k}[n^{\frac{1}{k}}]^{\frac{1}{k}}+\cdots+$$
$$\sum_{N^k\le i\le x<(N+1)^k}[n^{\frac{1}{k}}]^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$\sum_{j\le N}[(j+1)^k-j^k]j^{\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$\sum_{j\le N}\sum_{l=1}^{k}C_k^l j^{k-l+\frac{1}{k}}+O(N^{\frac{1}{k}})=$$
$$\frac{k^2}{k^2+1}x^{\frac{k^2+1}{k^2}}+O(x^{\frac{k^2-k+1}{k^2}})$$

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## 4 结束语

构筑企业网络安全防护系统,有效地保护企业信息系统的安全,是一个与病毒、黑客、不良管理斗争的过程.网络的安全不能单靠技术手段一劳永逸地解决,没有一种技术可以绝对保证网络安全,人的因素也很重要.因此,我们说,一个完整的网络安全解决方案应该是:高新的安全技术手段、周密的安全策略、良好的内部管理相结合.

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## Design of the Enterprises Network Security

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**Abstract** The design of network security should be taken as a whole, and security requirements considered under the general framework and every part and detail included. The security system should be realized in the design of firewall system, anti-virus system, physical isolation of the net, data preservation and network administration strategies.

**Keywords** computer network; network security; firewall; computer virus; data preservation

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## Generalized Some Asymptotic Formulars of F. Smarandache

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**Abstract** According to one sequence in problem 80, which was presented by number-theoretic expect F. Smarandache in his only problem not solution. Here the sequence is denoted by $a(n)$, using the Euler summation, the asymptatic formula of $\sum_{n\leqslant x} a(n)$ and its generalization are obtained, aseries of regular results is obtained.

**Key words** squence root; number-theortic function; asymptotic formula