A New Class Fusion Rule for solving Blackman's Association Problem

Albena Tchamova, Jean Dezert and Florentin Smarandache

Abstract—**This paper presents a new approach for solving the paradoxical Blackman's Association Problem. It utilizes the recently defined new class fusion rule based on fuzzy T-conorm/T-norm operators together with Dezert-Smarandache theory based, relative variations of generalized pignistic probabilities measure of correct associations, defined from a partial ordering function of hyper-power set. The ability of this approach to solve the problem against the classical Dempster-Shafer's method, proposed in the literature is proven. It is shown that the approach improves the separation power of the decision process for this association problem.**

***Index Terms*—Attribute Data Fusion, Data Association, Dezert-Smarandache Theory, Proportional Conflict Redistribution rules, Fuzzy operators.**

I. INTRODUCTION

Data association with its goal to select the most probable and correct associations between sensors' measurements and target tracks, from a large set of possibilities, is a fundamental and important content for each radar surveillance system. In general case the focus of tracking algorithms has centered on kinematics state estimation. However, targets' attribute information has the potential to not only estimate the identity/type information of the tracking targets, but it may also improve data association and kinematics tracking performance. Attribute data association can become a crucial and challenging problem in case when the sources of information are imprecise, uncertain, even conflicting and paradoxical. The specifics of the data association problem can vary according to both: the different fusion methods and the criteria to estimate the correct associations. There are various methods for combining such information and the choice of method depends on the richness of abstraction and diversity of sensor data. The most used until now Dempster-Shafer Theory (DST) ([3] and [4]) proposes a suitable mathematical framework for representation of uncertainty. Although very appealing, DST presents some weaknesses and limitations, related with the law of the third excluded middle. The Dempster's rule of combination can give rise to some paradoxes/anomalies and can fail to provide the correct solution for some specific association problems. This has been already pointed out by Samuel Blackman in [1], where the famous Blackman Association Problem (BAP) is formulated.

In this paper we focus our attention on the ability of one new, alternative class fusion rule, interpreting the fusion in terms of fuzzy T-Conorm and T-Norm operators (TCN rule), to solve efficiently the paradoxical Blackman's Association Problem on the base of relative variations of generalized pignistics probabilities measure, defined within recently developed Dezert-Smarandache Theory (DSmT) of plausible and paradoxical reasoning ([2] and [7]). It proposes a new general mathematical framework for solving fusion/association problems. This theory overcomes the practical limitations of DST, coming essentially from its inherent constraints, which are closely related with the acceptance of the law of the third excluded middle and can be interpreted as a general and direct extension of probability theory and the DST.

We first recall the BAP, then we browse the state of the art to find the correct solution through different approaches available in the literature. After a brief presentation of DSmT, DSmT based Proportional Redistribution Rule number 5 (PCR5), the new TCN combination rule and the DSmT based, relative variations of generalized pignistics probabilities measure, we provide a new solution of this problem, which is encountered in modern multisensor multitarget tracking and identification systems involved within defense applications. The last part of the paper provides a comparison of the performances of all the proposed approaches from Monte-Carlo simulation results.

II. THE BLACKMAN ASSOCIATION PROBLEM

The main purpose of information fusion/association is to produce reasonably aggregated, refined and/or completed granule of data obtained from a single or multiple sources of information with consequent adequate reasoning process. It means, that the main problem here consists not only in the way to aggregate correctly the sources of information, which in general are imprecise, uncertain, or/and conflicting, but it is also important to dispose of proper criterion to estimate the correct association. Actually, there is no a single, unique rule to deal simultaneously with such kind of information peculiarities, but a huge number of possible combinational rules, appropriate for a particular only application conditions, as well as a number of criterion to estimate the correct association.

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A. Original Blackman Association Problem.

The well known association problem provided by Samuel Blackman considers a very simple frame of discernments according to only two target's attribute types $\Theta = \{\theta_1, \theta_2\}$. It corresponds to a single attribute observation and two estimated targets tracks T_1 and T_2 associated with two predicted basic belief assignments (bba): $m_{T_1}(.)$ and $m_{T_2}(.)$ respectively:

$$m_{T_1}(.) = \{m_{T_1}(\theta_1) = 0.5;\ \ m_{T_2}(\theta_2) = 0.5;\ \ m_{T_1}(\theta_1 \cup \theta_2) = 0.0\}$$

$$m_{T_2}(.) = \{m_{T_2}(\theta_1) = 0.1;\ m_{T_2}(\theta_2) = 0.1;\ m_{T_2}(\theta_1 \cup \theta_2) = 0.8\}$$

It should be mentioned that both sources of information are independent and share one and the same frame of hypotheses, on which their basic belief assignment are defined.

During the next time moment, a single new attribute observation is detected. It is characterized with an associated bba, $m_Z(.)$, described within the same frame of discernments:

$$m_Z(.) = \{m_Z(\theta_1) = 0.5;\ \ m_Z(\theta_2) = 0.5;\ \ m_Z(\theta_1 \cup \theta_2) = 0.0\}$$

It is evident here, the new observation perfectly fits with the predicted bba of the first track, i.e. $m_Z(.) = m_{T_1}(.)$, whereas $m_Z(.)$ has some disagreement with the predicted bba of the second track $m_{T_2}(.)$. It should lead to a categorical decision about the correct assignment: $m_Z(.) = m_{T_1}(.)$. However, counter-intuitively, the solution, taken on the base of DST is just the opposite one: $m_Z(.) = m_{T_2}(.)$.

B. Second Blackman Association Problem.

In order to complete and compare all possible cases, we modify the first association problem into a second one with preserving the same predicted tracks' bbas: $m_{T_1}(.)$ and $m_{T_2}(.)$. In the opposite of the first case, we consider the new attribute measurement to fit with the second track's bba, i.e. $m_Z(.) = m_{T_2}(.)$. Because of perfect fitting, the correct decision here is apparently trivial : $m_Z(.) \leftrightarrow m_{T_2}(.)$.

III. State of the Art to Find a Correct Solution

In [2] there are described, examined and discussed several approaches to resolve the BAP. The first group includes approaches based on DST: *(i)* a minimum conflict criterion; *(ii)* a relative attribute likelihood function criterion, proposed by Blackman; *(iii)* minimum distance criterion; *(iiii)* Shubert's meta-conflict function criterion; *(iiiii)* entropy-based approaches. The results obtained via Monte Carlo simulations indicate that there is no reliable approach to solve the assignment problem based on DST for both cases described above. The numerical computation of the conflict for BAP 1 yields an unexpected, non-adequate, counter-intuitive result. The fusion/association process actually assigns the lower degree of conflict to the incorrect solution: $m_Z(.) \leftrightarrow m_{T_2}(.)$, providing a larger discrepancy between observation's bba $m_Z(.)$ with the predicted bba $m_{T_1}(.)$, than with the predicted bba $m_{T_2}(.)$, nevertheless $m_Z(.) = m_{T_1}(.)$. Therefore, the search for the minimum conflict between sources cannot be taken as a reliable solution for the general assignment problem since at least one example exists for which the method fails. The meta-conflict approach, proposed by Shubert [6], does not allow getting the optimal efficiency. The Blackman's approach gives the same performance. All entropy-based methods are less efficient than the min-conflict approach. The min-distance approach is the least efficient one. According to the combination rule used, it has been already reported in [2], [5], and [6] that the use of DST must usually be done with extreme caution if one has to take a final and important decision from the result of the Dempter's rule of combination. Always there is a need to be added some ad-hoc or heuristic techniques to the association process, in order to manage or reduce the possibility of high degree of conflict between sources. Otherwise, the fusion results lead to non-adequate conclusions, or cannot provide reliable results at all.

The second group approaches rely on the new DSmT of plausible and paradoxical reasoning. Its foundation is to allow imprecise/vague notions and concepts between elements of the frame of discernment. The main approaches to examine and estimate the correct data association within DSmT are based on the generalized pignistic transformation [2]: minimum variation of entropy-like measure, minimum variation of generalized pignistic entropy, minimum of relative variation of pignistic probabilities conditioned by the correct assignment.

The results obtained show that the method based on the relative variations of generalized pignistic probabilities conditioned by the correct assignment, yields adequate and proper decisions and outperforms all above approaches examined.

IV. Dezert-Smarandache Theory

The DSmT of plausible and paradoxical reasoning proposes a new general mathematical framework for solving fusion problems and a formalism to describe, analyze and combine all the available information, allowing the possibility for conflicts and paradoxes between the elements of the frame of discernment. DSmT differs from DST because it is based on the free Dedekind lattice. It works for any model (free DSm model and hybrid models - including Shafer's model as a special case) which fits adequately with the true nature of the fusion problem under consideration, expressed in terms of belief functions, with static and dynamic fusion problematics. The DSmT includes the possibility to deal with evidences arising from different sources of information, which don't have access to absolute interpretation of the elements θ under consideration and can be interpreted as a general and direct extension of probability theory and the DST.

A. Free-DSm model

Let $\theta = \{\theta_1, \ldots \theta_n\}$ be a set of n elements, which cannot be precisely defined and separated. A free-DSm model, denoted as $\mathrm{M}^f(\Theta)$, consists in assuming that all elements $\theta_i, i = 1, \ldots, n$ of Θ are not exclusive. The free-DSm model is an opposite to the Shafer's model $\mathrm{M}^0(\Theta)$, which requires the exclusivity and exhaustivity of all elements $\theta_i, i = 1, \ldots, n$ of Θ.

B. DSm hybrid Model

A DSm hybrid model $\mathrm{M}(\Theta)$ is defined from the free-DSm model $\mathrm{M}^f(\Theta)$ by introducing some integrity constraints on some elements $\theta_i \in D^\Theta$, if there are some certain facts in accordance with the exact nature of the model related to the problem under consideration. An integrity constraint on $\theta_i \in D^\Theta$ consists in forcing θ_i to be empty through the model $\mathrm{M}(\Theta)$, denoted as $\theta_i \overset{\mathrm{M}}{\equiv} \varnothing$. There are several possible kinds of integrity constraints:

- exclusivity constraints – when some conjunctions of elements $\theta_i, \ldots, \theta_k$ are truly impossible, i.e. $\theta_i \cap \ldots \cap \theta_k \overset{\mathrm{M}}{\equiv} \varnothing$;
- non-existential constraints – when some disjunctions of elements $\theta_i, \ldots, \theta_k$ are truly impossible, i.e. $\theta_i \cup \ldots \cup \theta_k \overset{\mathrm{M}}{\equiv} \varnothing$;
- mixture of exclusivity and non-existential constraints, for example $(\theta_i \cap \theta_j) \cup \theta_k$

The introduction of a given integrity constraint $\theta_i \overset{\mathrm{M}}{\equiv} \varnothing$ implies the set of inner constraints $B \overset{\mathrm{M}}{\equiv} \varnothing$ for all $B \subset \theta_i$. The Shafer's model $\mathrm{M}^0(\Theta)$ can be considered as the most constrained DSm hybrid model including all possible exclusivity constraints without non-existential constraint, since all elements in the frame are forced to be mutually exclusive.

C. Hyper-Power Set and Classical DSm Rule of Combination.

The hyper-power set D^θ is defined as the set of all composite possibilities build from θ with $\cup$ and $\cap$ operators such that:

1. $\varnothing, \theta_1, \ldots \theta_n \in D^\Theta$
2. $\forall A \in D^\theta, B \in D^\theta, (A \cup B) \in D^\theta, (A \cap B) \in D^\theta$.
3. No other elements belong to D^Θ, except those, obtained by using rules 1 or 2.

From a general frame of discernment θ with its free-DSm model, it is defined a map $m(.) : D^\theta \to [0,1]$, associated to a given source of evidence, which can support paradoxical, or conflicting information, as follows:

$$m(\varnothing) = 0 \text{ and } \sum_{A \in D^\theta} m(A) = 1$$

The quantity $m(A)$ is called A's general basic belief assignment (gbba) or the general basic belief mass for A. The belief and plausibility functions are defined for $\forall A \in D^\theta$:

$$Bel(A) = \sum_{B \in D^\theta, B \subseteq A} m(B)$$

$$Pl(A) = \sum_{B \in D^\theta, B \cap A \neq \varnothing} m(B)$$

The DSm classical rule of combination is based on the free-DSm model. For $k \geq 2$ independent bodies of evidence with gbbas $m_1(.)$, $m_2(.)$, …, $m_k(.)$ over D^Θ:

$$m_{\mathrm{M}^f(\Theta)}(A) = \sum_{\substack{X_1 \ldots X_k \in D^\Theta \\ X \cap \ldots \cap X_k = A}} \prod_{i=1}^{k} m_i(X_i)$$

with $m_{\mathrm{M}^f(\Theta)}(\varnothing) = 0$ by definition.

This rule is commutative and associative and requires no normalization procedure.

V. Proportional conflict redistribution rule no. 5

Instead of distributing equally the total conflicting mass onto elements of power set as within Dempster's rule through the normalization step, or transferring the partial conflicts onto partial uncertainties as within DSm hybrid rule, the idea behind the Proportional Conflict Redistribution rules is to transfer conflicting masses (total or partial) proportionally to non-empty sets involved in the model according to all integrity constraints. The general principle is to :

- calculate the conjunctive rule of the belief masses of sources;
- calculate the total or partial conflicting masses ;
- redistribute the conflicting mass (total or partial) proportionally on non-empty sets involved in the model according to all integrity constraints.

The way the conflicting mass is redistributed yields to several versions of PCR rules [7]. These PCR fusion rules work both in DST and DSmT frameworks and for static or dynamical fusion problematic, for any degree of conflict in [0, 1], for any DSm models (Shafer's model, free DSm model or any hybrid DSm model). The most sophisticated rule among them is the proportional conflict redistribution rule no. 5 (PCR5) The PCR5 combination rule for only two sources of information is defined by:

$m_{PCR5}(\varnothing) = 0$ and for $\forall X \in G^{\Theta} \setminus \{\varnothing\}$,

$$m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y \in G^{\Theta} \setminus \{X\} \\ c(X \cap Y) = \varnothing}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)}\right]$$

where G^{Θ} is the DSmT hyper power set; $m_{12}(X)$ corresponds to the conjunctive consensus on X between the two sources and where all denominators are different from zero and $c(X)$ is the canonical form of X.

No matter how big or small is the conflicting mass, PCR5 mathematically does a better redistribution of the conflicting mass than Dempster's rule and other rules since PCR5 goes backwards on the tracks of the conjunctive rule and redistributes the partial conflicting masses only to the sets involved in the conflict and proportionally to their masses put in the conflict, considering the conjunctive normal form of the partial conflict. PCR5 is quasi-associative and preserves the neutral impact of the vacuous belief assignment.

VI. The T-Conorm/T-Norm Combination Rule

The TCN rule of combination [8] represents a new class of combination rules based on specified fuzzy T-Conorm/T-Norm operators. This rule takes its source from the T-norm and T-conorm operators in fuzzy logics, where the AND logic operator corresponds in information fusion to the conjunctive rule and the OR logic operator corresponds to the disjunctive rule.

In this work we propose to interpret the fusion/association between the sources of information as a vague relation, characterized with the following two characteristics:

- *The way of association between the possible propositions build on the base of the frame of discernment.* It is based on operations union and intersection, and their combinations. These sets' operations correspond to logic operations Conjunction and Disjunction and their combinations.
- *The degree of association between the propositions.* It is obtained as a T-norm (for conjunction) or T-conorm (for disjunction) operators applied over the probability masses of corresponding focal elements. While the logic operators deal with degrees of truth and false, the fusion rules deal with degrees of belief of hypotheses. Within this work, we focus only on the Minimum T-norm based Conjunctive rule. It yields results very closed to conjunctive rule, which is appropriate for identification problems, restricting the set of hypotheses we are looking for. It has an adequate behavior in cases of total conflict presence. It is commutative and simply to apply.

The general principle of TCN rule consists in the following steps:

Step 1: Defining the min T-norm conjunctive consensus.

The min T-norm conjunctive consensus is based on the default min T-norm function. The way of association between the focal elements of the given two sources of information, $m_1(.)$ and $m_2(.)$, is defined as $X = \theta_i \cap \theta_j$ and the degree of association is as follows:

$$\tilde{m}_{12}(X) = \min\{m_1(\theta_i), m_2(\theta_j)\}$$

where $\tilde{m}_{12}(X)$ represents the basic belief assignments after the fusion, associated with the given proposition X by using T-norm based conjunctive rule. The TCN combination rule in Dempster Shafer theory framework is defined for $\forall X \in 2^{\Theta}$ by the equation:

$$m(X) = \sum_{\substack{X_i \cap X_j = X \\ X_i, X_j \in 2^{\Theta}}} \min\{m_1(X_i), m_2(X_j)\}$$

Step 2: Distribution of the mass, assigned to the conflict.

In some degree it follows the distribution of conflicting mass in the most sophisticated DSmT based Proportional Conflict Redistribution rule number 5 (PCR5) proposed in [7], but the procedure here relies on fuzzy operators. The total conflicting mass is distributed to all non-empty sets proportionally with respect to the *Maximum* between the elements of corresponding mass matrix's columns, associated with the given element X of the power set. It means the bigger mass is redistributed towards the element, involved in the conflict and contributing to the conflict with the maximum specified probability mass. The general procedure for fuzzy based conflict redistribution is as follows:

- Calculate all partial conflict masses separately;
- If $\theta_i \cap \theta_j = \varnothing$, then θ_i and θ_j are involved in the conflict; redistribute the corresponding masses $m_{12}(\theta_i \cap \theta_j) > 0$, involved in the particular partial conflicts to the non-empty sets θ_i and θ_j with respect to $\max\{m_1(\theta_i), m_2(\theta_j)\}$ and with respect to $\max\{m_1(\theta_j), m_2(\theta_i)\}$;
- Finally, for the given above two sources, the minimum T-Norm conjunctive consensus yields:

$$\tilde{m}_{12}(\theta_i) = \min(m_1(\theta_i), m_2(\theta_i)) + \min(m_1(\theta_i), m_2(\theta_i \cup \theta_j)) + \min(m_1(\theta_i \cup \theta_j), m_2(\theta_i))$$

$$\tilde{m}_{12}(\theta_j) = \min(m_1(\theta_j), m_2(\theta_i)) + \min(m_1(\theta_j), m_2(\theta_i \cup \theta_j)) + \min(m_1(\theta_i \cup \theta_j), m_2(\theta_j))$$

$$\tilde{m}_{12}(\theta_i \cup \theta_j) = \min(m_1(\theta_i \cup \theta_j), m_2(\theta_i \cup \theta_j))$$

Step 3: The basic belief assignment, obtained as a result of the applied TCN rule becomes:

$$\tilde{m}_{12PCR5}(\theta_i) = \tilde{m}_{12}(\theta_i) + m_1(\theta_i) \times \frac{\min\left(m_1(\theta_i), m_2(\theta_j)\right)}{\max\left(m_1(\theta_i), m_2(\theta_j)\right)} + m_2(\theta_i) \times \frac{\min\left(m_1(\theta_j), m_2(\theta_i)\right)}{\max\left(m_1(\theta_j), m_2(\theta_i)\right)}$$

$$\tilde{m}_{12PCR5}(\theta_j) = \tilde{m}_{12}(\theta_j) + m_2(\theta_j) \times \frac{\min\left(m_1(\theta_i), m_2(\theta_j)\right)}{\max\left(m_1(\theta_i), m_2(\theta_j)\right)} + m_1(\theta_j) \times \frac{\min\left(m_1(\theta_j), m_2(\theta_i)\right)}{\max\left(m_1(\theta_j), m_2(\theta_i)\right)}$$

Step 4: Normalization of the result. The final step of the TCN fusion rule concerns the normalization procedure:

$$\tilde{m}_{12PCR5}(X) = \frac{\tilde{m}_{12PCR5}(X)}{\sum_{\substack{X \neq \varnothing \\ X \in 2_\Theta}} \tilde{m}_{12PCR5}(X)}$$

TCN combinational rule does not belong to the general Weighted Operator Class. The nice features of the new rule could be defined as: very easy to implement, satisfying the impact of neutrality of Vacuous Belief Assignment; commutative, convergent to idempotence, reflecting majority opinion, assuring an adequate data processing and interpretation in case of total conflict. These main features make it appropriate for the needs of temporal data fusion.

VII. Measure of Estimation based on Generalized Pignistic Probabilities.

The minimum of relative variation of generalized pignistic probabilities within DSmT, conditioned by the correct assignment $\delta_i(P^*)$ is chosen as a measure of correct data association. It is defined from a partial ordering function of hyper-powerset, which is the base of DSmT.

It is proven that this measure outperforms all methods, examined in [2] for correct solving of Blackman's association problem. Our goal is to estimate and compare the performance of TCN combination rule on the base of that best criterion:

$$\delta_i(P^*) = \frac{\left|\Delta_i(P^*/Z) - \Delta_i(P^*/\hat{Z} = T_i)\right|}{\Delta_i(P^*/\hat{Z} = T_i)}$$

where:

$$\Delta_i(P^*) = \sum_{j=1}^{2} \frac{\left|P^*_{T_iZ}(\theta_j) - P^*_{T_iZ}(\theta_j) - P^*_{T_i}(\theta_j)\right|}{P^*_{T_i}(\theta_j)}$$

The term $P^*(.)$ represents a generalized pignistic probability, according to a given proposition; $\Delta_i(P^*)$ defines the relative variations of corresponding pignistic probabilities; $\Delta_i(P^*/\hat{Z} = T_i)$ is obtained as for $\Delta_i(P^*)$ by forcing the new measurement's bba to be equal to the given track's bba, i.e. $m_Z(.) = m_{T_i}(.)$ for pignistic probabilities, $P^*_{T_iZ}(\theta_j)$, derivation.

In the next section we will test its performance to resolve the BAP on the base of TCN rule and will compare it with the results, obtained by DST.

VIII. Simulation Results

In table 1 below the performance evaluation of several methods for solving the BAP are shown. We compare percentage of success in correct BAP resolving by the new TCN combination rule, DSmT based Proportional Conflict Redistribution rule number 5 and Dempster's rule with the corresponding measure of correct association as follows:

- TCN combinational rule and the best criterion based on the relative variations of generalized pignistic probabilities build from the DSmT (and the free DSm model)
- DSmT based Proportional Conflict Redistribution rule number 5 and the criterion based on the relative variations of generalized pignistic probabilities build from the DSmT
- Dempster's rule of combination and: *(i)* Dempster-Shafer theory based Blackman approach; *(ii) DST* based Min Conflict approach; *(iii) DST* based Meta conflict approach; *(iiii) DST* based Min Entropy approach.

The evaluation of methods' performances/efficiency is estimated through Monte-Carlo simulations. They are based on 10.000 independent runs. A basic run consists in generating randomly the two predicted bba: $m_{T_1}(.)$, $m_{T_2}(.)$ and the new observed bba $m_Z(.)$ according to a random assignment $m_Z(.) \leftrightarrow m_{T_1}(.)$ or $m_Z(.) \leftrightarrow m_{T_2}(.)$. Then we evaluate the percentage of right assignments for the given association criterion.

The evaluation of proposed here method for BAP's solving is performed on the base of the association criterion, proven to be the best among the investigated ones in [2].

The results show that all the methods, applied as measures of correct data associations within Dempster-Shafer theory lead to non-adequate and non-reliable decisions. Dempster's rule of combination can give rise to some paradoxes /anomalies and can fail to provide the correct solution for some specific association problems.

Monte Carlo simulations show that only methods based on the new TCN combination rule and DSmT based PCR5 rule with the minimum relative variations of generalized pignistic probabilities measure outperform all methods examined in this work.

TABLE I
PERFORMANCE EVALUATION OF METHODS FOR SOLVING BLACKMAN'S ASSOCIATION PROBLEM

Rule and Approach for solving BAP	***Percentage of success***
• TCN rule • relative variations of generalized pignistic probabilities build from the DSmT (and the free DSm model)	100%
• DSmT based PCR5 rule • relative variations of generalized pignistic probabilities build from the DSmT (and the free DSm model)	100 %
• Dempster's rule • *DST* based Blackman approach	70.31 %
• Dempster's rule • *DST* based Min Conflict approach	70.04 %
• Dempster's rule • *DST* based Meta conflict approach	70.04 %
• Dempster's rule • *DST* based Min Entropy approach	64.5 %

IX. CONCLUSIONS

We focused our attention on the paradoxical Blackman's association problem and propose a new approach to outperform Blackman's solution. The proposed approach utilizes the recently defined new class fusion rule based on fuzzy T-conorm/ T-norm operators. It is applied and tested together with a Dezert-Smarandache theory based, relative variations of generalized pignistics probabilities measure of correct association, defined from a partial ordering function of hyper-powerset. The ability of this approach to solve the problem against the classical Dempster-Shafer's method, proposed in the literature, is proven. It is shown that it assures an adequate data processing in case of high conflict between sources of information, when Dempster's rule yields counter-intuitive fusion results and improves the separation power of the decision process for the considered association problem.

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