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A New Methodology for Neutrosophic Multi-Attribute Decision-Making with Unknown Weight Information

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Abstract. In this paper, we present multi-attribute decision-making problem with neutrosophic assessment. We assume that the information about attribute weights is incompletely known or completely unknown. The ratings of alternatives with respect to each attributes are considered as single-valued neutrosophic set to catch up imprecise or vague information. Neutrosophic set is characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F). The modified grey relational analysis method is proposed to find out the best alternative for multi-attribute decision-making problem under neutrosophic environment. We establish a deviation based optimization model based on the ideal alternative to determine attribute weight in which the information about attribute weights is incompletely known. Again, we solve an optimization model with the help of Lagrange functions to find out the completely unknown attributes weight. By using these attributes weight we calculate the grey relational coefficient of each alternative from ideal alternative for ranking the alternatives. Finally, an illustrative example is provided in order to demonstrate its applicability and effectiveness of the proposed approach.

Keywords: Neutrosophic set; Single-valued neutrosophic set; Grey relational analysis; Multi-attribute decision making; Unknown weight information.

1 Introduction

In the real world problem, we often encounter different type of uncertainties that cannot be handled with classical mathematics. In order to deal differen types of uncertainty, Fuzzy set due to Zadeh [1] is very useful and effective. It deals with a kind of uncertainty known as "fuzziness". Each real value of [0, 1] represents the membership degree of an element of a fuzzy set i.e partial belongingness is considered. If $\mu_A(x) \in [0,1]$ is the membership degree of an element x of a fuzzy set *A*, then (1- $\mu_A(x)$) is assumed as the non-membership degree of that element. This is not generally hold for an element with incomplete information. In 1986, Atanassov [2] developed the idea of intuitionistic fuzzy set (IFS). An element of intuitionistic fuzzy set *A* characterized by the membership degree $\mu_A(x) \in [0,1]$ as well as non-membership degree $\nu_A(x) \in [0,1]$ with some restriction $0 \le \mu_A(x) + \nu_A(x) \le 1$. Therefoe certain amount of indeterminacy or incomplete information $1-(\mu_A(x)+\nu_A(x))$ remains by default. However, one may also consider the possibility $\mu_A(x)+\nu_A(x) > 1$, so that inconsistent beliefs are also allowed. In this case, an element may be regarded as both member and non-member at the same time. A set connected with this features is called Para-consistent Set [3]. Smarandache [3-5] introduced the concept of neutrosophic set (NS) which is actually generalization of different type of FSs and IFSs. Consider an example, if $\mu_A(x) \in [0,1]$ is a membership degree, $\nu_A(x) \in [0,1]$ is a non-membership degree of an element x of a set *A*, then fuzzy set can be expressed as *A*= $\langle x/(\mu_A(x), 0, 1-\mu_A(x))\rangle$ and IFS can be represented as *A* $=\langle x/(\mu_A(x), 1-\mu_A(x)-\nu_A(x), \nu_A(x))\rangle$

with $0 \le \mu_A(x) + \nu_A(x) \le 1$. The main feature of neutrosophic set is that every element of the universe has not only a certain degree of truth (T) but also a falsity degree (F) and indeterminacy degree (I). These three degrees have to consider independently from each other. Another interesting feature of neutrosophic set is that we do not even assume that the incompleteness or indeterminacy degree is always given by $1-(\mu_A(x)+\nu_A(x))$.

Multiple attribute decision-making (MADM) problem in the area of operation research, management science, economics, systemic optimization, urban planning and many other fields has gained very much attention to the researchers during the last several decades. These problems generally consist of choosing the most desirable alternative that has the highest degree of satisfaction from a set of alternatives with respect to their attributes. In this approach the decision makers have to provide qualitative and/or quantitative assessments for determining the performance of each alternative with respect to each attribute, and the relative importance of evaluation attribute.

There are many MADM methods available in the literature in crisp environment such as TOPSIS (Hwang & Yoon [6]), PROMETHEE (Brans et al. [7]), VIKOR (Opricovic [8-9]), and ELECTRE (Roy [10]) etc. However it is not always possible to evaluate the importance of attributes weights and the ratings of alternatives by using crisp numbers due to un-availability of complete information about attribute values. Chen [11] extended the classical TOPSIS by developing a methodology for solving multi-criteria decision-making problems in fuzzy environment. Zeng [12] solved fuzzy MADM problem with known attribute weight by using expected value operator of fuzzy variables. However, fuzzy set can only focus on the membership grade of vague parameters or events. It fails to handle non-membership degree and indeterminacy degree of imprecise parameters. Boran et al. [13] extended the TOPSIS method for multi-criteria intuitionistic decision-making problem. Xu and Hu [14] developed two projection models for MADM problems with intuitionistic fuzzy information. Xu [15] studied MADM problem with interval-valued intuitionistic fuzzy decision-making by using distance measure.

In IFS the sum of membership degree and non-membership degree of a vague parameter is less than unity. Therefore, a certain amount of incomplete information or indeterminacy arises in an intuitionistic fuzzy set. It cannot handle all types of uncertainties successfully in different real physical problems. Hence further generalizations of fuzzy set as well as intuitionistic fuzzy sets are required.

Neutrosophic set information is helpful to handling MADM for the most general ambiguity cases, including paradox. The assessment of attribute values by the decision maker takes the form of single-valued neutrosophic set (SVNS) which is defined by Wang et al. [16]. Ye [17] studied multi-criteria decision-making problem under SVNS environment. He proposed a method for ranking of alternatives by using weighted correlation coefficient. Ye [18] also discussed single-valued neutrosophic cross entropy for multi-criteria decision-making problems. He used similarity measure for interval valued neutrosophic set for solving multi-criteria decision-making problems. Grey relational analysis (GRA) is widely used for MADM problems. Deng [19-20] developed the GRA method that is applied in various areas, such as economics, marketing, personal selection and agriculture. Zhang et al. [21] discussed GRA method for multi attribute decision-making with interval numbers. An improved GRA method proposed by Rao & Singh [22] is applied for making a decision in manufacturing situations. Wei [23] studied the GRA method for intuitionistic fuzzy multi-criteria decision-making. Biswas et al. [24] developed an entropy based grey relational analysis method for multi-attribute decision-making problem under single valued neutrosophic assessments.

The objective of this paper is to study neutrosophic MADM with unknown weight information using GRA. The rest of the paper is organized as follows. Section 2 briefly presents some preliminaries relating to neutrosophic set and single-valued neutrosophic set. In Section 3, Hamming distance between two single-valued neutrosophic sets is defined. Section 4 is devoted to represent the new model of MADM with SVNSs based on modified GRA. In section 5, an illustrative example is provided to show the effectiveness of the proposed model. Finally, section 6 presents the concluding remarks.

2 Preliminaries of Neutrosophic sets and Single valued neutrosophic set

In this section, we provide some basic definition about neutrosophic set due to Smrandache [3], which will be used to develop the paper.

Definition 1 Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is characterized by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A, I_A and F_A are real standard or non-standard subsets of$]0^-, 1^+[$ that is $T_A : X \rightarrow]0^-, 1^+[$; $I_A : X \rightarrow]0^-, 1^+[$; $F_A : X \rightarrow]0^-, 1^+[$

It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$, $F_A(x)$ i.e. $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2 The complement of a neutrosophic set *A* is denoted by A^c and is defined by

$$T_{A^c}(x) = \{1^+\} - T_A(x) \text{ ; } I_{A^c}(x) = \{1^+\} - I_A(x) \text{ ; }$$
$$F_{A^c}(x) = \{1^+\} - F_A(x)$$

Definition 3 A neutrosophic set *A* is contained in the other neutrosophic set *B*, $A \subseteq B$ if and only if the following result holds.

$$\inf T_A(x) \leq \inf T_B(x) \text{ , } \sup T_A(x) \leq \sup T_B(x) \quad (1)$$

$$\inf I_A(x) \geq \inf I_B(x) \text{ , } \sup I_A(x) \geq \sup I_B(x) \quad (2)$$

$$\inf F_A(x) \geq \inf F_B(x) \text{ , } \sup F_A(x) \geq \sup F_B(x) \quad (3)$$

for all x in *X*.

3 Some basics of single valued neutrosophic sets (SVNSs)

In this section we provide some definitions, operations and properties about single valued neutrosophic sets due to Wang et al. [17]. It will be required to develop the rest of the paper.

Definition 4 (Single-valued neutrosophic set). Let *X* be a universal space of points (objects), with a generic element of X denoted by x. A single-valued neutrosophic set $\tilde{\mathcal{N}} \subset X$ is characterized by a true membership function $T_{\tilde{\mathcal{N}}}(x)$, a falsity membership function $F_{\tilde{\mathcal{N}}}(x)$ and an indeterminacy function $I_{\tilde{\mathcal{N}}}(x)$ with $T_{\tilde{\mathcal{N}}}(x)$, $I_{\tilde{\mathcal{N}}}(x)$, $F_{\tilde{\mathcal{N}}}(x) \in [0, 1]$ for all x in *X*.

When X is continuous a SVNSs, $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \int_x \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \right\rangle / x, \quad \forall x \in X.$$

and when X is discrete a SVNSs $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \sum_{i=1}^{m} \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \right\rangle / x \text{ , } \forall x \in X.$$

Actually, SVNS is an instance of neutrosophic set that can be used in real life situations like decision making, scientific and engineering applications. In case of SVNS, the degree of the truth membership $T_{\tilde{\mathcal{N}}}(x)$, the indeterminacy membership $I_{\tilde{\mathcal{N}}}(x)$ and the falsity membership $F_{\tilde{\mathcal{N}}}(x)$ values belong to [0, 1] instead of non-standard unit interval$]0^-, 1^+[$as in the case of ordinary neutrosophic sets.

It should be noted that for a SVNS $\tilde{\mathcal{N}}$,

$$0 \leq \sup T_{\tilde{\mathcal{N}}}(x) + \sup I_{\tilde{\mathcal{N}}}(x) + \sup F_{\tilde{\mathcal{N}}}(x) \leq 3 \text{ , } \forall x \in X. \quad (4)$$

and for a neutrosophic set, the following relation holds

$$0^- \leq \sup T_{\tilde{\mathcal{N}}}(x) + \sup I_{\tilde{\mathcal{N}}}(x) + \sup F_{\tilde{\mathcal{N}}}(x) \leq 3^+, \ \forall x \in X. \quad (5)$$

Definition 5 The complement of a neutrosophic set $\tilde{\mathcal{N}}$ is denoted by $\tilde{\mathcal{N}}^c$ and is defined by

$$T_{\tilde{\mathcal{N}}^c}(x) = F_{\tilde{\mathcal{N}}}(x) \text{ ; } I_{\tilde{\mathcal{N}}^c}(x) = 1 - I_{\tilde{\mathcal{N}}}(x) \text{ ; } F_{\tilde{\mathcal{N}}^c}(x) = T_{\tilde{\mathcal{N}}}(x)$$

Definition 6 A SVNS $\tilde{\mathcal{N}}_A$ is contained in the other SVNS $\tilde{\mathcal{N}}_B$, denoted as $\tilde{\mathcal{N}}_A \subseteq \tilde{\mathcal{N}}_B$, if and only if $T_{\tilde{\mathcal{N}}_A}(x) \leq T_{\tilde{\mathcal{N}}_B}(x)$; $I_{\tilde{\mathcal{N}}_A}(x) \geq I_{\tilde{\mathcal{N}}_B}(x)$; $F_{\tilde{\mathcal{N}}_A}(x) \geq F_{\tilde{\mathcal{N}}_B}(x)$ $\forall x \in X$.

Definition 7 Two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ are equal, i.e. $\tilde{\mathcal{N}}_A = \tilde{\mathcal{N}}_B$, if and only if $\tilde{\mathcal{N}}_A \subseteq \tilde{\mathcal{N}}_B$ and $\tilde{\mathcal{N}}_A \supseteq \tilde{\mathcal{N}}_B$.

Definition 8 (Union) The union of two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is a SVNS $\tilde{\mathcal{N}}_C$, written as $\tilde{\mathcal{N}}_C = \tilde{\mathcal{N}}_A \cup \tilde{\mathcal{N}}_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ by

$T_{\tilde{\mathcal{N}}_C}(x) = \max(T_{\tilde{\mathcal{N}}_A}(x), T_{\tilde{\mathcal{N}}_B}(x))$;
$I_{\tilde{\mathcal{N}}_C}(x) = \max(I_{\tilde{\mathcal{N}}_A}(x), I_{\tilde{\mathcal{N}}_B}(x))$;
$F_{\tilde{\mathcal{N}}_C}(x) = \min(F_{\tilde{\mathcal{N}}_A}(x), F_{\tilde{\mathcal{N}}_B}(x))$ for all x in X.

Definition 9 (Intersection) The intersection of two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is a SVNS $\tilde{\mathcal{N}}_C$, written as $\tilde{\mathcal{N}}_C = \tilde{\mathcal{N}}_A \cap \tilde{\mathcal{N}}_B$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ by $T_{\tilde{\mathcal{N}}_C}(x) = \min(T_{\tilde{\mathcal{N}}_A}(x), T_{\tilde{\mathcal{N}}_B}(x))$;
$I_{\tilde{\mathcal{N}}_C}(x) = \min(I_{\tilde{\mathcal{N}}_A}(x), I_{\tilde{\mathcal{N}}_B}(x))$;
$F_{\tilde{\mathcal{N}}_C}(x) = \max(F_{\tilde{\mathcal{N}}_A}(x), F_{\tilde{\mathcal{N}}_B}(x))$ for all x in X.

4 Distance between two neutrosophic sets.

Similar to fuzzy or intuitionistic fuzzy set, the general SVNS having the following pattern

$\tilde{\mathcal{N}} = \{(x/(T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)) : x \in X\}$. For finite SVNSs can be represented by the ordered tetrads:

$$\tilde{\mathcal{N}} = \{(x_1/(T_{\tilde{\mathcal{N}}}(x_1), I_{\tilde{\mathcal{N}}}(x_1), F_{\tilde{\mathcal{N}}}(x_1)), \ldots, x_m/(T_{\tilde{\mathcal{N}}}(x_m), I_{\tilde{\mathcal{N}}}(x_m), F_{\tilde{\mathcal{N}}}(x_m))\}, \quad \forall x \in X$$

Definition 10 Let

$$\tilde{\mathcal{N}}_A = \{(x_1/(T_{\tilde{\mathcal{N}}_A}(x_1), I_{\tilde{\mathcal{N}}_A}(x_1), F_{\tilde{\mathcal{N}}_A}(x_1)), \ldots, x_n/(T_{\tilde{\mathcal{N}}_A}(x_n), I_{\tilde{\mathcal{N}}_A}(x_n), F_{\tilde{\mathcal{N}}_A}(x_n))\}$$

and

$$\tilde{\mathcal{N}}_B = \{(x_1/(T_{\tilde{\mathcal{N}}_B}(x_1), I_{\tilde{\mathcal{N}}_B}(x_1), F_{\tilde{\mathcal{N}}_B}(x_1)), \ldots, x_n/(T_{\tilde{\mathcal{N}}_B}(x_n), I_{\tilde{\mathcal{N}}_B}(x_n), F_{\tilde{\mathcal{N}}_B}(x_n))\} \quad (6)$$

be two SVNSs in $X = \{x_1, x_2, \ldots, x_n)$.

Then the Hamming distance between two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is defined as follows:

$$d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) = \sum_{i=1}^{n} \left\{ \left|T_{\tilde{\mathcal{N}}_A}(x_1) - T_{\tilde{\mathcal{N}}_B}(x_1)\right| + \left|I_{\tilde{\mathcal{N}}_A}(x_1) - I_{\tilde{\mathcal{N}}_B}(x_1)\right| + \left|F_{\tilde{\mathcal{N}}_A}(x_1) - F_{\tilde{\mathcal{N}}_B}(x_1)\right| \right\} \quad (7)$$

and normalized Hamming distance between two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is defined as follows:

$$^{N}d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) = \frac{1}{3n} \sum_{i=1}^{n} \left\{ \left|T_{\tilde{\mathcal{N}}_A}(x_1) - T_{\tilde{\mathcal{N}}_B}(x_1)\right| + \left|I_{\tilde{\mathcal{N}}_A}(x_1) - I_{\tilde{\mathcal{N}}_B}(x_1)\right| + \left|F_{\tilde{\mathcal{N}}_A}(x_1) - F_{\tilde{\mathcal{N}}_B}(x_1)\right| \right\}$$

(8) with the following two properties

1. $0 \le d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) \le 3n$ (9)
2. $0 \le {}^{N}d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) \le 1$ (10)

5 GRA based single valued neutrosophic multiple attribute decision-making problems with incomplete weight information.

Consider a multi-attribute decision-making problem with *m* alternatives and *n* attributes. Let $A_1, A_2, \ldots, A_m$ be a discrete set of alternatives, and $C_1, C_2, \ldots, C_n$ be the set of attributes. The rating provided by the decision maker, describes the performance of alternative A_i against attribute C_j. The values associated with the alternatives for MADM problems can be presented in the following decision matrix

Table 1. Decision matrix of attribute values

$$D = \langle d_{ij} \rangle_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{22} & \ldots & d_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ d_{m1} & d_{m2} & \ldots & d_{mn} \end{bmatrix} \end{array} \quad (11)$$

The weight $w_j \in [0,1]$ (j = 1, 2, ..., n) reflects the relative importance of attribute C_j (j = 1, 2, ..., m) to the decision-making process such that $\sum_{j=1}^{n} w_j = 1$. S is a set of known weight information that can be represented by the following forms due to Park et al. [25], Park and Kim [26], Kim et al. [27], Kim and Ahn [28], and Park [29].

Form 1. A weak ranking: $w_i \ge w_j$, for i $\ne$ j;

Form 2. A strict ranking: $w_i - w_j \ge \varphi_i$, $\varphi_i > 0$, for i $\ne$ j;

Form 3. A ranking of differences: $w_i - w_j \geq w_k - w_l$, for $j \neq k \neq l$;

Form 4. A ranking with multiples: $w_i \geq \beta_j w_j$, $\beta_j \in [0,1]$, for $i \neq j$;

Form 5. An interval form: $\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i$, $0 \leq \alpha_i < \alpha_i + \varepsilon_i \leq 1$.

GRA is one of the derived evaluation methods for MADM based on the concept of grey relational space. The first step of GRA method is to create a comparable sequence of the performance of all alternatives. This step is known as data pre-processing. A reference sequence (ideal target sequence) is defined from these sequences. Then, the grey relational coefficient between all comparability sequences and the reference sequence for different values of distinguishing coefficient are calculated. Finally, based on these grey relational coefficients, the grey relational degree between the reference sequence and every comparability sequences is calculated. If an alternative gets the highest grey relational grade with the reference sequence, it means that the comparability sequence is most similar to the reference sequence and that alternative would be the best choice (Fung [30]). The steps of improved GRA method under SVNS are described below.

Step 1. Determine the most important criteria.

Generally, there are many criteria or attributes in decision-making problems, where some of them are important and others may not be so important. So it is crucial to select the proper criteria or attributes for decision-making situation. The most important attributes may be chosen with the help of experts' opinions or by some others method that are technically sound.

Step 2. Construct the decision matrix with SVNSs

Assume that a multiple attribute decision making problem have *m* alternatives and *n* attributes. The general form of decision matrix as shown in Table1 can be presented after data pre-processing. The original GRA method can effectively deal with quantitative attributes. However, there exist some difficulties in the case of qualitative attributes. In the case of a qualitative attribute, an assessment value may be taken as SVNSs. In this paper we assume that the ratings of alternatives A_i (i = 1, 2,…m) with respect to attributes C_j (j = 1, 2,…n) are SVNSs. Thus the neutrosophic values associated with the alternatives for MADM problems can be represented in the following decision matrix:

Table 2. Decision matrix with SVNS

$$D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$$

$$= \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & & C_n \end{array} \\ \begin{bmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \dots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \dots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \dots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{bmatrix} \end{array} \quad (12)$$

In this matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$, T_{ij}, I_{ij} and F_{ij} denote the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i with respect to attribute C_j. These three degrees for SVNS satisfying the following properties:

1. $0 \leq T_{ij} \leq 1$, $0 \leq I_{ij} \leq 1$, $0 \leq F_{ij} \leq 1$
2. $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3$.

Step 3. Determine the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS) for neutrosophic decision matrix.

The ideal reliability estimation can be easily determined due to Biswas et al. [24].

Definition 11 The ideal neutrosophic estimates reliability solution (INERS) $Q^{+}_{\tilde{\mathcal{N}}} = [q^{+}_{\tilde{\mathcal{N}}_1}, q^{+}_{\tilde{\mathcal{N}}_2}, \dots, q^{+}_{\tilde{\mathcal{N}}_n}]$ is a solution in which every component $q^{+}_{\tilde{\mathcal{N}}_j} = \langle T^{+}_j, I^{+}_j, F^{+}_j \rangle$, where $T^{+}_j = \max_i \{T_{ij}\}$, $I^{+}_j = \min_i \{I_{ij}\}$ and $F^{+}_j = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$.

Definition 12 The ideal neutrosophic estimates unreliability solution (INEURS) $Q^{-}_{\tilde{\mathcal{N}}} = [q^{-}_{\tilde{\mathcal{N}}_1}, q^{-}_{\tilde{\mathcal{N}}_2}, ..., q^{-}_{\tilde{\mathcal{N}}_n}]$ can be taken as a solution in the form $q^{-}_{\tilde{\mathcal{N}}j} = \langle T^{-}_j, I^{-}_j, F^{-}_j \rangle$, where

$T^{-}_j = \min_i \{T_{ij}\}$, $I^{-}_j = \max_i \{I_{ij}\}$ and $F^{-}_j = \max_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$.

Step 4. Calculate the neutrosophic grey relational coefficient of each alternative from INERS and INEURS.

Grey relational coefficient of each alternative from INERS is defined as:

$$\chi^{+}_{ij} = \frac{\min_i \min_j \Delta^{+}_{ij} + \rho \max_i \max_j \Delta^{+}_{ij}}{\Delta^{+}_{ij} + \rho \max_i \max_j \Delta^{+}_{ij}}, \text{ where}$$

$\Delta^{+}_{ij} = d\left(q^{+}_{\tilde{\mathcal{N}}j}, q_{\tilde{\mathcal{N}}ij}\right)$, i= 1, 2,…,m. and j=1, 2,…,n. (13)

Grey relational coefficient of each alternative from INEURS is defined as:

$$\chi^{-}_{ij} = \frac{\min_i \min_j \Delta^{-}_{ij} + \rho \max_i \max_j \Delta^{-}_{ij}}{\Delta^{-}_{ij} + \rho \max_i \max_j \Delta^{-}_{ij}}, \text{ where}$$

$\Delta^{-}_{ij} = d\left(q_{\tilde{\mathcal{N}}ij}, q^{-}_{\tilde{\mathcal{N}}j}\right)$, i = 1, 2,…,m. and j = 1, 2,…,n. (14)

ρ is the distinguishing coefficient or the identification coefficient, $\rho \in [0,1]$. Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. Generally, $\rho = 0.5$ is considered for decision- making situation.

Step 5. Determine the weights of criteria.

In the decision-making process, decision maker may often feel that the importance of the attributes is not same. Due to the complexity and uncertainty of real world decision-making problems, the information about attribute weights is usually incomplete. The estimation of the attribute weights plays an important role in MADM. Therefore, we need to determine reasonable attribute weight for making a reasonable decision. Many methods are available to determine the unknown attribute weight in the literature such as maximizing deviation method (Wu and Chen [31]), entropy method (Wei and Tang [32]; Xu and Hui [33]), optimization method (Wang and Zhang [34-35]) etc. In this paper, we use optimization method to determine unknown attribute weights for neutrosophic MADM.

The basic principle of the GRA method is that the chosen alternative should have the largest degree of grey relation from the INERS. Thus, the larger grey relational coefficient determines the best alternative for the given weight vector. To obtain the grey relational coefficient, we have to calculate weight vector of attributes if the information about attribute weights is incompletely known. The grey relational coefficient between INERS and itself is (1, 1, …, 1), similarly, coefficient between INEURS and itself is also (1, 1, …, 1). So the corresponding comprehensive deviations are

$$d^{+}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j \quad (15)$$

$$d^{-}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{-}_{ij}\right) w_j \quad (16)$$

Smaller value of (15) as well as (16) indicates the better alternative A_i. A satisfactory weight vector W= (w_1, w_2,…, w_n) is determined by making smaller all the distances $d^{+}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j$ and $d^{-}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{-}_{ij}\right) w_j$. We utilize the max-min operator developed by Zimmermann and Zysco [36] to integrate all the distances $d^{+}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j$ for i = 1, 2, …, m and $d^{-}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{-}_{ij}\right) w_j$ for i = 1, 2, …, m separately. Therefore, we can formulate the following programming model:

$$\text{(M-1a)} \begin{cases} \text{Min } \xi^{+} \\ \text{subject to}: \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j \le \xi^{+} \text{ for } i = 1, 2, ...m \\ W \in S \end{cases} \quad (17)$$

$$\text{(M-1b)} \begin{cases} \text{Min } \xi^- \\ \text{subject to}: \sum_{j=1}^{n}\left(1-\chi_{ij}^-\right)w_j \le \xi^- \text{for } i=1,2,...m \\ W \in S \end{cases} \quad (18)$$

Here $\xi^+ = \max_i \left\{ \sum_{j=1}^{n}\left(1-\chi_{ij}^+\right)w_j \right\}$ (19)

and $\xi^- = \max_i \left\{ \sum_{j=1}^{n}\left(1-\chi_{ij}^-\right)w_j \right\}$ for 1, 2, …, m. (20)

Solving these two model (M-1a) and (M-1b), we obtain the optimal solutions $W^+ = (w_1^+, w_2^+, ...w_n^+)$ and $W^- = (w_1^-, w_2^-, ...w_n^-)$ respectively. Combinations of these two optimal solutions will give us the weight vector of the attributes i.e. $W = \gamma W^+ + (1-\gamma)W^-$ for $\gamma \in [0,1]$. (21)

If the information about attribute weights is completely unknown, we can establish another multiple objective programming:

$$\text{(M-2)} \begin{cases} \min \ d_i^+(W) = \sum_{j=1}^{n}\left[\left(1-\chi_{ij}^+\right)w_j\right]^2, \quad i=1,...,m. \\ \text{subject to}: \sum_{j=1}^{n} w_j = 1 \end{cases} \quad (22)$$

Since each alternative is non-inferior, so there exists no preference relation between the alternatives. Then, we can aggregate the above multiple objective optimization models with equal weights in to the following single objective optimization model:

(M-3)

$$\begin{cases} \min \ d^+(W) = \sum_{i=1}^{m} d_i^+(W) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(1-\chi_{ij}^+\right)w_j\right]^2, \quad i=1,...,m. \\ \text{subject to}: \sum_{j=1}^{n} w_j = 1 \end{cases} \quad (23)$$

To solve this model, we construct the Lagrange function:

$$L(W,\lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(1-\chi_{ij}^+\right)w_j\right]^2 + 2\lambda\left(\sum_{j=1}^{n} w_j - 1\right) \quad (24)$$

Where λ is the Lagrange multiplier. Differentiating equation (24) with respect to w_j (j = 1, 2,…, n) and λ, and putting these partial derivatives equal to zero, we have the following set of equations:

$$\frac{\partial L(w_j,\lambda)}{\partial w_j} = 2\sum_{i=1}^{m} w_j (1-\chi_{ij}^+)^2 + 2\lambda = 0 \quad (25)$$

$$\frac{\partial L(w_j,\lambda)}{\partial \lambda} = \sum_{j=1}^{n} w_j - 1 = 0 \quad (26)$$

Solving equations (25) and (26), we obtain the following relation

$$w_j^+ = \left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left(1-\chi_{ij}^+\right)^2\right)^{-1}\right]^{-1} \Bigg/ \sum_{i=1}^{m}\left(1-\chi_{ij}^+\right)^2 \quad (27)$$

Then we get χ_i^+ for i = 1, 2,…, m.

Similarly, we can find out the attribute weight w_j^- taking into consideration of INERUS as:

$$w_j^- = \left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left(1-\chi_{ij}^-\right)^2\right)^{-1}\right]^{-1} \Bigg/ \sum_{i=1}^{m}\left(1-\chi_{ij}^-\right)^2 \quad (28)$$

Combining (27) and (28), we can determine the j-th attribute weight with the help of (21).

Step 6. Calculate of neutrosophic grey relational coefficient (NGRC).

The degree of neutrosophic grey relational coefficient of each alternative from INERS and INEURS are calculated by using the following equations:

$$\chi_i^+ = \sum_{j=1}^{n} w_j \chi_{ij}^+$$

(29)

and $\chi_i^- = \sum_{j=1}^{n} w_j \chi_{ij}^-$ for i= 1, 2,…,m. (30)

Step 7. Calculate the neutrosophic relative relational degree (NRD).

We calculate the neutrosophic relative relational degree of each alternative from INERS by employing the following equation:

$$R_i = \frac{\chi_i^+}{\chi_i^+ + \chi_i^-}, \text{ for } i = 1, 2, \ldots, m. \quad (31)$$

Step 8. Rank the alternatives.

Based on the neutrosophic relative relational degree, the ranking order of all alternatives can be determined. The highest value of R_i presents the most desired alternatives.

5 . Illustrative Examples

In this section, neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach. Let us consider the decision-making problem adapted from Ye [37]. Suppose there is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; and (4) A_4 is an arms company. The investment company must take a decision based on the following three criteria: (1) C_1 is the risk analysis; (2) C_2 is the growth analysis; and (3) C_3 is the environmental impact analysis. We obtain the following single-valued neutrosophic decision matrix based on the experts' assessment:

Table3. Decision matrix with SVNS

$$D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{4\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} \langle 0.4, 0.2, 0.3 \rangle & \langle 0.4, 0.2, 0.3 \rangle & \langle 0.2, 0.2, 0.5 \rangle \\ \langle 0.6, 0.1, 0.2 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.5, 0.2, 0.2 \rangle \\ \langle 0.3, 0.2, 0.3 \rangle & \langle 0.5, 0.2, 0.3 \rangle & \langle 0.5, 0.3, 0.2 \rangle \\ \langle 0.7, 0.0, 0.1 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.4, 0.3, 0.2 \rangle \end{bmatrix} \end{array} \quad (32)$$

Information about the attribute weights is partially known. The known weight information is given as follows: $S = \{.30 \le w_1 \le .35, .36 \le w_2 \le .48, .26 \le w_3 \le .30\}$ such that $w_j \ge 0$ for j = 1, 2, 3and $\sum_{j=1}^{3} w_j = 1$.

Step 1. Determine the ideal neutrosophic estimates reliability solution (INERS) from the given decision matrix (see Table 3) as:

$$Q^+_{\tilde{\mathcal{N}}} = [q^+_{\tilde{\mathcal{N}}_1}, q^+_{\tilde{\mathcal{N}}_2}, q^+_{\tilde{\mathcal{N}}_3}] = \left[\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\} \rangle, \langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\} \rangle, \langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\} \rangle \right]$$
$$= [\langle 0.7, 0.0, 0.1 \rangle, \langle 0.6, 0.1, 0.2 \rangle, \langle 0.5, 0.2, 0.2 \rangle]$$

Step 2. Similarly, determine the ideal neutrosophic estimates un-reliability solution (INEURS) as:

$$Q^+_{\tilde{\mathcal{N}}} = [q^-_{\tilde{\mathcal{N}}_1}, q^-_{\tilde{\mathcal{N}}_2}, q^-_{\tilde{\mathcal{N}}_3}] = \left[\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\} \rangle, \langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\} \rangle, \langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\} \rangle \right]$$
$$= [\langle 0.4, 0.2, 0.3 \rangle, \langle 0.4, 0.2, 0.3 \rangle, \langle 0.2, 0.3, 0.5 \rangle]$$

Step 3. Calculation of the neutrosophic grey relational coefficient of each alternative from INERS and INEURS

By using equation (13) the neutrosophic grey relational coefficient of each alternative from INERS can be obtained as:
$$[\chi^+_{ij}]_{4\times 3} = \begin{bmatrix} 0.3636 & 0.5000 & 0.4000 \\ 0.5714 & 1.0000 & 1.0000 \\ 0.3333 & 0.5714 & 0.8000 \\ 1.0000 & 1.0000 & 0.6666 \end{bmatrix}$$
(33)

and from equation (14), the neutrosophic grey relational coefficient of each alternative from INEUS is
$$[\chi^-_{ij}]_{4\times 3} = \begin{bmatrix} 1.0000 & 1.0000 & 0.7778 \\ 0.4667 & 0.4667 & 0.3333 \\ 0.7778 & 0.7778 & 0.3684 \\ 0.3333 & 0.4667 & 0.4111 \end{bmatrix} \quad (34)$$

Step 4. Determine the weights of attribute.

Case 1. Utilizing the model (M-1a) and (M-2b), we establish the single objective programming model:

Case 1a. Min ξ^+

subject to: $0.6364\, w_1 + 0.5000\, w_2 + 0.6000 w_3 \le \xi^+$;

$0.4286\, w_1 \le \xi^+$;

$$0.6667w_1 + 0.4286\, w_2 + 0.2000w_3 \leq \xi^+;$$

$$0.3334w_3 \leq \xi^+;$$

$$30 \leq w_1 \leq 35;\ 36 \leq w_2 \leq 48;\ 26 \leq w_3 \leq 30;$$

$$w_1 + w_2 + w_3 = 1;\ w_j \geq 0,\ j = 1, 2, 3.$$

Case 1b.

Similarly, $\text{Min}\ \xi^-$

subject to: $0.2222w_3 \leq \xi^-;$

$$0.5353w_1 + 0.5353\, w_2 + 0.6667\, w_3 \leq \xi^-;$$

$$0.2222w_1 + 0.2222\, w_2 + 0.6316w_3 \leq \xi^-;$$

$$0.6667\, w_1 + 0.5353w_2 + 0.5889w_3 \leq \xi^-;$$

$$30 \leq w_1 \leq 35;\ 36 \leq w_2 \leq 48;\ 26 \leq w_3 \leq 30;$$

$$w_1 + w_2 + w_3 = 1;\ w_j \geq 0,\ j = 1, 2, 3.$$

We obtain the same solution set $W^+ = W^- = (0.30, 0.44, 0.26)$ after solving *Case 1a* and *Case 1b* separately. Therefore, the obtained weight vector of attributes is W = (0.30, 0.44, 0.26).

Case 2. If the information about the attribute weights is completely unknown, we can use another proposed formula given in (27) and (28) to determine the weight vector of attributes. The weight vector W^+= (0.1851, 0.4408, 0.3740) is determined from equation (27) and W^-= (0.3464, 0.4361, 0.2174) from equation (28). Therefore, the resulting weight vector of attribute with the help of equation (21) (taking $\gamma = 0.5$) is W' = (0.2657, 0.4384, 0.2957). After normalizing, we obtain the final weight vector of the attribute as W= (0.2657, 0.4385, 0.2958).

Step 5. Determine the degree of neutrosophic grey relational co-efficient (NGRC) of each alternative from INERS and INEUS.

The required neutrosophic grey relational co-efficient of each alternative from INERS is determined by using equations (29) with the corresponding obtained weight vector W for Case-1 and Case-2 are presented in Table 4.

Similarly, the neutrosophic grey relational co-efficient of each alternative from INEURS is obtained with the help of equation (30) for all two cases are listed in Table 4.

Step 6. Neutrosophic relative degree (NRD) of each alternative from INERS can be obtained with the help of equation (31) and these are shown in Table 4

Table 4. Calculation of NGRC and NRD of each alternative from neutrosophic estimates reliability solution

Proposed method	Weight Vector	NGRC from INERS	NGRC from INEURS	NRD from INERS	Ranking Result	Selection
		0.4331	0.9422	0.3149		
		0.8714	0.4320	0.6686	$R_4>R_2>R_3>R_1$	A_4
Case-1	(0.30, 0.44, 0.26)	0.5594	0.6714	0.4545		
		0.9133	0.4122	**0.6890**		
		0.4342	0.9343	0.3173		
		0.8861	0.4272	0.6747	$R_4>R_2>R_3>R_1$	A_4
Case-2	(0.2657, 0.4385, 0.2958)	0.5758	0.6567	0.4672		
		0.9014	0.4149	**0.6847**		

Step 7. From Table 4, we can easily determine the ranking order of all alternatives according to the values of neutrosophic relational degrees. For case-1, we see that A_4 i.e. Arms company is the best alternative for investment purpose. Similarly, for case-2 A_4 i.e. Arms company also is the best alternative for investment purpose.

6 Conclusion

In this paper, we introduce single-valued neutrosophic multiple attribute decision-making problem with incompletely known or completely unknown attribute weight information based on modified GRA. In order to determine the incompletely known attribute weight minimizing deviation based optimization method is used. On the other hand, we solve an optimization model to find out the completely unknown attributes weight by using Lagrange functions. Finally, an illustrative example is provided to show the feasibility of the proposed approach and to demonstrate its practicality and effectiveness. However, we hope that the concept presented here will create new avenue of research in current neutrosophic decision-making arena. The main thrust of the paper will be in the field of practical decision-making, pattern recognition, medical diagnosis and clustering analysis.

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