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Efficient combination rule of Dezert-Smarandache theory*

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Abstract: The Dezert-Smarandache theory (DSmT) is a useful method for dealing with uncertainty problems. It is more efficient in combining conflicting evidence. Therefore, it has been successfully applied in data fusion and object recognition. However, there exist shortcomings in its combination rule. An efficient combination rule is presented, that is, the evidence's conflicting probability is distributed to every proposition based on remaining the focal elements of conflict. Experiments show that the new combination rule improves the reliability and rationality of the combination results. Although evidences conflict another one highly, good combination results are also obtained.

Keywords: DSmT, the focal element of conflict, object recognition.

1. Introduction

Decision layer information fusion shows high flexibility on the aspect of the information processing. The condition of the data transmission bandwidth is not required strictly to the system, therefore, the different kinds of environment information or each side can be reflected effectively, and asynchronous information can also be processed. Thus, at present, the achievements gained by information fusion mainly focus on the decision layer, and this forms a hot topic in the information fusion research. The management and combination of uncertain, imprecise, fuzzy, and even paradoxical or high conflicting sources of information has always been difficult and still remains today. Therefore, how to deal with the combination problems of this kind of information becomes important in the domain of artificial intelligence research. The methods adopted chiefly by the decision layer information fusion are Bayesian theory, Dempster-Shafer theory, and fuzzy set theory and expert system etc., and among all these methods, the Dempster-Shafer theory[1] is applied most extensively.

Dempster-Shafer theory is fit for information fusion without priori probability, and possesses superiority in the expression of uncertainty and measure survey and combination, and it is also fit for mankind's inference at the same time. Unfortunately, the evidence theory fails to manage the existing high conflicts between the various information sources at the step of normalization. How the highly conflicting evidences solve the fusion problems of several information sources is a prerequisite process for the multi-sensor object recognition and attributive fusion techniques. Especially, to improve the defect of inconsistent evidence combination, the Dezert-Smarandache theory (DSmT)[2–5] of plausible and paradoxical reasoning proposed by Dezert and Smarandache can be considered as an extension of the classical Dempster-Shafer theory (DST). It however includes fundamental differences with the DST. DSmT regards the elements of conflicting evidence as the focal elements of data fusion, and in this way, it is able to solve complex data/information fusion problems, especially when conflicts (paradoxes) between sources become large. However, the mass function of the main focal element is difficult to converge quickly in several cases while applying DSmT, and this increases the difficulty of object recognition. Thus, Ref. [6] and Ref. [7] present some improved methods based on DSmT to solve this problem. Al-

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though these methods show high efficiency, they also need to make some improvement.

This article presents an efficient combination rule based on DSmT, that is, the probability of conflicting evidence is distributed to every proposition based on remaining the focal elements of conflicting evidence. Comparing with the existing methods, the proposed approach is more efficient in combining conflicting evidence.

2. Foundations of the DSmT

The development of the Dezert-Smarandache theory of plausible and paradoxical reasoning (called DSmT for short) comes from the necessity to overcome the two following inherent limitations of the DST, which are closely related with the acceptance of the third middle excluded principle, i.e.,

(C1) the DST considers a discrete and finite frame of discernment U based on a set of exhaustive and exclusive elementary elements θ_i.

(C2) the bodies of evidence are assumed independent and provide their own belief function on the powerset 2^U but with the same interpretation for U.

The foundation of the DSmT is based on the refutation of the principle of the third excluded middle for a wide class of fusion problems owing to the nature of the elements of U. By accepting the third middle, we can easily handle the possibility to deal directly with paradoxes (partial vague overlapping elements/concepts) of the frame of discernment. This is the DSm model. In other words, we include the third exclude directly into the formalism to develop the DSmT and relax the (C1) and (C2) constraints of the Shafer's model. By doing this, a wider class of fusion problems can be solved by the DSmT.

The DSmT refutes also the excessive requirement imposed by (C2) in the Shafer's model, since it seems clear to us that the same frame U may be interpreted differently by the distinct bodies of evidence (experts). The DSmT includes the possibility to deal with evidences arising from different sources of information, which do not have access to absolute interpretation of the elements U under consideration. The DSmT can be interpreted as a general and direct extension of Bayesian theory and the Dempster-Shafer theory in the following sense.

Let $U = \{\theta_1, \theta_2\}$ be the simplest frame made of only two hypotheses (with no more additional assumptions on θ_1 and θ_2).

• The probability theory deals, under the assumptions on exclusivity and exhaustivity of hypotheses, with basic probability assignments (bpa) $m(\cdot) \in [0,1]$ such that

$$m(\theta_1) + m(\theta_2) = 1 \quad (1)$$

• The DST deals, under the assumptions on exclusivity and exhaustivity of hypotheses, with bpa $m(\cdot) \in [0,1]$ such that

$$m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) = 1 \quad (2)$$

• The DSmT deals, only under assumption on exhaustivity of hypotheses (i.e., the free DSm model), with the generalized bpa $m(\cdot) \in [0,1]$ such that

$$m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) + m(\theta_1 \cap \theta_2) = 1 \quad (3)$$

2.1 Notion of hyper-powerset D^U

One of the cornerstones of the DSmT is the notion of hyper-powerset, which is now presented. Let $U = \{\theta_1, \ldots, \theta_n\}$ be a set of n elements, which cannot be precisely defined and separated so that no refinement of U in a new larger set θ_{ref} of disjoint elementary hypotheses is possible. The hyper-powerset D^U is defined as the set of all composite propositions built from elements of U with $\cup$ and $\cap$ (U generates D^U under operators $\cup$ and $\cap$) operators such that

① $\phi, \theta_1, \ldots, \theta_n \in D^U$.

② If $A, B \in D^U$, then, $A \cap B \in D^U$ and $A \cup B \in D^U$.

③ No other elements belong to D^U, except those obtained using rules 1 or 2.

Examples of the hyper-power sets are given as follows.

(1) For the degenerate case ($n = 0$) where $U = \{\}$, one has $D^U = \{\phi\}$, and $|D^U| = 1$.

(2) When $U = \{\theta_1\}$, one has $D^U = \{\phi, \theta_1\}$, and $|D^U| = 2$.

(3) When $U = \{\theta_1, \theta_2\}$, one has $D^U = \{\phi, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$, and $|D^U| = 5$.

(4) When $U = \{\theta_1, \theta_2, \theta_3\}$, one has

$$D^U = \left\{ \begin{array}{l} \phi, \theta_1 \cap \theta_2 \cap \theta_3, \theta_1 \cap \theta_2, \theta_1 \cap \theta_3, \theta_2 \cap \theta_3, (\theta_1 \cup \theta_2) \cap \theta_3, (\theta_1 \cup \theta_3) \cap \theta_2 \\ (\theta_2 \cup \theta_3) \cap \theta_1, [(\theta_1 \cap \theta_2) \cup \theta_3] \cap (\theta_1 \cup \theta_2), \theta_1, \theta_2, \theta_3, (\theta_1 \cap \theta_2) \cup \theta_3 \\ (\theta_1 \cap \theta_3) \cup \theta_2, (\theta_2 \cap \theta_3) \cup \theta_1, \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \theta_2 \cup \theta_3, \theta_1 \cup \theta_2 \cup \theta_3 \end{array} \right\}, \text{ and } \left|D^U\right| = 19$$

From a general frame of discernment, we define a map associated to a given body of evidence which can support paradoxical information, as follows

$$m(\phi) = 0 \tag{4}$$

$$\sum_{A \in D^U} m(A) = 1 \tag{5}$$

The quantity $m(A)$ is called A's generalized basic belief assignment (gbba) or the generalized basic probability mass for A. The belief and plausibility functions are defined in almost the same manner as within the DST, i.e.,

$$Bel(A) = \sum_{\substack{B \subseteq A \\ B \in D^U}} m(B) \tag{6}$$

$$Pl(A) = \sum_{\substack{B \cap A \neq \phi \\ B \in D^U}} m(B) \tag{7}$$

These definitions are compatible with the DST definitions when the sources of information become uncertain but rational (they do not support paradoxical information). We still have $\forall A \in D^U, Bel(A) \leqslant Pl(A)$.

2.2 The DSm rule of combination

The DSm rule of combination $m(\cdot) = [m_1 \oplus m_2](\cdot)$ of two distinct (but potentially paradoxical) sources of evidences over the same general frame of discernment U with belief functions $Bel_1(\cdot)$ and $Bel_2(\cdot)$ associated with general information granules $m_1(\cdot)$ and $m_2(\cdot)$ is given by $\forall C \in D^U$,

$$m(C) = \sum_{A, B \in D^U, A \cap B = C} m_1(A) m_2(B) \tag{8}$$

Since D^U is closed under $\cup$ and $\cap$ operators, this new rule of combination guarantees that $m(\cdot) : D^U \to [0,1]$ is a proper general information granule. This rule of combination is commutative and associative, and can always be used for the fusion of paradoxical or rational sources of information (bodies of evidence). It is important to note that any fusion of sources of information generates either uncertainties, paradoxes, or more generally, both.

3. A new combination rule of DSmT

In Ref. [8], the author proposes other combination operators allowing an arbitrary redistribution of the conflicting mass on the propositions within the framework of Dempster-Shafer theory of evidence. In view of Ref. [8], we present a new method within the framework of DSmT.

Retaining the focal elements of conflict can increase its probability while applying DSmT, and at the same time, the probability of the main focal elements assigned by the rule correspondingly reduces. Thus, all these make the object recognition more complicated. The new method is to assign the local conflicting mass, which may exist on all the possible disjunctions of hypotheses from the sets involving the conflict within the framework of DSmT.

Since D^U is closed under $\cup$ and $\cap$ set operators, this new rule of combination guarantees that $m(\cdot)$ is a proper generalized belief assignment, i.e., $m(\cdot) : D^U \to [0,1]$. Each source $S_j (j = \{1, \ldots, J\})$, a mass function $m_j(\cdot)$ is defined by

$$m_j(\cdot) : D^U \to [0,1] \tag{9}$$

Let $m_j(A_i)$ be the belief assignments given by the $j\,(j = 1, \ldots, J)$ information sources to each subset A_i $(i = 1, \ldots, n)$. When $\cap_i A_i \neq \phi$, these subsets are compatible; we will assign this mass to the conjunction of the subsets A_i. If the subsets A_i are not compatible, we define the sets that take place in the redistribution of the conflicting mass. The sets of all propositions where the partial conflict masses have been redistributed are then defined by

$$Q^* = \{A / A \subseteq A_i, i = 1, \ldots, n\} \tag{10}$$

If the hypotheses A_i are incompatible, that is, their intersection is equal to the empty set, we have a partial conflicting mass called m^* given by the following relation

$$m^* = \sum_{\cap_{j=1}^{J} A_i = \phi} m_j(A_i), \quad i = 1, \ldots, n \tag{11}$$

The aim of the combination operators, proposed in this section, is to redistribute a partial conflicting mass m^* on a set of propositions.

The set of all propositions where the conflicting masses have been redistributed will be noted as Q, with

$$Q = 2^{Q^*} \tag{12}$$

At each set A, a mass equal to the sum of the masses assigned to the sets A_i is associated, such as A is included in the sets A_i. This mass is expressed as $\beta(\cdot)$ using Eq. (14). A part of the mass m^* will be assigned to each proposition A according to a weighting factor noted as w^*. From the masses function $\beta(\cdot)$, we define weighting factors given to each set A as follows

$$w^*(A) = \frac{\beta(A)}{\sum\limits_{A \subseteq Q} \beta(A)} \tag{13}$$

$$\beta(A) = \sum_{\substack{A \in Q^* \\ A \subseteq A_j}} m_j(A_i) \tag{14}$$

Thus, the total mass obtained after fusion for a proposition A will be the sum of two masses. It will be written as follows

$$m(A) = m_\cap(A) + m^c(A) \tag{15}$$

In Eq. (15), the first term, $m_\cap(\cdot)$ is derived from the conjunctive rule of combination defined by Eq. (8). The second one, noted as $m^c(\cdot)$, is the part of the conflict mass granted to the proposition A. This value can be written as

$$\forall A \subseteq Q, \quad m^c(A) = \sum m^{c*}(A) \tag{16}$$

where $m^{c*}(A)$ is the part of the partial conflicting masses m^* assigned to the proposition A.

$$\forall A \subseteq Q, \quad m^{c*}(A) = w^*(A) \cdot m^* \tag{17}$$

The principle of combination for two sources of information can be explained as follows. Let S_l represent a source supporting H' with a mass $m_l(H')$, and S_j represent a source supporting H'' with a mass $m_j(H'')$. If the propositions H' and H'' are in contradiction, that is to say, if $H' \cap H'' = \phi$, then one does not know which source is right and one has to consider that the solution is one of the two propositions. The local conflicting mass is

$$m^* = m_l(H') \cdot m_j(H'') \tag{18}$$

We will distribute this local conflicting mass proportionally to the mass affected to each source on the hypotheses H', H'', and $H' \cup H''$. This mass will be redistributed on the sets $Q = \{H', H'', H' \cup H''\}$. The distribution rule of the mass will be as follows. At first, we define the masses assigned to each subset according to Eq. (14). We obtain

$$\beta(H') = m_l(H') \tag{19}$$

$$\beta(H'') = m_j(H'') \tag{20}$$

We define $\beta(H' \cup H'') = m_l(H') + m_j(H'')$. The weighting factors will be defined according to Eq. (13). Thus, we obtain the following weighting factors,

$$w^*(H') = \frac{\beta(H')}{\beta(H') + \beta(H'') + \beta(H' \cup H'')} \tag{21}$$

$$w^*(H'') = \frac{\beta(H'')}{\beta(H') + \beta(H'') + \beta(H' \cup H'')} \tag{22}$$

$$w^*(H' \cup H'') = \frac{\beta(H' \cup H'')}{\beta(H') + \beta(H'') + \beta(H' \cup H'')} \tag{23}$$

Then, we obtain the distribution of the local conflicting mass m^* between the different propositions.

$$m^{c*}(H') = w^*(H') \cdot m^* \tag{24}$$

$$m^{c*}(H'') = w^*(H'') \cdot m^* \tag{25}$$

$$m^{c*}(H' \cup H'') = w^*(H' \cup H'') \cdot m^* \tag{26}$$

4. Simulation analysis

The simulation example of multi-sensor recognition system is given in this section. My plane and Hostile battleplane with type A 100 km apart are opposite to each other in the uniform flight, and the relative velocity of the planes is 1 km/s. Our plane has four kinds of sensors to provide the aero type information of the enemy in the automatic recognition system. The object recognition sensor provides data once per second. The accuracy of data that every sensor provides reduces along with the extended distances, and the accuracy of data that the sensor provides during 10 km is 80% while it is 40% during 100 km. One of the sensors is disturbed when the two machines are 40 km−50 km and 20 km−30 km apart, and the accuracy of data that the sensor provides during the interference is 5%.

Figure 1 shows the simulation results of the basic belief assignment function assigned to hostile battleplane with type A under different rules.

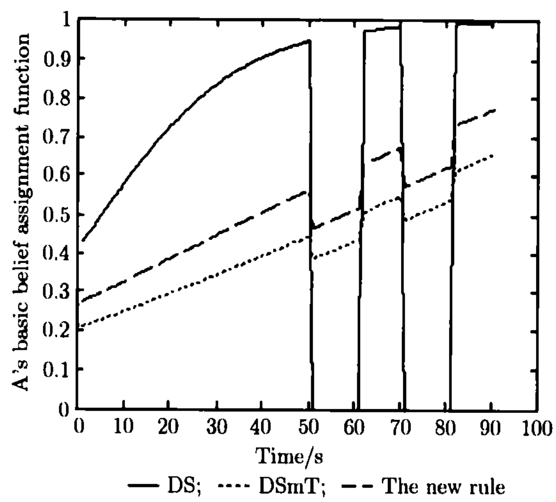

Fig. 1 The basic belief assignment function of hostile battleplane with type A

In Fig. 1, when evidences do not conflict one another, the method of D-S evidence theory is more effective in recognizing the object than DSmT and the new rule, and among all these methods, DSmT is the worst in solving non-conflict problems or low conflict problems. One of the reasons is that there are too many focal elements of conflict in DSmT framework. Another reason is that the basic belief assignment function assigned by DSmT is so scattered that the mass value of the main elements cannot quickly converge. All these make the method of DSmT not effective in solving data fusion problems when evidences do not conflict another one.

Although the D-S evidence theory can get false or even paradoxical results when evidences conflict another one highly, the decision based on DSmT can reach a satisfying conclusion especially when conflicts (paradoxes) between sources become large, which embodies the perfect properties of DSmT in solving uncertainty problems. The reason is that DSmT considers the focal elements of conflicting evidence as useful information, and retains them in the calculations, and therefore, it is able to solve complex data/information fusion problems which the evidence theory fails to solve, especially when conflicts (paradoxes) between sources become large and when the refinement of the frame of discernment is inaccessible because of the vague, relative, and imprecise nature of elements. The simulation result also shows that the mass function value of the main focal element given by the new rule is higher than that given by DSmT under the circumstance of highly conflicting evidence. Thus, the new method can recognize the object more quickly than DSmT, and can then obtain results that are considerably fit for the real world.

5. Conclusions

In this article, DSmT is introduced firstly. DSmT is more efficient in combining conflicting evidence than the D-S evidence theory; therefore, it has been successfully applied in data fusion and object recognition. However, there exist shortcomings when the calculation is very complicated. There are too many focal elements of conflict in DSmT framework, and therefore, the mass value of the main elements cannot converge quickly. Then, we propose the new combination rule based on DSmT. The simulation results of target recognition demonstrate that by use of the new rule, the task of target recognition is accomplished more precisely. It can increase the reliability of the main focal elements. Although evidences conflict another one highly, good combination results are also obtained.

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