Letter to the Editor

A Short Introduction of Cellular Automaton Universe *via* Cosmological KdV Equation

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ABSTRACT

It has been long known that the cosmic sound wave was there since the early epoch of the Universe. Signatures of its existence are abound. However, such a sound wave model of cosmology is rarely developed fully into a complete framework. This paper can be considered as our *second* attempt towards such a complete description of the Universe based on soliton wave solution of cosmological KdV equation. We submit that Robert Kurucz's hypothesis that *Big Bang should be replaced with a finite cellular automaton universe with no expansion*. Our model is preliminary but close in spirit to what Konrad Zuse envisaged long time ago. It is our hope that the new model can be verified with observation data.

Keywords: Solitary wave, cosmological KdV equation, nonlinear universe, cellular automata, PDE, Konrad Zuse.

1. Introduction

Konrad Zuse is probably the first scholar who imagine a *Computing Universe*. In recent years, there are a few researchers who suggest similar vision in terms of cellular automata. For example, Stephen Wolfram, Gerard 't Hooft, and Robert Kurucz. Nonetheless, it seems that there is no existing model which can be connected with a nonlinear PDE of the Universe. Thus, we try to offer a working model of Computing Universe which can be modelled and solved using computer algebra packages such as Mathematica.

Korteweg-de Vries (KdV) equation is a non-linear wave equation plays a fundamental role in diverse branches of mathematical and theoretical physics. Its significance to cosmology has been discussed by a number of authors, such as Rosu and recently Lidsey [3, 7]. It is suggested that the KdV equation arises in a number of important scenarios, including inflationary cosmology etc. Analogies can be drawn between cosmic dynamics and the propagation of the solitonic wave solution to the equation, whereby quantities such as the speed and amplitude profile of the wave

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can be identified with cosmological parameters such as the spectral index of the density perturbation spectrum and the energy density of the universe.

Then we advance further this KdV equation by virtue to Cellular Automaton method to solve the PDEs. We submit wholeheartedly Kurucz's hypothesis that *Big Bang should be replaced with a finite cellular automaton universe with no expansion*.[4][5]

Nonetheless, we are fully aware that our model is far from being complete, but perhaps the proposed cellular automaton model of the Universe is very close in spirit to what Konrad Zuse envisaged long time ago. However, we do not exercise possible link between our model and Cellular automaton model of Gerard 't Hooft; that is beyond the scope of this paper.[14]
It is our hope that the new proposed equation can be verified with observation data both at lab scale and also at large scale astronomy data. But we admit that our model is still in its infancy, more researches are needed to fill all the missing details.

2. Cosmological KdV Equation

The Korteweg-de Vries (KdV) equation is the completely integrable, third-order, non-linear partial differential equation (PDE):[3]

$$\partial_t u + \partial_x^3 u + \frac{3}{u_0} u \partial_x u = 0, \tag{1}$$

where u = u(x, t), $\partial_t = \partial/\partial t$, $\partial_x^3 = \partial^3/\partial x^3$, etc., u_0 is a constant and (x, t) represent space and time coordinates, respectively. This equation was originally derived within the context of small-amplitude, non-linear water wave theory and it is well known that it admits a solitonic wave solution of the form

$$u = u_0 \lambda^2 \sec h^2 \left[\lambda\left(x - \lambda^2 t\right)/2\right], \tag{2}$$

where the constant $\lambda/2$ represents the wavenumber of the soliton. The KdV soliton is characterized by the property that its speed and amplitude are proportional to the square of the wavenumber.

Rosu [7] and also Lidsey [3] both have considered some cosmological applications of KdV equation. We will consider here one application in inflationary universe model.
It can be shown that Friedmann equation after some steps which have been discussed in [3], yields to an equation which takes the form of (2), as follows:

$$H^2(\phi) = H_0^2 \lambda^2 \sec h^2 \left[\lambda A/2\right], \tag{3}$$

Where:

$$A = \frac{\sqrt{8\pi}}{m_p}\phi. \tag{4}$$

Therefore, it appears quite reasonable to consider this equation as originated from certain cosmological KdV physics.

3. Towards a Cellular Automaton Universe

Based on a paper by Steeb & Hardy [11], KdV equation can be written as a conservation law:

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left(-\frac{u^2}{2}-\frac{\partial^2 u}{\partial x^2}\right)=0, \tag{5}$$

It follows that, after the simplest discretization, we obtain the cellular automata:

$$u_j(t+1)=u_j(t)(u_{j+1}(t)-u_j(t))+u_{j+2}(t)-u_{j+1}(t)-u_{j-1}(t). \tag{6}$$

Thus $\sum_{j=0}^{N-1} u_j(t)$ is not an invariant.

In other words, it appears possible to discuss a Cellular Automaton-KdV Universe. But further analysis is required to study its potential applications.

4. Discussion

It has been long known that the cosmic sound wave was there since the early epoch of the Universe. Signatures of its existence are abound. However, such a sound wave model of cosmology is rarely developed fully into a complete framework. This paper can be considered as our *second* attempt towards such a complete description of the Universe based on soliton wave solution of cosmological KdV equation. We submit that Robert Kurucz's hypothesis that *Big Bang should be replaced with a finite cellular automaton universe with no expansion*. Our model is preliminary but close in spirit to what Konrad Zuse envisaged long time ago. It is our hope that the new model can be verified with observation data.

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